Faddeev null-plane model of the nucleon

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The nucleon is formulated as a relativistic system of three constituent quarks interacting via a zero-range two-body force in the null plane. The covariance of the null-plane Faddeev-like equation under kinematical front-form boosts is discussed. The nucleon wave function is obtained from the numerical solution of the Faddeev equation in the null plane. The proton electric form factor, obtained from the Faddeev wave function, reproduces the experimental data for low momentum transfers and qualitatively describes the asymptotic region.

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I. INTRODUCTION

Relativistic constituent quark models with null-plane wave functions [1] have received much attention lately. These wave functions are covariant under kinematical front-form boosts [2], as a consequence of the stability of the Fock-state decomposition under such transformations [3]. They include, for example, the boost from the bound-state center of mass system to a frame in which it has some specified momentum in the transverse direction [2]. Such a property is fundamental for the frame independence of the electromagnetic form factors of bound-state systems, in the limited sense of kinematical front-form transformations [4].

In particular, in such a framework, several studies have addressed nucleon properties, such as electromagnetic form factors [5–8] and the distribution function [9] with Gaussian wave functions. These models used constituent quarks as the relevant degrees of freedom.

We have also seen examples in the literature [10] in which the spontaneous breaking of chiral symmetry originate the constituent quark. The spontaneous breaking of chiral symmetry, which is an essential requirement of low-energy QCD, has been modeled using the Nambu–Jona-Lasinio (NJL) [11] contact interaction and applied with sucess to hadronic phenomenology [10]. Confinement is not included in these effective models. The NJL based models demands two physical notions: the relativistic constituent quark and a contact interaction.

Our aim in this work is to test the nucleon structure given in terms of these two concepts. We present a calculation of the proton electric form factor $[G_E(q^2)]$ by using the Faddeev null plane approach to obtain the three-quark nucleon wave function. The null plane gives the natural coordinates to investigate high energy processes [12–14]. The nucleon null-plane wave function is obtained from the solution of the Faddeev [15] equation with a zero-range force [16] acting between the constituent quarks. This is the first application of the Faddeev equation to the nucleon in the null plane with a contact interaction [17], in which $G_E(q^2)$ is obtained. No confinement is present in our effective model. The totally symmetric spatial part of the wave function is obtained numerically in a three-body calculation. We leave out the spin degrees of freedom in this first dynamical calculation of the null-plane wave function. We use the covariance of the proton model under kinematical front-form boosts to calculate $G_E(q^2)$, and compare our numerical results with the available experimental data. $G_E(q^2)$ scales with q^4 in the asymptotic region and describes the data for low momentum transfers.

This work represents a step in the study of Faddeev bound-state models of the nucleon in the null plane. The relativistic spin structure is beyond the scope of our present work but can be introduced through the Melosh rotation of the spins as discussed in Ref. [7], which is left for a future development.

This work is organized as follows. In Sec. II, we review the null-plane three body model with contact interaction and in Sec. III, we show explicitly that our model is covariant under kinematical front-form boosts. In Sec. IV, we construct the proton electric form-factor from the totally symmetric part of the three-quark wave function. And in Sec. V, we present our numerical results and compare it to the data. A brief discussion of the main findings is presented in the conclusion

II. THE MODEL

The three-body Faddeev equation with a pair-contact interaction in the null plane, has been discussed in detail in Ref. [16] and here we give just a brief discussion.

The null plane is defined by $x^+ = x^0+x^3 = 0$, and the kinematical momenta for each particle are given by k^+ and $\vec{k}_\perp$. It is usual to introduce the momentum fraction for each particle in a given system, $x = k^+/P^+$, where P^+ is the total + momentum component.

The bound-state two-body null-plane wave function for a constant vertex is given in Ref. [18]. The three-quark bound-state null-plane wave function, for the contact interaction, is constructed in terms of the Faddeev components of the vertex, $v(x,\vec{k}_\perp)$, as

$$\Psi(x_1,\vec{k}_{1\perp};x_2,\vec{k}_{2\perp}) = \frac{v(x_1,\vec{k}_{1\perp})+v(x_2,\vec{k}_{2\perp})+v(x_3,\vec{k}_{3\perp})}{\sqrt{x_1x_2x_3}(M_n^2 - M_0^2)}, \tag{1}$$

where M_n is the nucleon mass. Each quark has momentum fraction x_j and transverse momentum $\vec{k}_{j\perp}$ (j=1,3), satisfying $x_1+x_2+x_3 = 1$ and $\vec{k}_{1\perp}+\vec{k}_{2\perp}+\vec{k}_{3\perp} = 0$ in the nucleon center of mass. The free three-quark mass is

$$M_0^2 = \sum_{j=1,3} \frac{k_{j\perp}^2+M^2}{x_j} , \quad (2)$$

where M is the constituent quark mass.

The vertex component, v, satisfies a Weinberg-type [19] integral equation. It is derived from the Bethe-Salpeter equation, after integrating over k^- $(=k^0-k^3)$ in the internal momentum loop [16]. The subsystem scattering is summed up to all orders in the Faddeev construction of the three-body connected kernel equations. This corresponds to summing the two-body scattering process in the ladder approximation in our relativistic model. Such an amplitude is obtained after a renormalization process, in which the physical information input is the two-body bound-state mass, M_d, which corresponds to the diquark mass.

In our simplified model, it is important to observe that M_d is also equal to the meson mass, (μ), since the pole of the two-body amplitude in the S channel corresponds to a pole in the T channel, as one sees by exchanging an initial quark by a final one. One should observe that in a confining model such an identification is not possible, since colorless mesons are not confined in contrast to colored diquarks. But in our schematical model, diquarks and mesons are consequences of the nonconfining nature of the interaction. This should be understood only as the possibility of clustering inside the nucleon in more realistic models. Even in sophisticated models of the constituent quarks based on the Nambu–Jona-Lasinio approach, the diquark poles of the two quark amplitude are present, as a consequence of the contact interaction [10,20].

In a three quark bound-state system, it is sufficient to write the two-quark scattering amplitude for $M_2<2M$, where M_2 is the two-quark subsystem mass [16]:

$$\tau(M_2)=-i(2\pi)^2\left\{ \sqrt{\frac{M^2}{\mu^2}-\frac{1}{4}}\arctan\left(2\sqrt{\frac{M^2}{\mu^2}-\frac{1}{4}}\right)^{-1} - \sqrt{\frac{M^2}{M_2^2}-\frac{1}{4}}\arctan\left(2\sqrt{\frac{M^2}{M_2^2}-\frac{1}{4}}\right)^{-1}\right\}^{-1} . \quad (3)$$

The integral equation for $v(y,\vec{p}_\perp)$, in the nucleon center of mass system, is given by

$$v(y,\vec{p}_\perp)=\frac{i}{(2\pi)^3}\tau(M_2)\int_{\frac{M^2}{M_n^2}}^{1-y}\frac{dx}{x(1-y-x)}\int^{k_\perp^{\max}} d^2k_\perp \frac{v(x,\vec{k}_\perp)}{M_n^2-M_3^2} , \quad (4)$$

where the momentum y, x, $\vec{p}_\perp$, and $\vec{k}_\perp$ describe the spectator quark states. The virtual two quark subsystem mass is

$$M_2^2=(1-y)\left(M_n^2-\frac{p_\perp^2+M^2}{y}\right)-p_\perp^2 . \quad (5)$$

We constrain M_2 to assume real values [16], and it follows that the spectator transverse momentum attains a maximum value $k_\perp^{\max}=\sqrt{(1-x)(M_n^2x-M^2)}$.

The mass of the virtual three-quark intermediate state is

$$M_3^2=\frac{k_\perp^2+M^2}{x}+\frac{p_\perp^2+M^2}{y}+\frac{(\vec{p}+\vec{k})_\perp^2+M^2}{1-y-x}. \quad (6)$$

III. COVARIANCE UNDER KINEMATICAL FRONT-FORM BOOSTS

We show below that the null-plane Faddeev equation, Eq. (4), is covariant under kinematical front-form boosts. There are seven kinematical boosts, three of which include Lorentz boosts and are given by [2]

$$k'^+ = A^+ k^+ \quad , \vec{k'_\perp} = \vec{k_\perp}+\vec{A_\perp}\ k^+ , \quad (7)$$

with $k'_\nu k'^\nu = k_\nu k^\nu$. The other four kinematical boosts are three null-plane translations and rotation around the z direction.

The kinematical null-plane boost of the nucleon from its rest frame to a frame where it has a transverse momentum $\vec{P}_{n\perp}$ is defined by

$$A^+=\sqrt{1+P_{n\perp}^2/M_n^2} \text{ and } \vec{A}_\perp=\vec{P}_{n\perp}/M_n . \quad (8)$$

The null-plane Faddeev equation in the new frame given by Eq. (8) is written in terms of the new momentum variables related to the rest frame ones by Eq. (7). In Eq. (4), the momentum fraction remains the same, and the transverse momentum changes,

$$x'=x,\ y'=y,\ \vec{k}'_\perp=\vec{k}_\perp+\vec{P}_{n\perp}\ x, \text{ and } \vec{p'}_\perp=\vec{p}_\perp+\vec{P}_{n\perp}\ y, \quad (9)$$

where prime indicates quantities in the new frame. The maximum transverse momentum also changes according to the above equations.

The Faddeev component of the vertex in the new frame is given in terms of the center of mass vertex by

$$v'(x',\vec{k}'_\perp)=v(x,\vec{k}'_\perp-\vec{P}_{n\perp}\ x) , \quad (10)$$

since the transformation in the transverse direction can be eliminated by changing the integration variables. The mass of the nucleon, as obtained by the solution of Eq. (4), is the same in any frame related to the center of mass frame by a kinematical front-form boost.

IV. PROTON ELECTRIC FORM FACTOR

The electric form factor is calculated from I^+, the "good component" [12] of the electromagnetic current in the Breit frame, where the momentum transfer is such that

$$q^+=0 \text{ and } \vec{q}_\perp^{\,2}=-q^\nu q_\nu. \tag{11}$$

The transverse proton momenta are $\vec{P}^f_{n\perp}=-\vec{P}^i_{n\perp}=\vec{q}_\perp/2$, where the superscripts i and f stand for initial and final, respectively. In this particular frame, the pair-creation contribution to I^+ is suppressed [13,18,21], and only the constituent quark component of the wave function remains in the form-factor calculation, without violating kinematical front-form boost invariance.

In general, the truncation of the Fock space of the nucleon wave function to the constituent quark component, in the null-plane wave function, violates rotational and gauge invariance. The generators of rotations around the x and y directions are of nonkinematical nature in the null plane, and have off-diagonal matrix elements in the Fock space. The violation of the gauge condition comes from the fact that I^-, the "bad-component" of the current, also has off-diagonal matrix elements. Our model for the proton wave function is restricted only to the lowest Fock component, and is limited in that sense. But the key point in using this wave function in the calculation of the form factor is the use of the I^+ component of the current. This component is diagonal in the Fock space for $q^+=0$ [14] and no pair creation terms are present. It is also enough to calculate the matrix elements of I^+ to obtain all the form factors as has been demonstrated by Chung, Coester, Keister, and Polizou [4]. Only for a two-body bound state with spin one or higher are there limitations with such a scheme [4,13,22], which is of no relevance for our present study of the nucleon. At the present level of our phenomenological model, it only makes sense to calculate matrix elements that are diagonal in the null-plane Fock space.

The proton electric form factor is obtained from the three-body bound-state null-plane wave function. The conventions used for $G_E(q^2)$ are those of Ref. [23]. As the wave function is totally symmetric under particle permutation, we can reduce the electric form factor to the term in which only particle 3 absorbs the virtual photon:

$$G_E(q^2)=\int dx_1dx_2d^2k_{1\perp}d^2k_{2\perp}\Psi^f(x_1,\vec{k}_{1\perp};x_2,\vec{k}_{2\perp}) \times\Psi^i(x_1,\vec{k}_{1\perp};x_2,\vec{k}_{2\perp}) \ , \tag{12}$$

where $x_3=1-x_1-x_2$. The transverse momentum of particle 3 is different for the initial and final wave functions,

$$\vec{k}^i_{3\perp}=-\frac{\vec{q}_\perp}{2}-\vec{k}_{1\perp}-\vec{k}_{2\perp}$$

in the initial proton, and

$$\vec{k}^f_{3\perp}=\frac{\vec{q}_\perp}{2}-\vec{k}_{1\perp}-\vec{k}_{2\perp}$$

in the final proton. The wave-function normalization is such that $G_E(0) = 1$.

The initial and final proton wave functions in the Breit frame are given in terms of the center-of-mass wave function according to the boost transformation of the vertex, Eq. (10),

$$\Psi^i(x_1,\vec{k}_{1\perp};\ x_2,\vec{k}_{2\perp}) =\Psi\left(x_1,\vec{k}_{1\perp}+\frac{\vec{q}_\perp}{2}x_1;x_2,\vec{k}_{2\perp}+\frac{\vec{q}_\perp}{2}x_2\right)\ , \tag{13}$$

for the initial and

$$\Psi^f(x_1,\vec{k}_{1\perp}\ ;\ x_2,\vec{k}_{2\perp}) =\Psi\left(x_1,\vec{k}_{1\perp}-\frac{\vec{q}_\perp}{2}x_1;x_2,\vec{k}_{2\perp}-\frac{\vec{q}_\perp}{2}x_2\right)\ , \tag{14}$$

for the final proton. We observe that $G_E(q^2)$ is invariant under frame transformations related to the Breit frame by kinematical front-form boosts.

V. RESULTS

Our dynamical null-plane model of the proton wave function requires two parameters as input, the constituent quark mass (M) and the meson mass (μ). We restricted M and μ so that the nucleon mass is 938 MeV. For fixed nucleon mass, the ratio μ/M is free.

In Table I, we present the results of varying the ratio μ/M in the model. The quark mass decreases by increasing μ/M and the binding energy of the quarks in the nucleon also decreases. The diquark mass varies with μ/M and, for values around 1.8, it has a maximum corresponding to 695 MeV. The nucleon radius also attains its experimental value around 1.8, where it has a minimum as a function of μ/M. Curiously the model fits the experimental radius close to the minimum possible value of the radius in the model. The average of the experimental ω and ρ meson masses is 780 MeV.

We present in Table I, the values of the proton eletric form factor at 1 GeV^2, in order to study in more detail the gradual changes with μ/M. G_E presents a maximum as a function of μ/M, for values around 1.8. This is consistent with the minimum found for the radius. The experimental value for G_E suggests that for $1.6<\mu/M<1.95$, the model produces reasonable agreement with the form factor data. The meson mass in the model has values of $677<\mu<695$ MeV in this region, which is consistent with the experimental average value of 780 MeV. We observe that at the values of μ/M equal to 1.6 and 1.95, the form-factor is nearly the same, as shown in Table I. The quark binding energy changes with the ratio μ/M, as seen in Table I. For the above region of values of μ/M, the binding energy is within the values of $115 < 3M-M_n < 331$ MeV.

We present, in Fig. 1, the electric form-factor calculation below 2 GeV^2 for three representative values of the ratio $\mu/M=$ 1, 1.8, and 1.95, meaning a strong, average, and weakly bound meson, respectively. The values 1 and 1.95 are

TABLE I. Results for the binding energy of the quarks in the nucleon (third column), diquark or meson masses (fourth column), nucleon radius (fifth column), and form factor (last column) as a function of the ratio of the meson mass to the quark mass (μ/M) shown in the first columm. In the second column, we show the quark masses for a nucleon mass of 938 MeV. The experimental values of the nucleon radius and electric form factor at 1 GeV^2 [24] are shown in the last two lines.

μ/M	M (MeV)	$3M-M_n$ (MeV)	μ (MeV)	r_n (fm)	$G_E(q^2=1\text{GeV}^2)$
1.0	519	619	519	1.05	0.050
1.2	488	526	586	0.95	0.090
1.4	457	433	640	0.88	0.129
1.6	423	331	677	0.82	0.161
1.7	406	280	690	0.81	0.172
1.8	386	220	695	0.80	0.177
1.9	364	154	692	0.81	0.172
1.95	351	115	683	0.84	0.161
Expt.			780	0.81 ± 0.04[a]	0.179 ± 0.006
				0.862 ± 0.012[b]	

[a]Reference [25].
[b]Reference [26].

at the extremes of the interval for μ/M and 1.8 gives the minimum radius. The results for $\mu/M=1.6$ are close to those of $\mu/M=1.95$ in this region of momentum transfers. Comparision with the data [24] below 2 GeV^2 suggests that the model prefers the values of μ/M of about 1.6 and 1.95. The meson masses, in Table I, corresponding to these values are 677 MeV and 683 MeV, in reasonable agreement with the experimental meson mass.

The values of $q^4G_E(q^2)$ for squared momentum transfers below 6 GeV^2 are compared with the experimental data of Ref. [24] in Fig. 2, for several values of μ/M. To illustrate the change of the electric form factor with μ/M, we choose values ranging from 1 to 1.95. The form factor increases with μ/M and for values above 1.6, $q^4G_E(q^2)$ lies within the curves for 1.95 and 1.8. We observe a qualitative agreement with the avaliable data, for μ/M between 1.6 and 1.95 . The model shows a slight increase for $q^2 > 4$ GeV^2 and it saturates at the values of 0.33, 0.33, and 0.26 , for $\mu/M=$ 1.6, 1.8, and 1.95, respectively. The nonconfining nature of our wave function is the reason for the flatness of the product $q^4G_E(q^2)$ at high q^2. Models of the nucleon with confinement yield a form factor which decreases quickly in the asymptotic region [6], or they obtain a good agreement with the asymptotic data at the expense of a small constituent quark mass [7].

We compare our results with calculations in the Faddeev–Nambu–Jona-Lasinio approach [20]. These works obtained the binding energy of the constituents quarks in the nucleon. The values found for the binding energies were from 200 MeV to about 400 MeV. We conclude from the calculation of the proton electric form factor, that values of $\mu/M=$ 1.6 and 1.95 are favored by the low-energy data. This interval corresponds to binding energies between 115 and 331 MeV of the quarks in the nucleon. It is interesting that our result has an overlap with these previous calculations [20].

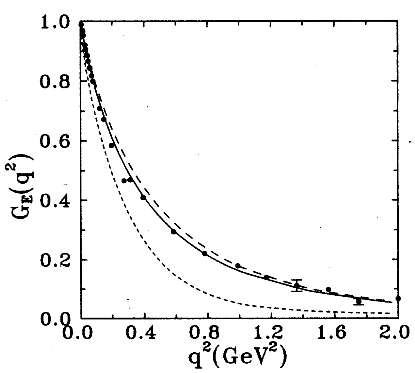

FIG. 1. Proton electric form factor for $q^2 < 2$ GeV^2. Constituent quark masses (M) and meson masses (μ) have been varied in the calculations, while the nucleon mass was kept fixed at 938 MeV. M = 519 MeV with μ = 519 MeV (short-dashed line); M = 386 MeV with μ = 695 MeV (dashed line) and M = 350.6 MeV with μ = 683.6 MeV (solid line). Experimental data from Ref. [24].

VI. CONCLUSIONS

We have developed a simple three-body model of the nucleon wave function in the null plane, in which the main

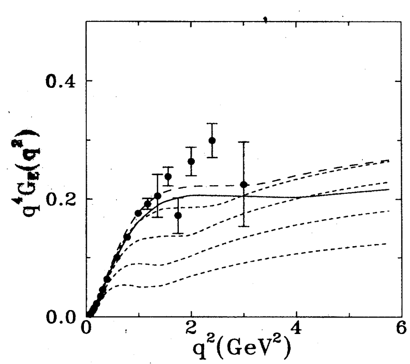

FIG. 2. Proton $q^4G_E(q^2)$ for $q^2 < 6$ GeV^2. The solid line is the result for $\mu/M=1.95$ and the dashed line for $\mu/M=1.8$. The short dashed lines with increasing values of $q^4G_E(q^2)$ have $\mu/M=$ 1, 1.2, 1.4, and 1.6, respectively.

physical ingredients are the constituent quarks and a contact interaction among them. We construct the null-plane Faddeev wave function of the proton from the values of the diquark and quark masses, the only inputs in our model.

We obtained the electric proton form factor, which is well described below 1 GeV for a reasonable meson mass, which we identified with the average ρ and ω meson masses. In the asymptotic region a qualitative agreement with experimental data is found. We have disconsidered the spin degree of freedom in the dynamical calculation. Nonetheless, the data for the proton electric form factor prefers binding energies of the constituent quarks in the interval between 115 MeV and 331 MeV, overlapping with the range of values from recent Faddeev–Nambu–Jona-Lasinio calculations [20]. The simplification of not including spin does not hinder us from obtaining meaningful physical results. Our results show that the explicit confinement of the constituent quarks is not of importance for the proton wave function once the constituent quarks are bound in the proton. This schematic dynamical model gives encouraging results and will be worth generalizing in the future to include the relativistic quark spin in the null plane.

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