PHYSICAL REVIEW C VOLUME 52, NUMBER 5 NOVEMBER 1995

Systematic features of signature inversion of $(h_{11/2})_p(i_{13/2})_n$ bands in doubly odd nuclei around $A\sim160$

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(Received 18 May 1995)

Based on the arguments of excitation energy systematics and alignment additivity rule, the spins of the lowest observed states of the $(h_{11/2})_p(i_{13/2})_n$ bands in ^{156}Tb, ^{158}Ho, ^{166}Lu, and ^{168}Ta are assigned as 8, 9, 8, and 9 instead of the previous assignments of 6, 6, 7, and 10, respectively. Taking these new spin assignments and the preliminary experimental results of ^{162,164}Lu into account, the systematic features of the signature inversion of the $(h_{11/2})_p(i_{13/2})_n$ bands of the doubly odd nuclei in the lighter rare-earth region (around $A\sim160$) are presented.

PACS number(s): 21.10.Hw, 27.70.+q

I. INTRODUCTION

The phenomenon of low-spin signature inversion of the $(h_{11/2})_p(i_{13/2})_n$ band in doubly odd nuclei was systematically observed in the neighborhood of $A\sim160$ ($Z=63$–73, $N=89$–95) and a recent systematic study of the experimental data of the yrast bands of these nuclei was given in [1]. This phenomenon has been extensively studied through various theoretical approaches, such as the cranked-shell model [2,3], the particle-rotor model [4–6], the angular-momentum projection method [7,8], and the interacting boson-fermion model [9]. However, the systematic and the theoretical studies both were bothered and complicated by the unreliable spin assignments of some of the nuclei in this region. For example, the spin assignment of the yrast band in ^{156}Tb was suspected and called for a reevaluation of it by Bengtsson *et al.* [2] as early as in 1984. However, since then no reevaluation has been made and experimental data with the suspected spin assignment of ^{156}Tb [10] have been fitted or quoted by almost all of the above-mentioned theoretical studies. In the present paper, we present the arguments to show that the spins (I_0's) of the lowest observed states of the $(h_{11/2})_p(i_{13/2})_n$ yrast bands in ^{156}Tb, ^{158}Ho, ^{166}Lu, and ^{168}Ta have to be assigned as 8, 9, 8, and 9 instead of the previous assignments of 6 [10], 6 [11], 7 [12], and 10 [13], respectively, and, by taking these new spin assignments and the preliminary data of ^{162,164}Lu [14] into account, the systematic features of the low-spin signature inversion of the doubly odd nuclei around $A\sim160$ are presented.

II. SPIN ASSIGNMENTS OF LOWEST OBSERVED STATES OF THE $(h_{11/2})_p(i_{13/2})_n$ YRAST BANDS IN ^{158}Ho, ^{156}Tb, ^{166}Lu, AND ^{168}Ta

In the lighter rare-earth region (around $A\sim160$), the phenomenon of low-spin signature inversion has been observed in about 17 doubly odd nuclei. The configurations of the yrast bands of these nuclei have all been assigned as $(h_{11/2})_p(i_{13/2})_n$ in their original papers (references will be given at appropriate places later). On the basis of these configuration assignments, the following natural assumptions are made to facilitate and simplify the description of the forthcoming spin assignments: (i) The excitation energy of the levels with the same spin in the $(h_{11/2})_p(i_{13/2})_n$ bands of a chain of isotopes (isotones) varies with neutron (proton)

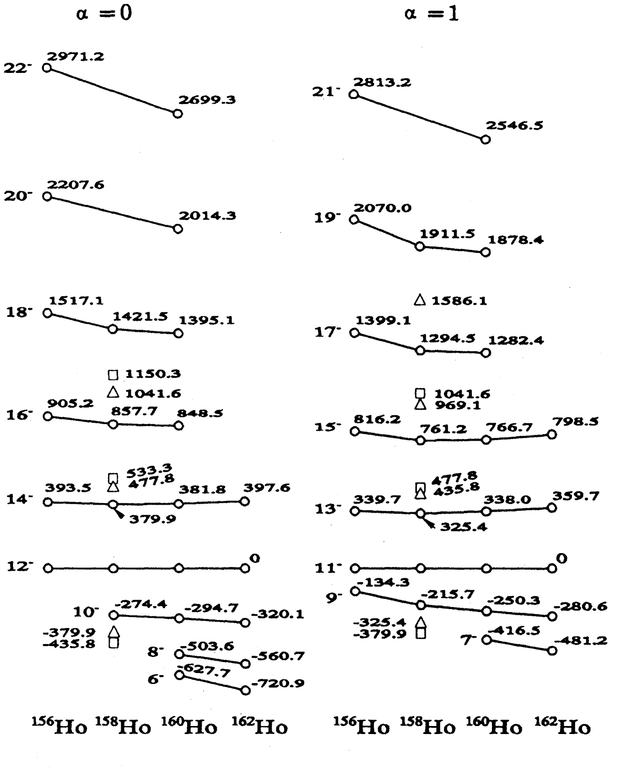

FIG. 1. Energy systematics of the $(h_{11/2})_p(i_{13/2})_n$ band in the $Z=67$ doubly odd isotopes. For ^{158}Ho, the symbols □, △, and ○ represent level positions when the spin of the lowest observed state of the $(h_{11/2})_p(i_{13/2})_n$ band is assigned as $I_0=6$, [11] $I_0=7$, [10,18], and $I_0=9$ (present work), respectively. The data sources are ^{156}Ho [21], ^{158}Ho [11], ^{160}Ho [22], and ^{162}Ho [17].

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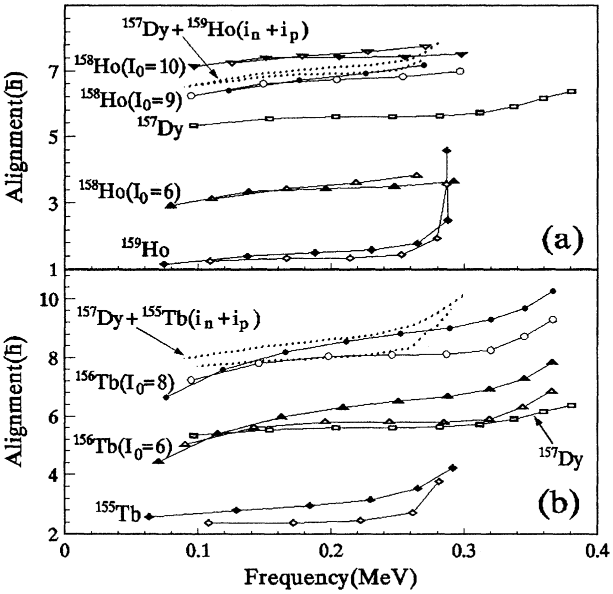

FIG. 2. (a) Comparison of alignments in ^{158}Ho under the assumptions of $I_0=6$, 9, and 10 with the sum of proton and neutron alignments in odd-A neighbors. The open symbols indicate $\alpha=\frac{1}{2}$ for the odd-A nuclei and $\alpha=1$ for the odd-odd nucleus; the filled symbols indicate $\alpha=-\frac{1}{2}$ for the odd-A nuclei and $\alpha=0$ for the odd-odd-nucleus. (b) The same as (a), but for ^{156}Tb under the assumptions of $I_0=6$ and 8.

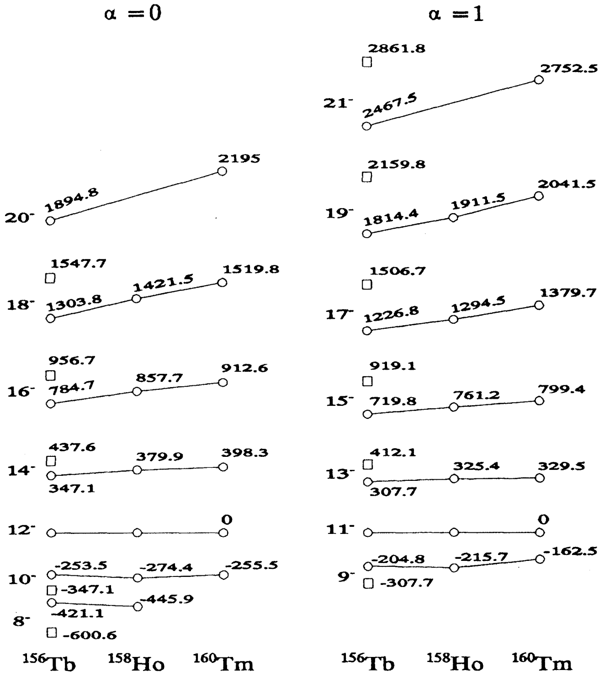

FIG. 3. Energy systematics of the $(h_{11/2})_p(i_{13/2})_n$ band in the $N=91$ doubly odd isotones. For ^{156}Tb, the symbols □ and ○ represent the level positions when the spin of the lowest observed state of $(h_{11/2})_p(i_{13/2})_n$ band is assigned as $I_0=6$ [10] and $I_0=8$ (present work), respectively. The data sources are ^{156}Tb [10], ^{158}Ho [11], and ^{160}Tm [15]. The spin of the lowest observed state of the $(h_{11/2})_p(i_{13/2})_n$ band in ^{158}Ho has been taken as $I_0=9$.

number in a smooth way. This assumption is, to certain extent, justified by the energy systematics of the $(h_{11/2})_p(i_{13/2})_n$ bands of the doubly odd isotopes $^{156-166}$Tm [15]. A deviation from the smooth trend may imply a questionable spin assignment. (ii) The alignment $\mathbf{i}_{pn}$ of the $(h_{11/2})_p(i_{13/2})_n$ band (for the undisturbed part of $I \geqslant j_p+j_n$ and below the first band crossing) in doubly odd nuclei approximately equals to the sum of the proton contribution $\mathbf{i}_p$ and the neutron contribution $\mathbf{i}_n$ as observed in the $(h_{11/2})_p$ and $(i_{13/2})_n$ bands of odd-A neighbors at the same rotational frequency. This assumption is, to certain extent, justified by the facts that in several cases, such as ^{152}Eu [16], ^{162}Ho [17], and ^{164}Tm [15], the I_0's assigned according to the alignment additivity rule are in agreement with those determined from experimental spectroscopy. In addition, this rule has often been used to assign the spin (I_0) of the lowest observed states of the $(h_{11/2})_p(i_{13/2})_n$ band in cases where there was not enough experimental information for assigning the I_0 through experimental spectroscopy [15]. (iii) The phenomenon of the signature inversion of the $(h_{11/2})_p(i_{13/2})_n$ band is similar for all the doubly odd nuclei discussed in the present paper. It means that, for all these nuclei, above the inversion point the signature splitting is normal and, thus the favored states (with signature $\alpha=0$, even spin) lie lower in energy, and below the inversion point the signature splitting is anomalous and, thus, the unfavored states (with signature $\alpha=1$, odd spin) lie lower in energy.

TABLE I. Harris parameters used in the present work. (References are [10] for ^{155,156}Tb, [19] for ^{157}Dy, [11] for ^{158}Ho, [23] for ^{165}Lu, [12] for ^{166}Lu, [24] for ^{167}Hf, and [13] for ^{167,168}Ta.)

	^{155}Tb	^{156}Tb	^{157}Dy	^{158}Ho	^{159}Ho
J_0 (MeV $^{-1}\hbar^2$)	27.4	26.9	24.8	31.2	31.8
J_1 (MeV $^{-3}\hbar^4$)	104	114	148	59	124
	^{165}Lu	^{166}Lu	^{167}Hf	^{167}Ta	^{168}Ta
J_0 (MeV $^{-1}\hbar^2$)	25.8	26.0	24.5	20.0	25.5
J_1 (MeV $^{-3}\hbar^4$)	90	95	97	150	104

A. Spin assignment of ^{158}Ho

$I_0=6$ was assigned to the lowest observed state of the $(h_{11/2})_p(i_{13/2})_n$ band of ^{158}Ho in [11] and, later, $I_0=7$ was adopted in [10,18]. Figure 1 presents the energy systematics of the $(h_{11/2})_p(i_{13/2})_n$ bands of the Ho doubly odd isotopes. The level with $I=12$ ($I=11$) is used as zero-energy reference for signature $\alpha=0$ ($\alpha=1$). It shows that both $I_0=6$ and $I_0=7$ result in the deviation from the smooth trend and $I_0=9$ fits to the smooth trend very well. Obviously, the energy systematics suggests that $I_0=9$ is the reasonable assignment. Figure 2(a) presents the alignments $\mathbf{i}_{pn}[(h_{11/2})_p(i_{13/2})_n]$ of ^{158}Ho with $I_0=6$, 9, and 10, and $\mathbf{i}_n[(i_{13/2})_n]$ of ^{157}Dy [19] and $\mathbf{i}_p[(h_{11/2})_p]$ of ^{159}Ho [20]. Harris parameters are taken from the original papers or extracted from the corresponding rotational band, according to the method as presented in [15], in cases where no such parameters are available. Harris parameters used in the present paper are listed in Table I. One sees, from Fig. 2(a), that the $\mathbf{i}_{pn}$ with $I_0=9$ is most close to the sum $\mathbf{i}_p+\mathbf{i}_n$ and hence the

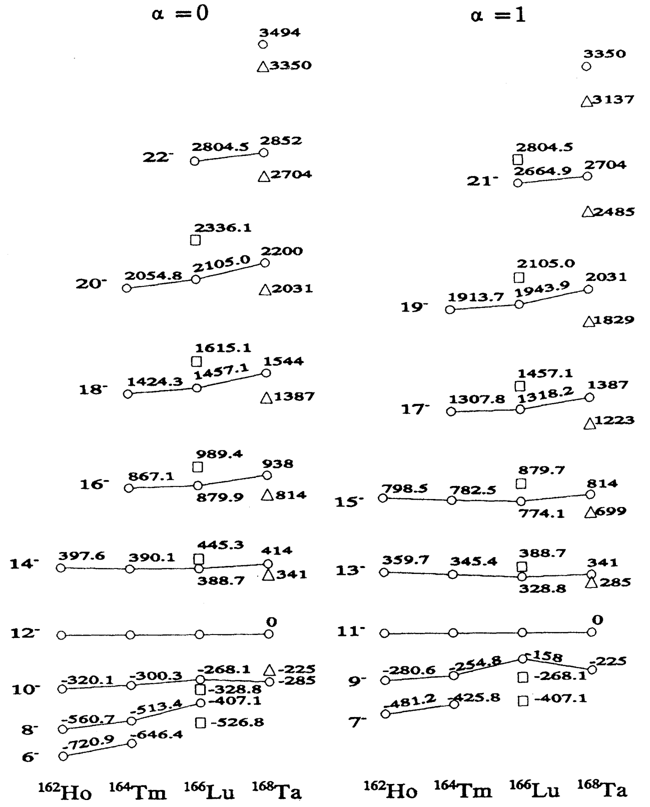

FIG. 4. Energy systematics of the $(h_{11/2})_p(i_{13/2})_n$ band in the $N=95$ doubly odd isotones. For ^{166}Lu, the symbols □, and ○ represent the level positions when the spin of the lowest observed state of the $(h_{11/2})_p(i_{13/2})_n$ band is assigned as $I_0=7$ [12] and $I_0=8$ (present work), respectively. For ^{168}Ta, the symbols △ and ○ represent the level positions when the spin of the lowest observed state of the $(h_{11/2})_p(i_{13/2})_n$ band is assigned as $I_0=10$ [13] and $I_0=9$ (present work), respectively. The data sources are ^{162}Ho [17], ^{164}Tm [15], ^{166}Lu [12], and ^{168}Ta [13].

alignment additivity rule also supports the spin assignment of $I_0=9$. The $\mathbf{i}_{pn}$ with $I_0=10$ is also quite close to the sum $\mathbf{i}_p+\mathbf{i}_n$, but the choice of $I_0=10$ will lead to the result that the favored states with even spin lie higher in energy above the inversion point and the unfavored states with odd spin lie higher in energy below the inversion point, i.e., the signature splitting is anomalous above inversion point and normal below the inversion point. This is in contradiction with the third assumption made earlier in this paper, and this is also not consistent with the energy systematics as shown in Fig. 1. Therefore it is concluded that both the energy systematics and the alignment additivity rule suggest the spin assignment of $I_0=9$ to the lowest observed state of the $(h_{11/2})p(i_{13/2})_n$ band in ^{158}Ho.

B. Spin assignment of ^{156}Tb

The spin of the lowest observed state of the $(h_{11/2})_p(i_{13/2})_n$ band in ^{156}Tb was assigned as $I_0=6$ in [10]. Figure 3 indicates that the assignment $I_0=6$ results in a deviation from the smooth trend and $I_0=8$ fits to the smooth trend very well. Figure 2(b) shows that i_{pn} with $I_0=8$ is most close to the sum $\mathbf{i}_p+\mathbf{i}_n$. Therefore, both the energy systematics and the alignment additivity rule suggest that $I_0=8$ is the reasonable spin assignment to the lowest observed state of $(h_{11/2})_p(i_{13/2})_n$ band in ^{156}Tb.

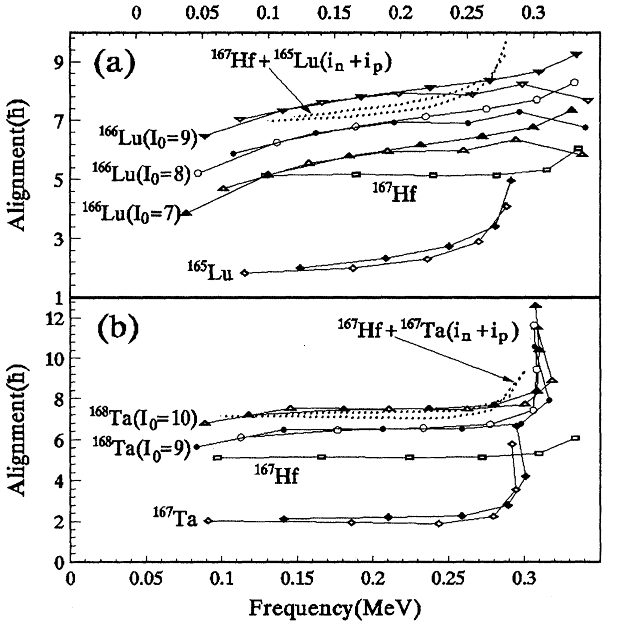

FIG. 5. (a) The same as Fig. 2(a), but for ^{166}Lu under the assumptions of $I_0=7$, 8, and 9. (b) The same as Fig. 2(a), but for ^{168}Ta under the assumptions of $I_0=9$ and 10.

C. Spin assignment of ^{166}Lu

$I_0=7$ was assigned to the lowest observed state of the $(h_{11/2})_p(i_{13/2})_n$ yrast band of ^{166}Lu in [12]. Figure 4 indicates that $I_0=7$ results in the deviation from the smooth trend and $I_0=8$ fits to the smooth trend very well. In Fig. 5(a), the sum $\mathbf{i}_p+\mathbf{i}_n$ is in between $\mathbf{i}_{pn}$ with $I_0=8$ and $\mathbf{i}_{pn}$ with $I_0=9$. The choice of $I_0=9$ will lead to the result which is in contradiction with the third assumption, and it is also not consistent with the energy systematics as shown in Fig. 4. Therefore both the energy systematics and the alignment additivity rule support the spin assignment of $I_0=8$ to the lowest observe state of the $(h_{11/2})_p(i_{13/2})_n$ band in ^{166}Lu.

D. Spin assignment of ^{168}Ta

$I_0=10$ was tentatively assigned to the lowest observed state of the $(h_{11/2})_p(i_{13/2})_n$ band of ^{168}Ta in [13]. Figure 4 shows that the spin assignment of $I_0=10$ will lead to the deviation from the smooth trend and $I_0=9$ fits to the smooth trend very well. Figure 5(b) indicates that the sum $\mathbf{i}_p+\mathbf{i}_n$ is in between $\mathbf{i}_{pn}$ with $I_0=9$ and $\mathbf{i}_{pn}$ with $I_0=10$. The choice of $I_0=10$ will lead to the result which is in contradiction with the third assumption, and it is also not supported by the energy systematics as shown in Fig. 4. Therefore both the energy systematics and the alignment additivity rule support the spin assignment of $I_0=9$ to the lowest observed state of the $(h_{11/2})_p(i_{13/2})_n$ band in ^{168}Ta.

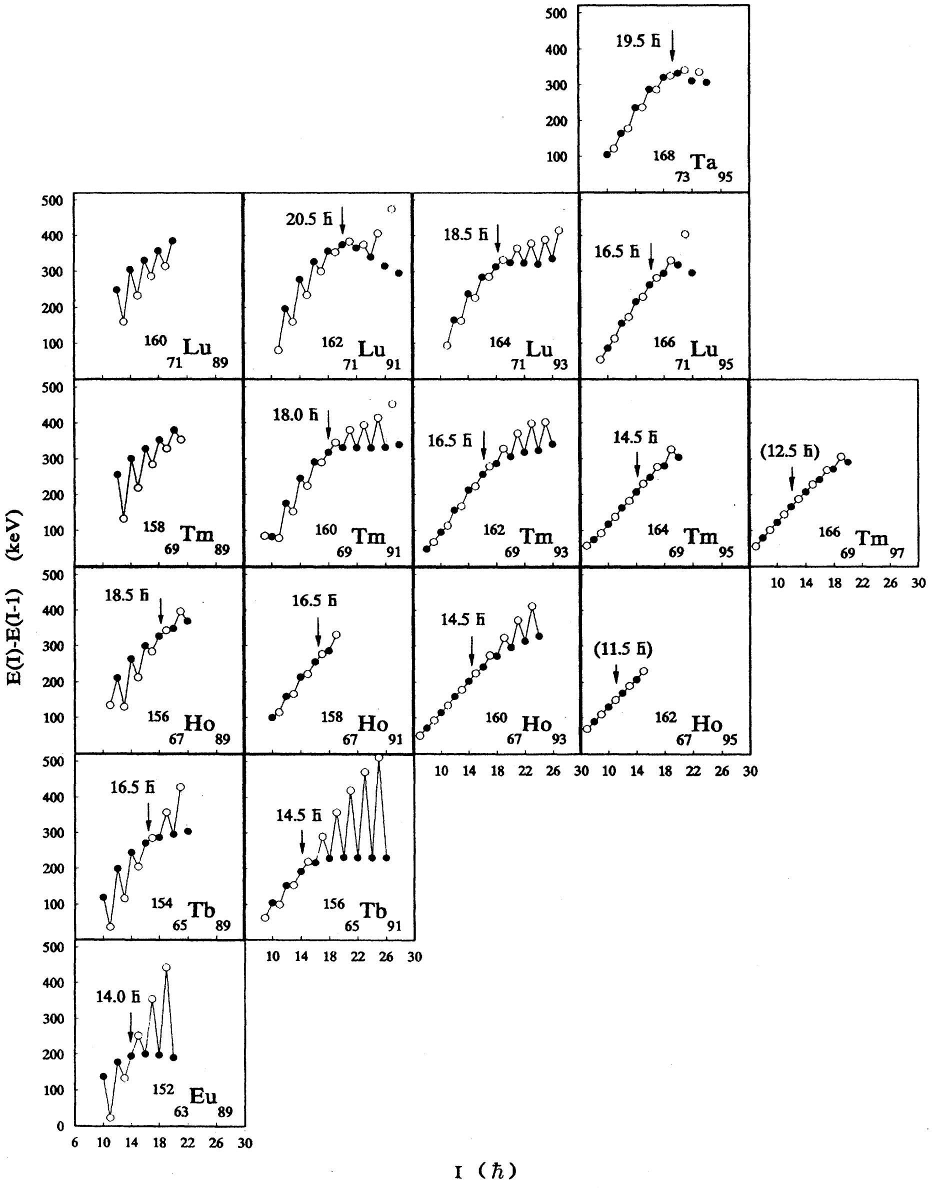

FIG. 6. $E(I)-E(I-1)$ (keV) vs $I(\hbar)$ plots of the $(h_{11/2})_p(i_{13/2})_n$ bands in the doubly odd nuclei around $A\sim160$. The solid circles and open circles indicate favored signature and unfavored signature, respectively. The data sources are ^{152}Eu [16], ^{154,156}Tb [10], ^{156}Ho [21], ^{158}Ho [11], ^{160}Ho [22], ^{162}Ho [17], $^{158-164}$Tm [15], ^{166}Tm [25], ^{160}Lu [26], ^{162,164}Lu [14], ^{166}Lu [12], and ^{168}Ta [13]. The I_0's of ^{156}Tb, ^{158}Ho, ^{166}Lu, and ^{168}Ta have been taken as 8, 9, 8, and 9, respectively.

III. SYSTEMATIC FEATURES OF THE SIGNATURE INVERSION OF THE $(h_{11/2})_p(i_{13/2})_n$ BANDS IN DOUBLY ODD NUCLEI AROUND $A\sim160$

The plots of $E(I)-E(I-1)$ versus I of the $(h_{11/2})_p(i_{13/2})_n$ bands of all the 17 known doubly odd nuclei around $A\sim160$ are presented in Fig. 6. The inversion point is indicated by an arrow and the corresponding spin. The spins at inversion points, indicated in Fig. 6, were approximately read from the plots of $E(I)-E(I-1)-[E(I+1)-E(I)+E(I-1)-E(I-2)]/2$ versus I. Taking ^{162}Tm [15] and ^{166}Lu [12] as an example, as shown in Fig. 7, the spin values at inversion points of these two nuclei have both been read as $16.5\hbar$ without taking their finer difference into account. Therefore, taking $16.5\hbar$ as the spin at inversion point in this paper is just to mean that the inversion point is at somewhere between $I=16$ and 17, i.e. for $I\geq17$ the favored states with even spin lie lower in energy and for $I\leq16$ the unfavored states with odd spin lie lower in energy. The uncertainty of the spin of the inversion point obtained in this way is not greater than $0.5\hbar$.

From Fig. 6, the systematic features are summarized as follows.

(i) In a chain of isotopes, with increasing neutron number, the inversion point shifts to lower spin regularly (with a step of approximately $2\hbar$ between two consecutive doubly odd isotopes).

(ii) In a chain of isotones, with increasing proton number, the inversion point shifts to higher spin regularly (with a step of approximately $2\hbar$ between two consecutive doubly odd isotones).

(iii) In a chain of isotopes, the staggering magnitude of the signature dependence below the inversion point decreases with increasing neutron number. This is true for all chains of isotopes with $Z=65$, 67, 69, and 71. No exception is observed.

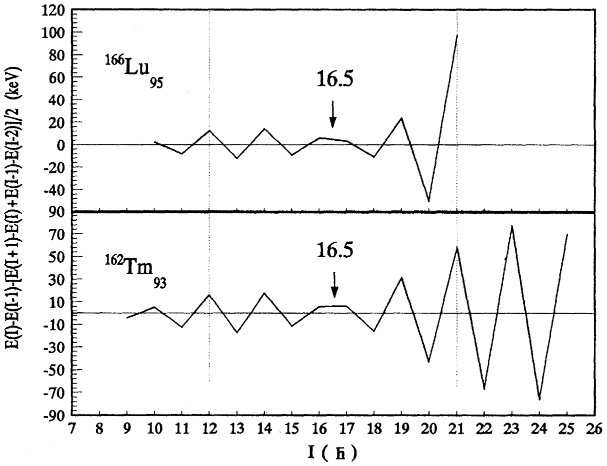

FIG. 7. $E(I)-E(I-1)-[E(I+1)-E(I)+E(I-1)-E(I-2)]/2$ (keV) vs $I(\hbar)$ plots of the $(h_{11/2})_p(i_{13/2})_n$ bands in ^{162}Tm [15] and ^{166}Lu [12]. The I_0 of ^{166}Lu has been taken as 8. The inversion point is indicated by an arrow and the corresponding spin.

(iv) The variation trend of staggering magnitude with increasing proton number is not as simple as that with increasing neutron number. For the known isotones of $N=93$ and 95, the staggering magnitude below inversion point increases with increasing proton number. For $N=91$, the low-spin staggering magnitude of ^{156}Tb ($Z=65$) is larger than that of ^{158}Ho ($Z=67$) and then it starts to increase from $Z=67$ with increasing proton number. For $N=89$, no clear variation trend is observed.

IV. DISCUSSIONS

(a) In Fig. 6, the original I_0's 6, 6, 7, and 10 have been replaced by 8, 9, 8, and 9 for ^{156}Tb, ^{158}Ho, ^{166}Lu, and ^{168}Ta, respectively. As the results of the replacements, two kinds of irregularities were removed from Fig. 6. First, if the original I_0's were used, the spin values of inversion points would be 12.5, 13.5, 15.5, and 20.5 instead of the present values of 14.5, 16.5, 16.5, and 19.5 for ^{156}Tb, ^{158}Ho, ^{166}Lu, and ^{168}Ta, respectively, resulting in the irregularities of the variation of the spin (at inversion point) with increasing proton and neutron number at the positions of these nuclei. Second, if the replacements were not made, the favored states (with even spin) would lie higher in energy above the inversion point and the unfavored states (with odd spin) would lie higher in energy below the inversion point, i.e., the signature splitting would be anomalous above the inversion point and normal below the inversion point for the nuclei ^{158}Ho, ^{166}Lu, and ^{168}Ta, while the signature splitting of all the other doubly odd nuclei in this mass region is normal above the inversion point and anomalous below the inversion point. The removal of the two kinds of irregularities further justifies the new spin assignments and thus the assumptions made earlier in this paper.

(b) The new spin values of ^{156}Tb and ^{158}Ho are just what Bengtsson *et al.* expected and needed for improving the agreement between the experimental data and their theoretical predictions as described in their comprehensive study on the signature inversion [2].

(c) The alignment additivity rule is based on the assumption that the proton-neutron residual interaction is negligible. In fact, generally this is not the case. In the cases of ^{158}Ho, ^{166}Lu, and ^{168}Ta, the $\mathbf{i}_{pn}$ deviates from the sum $\mathbf{i}_p+\mathbf{i}_n$ appreciably and especially in the case of ^{168}Ta, where the sum $\mathbf{i}_p+\mathbf{i}_n$ is even more close to the $\mathbf{i}_{pn}$ with $I_0=10$ than to that with $I_0=9$ while the latter is adopted in the present work. Therefore it is not safe to rely on the alignment additivity rule as the sole argument and, thus, other arguments have to be considered at the same time.

(d) With increasing neutron number, the decreasing trend of the staggering magnitude below inversion point is consistent with the variation trend of γ deformation, since the deformation driving effect towards positive γ deformation is decreased when the quasineutron is placed higher up in the $i_{13/2}$ shell with increasing neutron number. This systematic feature seems to favor the assumption that low-spin signature inversion is the consequence of the triaxiality [2].

(e) It was predicted that low-spin signature inversion of doubly odd nuclei in the lighter rare-earth region can only be observed in the region of $62<Z<70$ [2]. This is not consistent with the recent observations of low-spin signature inversion in the nuclei with $Z=71$ and 73 (see Fig. 6) and the increasing trend of staggering magnitude below inversion point for $N=91$ ($Z\geqslant 67$), 93 and 95. According to the band-crossing mechanism of signature inversion proposed by Hara and Sun [7] the low-spin signature inversion of the high-K $(h_{11/2})_p(i_{13/2})_n$ band comes from the influence (through band coupling) of the low-K $(h_{9/2})_p(i_{13/2})_n$ band and the increasing trend of low-spin staggering magnitude with increasing proton number for N=93 was predicted [7]. Although it seems to be difficult for this mechanism to predict the variation trend of the staggering magnitude of isotones with $N=89$ and 91 (for $Z<67$), its success in understanding the increasing trend of low-spin staggering magnitude for $N=91$ ($Z\geqslant 67$), 93, and 95 suggests that it is important to take the contribution of $(h_{9/2})_p$, in a proper way, into consideration for the understanding of the systematic occurrence of low-spin signature inversion outside of the predicted region of [2].

ACKNOWLEDGMENT

This work was supported by the National Natural Science Foundation of China.

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