

Isovector response function of hot nuclear matter with Skyrme interactions

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We investigate the role of the effective nucleon-nucleon interaction in the description of giant dipole resonances in hot nuclei. For this purpose we calculate the response function of hot nuclear matter to a small isovector external perturbation using various effective Skyrme interactions. We find that for Skyrme forces with an effective mass close to unity an undamped zero sound mode occurs at zero temperature. This mode gives rise in finite nuclei (calculated via the Steinwedel-Jenssen model) to a resonance whose energy agrees with the observed value. We find that zero sound disappears at a temperature of a few MeV, leaving only a broad peak in the dipole strength. For Skyrme forces with a small value of the effective mass (0.4), there is no zero sound at zero temperature but only a weak peak located too high in energy. The strength distribution in this case is nearly independent of temperature and shows small collective effects. The relevance of these results for the saturation of photon multiplicities observed in recent experiments is pointed out.

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I. INTRODUCTION

Giant dipole resonances built on nuclear excited states have been the subject of numerous studies since their first observation at the Berkeley 88" cyclotron in 1981 [1]. By now a significant amount of information is available concerning their evolution with increasing excitation energy (which is mainly stored in rotational and thermal degrees of freedom) [2–5]. The energy of the resonance is known to a good accuracy to be nearly independent of the excitation energy (E^*) while the width increases up to approximately $E^* \sim 130$ MeV in the case of nuclei with $A \approx 110$. Beyond this value there are indications that the width saturates [4–6] although some experiments suggest in contrast a steady increase [7,8]. In the recent experiments performed at the GANIL facility with a 37 MeV per nucleon argon beam, the saturation of the width was observed at excitation energies greater than about 250 MeV [6] together with a saturation of the photon multiplicity. At such high excitation energies, it is not clear, however, whether or not thermalization is actually reached.

There are experimental indications that angular momentum contributes significantly to the width of the resonance up to the point where scission occurs [4]. Although some calculations of this effect [9–11] show a rapid and continuous increase of the width with angular momentum, they do not seem to account either for the observed variation at mean and high excitation energies or for the saturation.

At zero temperature the dominant contribution to the width is the spreading over two-particle–two-hole configurations [12]. It shows, however, a negligible variation with temperature [13,14]. In contrast, in Ref. [15], a rapid increase of the width at small temperatures and a saturation beyond

$T = 4$ MeV was obtained by focusing explicitly on those additional configurations (particle-particle and hole-hole states) which begin to contribute in the random phase approximation (RPA) as soon as the temperature is no longer zero.

RPA calculations generally give resonance energies in agreement with each other (and with experimental data). In contrast, rather different predictions can be found in the literature about the width dependence on temperature. Some RPA results for the giant dipole resonance for ^{40}Ca predict a sharp peak at zero temperature and an increase of the width with temperature [16] whereas such is not the case for the self-consistent RPA calculations of Ref. [17]. In this last case, one already has at zero temperature a fragmented resonance displaying little evolution with temperature. In the nuclear matter calculations of Refs. [18,19] it was found that for specific values of the particle-hole interaction strength a disappearance of the collective (zero sound type) mode occurs at a temperature of a few MeV. This result was, however, obtained in the case of a schematic Skyrme-type interaction which contains no momentum-dependent terms and therefore cannot be considered as a reliable effective force.

The purpose of the present paper is to investigate what can be learned from nuclear matter calculations of the isovector response function at finite temperature using more appropriate effective forces. In what follows we consider the standard effective Skyrme interactions which have been successful at describing accurately nuclear ground state properties while retaining contact with the standard fundamental description of nuclear matter based on the Brueckner reaction matrix [20]. Calculations with such Skyrme forces are more cumbersome but analytical formulas can still be obtained and interpreted in transparent way. These formulas generalize those of Garcia-Recio *et al.* [21] for the zero temperature response function. These formulas will allow us to discuss in rather general terms the evolution of the collective behavior of nuclear matter with temperature. In particular we will see that zero sound is not as robust in nuclear matter as

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in usual Fermi liquids and disappears at temperatures of a few MeV for all the standard Skyrme forces investigated in the present work.

II. RESPONSE FUNCTION OF HOT NUCLEAR MATTER

We calculate the response function of nuclear matter at finite temperature to an infinitesimal external field of the form

$$V_{\text{ext}} = \varepsilon \tau_3 e^{-i\mathbf{q}\cdot\mathbf{r}} e^{-i(\omega+i\eta)t}, \quad (2.1)$$

where τ_3 is the third isospin Pauli matrix and η a vanishingly small positive number corresponding to an adiabatic switching of the field from the time $t = -\infty$. The temporal evolution of the one-body density matrix ρ is determined by the time-dependent Hartree-Fock equation in the presence of the perturbation term, i.e.,

$$i\hbar \partial_t \rho = [W + V_{\text{ext}}, \rho]. \quad (2.2)$$

We consider effective Skyrme forces (see Appendix A for details concerning the Skyrme force). In this case, the energy density can be written in terms of the one-body density, kinetic energy density τ , and momentum density \mathbf{j} .

A. Construction of the response function

For a small enough external field it is legitimate to linearize the mean field evolution equation (2.2) around its static solution. This procedure leads to the following approximate equation in momentum space for the difference $\delta\rho = \rho_n - \rho_p$ between the neutron and proton density matrices:

$$i\hbar \partial_t \langle \mathbf{k} | \delta\rho | \mathbf{k}' \rangle = [\epsilon(\mathbf{k}) - \epsilon(\mathbf{k}')] \langle \mathbf{k} | \delta\rho | \mathbf{k}' \rangle + 2[f(\mathbf{k}') - f(\mathbf{k})] \langle \mathbf{k} | (W_n - W_p) | \mathbf{k}' \rangle + 4\varepsilon [f(\mathbf{k}') - f(\mathbf{k})] \delta(\mathbf{k}' - \mathbf{k} - \mathbf{q}) e^{-i(\omega+i\eta)t}. \quad (2.3)$$

In this equation $\epsilon(\mathbf{k}) = \hbar^2 k^2 / 2m^*$ (with m^* being the effective mass; see Appendix A) is the kinetic energy of a single-particle state with momentum \mathbf{k} in symmetric nuclear matter and $f(\mathbf{k})$ is the corresponding occupation number,

$$f(\mathbf{k}) = 1 / (1 + e^{\beta[\epsilon(\mathbf{k}) - \mu]}), \quad (2.4)$$

where $\beta = 1/T$ is the inverse temperature. Note that since $\epsilon(\mathbf{k})$ contains the kinetic energy only, the quantity μ in the previous equation is not quite the chemical potential $\bar{\mu}$ but rather $\bar{\mu} - U$, where U is the mean field defined in Appendix A. From here on we will adopt standard units in which $\hbar = c = 1$.

The external field (2.1) induces a difference between neutron and proton density distributions for which we consider the following time-dependent *Ansätze* suggested by the form of V_{ext} :

$$\langle \mathbf{r} | \delta\rho | \mathbf{r} \rangle = \alpha e^{-i\mathbf{q}\cdot\mathbf{r}} e^{-i(\omega+i\eta)t}, \quad \langle \mathbf{r} | \delta\tau | \mathbf{r} \rangle = \beta e^{-i\mathbf{q}\cdot\mathbf{r}} e^{-i(\omega+i\eta)t}, \quad \langle \mathbf{r} | \delta\mathbf{j} | \mathbf{r} \rangle = \gamma \mathbf{q} e^{-i\mathbf{q}\cdot\mathbf{r}} e^{-i(\omega+i\eta)t}. \quad (2.5)$$

From the definitions of the densities given in Appendix A the coefficients α , β and γ must satisfy

$$(\alpha, \beta, \gamma) = \int \frac{d^3k}{(2\pi)^3} \left(1, \mathbf{k} \cdot (\mathbf{k} + \mathbf{q}), \frac{1}{q^2} (2\mathbf{k} + \mathbf{q}) \cdot \mathbf{q} \right) \langle \mathbf{k} | \delta\rho(t=0) | \mathbf{k} + \mathbf{q} \rangle. \quad (2.6)$$

The variation in the energy density can be written in terms of the quantities (2.5), yielding

$$W_n(t) - W_p(t) = 2V_0 \delta\rho(\mathbf{r}, t) + 2V_1 \nabla \cdot \delta\rho(\mathbf{r}, t) \nabla + 2V_1 \delta\tau(\mathbf{r}, t) + 2iV_1 (\nabla \cdot \delta\mathbf{j} + \delta\mathbf{j} \cdot \nabla). \quad (2.7)$$

In this formula V_0 and V_1 are related to the parameters of the Skyrme interaction via the following expressions (see Appendix A):

$$V_0 = -\frac{t_0}{2} (x_0 + \frac{1}{2}) - \frac{t_3}{12} (x_3 + \frac{1}{2}) \rho_0^\alpha - \frac{q^2}{16} [3t_1(1+2x_1) + t_2(1+2x_2)], \quad V_1 = \frac{1}{16} [t_2(1+2x_2) - t_1(1+2x_1)], \quad (2.8)$$

where ρ_0 is the equilibrium density of nuclear matter.

The retarded response function is determined by the corresponding polarizability, i.e., the ratio of the density change to the field strength,

$$\Pi(\omega, \mathbf{q}) = \alpha / \varepsilon. \quad (2.9)$$

In the case of the Skyrme effective force, this function is found by noting that Eq. (2.3) is solved by an *Ansatz* of the form

$$\delta\rho(t) = \delta\rho(t=0) e^{-i(\omega+i\eta)t}, \quad (2.10)$$

provided the matrix elements at time zero satisfy

$$\langle \mathbf{k} | \delta\rho(t=0) | \mathbf{k} + \mathbf{q} \rangle = 4 \frac{f(\mathbf{k} + \mathbf{q}) - f(\mathbf{k})}{\epsilon(\mathbf{k} + \mathbf{q}) - \epsilon(\mathbf{k}) + \omega + i\eta} [V_0\alpha + V_1\mathbf{k} \cdot (\mathbf{k} + \mathbf{q})\alpha + \epsilon + V_1\beta - V_1\gamma(2\mathbf{k} + \mathbf{q}) \cdot \mathbf{q}]. \quad (2.11)$$

By multiplying the previous equation respectively by 1, $\mathbf{k} \cdot (\mathbf{k} + \mathbf{q})$ and $(2\mathbf{k} + \mathbf{q}) \cdot \mathbf{q}$ and by integrating over \mathbf{k} we obtain the following set of linear equations: for α , β , and γ

$$\alpha = (V_0 + V_1\Pi_2)\alpha + V_1\beta\Pi_0 + 2m^*\omega V_1\Pi_0\gamma + \epsilon\Pi_0, \quad \beta = (V_0\Pi_2 + V_1\Pi_4)\alpha + V_1\Pi_2\beta + 2\gamma m^*\omega V_1\Pi_2, \quad (2.12)$$

$$\gamma = -\frac{2m^*\omega}{q^2(1 - 2V_1m^*\rho_0)} \alpha.$$

The last equation for γ can be checked to be a mere consequence of the equation of continuity for neutrons:

$$\partial_t \rho_n + \frac{1}{m} \nabla \cdot \mathbf{j} = \frac{1}{2}(t_1 + t_2)(\rho_n \mathbf{j} - \rho \mathbf{j}_n), \quad (2.13)$$

with an analogous equation for protons.

Solving the above linear system for α leads to the following expression of the retarded response function:

$$\Pi(\omega, \mathbf{q}) = \frac{\Pi_0(\omega, \mathbf{q})}{1 - \bar{V}_0\Pi_0(\omega, \mathbf{q}) - 2V_1\Pi_2(\omega, \mathbf{q}) - V_1^2\Pi_2^2(\omega, \mathbf{q}) - V_1^2\Pi_4\Pi_0}. \quad (2.14)$$

In this equation Π_0 is the unperturbed response function (often referred to as the Lindhard function [22]). In what follows the quantities Π_2 and Π_4 will be referred to as generalized Lindhard functions. They are defined as

$$\Pi_{2N}(\omega, \mathbf{q}) = \frac{4}{(2\pi)^3} \int d^3k \frac{f(\mathbf{k} + \mathbf{q}) - f(\mathbf{k})}{\omega + i\eta - \epsilon(\mathbf{k}) + \epsilon(\mathbf{k} + \mathbf{q})} [\mathbf{k} \cdot (\mathbf{k} + \mathbf{q})]^N, \quad (2.15)$$

where the limit $\eta \rightarrow 0^+$ is implicit. Analytical expressions for the real and imaginary parts of $\Pi_{2N}(\omega, q)$ are given in Appendix B as well as the relation between our definitions and those of other authors [21]. In Eq. (2.14) we have used a modified coefficient \bar{V}_0 defined by

$$\bar{V}_0 = V_0 - \left(\frac{m^*\omega}{q}\right)^2 \frac{2V_1}{1 - 2V_1m^*\rho_0}. \quad (2.16)$$

This modified coefficient arises because of the change in the momentum density induced by the external field. For interactions with no momentum dependence, i.e., $t_1 = t_2 = 0$, we recover the results of Ref. [18].¹

Our formula for the response function (2.14) generalizes that of Garcia-Recio *et al.* [21] to which it reduces at zero temperature.

B. Some properties of the response function

A collective behavior will be observed when the response function exhibits a peak. Although this may be the case when the denominator in the response function goes through a minimum, the most familiar situation corresponds to the case where there is a pole in Eq. (2.14). This occurs when the value ω_0 of the frequency is such that

$$1 = \bar{V}_0\Pi_0 + 2V_1\Pi_2 + V_1^2(\Pi_2^2 + \Pi_0\Pi_4). \quad (2.17)$$

The real part of the frequency ω_0 determines the energy of the resonance and its imaginary part the width. The corresponding relation is easily constructed when the imaginary part of ω is small so that a linearization of the previous equation near $\text{Im}(\omega_0) = 0$ can be made [22]. In other cases a numerical construction of the strength function is necessary, as performed in some of the examples below.

It is interesting to note that the symmetry energy coefficient a_τ of nuclear matter is, as expected, an important ingredient to determine whether or not there is a pole. Indeed one has the following relation between a_τ and the coefficients occurring in Eq. (2.17):

$$a_\tau = \frac{k_F^2}{6m^*} + \frac{\rho_0}{2} \left(V_0 + \frac{q^2}{16} [t_2(1+x_2) + 3t_1(1+x_1)] + 2V_1k_F^2 \right), \quad (2.18)$$

where $k_F = (3\pi^2\rho_0/2)^{1/3}$ is the Fermi momentum.

The formula we have derived for the response function looks somewhat cumbersome and thus it seems worthwhile to explore whether approximations to the previous scheme can be derived. We have found that one such useful scheme is provided by a Thomas-Fermi-type approximation in which one assumes that the change in the kinetic energy density τ can be calculated from the proportionality relation $\tau \sim \rho^{5/3}$, which leads to

¹A factor of 2 is, however, missing in Eq. (13) of this reference, which implies that the interaction strengths in Figs. 1 and 2 of this reference must be multiplied by a factor of 2. This factor is also missing in Ref. [19].

$$\delta\tau(\mathbf{r},t)/\tau(\mathbf{r},t)=5\delta\rho(\mathbf{r},t)/3\rho(\mathbf{r},t). \quad (2.19)$$

Inserting this value into Eq. (2.11) one obtains the approximate expression for the response function,

$$\Pi(\omega, \mathbf{q}) = \frac{\Pi_0(\omega, \mathbf{q})}{1 - \tilde{V}_0 \Pi_0(\omega, \mathbf{q}) - 2V_1 \Pi_2(\omega, \mathbf{q})}, \quad (2.20)$$

in which the coefficient \tilde{V}_0 is defined by

$$\tilde{V}_0 = V_0 - 2V_1 k_F^2. \quad (2.21)$$

This approximate formula was explored in Ref. [19]. It was found to reproduce correctly the main features of the exact formula derived in the previous section: the existence of a zero sound at zero temperature with a disappearance of this mode at temperatures of a few MeV.

In the limit $\omega \rightarrow 0$ and $q \rightarrow 0$, the response function becomes the static polarizability. In the isovector channel, the latter is related to the symmetry energy coefficient a_τ at temperature T . Indeed for $q = \omega = 0$ the energy density of nuclear matter in the presence of a small external field is given by

$$\mathcal{H} = \mathcal{H}_0 + \frac{a_\tau(T)}{\rho_0} (\delta\rho)^2 + \epsilon \delta\rho = \mathcal{H}_0 + \frac{a_\tau}{\rho_0} \alpha^2 + \epsilon \alpha. \quad (2.22)$$

By minimizing with respect to α we find

$$\alpha = -\frac{\epsilon \rho_0}{2a_\tau}, \quad (2.23)$$

yielding

$$\Pi(\omega=0, q=0) = -\frac{\rho_0}{2a_\tau}. \quad (2.24)$$

This formula can also be obtained by performing the limit $\omega \rightarrow 0$ and $q \rightarrow 0$ in the expression of the RPA response function (2.14), which provides a check of our formula for this quantity.

The value of the strength distribution per unit volume $S(\omega)$ for the operator $\tau_3 \exp(i\mathbf{q} \cdot \mathbf{r})$ is proportional to the imaginary part of the response function:

$$S(\omega) = -\frac{1}{\pi} \text{Im} \Pi(\omega, q). \quad (2.25)$$

It is also related to the photoabsorption strength distribution S_{abs} [23]:

$$S_{\text{abs}}(\omega) = -\frac{1}{\pi} \frac{1}{1 - e^{-\beta\omega}} \text{Im} \Pi(\omega, q). \quad (2.26)$$

Furthermore, it satisfies the energy weighted sum rule [23]

$$\int_0^\infty d\omega \omega S(\omega) = \frac{q^2}{2m^*} \rho_0 (1 + \kappa), \quad (2.27)$$

where

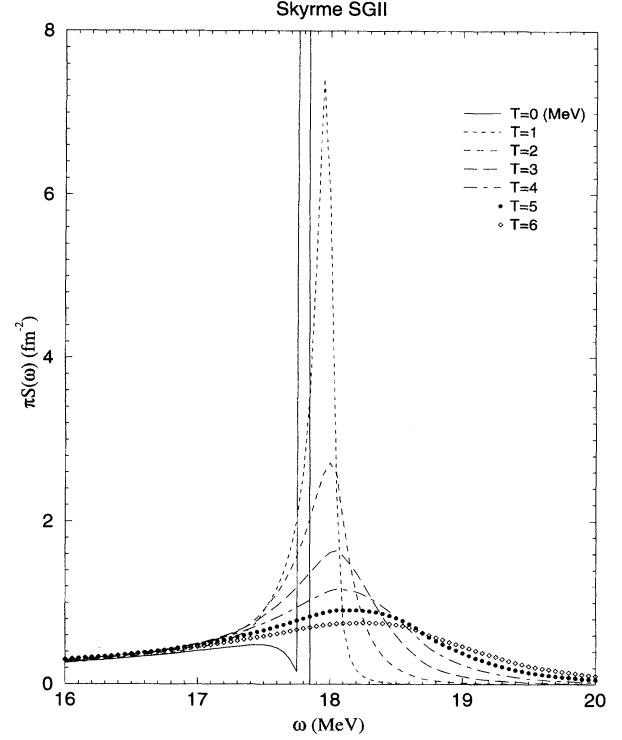


FIG. 1. Distribution of strength per unit volume for the operator $\exp(i\mathbf{q} \cdot \mathbf{r})$ (in fm^{-2}) as a function of the energy ω (in MeV) for a momentum $q=0.23 \text{ fm}^{-1}$ and for different values of the temperature $T=0 \rightarrow 6$ MeV, in the case of the interaction SGII.

$$\kappa = -\frac{m^* \rho_0}{8} [t_2(1+2x_2) - t_1(1+2x_1)] \quad (2.28)$$

is the enhancement factor arising from the momentum-dependent terms of the Skyrme interaction.

III. RESULTS AND DISCUSSION

The strength function is plotted for the Skyrme forces SGII [24], SkM [25], SIII, SI, and SV [26,27] in Figs. 1, 2, 3, 4, and 5, respectively, as a function of the excitation energy ω for various values of the temperature. The corresponding values of the Skyrme parameters are given in Appendix A. In these figures we have chosen the value of the momentum transfer q in such a way that it corresponds to the dipole mode in lead-208 described by the Steinwedel-Jensen model [28]. In this model neutrons oscillate against protons inside a sphere of radius R according to the formula

$$\langle \mathbf{r} | \delta\rho | \mathbf{r} \rangle = \epsilon \sin(\mathbf{q} \cdot \mathbf{r}) \sin(\omega t), \quad (3.1)$$

where

$$q = \frac{\pi}{2R}. \quad (3.2)$$

Taking $R=6.7 \text{ fm}$ for lead-208 we find $q=0.23 \text{ fm}^{-1}$.

We have checked in our numerical calculations that the energy-weighted sum rule is well satisfied. Some examples

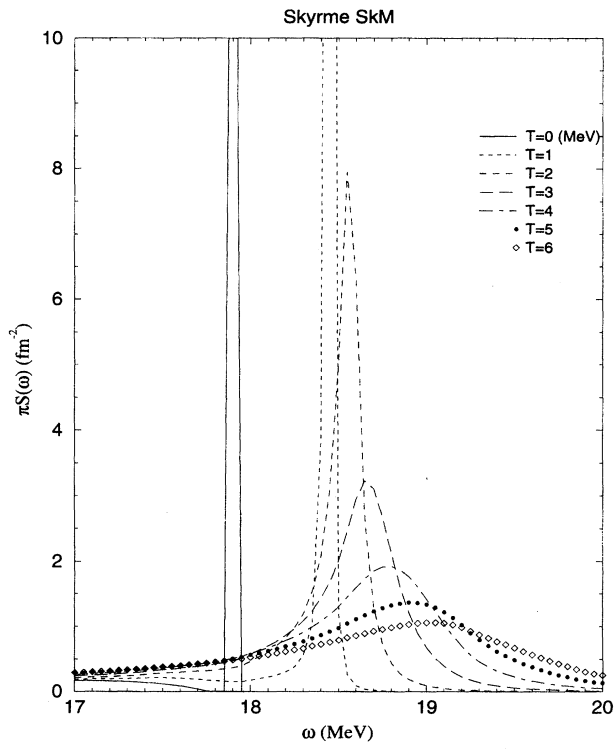


FIG. 2. Same as Fig. 1 for the force SkM.

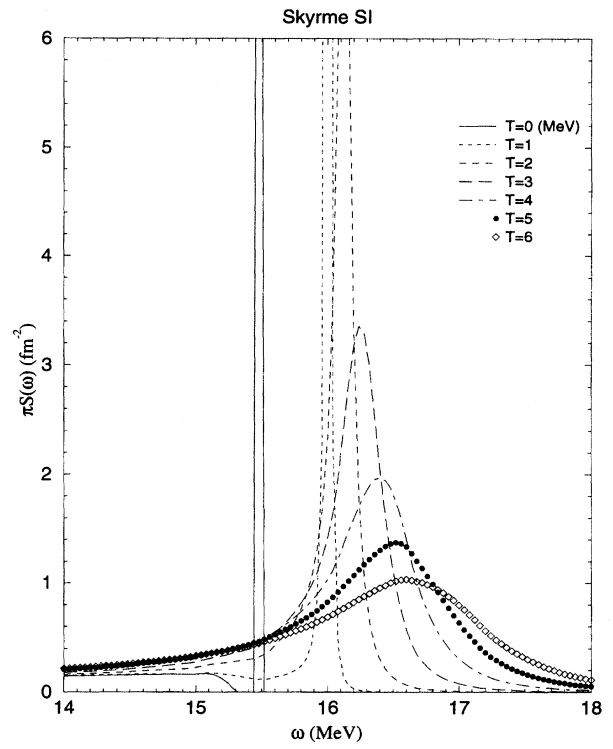


FIG. 4. Same as Fig. 1 for the force SI.

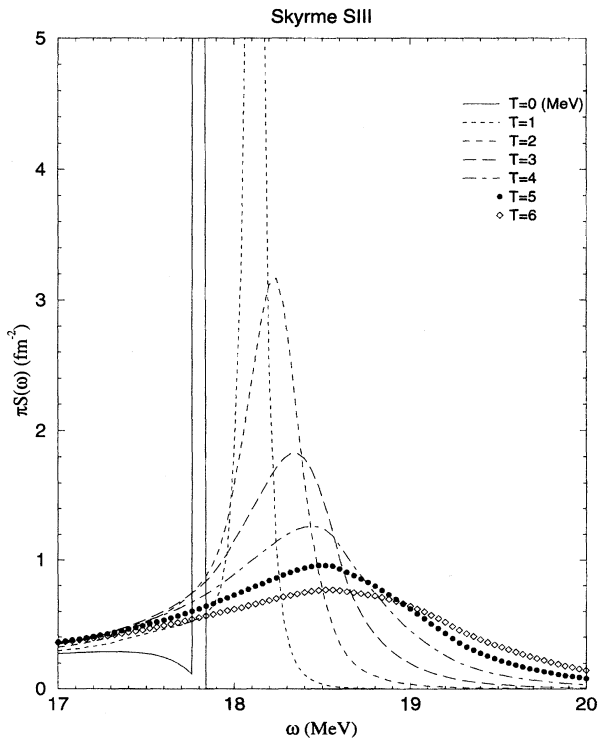


FIG. 3. Same as Fig. 1 for the force SIII.

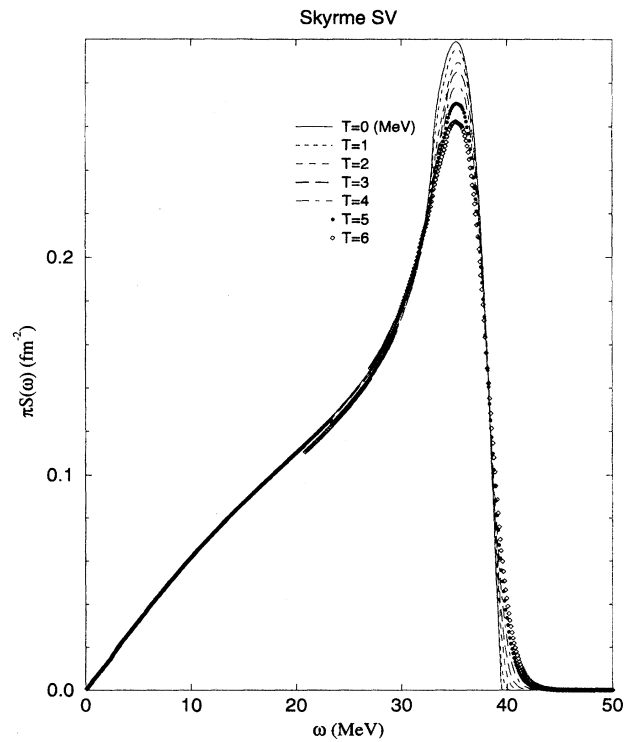


FIG. 5. Same as Fig. 1 for the force SV.

TABLE I. Right hand side (RHS) of the energy-weighted sum rule (MeV fm^{-3}) compared to the integrated value of the strength m_1 for $T = 0, 3,$ and 6 MeV.

	RHS	$m_1(T=0)$	$m_1(T=3)$	$m_1(T=6)$
SGII	52	25	52	52
SkM	54	15	54	54
SIII	49	18	49	49
SI	43	11	43	43
SV	121	122	121	120

are shown in Table I, which compares the right hand side of the sum rule [c.f. Eq. (2.27)] to the value of the integrated strength using Simpson's rule. It can be checked that there is an excellent agreement. There are some apparent exceptions, however, which correspond to the presence of a sharp zero sound peak which falls in between two mesh points of the integration method. We have checked that the contribution of the pole just provides the missing strength in these cases.

For interactions SGII, SkM, and SIII the strength function shows a strong temperature dependence and exhibits a sharp peak at zero temperature. This peak occurs at a value of 18 MeV which is slightly higher than the observed one (we assume that we can rely on the Steinwedel-Jenssen model). One common feature of the three interactions SGII, SkM, and SIII is that their effective masses are very close (respectively, 0.78, 0.79, and 0.76, in units of the bare mass).

For a Skyrme interaction with an effective mass closer to unity, such as SI (see Fig. 4), the resonance is found at a lower energy (15.5 MeV). For Skyrme forces with a very small effective mass, such as SV ($m^*/m=0.36$), the energy of the dipole resonance (see Fig. 5) is much higher (about 35 MeV) than the observed value and the dipole strength is nearly independent of temperature. It still exhibits a weak collective behavior but for most of the energy range, the form of the response function is not very different from the imaginary part of the bare Lindhard function Π_0 , which is shown in Fig. 6. The imaginary part of Π_0 exhibits a nearly linear growth with energy which is also present in the imaginary part of the RPA response function, with moderate deviations occurring only in the resonance region.

Let us now try to understand qualitatively the previous results about the position of the resonance and its evolution with temperature. The imaginary part of the bare response function Π_0 is known to show at zero temperatures two angular points at the following values of the energy (see Appendix B and Fig. 6)

$$\omega_{\pm} = \frac{qk_F}{m^*} \pm \frac{q^2}{2m^*}. \quad (3.3)$$

As temperature rises the corresponding discontinuities are smeared by the presence of the Fermi occupation numbers (Fig. 6). At these points the real part of the bare response function has a vertical slope (at zero temperature) and a maximum in between these points. For the particular value of the momentum transfer we are considering ($q=0.23 \text{ fm}^{-1}$) and for a Fermi momentum $k_F=1.36 \text{ fm}^{-1}$ one finds

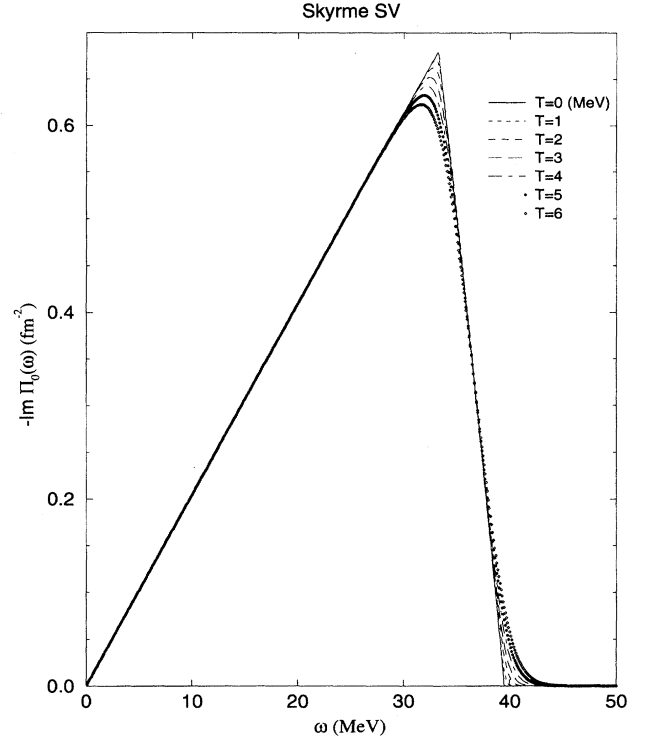


FIG. 6. Imaginary part of Π_0 (in fm^{-2}) as a function of the energy ω (in MeV), in the case of SV force, for $q=0.23 \text{ fm}^{-1}$; the temperatures are from $T=0$ to $T=6$ MeV.

$$\omega_{\pm} = \frac{m}{m^*} \times (13. \pm 1.1) \text{ MeV}. \quad (3.4)$$

Let us now assume for more simplicity (which is supported by numerical estimates) that the Lindhard function of order zero is the most important one for our discussion, so that the RPA response (2.14) can be approximated as

$$\Pi(\omega, \mathbf{q}) \approx \frac{\Pi_0(\omega, \mathbf{q})}{1 - V_0 \Pi_0(\omega, \mathbf{q})}. \quad (3.5)$$

At this point we should note that the maximum value of the real part of the bare response is proportional to the effective mass,

$$\max(\text{Re}\Pi_0) = m^* k_F A(k_F, q, T), \quad (3.6)$$

as can be seen from the equations given in Appendix B. Since the maximum is smeared and reduced at high temperatures, the function A decreases as temperature increases. In terms of this function the condition for the existence of a zero sound is

$$1 \leq V_0 m^* k_F A(T). \quad (3.7)$$

It turns out that for interactions with an effective mass close to unity and at zero temperature the condition (3.7) is just satisfied. If this is the case, we expect the response function to have a maximum when the real part of the bare response is near its maximum, i.e., between the two points defined by

TABLE II. Velocity c_0 of the collective mode compared to the Fermi velocity v_F for the Skyrme forces SI, SGII, and SV.

	SI	SV	SGII
c_0	0.35	0.77	0.40
v_F	0.30	0.77	0.35

Eq. (3.3). Since these points are quite close, a reasonable estimate of the resonance energy is

$$\omega_R \simeq \frac{m}{m^*} \times 13 \text{ MeV}. \quad (3.8)$$

By comparing this formula with the results in Figs. 1–5, it can be seen to provide a good description of the resonance energies. Note that for other values of the momentum transfer q , a similar construction would give the dispersion relation

$$\omega_R \simeq c_0 \times q, \quad (3.9)$$

with

$$c_0 \simeq \frac{k_F}{m^*} = v_F = 0.3 \times c \times \frac{m}{m^*}, \quad (3.10)$$

where we have taken $k_F = 1.36 \text{ fm}^{-1}$. This formula exhibits the important role played by the value of the effective mass. In actual calculations the sound velocity c_0 is slightly larger than the Fermi velocity as can be seen from Table II. The similarity between the two velocities shows that nuclear matter exhibits weaker collective effects than helium-3. Indeed, in this last case theoretical estimates of the zero sound velocity are between 2 and 3.5 times the Fermi velocity [29].

The dispersion relation $\omega_R = c_0 q$ implies that the resonance in light nuclei is located higher in energy according to the relation $q = 2\pi/R$, i.e., evolves with mass number as $\omega_R \sim A^{-1/3}$ in agreement with the empirical formula [28]. Since in our model the strength is concentrated in a single region, this result also means weaker collectivity in light nuclei.

Let us now show that the previous formulas also provide an explanation for the presence or not of a zero sound at zero temperature and for the different temperature dependences of the strength obtained for various interactions. Indeed from the relation between V_0 , a_τ , and m^* we have

$$\frac{m^*}{m} \frac{V_0}{2} \rho_0 = \frac{m^*}{m} a_\tau - \frac{k_F^2}{6m}. \quad (3.11)$$

Taking $a_\tau = 30 \text{ MeV}$ and $k_F = 1.36 \text{ fm}^{-1}$ this relation shows that the quantity $m^* V_0$, which is the relevant one for our discussion, is much smaller for interactions with a small effective mass (0.4) than for interactions with $m^*/m \simeq 1$. Therefore, if the condition (3.7) is just satisfied at zero temperature for $m^*/m = 1$, such will not be the case for an inter-

action with $m^*/m = 0.4$. Similarly, since the function A decreases with temperature, it is also clear that the condition for the existence of a zero sound will eventually no longer hold at high enough temperatures (in actual calculations a few MeV). It is also interesting to note that the coefficient V_0 is proportional to the potential part of the symmetry energy coefficient which is thus also important for the existence of the zero sound mode.

IV. CONCLUSION

In conclusion we have found that the standard Skyrme forces SGII, SII, and SkM give rise at zero temperature to a zero sound type collective mode exhibiting the usual dispersion relation $E_{\text{res}} = c_0 q$. The sound velocity for these forces is just slightly greater than the Fermi velocity, which implies a rather weak collective behavior as compared to helium-3 for which there is a factor of 3 between the two velocities. For values of the momentum transfer corresponding in the Steinwedel-Jenssen model to the giant dipole mode in lead-208, we have found that the zero sound mode disappears at temperatures of a few MeV. This may be related to the saturation of photon multiplicities observed in some recent experiments [6]. For Skyrme forces with a small value of the effective mass such as SV, we have found no zero sound at zero temperature and a weak variation of the strength with temperature. It is worthwhile noting that the previous forces all provide good descriptions of nuclear properties, such as binding energies and radii, all over the periodic table. In spite of this common property, they do give rather different predictions for the temperature evolution of giant resonances and also for their positions. Collective properties thus appear to give useful information on the effective nucleon-nucleon interaction.

One limitation of our calculations is that although they do include the effect of the volume symmetry energy, they ignore the effect of the surface symmetry energy which is known to play a role, especially in light nuclei [30]. They ignore as well shell effects which are also important in light nuclei. These effects indeed produce in this case a fragmentation of the strength in RPA calculations (see for instance the results of Sagawa and Bertsch [17] using the SGII force) whereas our model produces only a broad peak. Complete RPA calculations in finite nuclei (including heavy nuclei) would thus be of interest to complete the results of the present work. For such calculations we believe that the discussion we have presented would be a useful guide.

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TABLE III. Numerical values of the parameters $t_0, t_1, t_2, t_3, x_0, x_1, x_2, x_3$, and α corresponding to the five Skyrme interactions SGII, SkM, SIII, SI, and SV considered in this work.

	t_0 (MeV fm ³)	t_1 (MeV fm ⁵)	t_2 (MeV fm ⁵)	t_3 (MeV fm ⁶)	x_0	x_1	x_2	x_3	α
SGII ^a	-2645.0	340.0	-41.9	15595.0	0.09	-0.588	1.425	0.0604	1/6
SkM ^b	-2645.0	410.0	-135.0	15595.0	0.09	0	0	0	1/6
SIII ^c	-1128.75	395.0	-95.0	14000.0	0.45	0	0	1	1
SI ^c	-1057.3	235.9	-100.0	14463.5	0.56	0	0	1	1
SV ^c	-1248.29	970.56	107.22	0.0	-0.17	0	0	1	1

^aFrom Ref. [24].

^bFrom Ref. [25].

^cFrom Refs. [26,27].

APPENDIX A: PARAMETERS OF THE INTERACTIONS USED

We consider an effective Skyrme interaction of the following form:

$$v_{12} = t_0(1 + x_0 P_\sigma) \delta(\mathbf{r}_1 - \mathbf{r}_2) + \frac{t_1}{2}(1 + x_1 P_\sigma) [\delta(\mathbf{r}_1 - \mathbf{r}_2) k^2 + k'^2 \delta(\mathbf{r}_1 - \mathbf{r}_2)] \\ + t_2(1 + x_2 P_\sigma) \mathbf{k}' \cdot \delta(\mathbf{r}_1 - \mathbf{r}_2) \mathbf{k} + \frac{t_3}{6}(1 + x_3 P_\sigma) \rho^\alpha \delta(\mathbf{r}_1 - \mathbf{r}_2), \quad (\text{A1})$$

where P_σ is the spin exchange operator; see Table III for numerical values.

Denoting the single-particle wave functions by $\phi_i(\mathbf{r}, \sigma, q)$, σ and q being the labels for spin and isospin, the nucleon density $\rho_q(\mathbf{r})$, the kinetic energy density $\tau_q(\mathbf{r})$, and the momentum density $\mathbf{j}(\mathbf{r})$ can be expressed as

$$\rho_q(\mathbf{r}) = \sum_{i,\sigma} |\phi_i(\mathbf{r}, \sigma, q)|^2, \quad \tau_q(\mathbf{r}) = \sum_{i,\sigma} |\nabla \phi_i(\mathbf{r}, \sigma, q)|^2, \quad \mathbf{j}(\mathbf{r}) = \sum_{i,\sigma} \frac{1}{2i} (\nabla - \nabla') \phi_i^*(\mathbf{r}', \sigma, q) \phi_i(\mathbf{r}, \sigma, q) |_{\mathbf{r}=\mathbf{r}'}. \quad (\text{A2})$$

The Hartree-Fock mean field Hamiltonian reads

$$W_q(\mathbf{r}) = \nabla \frac{1}{2m_q^*} \nabla + U_q(r), \quad (\text{A3})$$

where U_q is a local potential given in the case of neutrons ($q=n$) by

$$U_n(\mathbf{r}, t) = t_0 \left\{ (1 + x_0 \frac{1}{2}) \rho(\mathbf{r}, t) - (x_0 + \frac{1}{2}) \rho_n(\mathbf{r}, t) \right\} + t_3 \frac{1}{12} \left\{ (2 + \alpha) \left(1 + \frac{x_3}{2} \right) \rho^{(\alpha+1)}(\mathbf{r}, t) \right\} - t_3 \left\{ (x_3 + \frac{1}{2}) [2\rho_n(\mathbf{r}, t) \rho^\alpha(\mathbf{r}, t)] \right. \\ \left. + \alpha [\rho_p^2(\mathbf{r}, t) + \rho_n^2(\mathbf{r}, t)] \rho^{(\alpha-1)} \right\} + \left\{ \frac{1}{4} t_1 \left(1 + \frac{x_1}{2} \right) + \frac{1}{4} t_2 \left(1 + \frac{x_2}{2} \right) \right\} \left[\tau - \frac{1}{2i} (\nabla \cdot \mathbf{j} + \mathbf{j} \cdot \nabla) \right] + \left\{ -\frac{1}{4} t_1 (x_1 + \frac{1}{2}) + \frac{1}{4} t_2 (x_2 + \frac{1}{2}) \right\} \\ \times \left[\tau_n - \frac{1}{2i} (\nabla \cdot \mathbf{j}_q + \mathbf{j}_q \cdot \nabla) \right] + \left\{ -\frac{3}{8} t_1 (\frac{1}{2} + x_1) + \frac{1}{8} (\frac{1}{2} + x_2) \right\} \nabla^2 \rho + \left\{ \frac{3}{8} t_1 (\frac{1}{2} + x_1) + \frac{1}{4} t_2 (\frac{1}{2} + x_2) \right\} \nabla^2 \rho_n, \quad (\text{A4})$$

with a similar expression for protons. The effective mass m_q^* is given by

$$\frac{1}{2m_q^*} = \frac{1}{2m} + \frac{1}{4} \left[t_1 \left(1 + \frac{x_1}{2} \right) + t_2 \left(1 + \frac{x_2}{2} \right) \right] \rho + \frac{1}{8} [t_2 (x_2 + \frac{1}{2}) - t_1 (x_1 + \frac{1}{2})] \rho_q. \quad (\text{A5})$$

APPENDIX B: EXPRESSION OF THE RESPONSE FUNCTION

In this appendix we give the explicit expressions of the real and imaginary parts of the generalized Lindhard functions defined in Eqs. (2.15). For this purpose we need to define the following integrals:

$$I_{2N} = \frac{2}{(2\pi)^3} \int d^3k k^{(2N)} \frac{f(\mathbf{k}+\mathbf{q}) - f(\mathbf{k})}{\omega + i\eta + \epsilon(\mathbf{k}+\mathbf{q}) - \epsilon(\mathbf{k})}. \quad (\text{B1})$$

The imaginary parts of the generalized Lindhard functions are given by

$$\begin{aligned} \text{Im}\Pi_0(\omega, \mathbf{q}) &= -\frac{m^*{}^2}{\pi q \beta} \ln \frac{1 + e^{\beta(\mu - E_-)}}{1 + e^{\beta(\mu - E_+)}} , \\ \text{Im}\Pi_2(\omega, \mathbf{q}) &= -\frac{2m^*{}^3}{\pi \beta^2 q} \left(\beta \sqrt{E_+ E_-} \ln \frac{1 + e^{\beta(\mu - E_-)}}{1 + e^{\beta(\mu - E_+)}} + \text{Li}_2(1 + e^{\beta(\mu - E_+)}) - \text{Li}_2(1 + e^{\beta(\mu - E_-)}) \right) , \\ \text{Im}\Pi_4(\omega, \mathbf{q}) &= -q \sqrt{2m^* E_+} \text{Im}\Pi_2 + 2 \text{Im}I_4 - 2 \sqrt{2m^* E_+} q \text{Im}I_2 , \end{aligned} \quad (\text{B2})$$

where Li_2 is the Euler dilogarithmic function [31],

$$\text{Li}_2(x) = \int_1^x \frac{\ln(t)}{t-1} dt, \quad (\text{B3})$$

and

$$E_{\pm} = \frac{m^*}{2q^2} \left(\omega \pm \frac{q^2}{2m^*} \right)^2. \quad (\text{B4})$$

The expressions of the functions $\text{Im}I_2$ and $\text{Im}I_4$ are

$$\begin{aligned} \text{Im}I_2(\omega, q) &= -\frac{m^*{}^3}{\pi q \beta^2} \left(\beta E_+ \ln \frac{1 + e^{\beta(\mu - E_-)}}{1 + e^{\beta(\mu - E_+)}} + \text{Li}_2(1 + e^{\beta(\mu - E_+)}) - \text{Li}_2(1 + e^{\beta(\mu - E_-)}) \right) , \\ \text{Im}I_4(\omega, q) &= -\frac{2m^*{}^4}{\pi q \beta} E_+^2 \left(\ln \frac{1 + e^{\beta(\mu - E_-)}}{1 + e^{\beta(\mu - E_+)}} + 2 \int_1^{\infty} dz z \ln \frac{1 + e^{-\beta(zE_+ - \omega - \mu)}}{1 + e^{-\beta(zE_+ - \mu)}} \right). \end{aligned} \quad (\text{B5})$$

At zero temperature the real parts of the Π_{2N} are given by

$$\begin{aligned} \text{Re}\Pi_0(T=0) &= -\frac{m^* k_F}{\pi^2} \left(-1 + \frac{k_F}{2q} [\phi(x_+) + \phi(x_-)] \right) , \\ \text{Re}\Pi_2(T=0) &= \frac{m^* k_F^3}{2\pi^2} \left(-3 + x_+ x_- + x_+^2 + x_-^2 + \frac{k_F}{2q} [(1 - x_+^2 - 2x_+ x_-) \phi(x_+) + (1 - x_-^2 - 2x_+ x_-) \phi(x_-)] \right) , \\ \text{Re}\Pi_4(T=0) &= 2 \left(\text{Re}I_4(T=0) - 2q \sqrt{2m^* E_+} \text{Re}I_2(T=0) + q^2 m^* E_+ \text{Re}\Pi_0(T=0) + \frac{1}{3\pi^2} m^* q^2 k_F^3 \right) , \end{aligned} \quad (\text{B6})$$

where

$$\begin{aligned} \text{Re}I_2(T=0) &= \frac{m^* k_F^3}{4\pi^2} \left\{ -3 - x_+ x_- - x_+^2 + x_-^2 + \frac{k_F}{2q} \left[(1 + x_+^2) \phi(x_+) + \left(1 + x_-^2 + \frac{4m^* \omega}{k_F^2} \right) \phi(x_-) \right] \right\} , \\ \text{Re}I_4(T=0) &= -\frac{m^* k_F^6}{2\pi^2 q} \left[\left(\frac{1}{6} (1 + x_-^2 + x_-^4) + \frac{m^* \omega}{k_F^2} (1 + x_-^2) + \frac{2m^*{}^2 \omega^2}{k_F^4} \right) \phi(x_-) + \frac{1}{6} (1 + x_+^2 + x_+^4) \phi(x_+) + \frac{5q}{3k_F} \right. \\ &\quad \left. + \frac{1}{3} \left(\frac{1}{3} x_-^3 + \frac{1}{3} x_+^3 + \frac{2m^* \omega x_-}{k_F^2} + \frac{8q^2 x_+}{k_F^2} \right) + \frac{x_-^5}{3} + \frac{x_+^5}{3} + \frac{2m^* \omega x_-^3}{k_F^2} + \frac{4m^*{}^2 \omega^2 x_-}{k_F^4} \right] , \end{aligned} \quad (\text{B7})$$

with $x_{\pm} = q2k_F \pm m^* \omega q k_F$ and $\phi(x) = (1 - x^2) \ln |(x-1)(x+1)|$.

For nonzero temperature the expression of $\text{Re}\Pi_{2N}$ ($N=0,1,2$) is an average of the zero temperature functions calculated for the same values of ω and q , but with various values of the Fermi momentum k_F distributed with a weight factor which is just the derivative of the Fermi occupation number. Explicitly one has the following formulas:

$$\begin{aligned}
\text{Re}\Pi_0(\omega, q, T) &= - \int \text{Re}\Pi_0(\omega, q, T=0, k_F=k) df(k, T), \\
\text{Re}\Pi_2(\omega, q, T) &= - \int \text{Re}\Pi_2(\omega, q, T=0, k_F=k) df(k, T), \\
\text{Re}\Pi_4(\omega, q, T) &= - \int \text{Re}\Pi_4(\omega, q, T=0, k_F=k) df(k, T),
\end{aligned} \tag{B8}$$

where $f(k, T)$ is the occupation number. For the case of zero temperature we have

$$df(k) = -\delta(k - k_F)dk,$$

yielding the above expressions for these functions.

We would like also to show the relation between our definition of generalized Lindhard functions (Π_{2i}) and those of Garcia-Recio *et al.* [21] ($\bar{\Pi}_{2N}$) defined in the limit $T=0$:

$$\Pi_0 = 4\bar{\Pi}_0, \quad \Pi_2 = 4\bar{\Pi}_2 - 2q^2\bar{\Pi}_0, \quad \Pi_4 = 4\bar{\Pi}_4 - 4q^2\bar{\Pi}_2 - q^2m^*\rho_0 + q^4\bar{\Pi}_0 + (2m^*\omega q)^2\bar{\Pi}_0. \tag{B9}$$

Our definition of V_0 and V_1 and their W_1 and W_2 are related by

$$V_0 = \frac{W_1}{4} + \frac{q^2}{8} W_2, \quad V_1 = \frac{W_2}{4}. \tag{B10}$$

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- [1] J. O. Newton *et al.*, Phys. Rev. Lett. **46**, 1383 (1981); J. E. Draper *et al.*, *ibid.* **49**, 434 (1982).
- [2] J. J. Gaardhoje, Annu. Rev. Nucl. Part. Sci. **42**, 483 (1992).
- [3] N. Vinh Mau, lectures given at the II TAPS Workshop, Guardamar, 1993.
- [4] A. Bracco *et al.*, Phys. Rev. Lett. **62**, 2080 (1989).
- [5] N. Alamanos and F. Auger, in *Proceedings of the third IN2P3-Riken Symposium on Heavy Ion Collision*, Shinrin-Koen, Saitama, 1994, edited by N. Frascaria and T. Motobayashi (World Scientific, Singapore, 1995).
- [6] H. Lefaou *et al.*, Phys. Rev. Lett. **72**, 3321 (1994).
- [7] K. Yoshida *et al.*, Phys. Lett. B **245**, 7 (1990).
- [8] J. Kasagi *et al.*, Nucl. Phys. **A538**, 585c (1992).
- [9] R. A. Broglia, T. Dossing, B. Lauritzen, and B. R. Mottelson, Phys. Rev. Lett. **58**, 326 (1987).
- [10] Y. Alhassid and B. Bush, Nucl. Phys. **A509**, 461 (1990).
- [11] M. Gallardo, M. Diebel, T. Dossing, and R. A. Broglia, Nucl. Phys. **A443**, 415 (1985).
- [12] S. Adachi and N. Van Giai, Phys. Lett. **149B**, 447 (1984).
- [13] P. F. Bortignon, R. A. Broglia, B. Bertsch, and J. Pacheco, Nucl. Phys. **A460**, 149 (1986).
- [14] N. Dinh Dang, Nucl. Phys. **A504**, 143 (1989).
- [15] N. Vinh Mau, Nucl. Phys. **A548**, 381 (1992).
- [16] D. Vautherin and N. Vinh Mau, Nucl. Phys. **A422**, 140 (1984).
- [17] H. Sagawa and G. F. Bertsch, Phys. Lett. **146B**, 140 (1984).
- [18] F. L. Braghin and D. Vautherin, Phys. Lett. B **333**, 289 (1994).
- [19] A. Abada, F. L. Braghin, and D. Vautherin, in *Proceedings of the third IN2P3-Riken Symposium on Heavy Ion Collision* [5].
- [20] J. W. Negele and D. Vautherin, Phys. Rev. C **5**, 1472 (1972).
- [21] C. Garcia-Recio, J. Navarro, N. Van Giai, and L. L. Salcedo, Ann. Phys. (N.Y.) **214**, 293 (1992).
- [22] A. L. Fetter and J. D. Walecka, *Quantum Theory of Many Particle Systems* (McGraw-Hill, New York, 1971).
- [23] Ph. Chomaz, D. Vautherin, and N. Vinh Mau, Phys. Lett. B **242**, 313 (1990).
- [24] N. Van Giai and H. Sagawa, Phys. Lett. **106B**, 379 (1981).
- [25] H. Krivine, J. Treiner, and O. Bohigas, Nucl. Phys. **A336**, 155 (1980).
- [26] D. Vautherin and D. M. Brink, Phys. Rev. C **5**, 626 (1972).
- [27] M. Beiner, H. Flocard, N. Van Giai, and P. Quentin, Nucl. Phys. **A238**, 29 (1975).
- [28] P. Ring and P. Schuck, *The Nuclear Many Body Problem* (Springer, Berlin, 1982).
- [29] J. W. Negele and H. Orland, *Quantum Many Particle Systems* (Addison-Wesley, Redwood City, CA, 1988), Sec. 6.2.
- [30] B. K. Jennings and A. D. Jackson, Nucl. Phys. **A342**, 23 (1980).
- [31] *Handbook of Mathematical Functions*, edited by S. Abramowitz and I. Stegun (Dover, New York, 1950).