Repulsive short-range three-nucleon interaction

S. A. Coon,[1] M. T. Peña,[2] and D. O. Riska[3]
[1]Physics Department, New Mexico State University, Las Cruces, New Mexico 88033
[2]Centro de Fisica Nuclear, 1699 Lisboa and Instituto Superior Técnico, 1096 Lisboa, Portugal
[3]Department of Physics, University of Helsinki, 00014 Finland
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The three-nucleon interaction that is implied by the pion and the effective scalar and vector meson exchange components of the nucleon-nucleon interaction arises from the excitation of intermediate nucleon-antinucleon pairs. Using several wave-function models we show that this interaction is repulsive, reducing the calculated binding energy of the trinucleons by about 200–300 keV. The contributions of intermediate $N(1440)$ resonances (and the same meson exchanges) to this three-nucleon interaction are also estimated and shown to be small.

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I. INTRODUCTION

Although the nuclear three-nucleon interaction (TNI) is very weak in comparison to the two-nucleon interaction [1], it nevertheless has been found that the binding energies of the bound three- and four-nucleon systems cannot be understood without taking into account the attraction caused by the TNI [2]. The main component of the nuclear TNI is that associated with two-pion exchange, which includes pion rescattering through an intermediate virtual Δ_{33} resonance. The standard model for the two-meson exchange component of the TNI is the so-called Tucson-Melbourne model, which also includes ρ-meson exchange in addition to pion exchange [3–5]. It has been found that when realistic models for the nucleon-nucleon interaction are employed, the Tucson-Melbourne π-exchange model for the TNI leads to an overbinding of a few hundred keV in the trinucleons [6] and of 2–4 MeV in the case of the alpha particle [7]. To compensate for this overbinding an additional repulsive spin-independent phenomenological TNI of short range has been proposed [8]. On the other hand, the Tucson-Melbourne model (*with both π and ρ exchange*) has recently been shown to give the correct binding of the trinucleon with the Tucson-Melbourne meson-baryon-baryon vertex functions ("form factors") [9]. These form factors, however, differ in their short range behavior from those of the (older) realistic models of the nucleon-nucleon interaction. While this form factor discrepancy [10] is further studied in the case of the NN interaction [11], there remains a need for further investigation of the short-range aspects of the TNI.

There is no known dynamical mechanism that would lead to a spin-independent short-range TNI. The short-range three-nucleon interaction that arises from scalar and vector meson exchanges with an intermediate nucleon-antinucleon pair were considered in Ref. [12], but were found to be both insignificantly small and spin dependent (the different numerical results in these references were due to the effect of a nonlocal term associated with the momentum dependence of the intermediate antinucleon that was dropped in the former and retained in the latter). We here consider another set of related three-nucleon interactions—those that arise from pion and "effective" scalar and vector meson exchanges and that involve excitation of intermediate nucleon-antinucleon pairs and $N(1440)$ resonances (Fig. 1). The presence and form of the former [Fig. 1(a)] — and more important one of these interactions — is implied by the pion and scalar and vector meson exchange components of the nucleon-nucleon interaction, and can be derived directly from a given complete model for the two-nucleon interaction. The derivation is especially straightforward if the nucleon-nucleon interaction has explicit scalar and vector meson exchanges with associated meson-baryon-baryon vertex functions, but the former TNI can also be constructed from "effective" scalar and vector meson exchanges obtained from any complete energy-independent model of the nucleon-nucleon interaction [13]. The magnitude of the latter TNI [Fig. 1(b)] is less certain because of the wide uncertainty in the meson-nucleon-$N(1440)$ ("Roper resonance") coupling strengths. Both of these sets of three-nucleon interactions nevertheless have a very simple form, although spin dependent, and when combined provide an amount of additional repulsion that approximately corresponds to that required and hitherto ascribed to the purely phenomenological spin-independent short-range TNI.

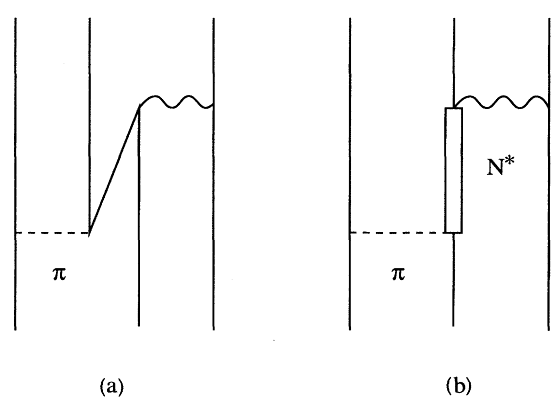

FIG. 1. (a) π–short-range exchange three-nucleon interaction that involves an intermediate nucleon-antinucleon pair, (b) π–short-range exchange three-nucleon interaction that involves excitation of an intermediate $N(1440)$ resonance. The wavy lines symbolize scalar and vector meson exchange.

In Sec. II of this paper we derive the effective π-scalar and π-vector meson exchange three-nucleon interactions which arise from excitation of intermediate nucleon-antinucleon pairs and show how the potentials which describe these interactions can be constructed from realistic models for the nucleon-nucleon interaction. It is this construction method which distinguishes the TNI's of the present paper from the Brown-Green [1], Tucson-Melbourne [3–5], Brazil [14], and the Fujita-Miyazawa [15] TNI's (the last being extensively used in a program of studies with light nuclear systems [16]) The latter TNI's did not attempt such a tight connection with a nucleon-nucleon interaction but relied on other aspects of hadronic phenomenology for motivation and parameter fixing. In Sec. III we derive the corresponding three-nucleon interactions that arise from excitation of $N(1440)$ resonances on the intermediate nucleon and derive the corresponding coupling constants from the partial decay widths. In Sec. IV we present numerical results for the contribution to the binding energy of the trinucleons, which arises from these interactions using oscillator and Malfliet-Tjon [17] wave functions as well as Paris and Bonn "OBEPQ" [9,18] wave functions. Finally Sec. V contains a concluding discussion. The partial wave decomposition of the TNI is described in the Appendix.

II. PION-SCALAR AND -VECTOR MESON EXCHANGE THREE-NUCLEON INTERACTIONS

Realistic models for the nucleon-nucleon interaction as, e.g., the Bonn [19], Nijmegen [20], and Paris [21] models are based on meson exchange models, in which the Lorentz invariant on-shell nucleon-nucleon scattering amplitude is constructed from phenomenological meson-nucleon Lagrangians, and then used as an off-shell kernel in a wave equation. In practice a nonrelativistic "adiabatic" approximation is also involved [22,23]. The important point is that a given model for the on-shell amplitude that is extrapolated off shell in this way automatically implies a model for the $NN \rightarrow NNN\bar{N}$ amplitude as well. This amplitude is the central component in the three-nucleon interactions that arise from excitation of virtual nucleon-antinucleon pairs on the intermediate nucleon [Fig. 1(a)]. Thus any meson exchange model for the nucleon-nucleon interaction will by construction imply the presence of three-nucleon interactions of this type, which formally can be derived directly from the NN interaction model without any need for further assumptions.

We shall here consider the π-scalar and π-vector meson exchange TNI's of this form. An important part of the motivation for this is the recent observation that the off shell $\pi + N \rightarrow N +$ scalar and $\pi + N \rightarrow N +$ vector amplitudes are "almost" observable, in that they successfully describe most of the cross section for the reaction $pp \rightarrow pp\pi^0$ near threshold [24]. Thus the theoretical model for these amplitudes can be viewed as having a good empirical foundation. In the derivation of these three-nucleon amplitudes we shall use the usual phenomenological scalar ("σ," "a_0") and vector meson ("ω," "ρ") Lagrangians, but in the end will replace the pure boson exchange interactions by the corresponding "effective" scalar and vector meson exchange interactions that form the most important short-range components of the nucleon-nucleon interaction.

To construct the π-scalar ("σ")-meson three-nucleon interaction that corresponds to the Feynman diagram in Fig. 1(a) we employ the πNN and σNN couplings

$$\mathscr{L}_{\pi NN} = i \frac{f_{\pi NN}}{m_\pi} \bar{\psi} \gamma_5 \gamma_\mu \partial_\mu \vec{\phi}_\pi \cdot \vec{\tau} \psi, \tag{2.1a}$$

$$\mathscr{L}_{\sigma NN} = g_\sigma \bar{\psi} \phi_\sigma \psi. \tag{2.1b}$$

Here $\vec{\phi}_\pi$ is the isovector pion and ϕ_σ the isoscalar scalar meson field and $f_{\pi NN}$ ($f^2_{\pi NN}/4\pi \simeq 0.08$) and g_σ the corresponding coupling constants.

The three-nucleon interaction that corresponds to the Feynman diagram in Fig. 1(a) is obtained by retaining only the negative energy part of the fermion propagator for the intermediate nucleon and adding the term with the pion and scalar meson couplings in reversed order. In the actual construction of the operator we exploit the fact that the nucleons are nearly on shell, and use the Dirac equation to simplify the algebra. In this way a contact term operator arises, which has to be retained along with the pair term in the three-nucleon interaction operator. This contact term operator alters the nature of the πNN coupling from the pseudovector coupling of Eq. (2.1a) to a chiral rotated representation which will be discussed later. The resulting three-nucleon interaction has the simple form

$$V_{\pi\sigma} = \frac{g_\sigma^2}{m_N} \left(\frac{f_{\pi NN}}{m_\pi} \right)^2 \frac{\vec{\sigma}^1 \cdot \vec{k}_\pi \vec{\sigma}^2 \cdot \vec{k}_\sigma}{(k_\pi^2 + m_\pi^2)(k_\sigma^2 + m_\sigma^2)} \vec{\tau}^1 \cdot \vec{\tau}^2 + (\text{permutations}). \tag{2.2}$$

Here the direction of the meson momenta are taken so that they point away from the intermediate nucleon, that is indicated by the superscript 2 on the spin and isospin operators. The symbol "permutations" stands for first adding a term, in which the nucleon coordinates 1 and 2 are exchanged and then taking into account the additional (4) terms in which the intermediate nucleon-nucleon pair is excited on nucleon 1 and 3 in turn.

In order to make this expression for the pion-scalar TNI consistent with a realistic model for the nucleon-nucleon interaction it is natural to replace the term $-g_\sigma^2/(k_\sigma^2 + m_\sigma^2)$ in the expression (2.2) by the corresponding general "effective" isospin independent scalar component $v_S^+(\vec{k}_\sigma)$ of the nucleon-nucleon interaction, which may be constructed using the method of Ref. [13]. In a similar way it is natural to replace the term $(f_{\pi NN}/m_\pi)^2/(k_\pi^2 + m_\pi^2)$ by the effective isospin dependent pseudoscalar exchange potential $v_P^-(\vec{k}_\pi)/4m_N^2$ of the nucleon-nucleon interaction. In this way the short-range modifications — i.e., form factors — of the simple meson exchange interactions are determined by the corresponding components of the nucleon-nucleon interaction model.

Specifically, the method of Ref. [13] rewrites a nonrelativistic nucleon-nucleon potential model on the energy shell in terms of five nonrelativistic spin amplitudes which can be viewed as nonrelativistic limits of five relativistic Fermi invariants. For the construction of the TNI's in this paper we need the isospin independent ($^+$) and isospin dependent

($^-$) Fermi invariants: scalar (S), pseudoscalar (P), and vector (V). The Fermi invariant potential coefficients $v_j^{\pm}(\vec{k})$, $j=S,P,V$ are obtained as linear combinations of the nonrelativistic components of a given potential. The Fermi invariant potential coefficients $v_j^{\pm}$ are functions of k^2 only, which means that the underlying interactions have no energy dependence. The procedure and results for carrying out this program for a potential (such as Paris), which has a short-range behavior determined in coordinate space, are displayed in [13]. If the potential is already expressed in terms of relativistic invariants corresponding to the exchange of scalar, vector, and pseudoscalar bosons (such as the Bonn or Nijmegen potentials) the procedure amounts to the replacement (for example)

$$\frac{(f_{\pi NN}/m_\pi)^2}{m_\pi^2+k_\pi^2}\to\frac{v_P^-(\vec{k}_\pi)}{4m_N^2}\to\frac{(f_{\pi NN}/m_\pi)^2}{k_\pi^2+m_\pi^2}\left(\frac{\Lambda_\pi^2-m_\pi^2}{\Lambda_\pi^2+k_\pi^2}\right)^2, \tag{2.3}$$

where the meson-nucleon-nucleon vertex $(\Lambda_\pi^2-m_\pi^2)/(\Lambda_\pi^2+k_\pi^2)$, is of the form chosen for the Bonn OBEPQ potential used in our numerical investigations. In either case, the short-range behavior of the TNI so constructed is fully determined by the short-range behavior of the corresponding nucleon-nucleon interaction, which is the issue at hand.

In the case of an isospin-1 scalar meson exchange (a_0 channel) the TNI that corresponds to the expression (2.2) is

$$V_{\pi a}=\frac{g_a^2}{m_N}\left(\frac{f_{\pi NN}}{m_\pi}\right)^2\frac{\vec{\sigma}^1\cdot\vec{k}_\pi}{(k_\pi^2+m_\pi^2)(k_a^2+m_a^2)}\times\{\vec{\sigma}^2\cdot\vec{k}_a\vec{\tau}^1\cdot\vec{\tau}^3+2i\vec{\sigma}^2\cdot\vec{P}_2\vec{\tau}^1\cdot\vec{\tau}^2\times\vec{\tau}^3\}. \tag{2.4}$$

Here g_a is the a_0NN coupling constant and m_a and $\vec{k}_a$ the mass and momentum of the exchanged a_0, respectively. In view of the smallness of the nucleon momenta in the bound states we shall not consider the nonlocal term in this TNI, which contains the momentum $\vec{P}_2=(\vec{p}_2+\vec{p}_2')$ of the intermediate nucleon. In order to make the πa_0 TNI (2.4) consistent with the models for the nucleon-nucleon interaction we shall replace the simple a_0 exchange interaction in (2.4) by the corresponding isospin dependent scalar exchange component of the nucleon-nucleon interaction: $g_a^2/(m_a^2+k_a^2)\to-v_S^-(\vec{k}_a)$.

To construct the $\pi\omega$ TNI we employ the ωNN Lagrangian

$$\mathscr{L}=ig_\omega\bar{\psi}\gamma_\mu\omega_\mu\psi, \tag{2.5}$$

where g_ω is the ωNN coupling constant in addition to (2.1a). The resulting TNI potential is

$$V_{\pi\omega}=-\frac{g_\omega^2}{m_N}\left(\frac{f_{\pi NN}}{m_\pi}\right)^2\frac{\vec{\sigma}^1\cdot\vec{k}_\pi\vec{\sigma}^2\cdot\vec{k}_\omega}{(k_\pi^2+m_\pi^2)(k_\omega^2+m_\omega^2)}\vec{\tau}^1\cdot\vec{\tau}^2+(\text{permutations}). \tag{2.6}$$

This interaction, which arises from the charge component of the ω field is similar in form to the $\pi\sigma$ TNI (2.2), but has the opposite sign. As in the case of the two-nucleon interaction there will therefore be a strong partial cancellation between the $\pi\sigma$ and $\pi\omega$ three-nucleon interactions, such that the $\pi\sigma$ typically is the stronger interaction, because of the somewhat longer (intermediate) range of the effective scalar meson exchange interaction.

The $\pi\rho$ exchange TNI has one component that arises from the charge component and one that arises from the spatial component. The first one of these has the form

$$V_{\pi\rho}^C=-\frac{g_\rho^2}{m_N}\left(\frac{f_{\pi NN}}{m_\pi}\right)^2\frac{\vec{\sigma}^1\cdot\vec{k}_\pi\vec{\sigma}^2\cdot\vec{k}_\rho}{(k_\pi^2+m_\pi^2)(k_\rho^2+m_\rho^2)}\vec{\tau}^1\cdot\vec{\tau}^3, \tag{2.7}$$

and the expression for the latter is

$$V_{\pi\rho}^S=-i\frac{g_\rho^2}{m_N}\left(\frac{f_{\pi NN}}{m_\pi}\right)^2\frac{\vec{\sigma}^1\cdot\vec{k}_\pi}{(k_\pi^2+m_\pi^2)(k_\rho^2+m_\rho^2)}\times\sigma^2\cdot[2\vec{P}_3+i\vec{\sigma}^3\times\vec{k}_\rho]\vec{\tau}^1\cdot\vec{\tau}^2\times\vec{\tau}^3. \tag{2.8}$$

The local part of the $\pi\rho$ exchange three-nucleon interaction $V_{\pi\rho}^S$ is referred to as the "$\pi\rho$ Kroll-Rudermann interaction" [5] or the "seagull" [14] in the literature, and has been considered in the trinucleon before [9,25,26]. In the treatment of the Tucson-Melbourne TNI (an expansion of the $\rho N\to\pi N$ amplitude) $V_{\pi\rho}^C$ is small because of a near cancellation between the intermediate nucleon-antinucleon part and the ρ analog Fubini-Furlan-Rossetti contribution to pion photoproduction. The isoscalar $V_{\pi\rho}^C$ corresponds to the remaining lead term in Eqs. (2.13b) and (2.14a) of [4]. It was estimated in nuclear matter in [4], derived and then neglected altogether in the Brazil TNI [14], and is evaluated in the trinucleon for the first time in the present study. To make the present expressions for the $\pi\omega$ and $\pi\rho$ exchange three-nucleon interactions consistent with the nucleon-nucleon interaction model we shall replace the bare vector meson interactions $g_\omega^2/(k_\omega^2+m_\omega^2)$ and $g_\rho^2/(k_\rho^2+m_\rho^2)$ with the corresponding isospin independent $[v_V^+(\vec{k}_\omega)]$ and isospin dependent $[v_V^-(\vec{k}_\rho)]$ vector exchange components of the nucleon interaction as suggested in Ref. [13].

We now relate the TNI's just derived to the corresponding TNI's due to nucleon-antinucleon pairs in the two-pion exchange case [3,4], and to the corresponding two-pion exchange TNIs which arise from an expansion of a chiral (effective) Lagrangian in powers of the inverse nucleon mass [27] or the two-pion exchange TNI's obtained by a chiral perturbation theory approach [28]. The latter three TNI's are identical to the order considered in the various expansion philosophies [27,29]. These sets of potentials depend upon arbitrary parameters μ and ν, just as do two-body relativistic corrections in the one-pion-exchange potential (OPEP). Since realistic two-nucleon potentials potentials contain OPEP a further consistency demanded of a sensible TNI is a consistency in μ and ν between the TNI and the realistic NN potential with which it is to be used. The parameter ν of the operators of this paper is always $\nu=1/2$, corresponding to no meson retardation, which is the case with the Bonn and Paris potentials. The value of μ labels an ambiguity in the relativistic corrections (i.e., from $1/m_N$-expansion methods) to operators involving pion exchange [30]. The same continuous parameter also acts as a chiral rotation to determine the pseudoscalar ($\mu=0$) or pseudovector ($\mu=1$) content of the πNN coupling in models of the nucleon-nucleon interaction or TNI's which obey approximate chiral symmetry [27]. Fixing arbitrarily this parameter means choosing a special representation of a unitarily equivalent class of operators. Because of this unitary equivalence, the observables should not depend on the value of μ or ν, i.e., on the choice of representation, provided that all operators and wave functions are chosen consistently [31]. The derivation of the TNI's of Eqs. (2.2)–(2.7) (and of the nucleon-antinucleon pair term of the Tucson-Melbourne TNI [3]) corresponds to $\mu=-1$, as does the one-pion-exchange part of the Bonn OBEPQ [32]. Thus it is completely consistent to determine the short-range modifications of the simple meson exchange interactions of Eqs. (2.2) and (2.6) by the corresponding components of the Bonn OBEPQ nucleon-nucleon interaction model. The same replacement procedure with the Paris potential is, however, more problematic, as the parameter of that potential has been identified to be $\mu=0$ [32,33] or to be a different value by other authors.

Finally, we make qualitative remarks about the order of the TNI's of this section in a power counting scheme which has been used to classify two-pion exchange TNI's [29] and can be connected to the power counting of chiral perturbation theory [34]. Intermediate state nucleon-antinucleon pair terms generate effective two-pion seagull couplings of order $f_{\pi NN}^2/m_\pi^2$ and of order $f_{\pi NN}^2/(m_\pi^2 m_N)$. These seagulls can be connected with single pion-nucleus vertices [of order $f_{\pi NN}/m_\pi$ and $f_{\pi NN}/(m_\pi m_N)$] using time-dependent perturbation theory to form physical amplitudes from pion exchange [30]. It was shown in Ref. [27] that the seagull contribution to the lowest order two-pion exchange TNI involves only the latter seagull and is therefore [let $f_0^2=f_{\pi NN}^2/(4\pi)\sim 0.08$] of order $(f_0^4 m_\pi^2/m_N)$ or ~ 100 keV/triplet, where the latter estimate is made by treating the dimensionless (coordinate space) factors as order (1). The two-pion exchange TNI terms, analogous to the Feynman diagram of Fig. 1(a), in the Tucson-Melbourne TNI's are identical to the seagull generated TNI's of Ref. [27]. Careful calculations [35,9] of the effect of these terms on the triton binding energy are indeed consistent (to a factor of 2 or 3) with the power counting estimate of ~ 100 keV/triplet. An important conclusion of Ref. [27] was that there can be no TNI terms of order $f_0^4 m_N$ or ~ 5 MeV/triplet in a consistent expansion in powers of v^2/c^2 if one insists on an energy independent OPEP (such as one finds in many realistic NN interactions). That is, the sum of TNI terms from the $(f_{\pi NN}^2/m_\pi^2)$ two-pion seagull is identically zero provided the OPEP underlying the calculation is energy independent (and is not zero, as found in the early papers of Ref. [28], if the OPEP is not energy independent). In particular, there are no terms of order $f_0^4 m_N$ from nucleon-antinucleon pairs in the two-pion exchange Tucson-Melbourne TNI. The Tucson-Melbourne pair terms were obtained from a subtraction process [3] which we have determined equivalent to that used to derive the nucleon-antinucleon pair terms of this section.

With this background, one can ask of the potentials of Eqs. (2.2)–(2.7) (i) can they also be identified with an effective pion-scalar or pion-vector seagull from chiral couplings, and (ii) are they the lowest leading order TNI's in the power counting scheme? A conclusive answer requires a detailed (and tedious) calculation of all TNI terms up to order V^2/m_N as was done in Ref. [27] for $V=V_\pi$(OPEP). This we have not done but have some confidence that the answer to both question is yes. The first TNI derived in this section is of the schematic form $V_\sigma V_\pi/m_N$, where V_σ is the NN potential from single sigma exchange according to the coupling (2.1b). This form would be expected from the pion-scalar seagull derived in [30]

$$\frac{f_{\pi NN} g_\sigma(\mu-1)}{m_\pi 2m_N}\vec{\phi}_\pi\cdot\vec{\tau}\vec{\sigma}\cdot\vec{\nabla}\phi_\sigma,$$

where we have adapted a term from Eq. (6f) of [30] to our notation. If the effective chiral coupling were indeed pure pseudovector ($\mu=1$), as implied by Eq. (2.1a), the seagull would disappear. However, the effective chiral coupling of the TNI of eq. (2.2) corresponds to $\mu=-1$. Now visualize that the scalar meson of this seagull connects to one nucleon with coupling g_σ and the pion connects to the other nucleon with coupling $f_{\pi NN}/m_\pi$ to obtain a TNI of the strength and structure of Eq. (2.2), including the $\vec{\sigma}^2\cdot\vec{k}_\sigma$ structure of the intermediate nucleon. Because of the $1/m_N$ structure of the pion-scalar seagull the TNI of Eq. (2.2) is the lowest order of this type of TNI's which can occur in a v^2/c^2 expansion.

III. INTERMEDIATE $N(1440)$ RESONANCES

Excitation of virtual $N(1440)$ (Roper) resonances on the intermediate nucleon also contributes a weak but still significant π-scalar and π-vector meson exchange three-nucleon interaction [Fig. 1(b)]. Naturally a contribution to the $\pi\pi$ three-nucleon interaction also arises from intermediate $N(1440)$ excitation, but this is effectively included in the Tucson-Melbourne TNI, which is based on an off shell extrapolation of the complete πN scattering amplitude.

To construct the $\pi\sigma$ and $\pi\omega$ three nucleon interactions that are associated with intermediate $N(1440)$ resonance excitation we employ the effective Lagrangians

$$\mathscr{L}_{\pi NN^*}=i\frac{f_\pi^*}{m_\pi}\bar{\psi}_*\gamma_5\gamma_\mu\partial_\mu\vec{\phi}\cdot\vec{\tau}\phi+\text{H.c.},\tag{3.1a}$$

$$\mathscr{L}_{\sigma NN^*}=g_\sigma^*\bar{\psi}_*\phi_\sigma\psi+\text{H.c.},\tag{3.1b}$$

$$\mathscr{L}_{\omega NN^*}=ig_\omega^*\bar{\psi}_*\gamma_\mu\omega_\mu\psi+\text{H.c.}\tag{3.1c}$$

Here ψ_* denotes the Roper resonance spinor field and f_π^*, g_σ^*, and g_ω^* are the πNN^*, σNN^*, and ωNN^* coupling strengths, N^* being an abbreviation for the $N(1440)$. The expressions for the $\pi\sigma$ and $\pi\omega$ TNI potentials that are associated with intermediate $N(1440)$ excitation can then be derived in a straightforward way, the results being

$$V_{\pi\sigma}^*=-\frac{2g_\sigma g_\sigma^*}{m^*-m_N}\frac{f_{\pi NN}f_\pi^*}{m_\pi^2}\frac{\vec{\sigma}^1\cdot\vec{k}_\pi\vec{\sigma}^2\cdot\vec{k}_\pi}{(k_\pi^2+m_\pi^2)(k_\sigma^2+m_\sigma^2)}\vec{\tau}^1\cdot\vec{\tau}^2$$
$$+(\text{permutations}),\tag{3.2a}$$

$$V^*_{\pi\omega}=\frac{2g_\omega g^*_\omega}{m^*-m_N}\frac{f_{\pi NN}f^*_\pi}{m^2_\pi}\frac{\vec{\sigma}^1\cdot\vec{k}_\pi\vec{\sigma}^2\cdot\vec{k}_\pi}{(k^2_\pi+m^2_\pi)(k^2_\omega+m^2_\omega)}(\vec{\tau}^1\cdot\vec{\tau}^2). \tag{3.2b}$$

Here m^* is the mass of the $N(1440)$ resonance. It is worth noting that the terms that depend on the pion momentum $\vec{k}_\pi$ in these three-nucleon interactions have the same form as the one-pion exchange interaction between two nucleons.

The πNN^* coupling constant f^*_π may be calculated from the $N^*\to N\pi$ partial decay width as

$$\frac{f^{*2}_\pi}{4\pi}=\frac{m^*m_\pi}{(m^*+m_N)p(E_N-m_N)}\Gamma(N^*\to N\pi). \tag{3.3}$$

Here p is the nucleon momentum and E_N the nucleon energy in the rest frame of the decaying N^*. With the $N\pi$ branching ratio of the total decay width 350 MeV being 60% we obtain $f^*_\pi/4\pi\simeq 0.031$, which is somewhat less than one half of the corresponding value $f^2_{\pi NN}/4\pi\simeq 0.08$ for the πNN coupling strength.

The determination of the σNN^* and ωNN^* coupling strength is associated with considerably larger uncertainties. To obtain an estimate for the σNN^* coupling constant we calculate it from the decay width for $N^*\to N(\pi\pi)^{I=0}_{S\text{ wave}}$ as

$$\frac{g^{*2}_\sigma}{4\pi}=\frac{m^*}{p(E_N+m_N)}\Gamma[N^*\to N(\pi\pi)^{I=0}_{S\text{ wave}}]. \tag{3.4}$$

Here we assume that all of the $I=0$ S-wave part of the $\pi\pi$ continuum can be interpreted as a broad effective σ meson. The branching ratio for this decay channel is 5–15 %. Assuming m_σ = 410 MeV at the midpoint between the $\pi\pi$ threshold and kinematical phase space cutoff we obtain

$$\frac{g^{*2}_\sigma}{4\pi}\simeq 0.1 \tag{3.5}$$

for $\Gamma[N^*\to N(\pi\pi)^{I=0}_{S\text{ wave}}]$ = 35 MeV. This value for $g^{*2}_\sigma/4\pi$ is expected to have an uncertainty of about a factor 2.

As the N^* cannot decay into a $N\omega$ state, the coupling constant g_ω cannot be determined directly from empirical data. We shall here assume that $g^*_\omega/g_\omega=g^*_\sigma/g_\sigma$ as suggested by the constituent quark model. In the Bonn boson exchange model OBEPQ for the nucleon-nucleon interaction this ratio is 1.55 [19]. This would then suggest that

$$\frac{g^{*2}_\omega}{4\pi}=0.24, \tag{3.6}$$

a value with which a substantial uncertainty margin also has to be associated.

The bare meson exchange potentials in the TNI's (3.2a) and (3.2b) will be modified at high values of momentum transfer by shorter-range dynamics in the same way as the NN interaction. To describe this short-range modification in a way that is consistent with that of the NN interaction we shall introduce the same vertex factors as in the Bonn boson exchange model OBEPQ for the NN interaction [19] by means of the substitutions

$$\frac{1}{m^2_\pi+k^2_\pi}\to\frac{1}{k^2_\pi+m^2_\pi}\left(\frac{\Lambda^2_\pi-m^2_\pi}{\Lambda^2_\pi+k^2_\pi}\right)^2, \tag{3.7a}$$

$$\frac{1}{m^2_\sigma+k^2_\sigma}\to\frac{1}{k^2_\sigma+m^2_\sigma}\left(\frac{\Lambda^2_\sigma-m^2_\sigma}{\Lambda^2_\sigma+k^2_\sigma}\right)^2, \tag{3.7b}$$

$$\frac{1}{m^2_\omega+k^2_\omega}\to\frac{1}{k^2_\omega+m^2_\omega}\left(\frac{\Lambda^2_\omega-m^2_\omega}{\Lambda^2_\omega+k^2_\omega}\right)^2. \tag{3.7c}$$

For the form factor mass scale parameters we use the values Λ_π = 1.3 GeV/c^2, $\Lambda_\sigma=\Lambda_\omega$ = 2.0 GeV/c^2.

IV. NUMERICAL ESTIMATES

An explicit calculation of the contribution of the π-scalar and π-vector exchange TNI to the binding energy of the trinucleons is necessary to evaluate the importance of these potentials. The power counting estimates which work so well for the pair terms of the π-π exchange TNI fail completely for these short-range TNI's. They should because the heavy meson exchange shrinks to a point in the power counting scheme and is overwhelmed by the hole in the wave function that heavy meson exchange produces in the NN force. For example, the power counting argument applied to the π-ρ "Kroll-Ruderman" term (i.e., the nucleon-antinucleon pair term) of the Tucson-Melbourne TNI would give a repulsive contribution of about 25 MeV/triplet. Complete calculations with realistic triton wave functions [9] find that the true contribution to the triton binding energy is about 0.1 to 0.5 MeV. Even simple S-state triton wave functions such as a harmonic oscillator or the Malfliet-Tjon I-III model wave function alter the power counting estimate by two orders of magnitude to about 0.15–0.2 MeV [5].

We shall first estimate the contribution of the π-scalar and π-vector exchange TNI to the binding energy of the trinucleons considering only the S-state component of the three-nucleon bound state wave function using several different wave-function models. As this wave-function component gives the dominant contribution to the norm of the wave function, this will be sufficient to establish the overall magnitude of the matrix elements of the three-nucleon interactions considered. The correction due to the D states are considered with the aid of realistic triton wave functions.

In the first estimates presented, the orbital part of the wave function is described by the harmonic oscillator and three-channel Malfliet-Tjon I-III model wave function [17]. The use of the harmonic oscillator model is motivated by the fact that the resulting matrix elements can be reduced to quadrature of very simple expressions, which allows the qualitative features to be elucidated, and provides good reference tests for the calculation with wave-function models that are based on realistic nucleon-nucleon interaction models — in this case the Paris and Bonn OBEPQ potentials [19,21]. The harmonic oscillator and three-channel Malfliet-Tjon III model wave functions do not take into account p- and d-wave nucleon-nucleon correlations, and therefore the results that are obtained with such schematic model wave functions are *a priori* only suggestive, although the numerical values for the matrix elements of the three-nucleon interactions considered here that are obtained with the harmonic oscillator and Malfliet-Tjon wave-function models turn out to be similar to those obtained in the end with the realistic wave functions.

In the case of a wave function with only a completely symmetric S-state component the radial matrix element may be expressed as

$$\langle S|V_3|S\rangle \equiv \langle V_3\rangle = \int \frac{d^3\tau}{(2\pi)^3}\int \frac{d^3v}{(2\pi)^3} g(\vec{\tau},\vec{v})\,\phi_0^\dagger V_3(\vec{\tau},\vec{v})\,\phi_0 . \tag{4.1}$$

Here ϕ_0 is the totally antisymmetric spin-isospin vector and $\vec{\tau}$ and $\vec{v}$ are differences of nucleon Jacobi coordinates, which are related to the meson momenta in the three-nucleon interactions as

$$\vec{k}_\pi = \left(\vec{\tau} + \frac{\vec{v}}{2}\right), \quad \vec{k}_{\sigma,\omega} = \vec{v}. \tag{4.2}$$

In (4.1) the function $g(\vec{\tau},\vec{v})$ is the Fourier transform of the nucleon density function in coordinate space, which in the case of the harmonic oscillator model takes the form

$$g(\vec{\tau},\vec{v}) = e^{-\tau^2/2\alpha^2} e^{-3v^2/8\alpha^2}, \tag{4.3}$$

α being the oscillator parameter $\sqrt{m\omega_0}$ for which we use the value 0.60 fm^{-1}.

The oscillator model leads to the following expression for the matrix element of the $\pi\sigma$ three-nucleon interaction (2.2) in ^{3}H and ^{3}He:

$$\langle V_{\pi\sigma}\rangle = \frac{3}{2\pi^4}\frac{g_\sigma^2}{m_N}\left(\frac{f_{\pi NN}}{m_\pi}\right)^2 \int_0^\infty dv\, e^{-v^2/2\alpha^2} \frac{v^3}{v^2+m_\sigma^2} \times \int_0^\infty dk\, e^{-k^2/2\alpha^2} \frac{k^3}{k^2+m_\pi^2} \sqrt{\frac{\pi\,\alpha^2}{kv}}\, I_{3/2}\left(\frac{kv}{2\alpha^2}\right), \tag{4.4}$$

where $I_{3/2}$ is a modified Bessel function. The corresponding expression for the matrix element of the πa_0 three-nucleon interaction (2.4) is obtained by the substitutions $g_\sigma \rightarrow g_a$ and $m_\sigma \rightarrow m_a$ and an overall change of sign in the expression (4.4), if the terms proportional to the total momentum of the intermediate nucleon in (2.4) are neglected. The sign change arises from different isospin dependence in (2.2) and (2.4).

The similarity in form between the $\pi\sigma$ (2.2) and $\pi\omega$ (2.6) three-nucleon interactions makes it obvious that the expression for the matrix element of $V_{\pi\omega}$ is given by an expression of the form (4.4) with g_σ, m_σ replaced by g_ω and m_ω, and with an overall minus sign. The expression for the matrix element of the $\pi\rho$ three-nucleon interaction $V^C_{\pi\rho}$ (2.7) is obtained by the corresponding substitution in (4.4) of $g_\sigma \rightarrow g_\omega$, $m_\sigma \rightarrow m_\rho$, but without any change of the overall sign.

The harmonic oscillator model of a three-body bound state has special properties which allow a practical implementation of Eq. (4.1) as shown above. The numerical calculation using the Malfliet-Tjon I-III model calculation was made with the methods described in Refs. [5,26]. The S-state wave function that corresponds to the Malfliet-Tjon I-III interaction is given on a mesh over the configuration space Jacobi variables described in [17]. The calculation of the matrix elements then reduces to three-dimensional quadrature over the Fourier transforms of the three-nucleon interaction potentials given in Secs. II and III above. The numerical values of the harmonic oscillator and Malfliet-Tjon model wave functions turn out to be very similar.

TABLE I. Matrix elements in MeV of the π-scalar and π-vector exchange interactions which involve an intermediate nucleon-antinucleon pair. The three-nucleon interactions are constructed from the Bonn boson exchange model OBEPQ for the NN interaction [19]. The matrix elements are obtained with S-state oscillator and Malfliet-Tjon I-III wave functions.

	H.O.	MF I-III
$V_{\pi\sigma}$	0.392	0.457
$V_{\pi a}$	−0.012	−0.010
$V_{\pi\omega}$	−0.301	−0.337
$V^C_{\pi\rho}$	0.020	0.016
Tot	0.099	0.126

In Table I we give the numerical values for the matrix elements of the three-nucleon π short-range three-nucleon interactions that are associated with excitation of intermediate nucleon-antinucleon pairs. The numerical values have been given both for the case of the oscillator model wave function and the S-state wave function that corresponds to the Malfliet-Tjon (MT) I-III interaction [17]. The numbers in the table correspond to the case when the Bonn boson exchange potential model OBEPQ (Table V, [19]) parameterization has been used to construct the "effective" pion (pseudoscalar) and scalar and vector exchange potentials as described in Sec. II above.

The wave functions that are obtained from realistic interaction models call for a different calculational method. As the matrix elements involve the Jacobi coordinates of the initial and final wave functions separately the momentum transfer variables in Eq. (4.2) have to be supplemented by sums of Jacobi variables, which in the case of the harmonic oscillator model cancel the normalization integral and therefore do not need to be considered in Eq. (4.1). Employment of wave functions that are obtained from momentum space solutions of the Faddeev equations require a partial wave decomposition of the TNI [4]. At the same time it provides an alternate way to calculate the matrix element of the expression (4.4), which will be used to check the numerical calculation. This partial wave decomposition of the TNIs above is described in the Appendix A.

The realistic wave functions that were used in the calculation were calculated by solving Faddeev equations retaining in the two-body t matrix all two-body partial waves with total angular momentum $j \leq 2$. Then, the orbital angular momenta include besides the s waves of the schematic interactions such as MT-III, also p,d, and some f waves. The calculation yields Faddeev amplitudes in 18 three-nucleon partial waves in the jj coupling scheme (i.e., all the three-

TABLE II. Matrix elements in MeV of the π-σ and π-ω exchange interactions which involve an intermediate nucleon-antinucleon pair. The three-nucleon interactions are constructed from the Bonn boson exchange model OBEPQ for the NN interaction [19]. The matrix elements are obtained with components of the Paris and Bonn (OBEPQ) wave functions that comprise S, D, and even P three-nucleon states. We display the individual contributions from these components.

	Paris				OBEPQ			
	$\langle S\|V_3\|S\rangle$	$\langle S\|V_3\|D\rangle$	$\langle S\|V_3\|P\rangle$	Tot	$\langle S\|V_3\|S\rangle$	$\langle S\|V_3\|D\rangle$	$\langle S\|V_3\|P\rangle$	Tot
$V_{\pi\sigma}$	0.409	0.617	−0.023	1.003	0.689	0.852	−0.025	1.516
$V_{\pi\omega}$	−0.306	−0.484	0.020	−0.770	−0.528	−0.678	0.022	−1.184
Tot	0.103	0.133	-0.003	0.233	0.161	0.174	−0.003	0.332

body partial waves with the quantum numbers of the triton that can be formed with the above restriction of two-body channels). From the Faddeev amplitudes the three-body wave function is calculated in the LS coupling scheme. Even though the number of channels in the Faddeev amplitudes is finite, the exact wave function obtained from them has infinitely many three-body channels, as a consequence of the action of the permutation operators on the Faddeev amplitudes. For the expectation value of the TNI force we truncate these three-body channels to the most important eight LS channels. These eight three-body channels are all one can consider corresponding to the triton quantum numbers if, beyond the s waves, the two-body pair or relative orbital angular momentum of the spectator or even both are allowed to be in a d wave. Because of this truncation, no three-body channels with two-body p waves are included in the wave function actually used, although a three-body P-wave contribution *is* generated by the coupling of the two-body d waves. Two of these eight channels are pure s-waves, both in the two-body pair and in the relative orbital angular momentum of the spectator. The corresponding three-body amplitudes from these two channels alone differ from the ones obtained with a two-body t matrix in pure s waves to begin with. This fact makes the effect of these two channels qualitatively different from its effect in the simple calculations with the Gaussian and Malfliet-Tjon models: by construction, the Paris and Bonn three-body wave functions components in these channels do contain already "correlations" due to higher partial waves. As for the other six channels, three build up the triton D-state wave function (by having the spectator, the pair, and both in a two-body d wave) . Through the coupling of two-body d waves in both the pair and spectator, the other three channels generate a triton P state and an additional contribution to the S state. The eight channels together comprise 94% and 96% of the total norm of the full wave function, respectively, for the Paris and OBEPQ cases (in the actual calculation we did not renormalize the truncated wave function to 1).

In Table II we give also for the OBEPQ short-range parameterization of the TNI's the expectation values for the Bonn OBEPQ and Paris triton wavefunction models. Table II displays the π-ω and π-σ exchange force effects only (according to Table I they dominate the overall π-scalar and π-vector exchange TNI). In the truncation of the wave function we use, the diagonal expectation values $\langle D|V_3|D\rangle$ and $\langle P|V_3|P\rangle$ are identically zero, so the dominant correction to the $\langle S|V_3|S\rangle$ term is the strong off-diagonal $\langle S|V_3|D\rangle$ coupling induced by the rank-2 spatial tensor component of the π−short-range three-nucleon interactions. A cancellation effect between the scalar and vector exchanges lessens the overall D-state effect from that of the individual π-σ and π-ω pieces. A comparison with more complete calculations [36] in progress does not indicate the presence of any more significant D-state effects, even in a nonperturbative calculation. The off-diagonal $\langle S|V_3|P\rangle$ matrix element to the three-body P state is rather small and the cancellation between the scalar and vector exchanges is nearly complete. Table II confirms the trend of repulsion obtained with the more schematic wave function models used in the calculations of Table I. It also stresses once more the traditional extreme results of the OBEPQ potential for the triton binding energy.

All wave-function models indicate that the S-state matrix elements of the π−short-range three-nucleon interactions that are associated with intermediate $N\bar{N}$ pair excitation are about 100–300 keV [99 keV, 126 keV, 233 keV, 332 keV, respectively for the harmonic oscillator (H.O.), MT, Paris and Bonn model wave functions], when these three nucleon interactions are constructed so as to be consistent with the Bonn OBEPQ model for the NN interaction. This 100–300 keV result has a substantial theoretical uncertainty margin that is due to the remaining uncertainty in the different short-range behavior of the nucleon-nucleon interactions. This ultimately relates to the consistency problem of how to deal with the ambiguities of a nonrelativistic reduction. To illustrate this uncertainty we have also constructed these π-scalar and π-vector exchange three-nucleon interactions from the Paris model for the nucleon-nucleon interaction [21]. In that potential model the isospin independent scalar exchange component is much weaker than in other realistic phenomenological potential models, and is in fact repulsive at short range [13]. As a consequence that matrix element $\langle V_{\pi\sigma}\rangle$ calculated with the oscillator wave-function model is only 0.143 MeV as compared to the corresponding value 0.392 MeV obtained with the TNI consistent with the Bonn OBEPQ NN potential. Replacing the oscillator wave function by the Paris wave function reduces the matrix element to the very small value of 0.067 MeV. This reduction of $\langle V_{\pi\sigma}\rangle$ makes the net matrix element of the π−short-range three-nucleon interaction attractive, when constructed from the parametrized Paris potential ($\simeq -0.200$ MeV for the oscillator wave function, or $\simeq -0.174$ MeV for the Paris wave function). The difference of almost 0.5 MeV (and a change of sign) between these two most consistent calculations presented here (a Bonn wave function and Bonn TNI compared to the Paris wave function and Paris TNI) is clearly due to the weak isospin independent scalar exchange component of the Paris potential. It may also be due in part to the μ parameter inconsistency between the derivation of the TNI and the Paris choice of short-range modification, discussed toward the end of Sec. II.

The TNI's that are associated with $N(1440)$ intermediate states lead to much smaller binding energy effects than those just discussed, a fortunate result for our goal of building TNI's consistent with a given nucleon-nucleon interaction. For example, with the oscillator model density function (4.3) the matrix element of the $N(1440)$ intermediate state $\pi\sigma$ three-nucleon interaction (3.2a) takes the form

$$\langle V^*_{\pi\sigma}\rangle = \frac{3}{\pi^4}\frac{g_\sigma g^*_\sigma}{m^*-m_N}\frac{f_{\pi NN}f^*_\pi}{m_\pi^2}\int_0^\infty dv\, e^{-v^2/2\alpha^2}\frac{v^2}{v^2+m_\sigma^2} \times \int_0^\infty dk e^{-k^2/2\alpha^2}\frac{k^4}{k^2+m_\pi^2}\sqrt{\frac{\pi\alpha}{kv}}I_{1/2}\left(\frac{kv}{2\alpha^2}\right). \quad (4.5)$$

The corresponding expression for the matrix element of the $\pi\omega$ exchange three-nucleon interaction $V^*_{\pi\omega}$ (3.2b) can be obtained simply from (4.5) by means of the substitutions $g^*_\sigma \rightarrow g^*_a$ and $m_\sigma \rightarrow m_\omega$ and an accompanying overall sign change.

The matrix element of the $N(1440)$ intermediate state $\pi\sigma$ three-nucleon interaction $V^*_{\pi\sigma}$ (4.5) for the S-state oscillator model for the bound trinucleon states is $+0.200$ MeV. Here we have used the parameters m_σ = 530 MeV and $g^2_\sigma/4\pi$ = 8.2797 for the mass and (nucleon) coupling constants for the exchanged σ meson as suggested by the Bonn OBEPQ potential [19], and the short-range form factors (3.7). The values of the meson-NN^* coupling constants are those derived in Sec. III. The corresponding matrix element of the $\pi\omega$ TNI $V^*_{\pi\omega}$ (3.2b) is -0.150 MeV so that the net contribution of the TNI associated with the $N(1440)$ is repulsive and $\simeq 0.050$ MeV. When the same calculations are made with the substitution of the Malfliet-Tjon wave function for the oscillator wave function, the individual contributions are $\simeq +0.065$ MeV from $V^*_{\pi\sigma}$ and $\simeq -0.045$ MeV from $V^*_{\pi\omega}$, for a total repulsion of $\simeq 0.020$ MeV. For both these schematic triton wave functions the sum of π–short-range TNI's produces a small, but non-negligible, fraction of the repulsive contributions in Table I due to the TNI associated with intermediate $N\bar{N}$ pair excitation. For the more realistic wave functions, including D-state effects, this fraction becomes very small indeed. The OBEPQ parameterization of the TNI evaluated with the Paris triton wave function yields only $+0.004$ MeV. In our most consistent calculation (Bonn OBEPQ triton wave function and the OBEPQ parameterization of the TNI), the total repulsion from the TNI associated with the $N(1440)$ is about one-tenth (+0.034:+0.332 MeV) of that of the intermediate $N\bar{N}$ pair excitation.

With the S-state oscillator trinucleon model and the semirealistic, Malfliet-Tjon NN potential wave function, the combined contribution of all the π–short-range TNI's (constructed from OBEPQ for the NN interaction) is thus about 150 keV. More realistic triton wave functions give significant contributions from the three-body D state. The net result increases the expectation value from the TNI associated with intermediate $N\bar{N}$ pair excitations, and decreases the expectation value from the TNI associated with the $N(1440)$ to a negligible amount. The final numbers range from 200 keV to 350 keV, which corresponds to a significant fraction of the repulsive TNI contribution postulated [8] to explain the binding energies of the three-nucleon bound states.

The need for a phenomenological TNI such as that of Ref. [8] is lessened by other mechanisms, not considered there, which also give a slight repulsive contribution to the triton binding energy. The repulsive contribution of 200 keV to 350 keV, derived and evaluated in this paper, lies between perturbative estimates of the intermediate $N\bar{N}$ pair excitations in the Tucson-Melbourne TNI. That is, the two-pion exchange pair term gives a repulsion of 200 keV and the "Kroll-Ruderman" term [$V^S_{\pi\rho}$ of Eq. (2.7)] gives a repulsion of about 500 keV. The latter perturbative estimates [9] were made with a three-nucleon wave function obtained from *both* the Bonn OBEPQ NN interaction and the Tucson-Melbourne TNIs (with the Tucson-Melbourne short-range behavior). Estimates of pair excitations made with other realistic NN interactions such as the Paris or Nijmegen ones are, as usual, much smaller than those with OBEPQ. These pair excitations are an easily visualized form of relativistic correction [30] and can thus be qualitatively compared with recent calculations of another relativistic correction in the triton. This relativistic correction, which corresponds to an interacting pair of nucleons whose center of mass is not the nuclear center of mass, produces $\sim$250–300 keV repulsion in two very different calculations [37,38]. This effect is comparable in magnitude to either of the pair excitations calculated here.

V. DISCUSSION

The present results demonstrate that the pion-scalar and pion-vector meson exchange three-nucleon interactions are important on the general scale of three-nucleon interactions. At the level of precision attained by calculations with the present realistic semiphenomenological nucleon-nucleon interactions, which also contain three-nucleon interactions, this TNI has to be included in the calculation of nuclear binding energies. The repulsive contribution of this π–short-range exchange TNI appears able to explain most of the repulsion hitherto ascribed to the purely phenomenological spin-independent TNI of short range, which was introduced to achieve agreement with the empirical binding energies of the few-nucleon systems [8].

The numerical values of the matrix elements of the π–short-range three-nucleon interactions presented here show that the occurrence of cancellations decreases somewhat the importance of the three-nucleon D state in the effect of the TNI, but it remains important. A comparison with more complete calculations [36] corroborates our conclusion.

The most uncertain in magnitude of the three-nucleon interactions considered here is that associated with the excitation of intermediate $N(1440)$ resonances. The main uncertainty in this interaction is due to the unknown $\omega NN^*(1440)$ coupling constant. Fortunately, this interaction has much smaller effects on the triton binding energy than the interaction due to nucleon-antinucleon pair terms which can be directly related to realistic NN interactions in the manner shown here.

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APPENDIX: PARTIAL WAVE DECOMPOSITION

While the calculations that employ the semirealistic Gaussian and Malfliet-Tjon wave functions provide results that are qualitatively indicative, present state-of-the-art three-nucleon calculations have provided several examples of delicate cancellations that may be missed by schematic wave-function models. This motivated a calculation based on more realistic wave functions, such as the ones obtained with the Paris and Bonn OBEPQ potentials (in Refs. [9,18]). These two potentials lead to quite different answers for the binding energy of the triton, and different probabilities for the S and D state components of the wave function.

The expectation value of the TNI can be calculated by means of Eq. (4.1). Although this equation is general, it requires the knowledge of $g(\vec{\tau},\vec{v})$. Since this last function is not directly supplied by any standard Faddeev code, it demands extra computational effort. The algorithm that is usually employed with realistic interaction models is to evaluate the matrix element of the TNI in the partial wave decomposed basis used in the wave-function calculation. The partial wave decomposition has been presented in a fairly general way in Ref. [3]. We here review its main steps, keeping the notation close to the one introduced in that work.

In the LS coupling scheme the notation for the partial wave decomposed wave function is

$$|pq\alpha\rangle_2=|pq,[(l\lambda)L(s1/2)S]JJ_z,(t1/2)TT_z\rangle_2,$$

where the index 2 specifies the particle that is taken to be the spectator in the definition of $(\vec{p},\ \vec{q})$. To calculate the TNI matrix element in this basis, we start by separating explicitly the spin dependence of the TNI from the orbital one. For this purpose the TNI is decomposed in spherical components and consequently in spherical harmonics. Subsequently closed form quadrature over the angular arguments of these spherical harmonics is performed. This requires expansion in Legendre polynomials of the angle dependence of the form factors and propagators. These expansions (in three angles) are done numerically, through a Gaussian mesh of at least 20 points for each angle.

To illustrate the technique we consider the intermediate $N\bar{N}$ π-σ TNI. For this particular case we have (leaving out the trivial isospin dependence),

$$(\vec{\sigma}^2\cdot\vec{k}_\sigma)(\vec{\sigma}^1\cdot\vec{k}_\pi)=\frac{4\pi}{3}k_\sigma k_\pi\sum_k\sqrt{2k+1}\times[\sigma(1)\times\sigma(2)]^kY^k_{11}(\hat{k}_\pi,\hat{k}_\sigma). \tag{A1}$$

Here the $Y^k_{11}(\hat{k}_\pi,\hat{k}_\sigma)$ function stands for the two (coupled) spherical harmonics, which depend separately on the momenta of each meson, i.e.,

$$[Y^1\times Y^1]^k=\sum_{m_1,m_2}C^{11k}_{m_1m_2m}Y^1_{m_1}(\hat{k}_\pi)Y^1_{m_2}(\hat{k}_\sigma). \tag{A2}$$

After expressing the exchanged meson momenta in terms of the Jacobi coordinates $\vec{p},\vec{q}$ (for the initial three-nucleon state) and $\vec{p}\,',\vec{q}\,'$ (for the final three-nucleon state),

$$\vec{k}_\pi=(\vec{p}-\vec{p}\,')-\frac{1}{2}(\vec{q}-\vec{q}\,'),$$

$$\vec{k}_\sigma=(\vec{p}-\vec{p}\,')+\frac{1}{2}(\vec{q}-\vec{q}\,'),$$

the function $Y^k_{11}(\hat{k}_\pi,\hat{k}_\sigma)$ can be decomposed in coupled spherical harmonics of simpler arguments:

$$Y^k_{11}(k_\pi,k_\sigma)=3\sum_{r_1+r_2=1}\sum_{s_1+s_2=1}(-1)^{r_2}F(1,r_1,r_2)F(1,s_1,s_2)\times\left(\frac{1}{2}\right)^{r_2+s_2}\frac{|\vec{p}-\vec{p}\,'|^{r_1+s_1}|\vec{q}-\vec{q}\,'|^{r_2+s_2}}{k_\pi k_\sigma}\sum_{t_1t_2}\begin{Bmatrix} r_1 & r_2 & 1\\ s_1 & s_2 & 1\\ t_1 & t_2 & k_1\end{Bmatrix}C^{r_1s_1t_1}_{000}C^{r_2s_2t_2}_{000}Y^k_{t_1t_2}(\widehat{\vec{p}-\vec{p}\,'},\widehat{\vec{q}-\vec{q}\,'}), \tag{A3}$$

where

$$F(a,b,c)=\sqrt{\frac{(2a+1)!}{(2b)!(2c)!}}.$$

Denote by $f_a(k_a)$ the product of the propagator of meson a by the two NNa couplings, including the form factor function that is introduced at the vertices. The momentum k_a depends on three angles: the angle between $\vec{p}-\vec{p}\,'$ and $\vec{q}-\vec{q}\,'$, whose cosine is x_1, the angle between $\vec{p}$ and $\vec{p}\,'$, the cosine of which is x_2, and the angle between $\vec{q}$ and $\vec{q}\,'$, the cosine of which is x_3.

To prepare the angular integrations we can start by doing a decomposition in Legendre polynomials in the angular variable x_1:

$$f_\pi(k_\pi)f_\sigma(k_\sigma)=\sum_{l_1}\frac{2l_1+1}{2}g_{l_1}(|\vec{p}-\vec{p}\,'|,|\vec{q}-\vec{q}\,'|)P_{l_1}(x_1). \tag{A4}$$

The two additional Legendre polynomial decompositions — in x_2 and x_3 — involve the function $g_{l_1}(|\vec{p}-\vec{p}\,'|,|\vec{q}-\vec{q}\,'|)$ of Eq. (A4) as well as powers of $|\vec{p}-\vec{p}\,'|$ and $|\vec{q}-\vec{q}\,'|$, originated by contracting the function $Y^k_{t_1t_2}(\hat{\vec{p}-\vec{p}\,'},\hat{\vec{q}-\vec{q}\,'})$ from Eq. (A3) with P_{l_1} from Eq. (A4). The final result of the three decompositions is

$$H_{l_1l_2l_3\alpha_1t_3\alpha_2t_4}(p,p',q,q')=\int_{-1}^{1}dx_1\int_{-1}^{1}dx_2\int_{-1}^{1}dx_3P_{l_1}(x_1)P_{l_2}(x_2)P_{l_3}(x_3)|\vec{p}-\vec{p}\,'|^{\alpha_1-t_3}|\vec{q}-\vec{q}\,'|^{\alpha_2-t_4}f_\pi(k_\pi)f_\sigma(k_\sigma). \tag{A5}$$

Using Eq. (A1), each term of the Σ_k, $V^k_{\pi\sigma}$, gives for the orbital part of the matrix element,

$$\langle(l'\lambda')L'M'|V^k_{\pi\sigma}|(l\lambda)LM\rangle=\frac{(4\pi)^2}{8}\hat{k}^2\hat{L}C^{kLL'}_{\mu MM'}(-)^{k+l+\lambda}\sum_{l_1l_2l_3}(-)^{l_1+l_2+l_3}\hat{l}_1^2\hat{l}_2^2\hat{l}_3^2\sum_{t_1t_2t_3t_4}\hat{t}_1\hat{t}_2\hat{t}_3\hat{t}_4\xi^{l_1l_2l_3t_3t_4}_{t_1t_2k}\begin{Bmatrix} t_2 & t_1 & k\\ t_3 & t_4 & l_1\end{Bmatrix}$$
$$\times\begin{Bmatrix} l & l' & t_3\\ \lambda & \lambda' & t_4\\ L & L' & k\end{Bmatrix}C^{t_1l_1t_3}_{000}C^{t_2lt_4}_{000}\phi_{t_3l_2ll'}(p,p')\phi_{t_4l_3\lambda\lambda'}(q,q'), \tag{A6}$$

with

$$\phi_{abcd}(k,k')=\sum_{f_1+f_2=a}F(a,f_1,f_2)k^{f_1}k'^{f_2}(-)^{f_2}C^{bf_1c}_{000}C^{bf_2d}_{000}\begin{Bmatrix} d & c & a\\ f_1 & f_2 & b\end{Bmatrix} \tag{A7}$$

and

$$\xi^{l_1l_2l_3t_3t_4}_{t_1t_2k}=\sum_{r_1+r_2=1}\sum_{s_1+s_2=1}C^{r_1s_1t_1}_{000}C^{r_2s_2t_2}_{000}F(1,r_1,r_2)F(1,s_1,s_2)(-)^{r_2}\left(\frac{1}{2}\right)^{r_2+s_2}\begin{Bmatrix} r_1 & r_2 & 1\\ s_1 & s_2 & 1\\ t_1 & t_2 & k\end{Bmatrix}H_{l_1l_2l_3\alpha_1t_3\alpha_2t_4}(p,p',q,q') \tag{A8}$$

($\alpha_1=r_1+s_1$; $\alpha_2=r_2+s_2$).

Applying the Wigner-Eckart theorem, we extract from Eq. (A6) the orbital reduced matrix element, which ultimately we reconnect to the spin one, generating the desired final result.

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