Collisional damping in heated nuclei within the Landau-Vlasov kinetic theory

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Collisional damping of nuclear collective vibrations is studied within the framework of the Vlasov-Landau equation including retardation (memory) effects in the collision integral. Expressions for the nuclear two-body viscosity and the collisional width Γ of the giant multipole resonances in heated nuclei are obtained in the case of the quadrupole dynamic distortion of the Fermi surface. An improved formula for Γ is also proposed in which all multipolarities of the distortion of the Fermi sphere are taken into account. The collisional width of the giant dipole resonance is calculated as a function of excitation energy for the Sn nuclei region using this formula. It shows a weak variation with temperature. In the temperature range of $T \approx 2.5-7$ MeV, the contribution of collisional damping to GDR width does not exceed 50% of the experimental values.

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I. INTRODUCTION

In recent years experimental and theoretical [1–6] investigations of the giant multipole resonances (GMR) built on excited states of nuclei have been very extensive. Experimental data for the giant dipole resonance (GDR) in the Sn region demonstrate a stability of the energy $E=\hbar\omega_0$ of the resonance centroid with respect to temperature T, whilst at low excitation energies the width increases as T^2 and shows much weaker variation at higher temperature. So far there is no satisfactory description of the behavior of the observed width. The interplay between different relaxation mechanisms and dependence on the temperature of the different contributions to the total width are still questions of debate.

In general the total width depends on the intrinsic width and deformation of the nuclear shape, as well as on thermodynamic fluctuations of the nuclear shape and orientation angles. The intrinsic width is determined by coupling of both particle and hole to more complicated states lying at the same excitation energy [collisional damping (CD)]. It also depends on the escape probability of particles in the continuum and density fluctuations.

Comparisons of experimental data with calculations based on the theoretical model [5], which include rotation effects and the coupling to time-dependent thermal fluctuations of the nuclear surface, indicate a weak variation (20−30%) of the GDR intrinsic width with temperature. In Ref. [6] the first microscopical calculation of intrinsic width in ^{16}O and ^{40}Ca was performed taking into account all main relaxation mechanisms. The widths found in [6] were also almost independent of temperature at high excitation energy.

In this article we will discuss a special feature of the temperature dependence of collisional damping using a transport theory. In particular, we will show that the collisional contribution to the intrinsic width exhibits a weak variation with temperature.

The CD relaxation mechanism in cold nuclei was investigated in Refs. [7–13]. In particular, its contribution to the GMR widths with multipolarities $L \geqslant 1$ was estimated as $\leqslant 30\%$. The retardation (memory) effects in a collision integral should be taken into account for a proper consideration of the CD relaxation channel in the presence of fast collective motion, at least at low excitation energies.

Previously the CD channel of the GMR decay in heated nuclei was investigated within a transport theory in Refs. [8–10,13,14]. In Refs. [10,13] a relaxation rate approximation (RRA) for the calculation of the damping width was used. In this approach the width is proportional to the quasiparticle collisional frequency $\omega_c \equiv 1/\tau$, where τ is the relaxation time. As a result the width has the same behavior with temperature as the collisional frequency, namely, it increases as T^2. As is known from Fermi liquid theory [15], the RRA method is applicable for the evaluation of the damping width only in a regime of rare collisions. In this case, i.e., when the frequency ω_0 of the collective vibration is much greater than the collision frequency ω_c ($\omega_0 \gg \omega_c$), a zero sound propagates in the Fermi system. But with increasing temperature the collision frequency ω_c also increases. The condition $\omega_0 \gg \omega_c$ for the propagation of the zero sound may be violated and the RRA method for the evaluation of damping width will be invalid. For example, in the frequent collision regime ($\omega_0 \ll \omega_c$) the CD width is inversely related to the quasiparticle collisional frequency [11,12,14,15].

In what follows we concentrate on the damping properties of giant resonances. We calculate the CD width of the giant resonances starting from a Landau-Vlasov transport equation with a memory-dependent collision term. In Sec. II we derive the basic local equations of motion taking into account the memory and temperature effects in the collision integral. In Sec. III we obtain simple analytical expressions for the viscosity, amplitude attenuation coefficient, and width of the GMR in the case of dynamic quadrupole distortion of the Fermi surface. These expressions are valid for an arbitrary relation between ω_0 and ω_c. Particularly, they are valid in the transition region from the zero sound (collisionless) regime to first sound (hydrodynamic) regime in excited nuclei. The general form of the expression for the attenuation coefficient agrees with the interpolation formula assumed in [16]. In Sec. IV we also propose an approximate formula for the CD width deduced by taking into account all multipolarities of the Fermi sphere in the zero sound regime. Using this formula we investigate the general behavior of the GDR col-

lisional width as a function of the excitation energy in the region of Sn nuclei.

II. THE MOMENT EQUATIONS

We start from the Landau-Vlasov equation with the memory-dependent collision integral $St(t)$,

$$\frac{\partial f}{\partial t}+\frac{\vec{p}}{m}\frac{\partial f}{\partial \vec{r}}-\frac{\partial V}{\partial \vec{r}}\frac{\partial f}{\partial \vec{p}}=St(t). \tag{1}$$

Here V is the self-consistent mean field and f is the Wigner distribution function in which we take into account only the deformation of the Fermi sphere with multipolarities $\ell\leq 2$

$$f=f_s+\delta f,\quad \delta f=\sum_{\ell=1}^{2}\delta f_\ell. \tag{2}$$

Here $f_s\equiv f_s(\vec{r},\vec{p},t)$ corresponds to the spherical Fermi surface and δf represents both quadrupole deformation and displacement of the Fermi surface. For small deviations from a Fermi sphere the right-hand side (RHS) $St(t)$ of (1) is a collision integral linearized in δf and it may be represented in the form [17,18]

$$St(t)\equiv \delta St(\vec{p},t)=\int_{-\infty}^{t}dt' A(t-t')\,\delta f(t'), \tag{3}$$

which takes into account the retardation effects. Here $A(t-t')$ allows for the memory effects. In this paper we will not use an explicit form of A. Below we will need $\delta St(\vec{p},t)$ only for periodic oscillation of δf and we will use some extension of results which were obtained earlier in [11,12] [see Eqs. (21) and (22)].

We begin by calculating the first three moments of Eq. (1) in $\vec{p}$ space. As a result we have (for details, see [19,20])

$$\frac{\partial \rho}{\partial t}=-\frac{\partial}{\partial r_\nu}\rho u_\nu, \tag{4}$$

$$\frac{\partial}{\partial t}m\rho u_\alpha+\frac{\partial}{\partial r_\nu}m\rho u_\nu u_\alpha+\frac{\partial}{\partial r_\nu}P_{\nu\alpha}+\rho\frac{\partial}{\partial r^\alpha}V=0, \tag{5}$$

$$\frac{\partial}{\partial t}P_{\alpha\beta}+\frac{\partial}{\partial r_\nu}u_\nu P_{\alpha\beta}+P_{\nu\beta}\frac{\partial}{\partial r_\nu}u_\alpha+P_{\nu\alpha}\frac{\partial}{\partial r_\nu}u_\beta=Q_{\alpha\beta}, \tag{6}$$

where

$$Q_{\alpha\beta}=\frac{g}{m}\int\frac{d\vec{p}}{(2\pi\hbar)^3}(p_\alpha-mu_\alpha)(p_\beta-mu_\beta)\,\delta St(\vec{p},t), \tag{7}$$

with g being the degeneracy factor. In (4)–(6) and in the following expressions repeated greek indices are to be understood as summed over.

In (4)–(6) ρ is the local density of the nucleons

$$\rho=g\int\frac{d\vec{p}}{(2\pi\hbar)^3}f, \tag{8}$$

$\vec{u}$ is the velocity field

$$\vec{u}=\frac{g}{\rho}\int\frac{d\vec{p}}{(2\pi\hbar)^3}\frac{\vec{p}}{m}f, \tag{9}$$

and $P_{\alpha\beta}$ is the pressure tensor

$$P_{\alpha\beta}=\frac{g}{m}\int\frac{d\vec{p}}{(2\pi\hbar)^3}(p_\alpha-mu_\alpha)(p_\beta-mu_\beta)f. \tag{10}$$

The tensor $P_{\alpha\beta\nu}$ of third rank

$$P_{\alpha\beta\nu}=\frac{g}{m}\int\frac{d\vec{p}}{(2\pi\hbar)^3}(p_\alpha-mu_\alpha)(p_\beta-mu_\beta)(p_\nu-mu_\nu)f$$

vanishes,

$$P_{\alpha\beta\nu}=0, \tag{11}$$

because we take into account only the deformation of the Fermi sphere with multipolarities $\ell\leq 2$ and the deformation with $\ell=1$ corresponds to a displacement of the Fermi sphere in $\vec{p}$ space by a vector $m\vec{u}$.

The pressure tensor $P_{\alpha\beta}$ can be written as

$$P_{\alpha\beta}=P\delta_{\alpha\beta}+P'_{\alpha\beta}, \tag{12}$$

where the component $P'_{\alpha\beta}$, associated with dissipative processes, is

$$P'_{\alpha\beta}=\frac{g}{m}\int\frac{d\vec{p}}{(2\pi\hbar)^3}(p_\alpha-mu_\alpha)(p_\beta-mu_\beta)\,\delta f_2 \tag{13}$$

and P is the pressure due to motion of nucleons without distortion of the Fermi sphere

$$P=2\epsilon_{\rm kin}/3. \tag{14}$$

Here $\epsilon_{\rm kin}$ is the kinetic energy density in the case of a spherical Fermi surface. For example, we have in the Thomas-Fermi approximation

$$\epsilon_{\rm kin}=g\int\frac{d\vec{p}}{(2\pi\hbar)^3}\frac{p^2}{2m}f_s=\frac{3}{10}\frac{\hbar^2}{m}\left(\frac{3\pi^2}{g}\right)^{2/3}\rho^{5/3}. \tag{15}$$

In the following we restrict our consideration to the case of small amplitude vibrations around the equilibrium value of the density $\rho_{\rm eq}$ and the pressure $P_{\rm eq}$, where $P_{\rm eq}$ is given by the relations (14) and (15) at $\rho=\rho_{\rm eq}$. We will thus consider a periodic time dependence of the form $\exp(i\omega t)$ for the quantities involved. We will also omit terms proportional to u^2 in Eq. (5), since they are zero in the linear approximation of δu.

Let us now introduce the displacement field $\vec{\chi}$,

$$\vec{u}=\partial\vec{\chi}/\partial t, \tag{16}$$

and write down Eq. (5) in the following way [20]:

$$m\rho\frac{\partial^2}{\partial t^2}\chi_\nu+\left(\frac{\partial}{\partial r_\nu}P+\rho\frac{\partial}{\partial r_\nu}V\right)+\frac{\partial}{\partial r_\mu}P'_{\nu\mu}=0. \tag{17}$$

Here the terms in parentheses are the conservative forces per unit volume calculated with spherical surface. The last term

on the left-hand side of Eq. (17) represents the dissipative force. It takes into account dynamic distortion of the Fermi surface in a moving Fermi liquid.

Using relations (14) and (15), we can rewrite the expression in the parentheses in (17) as

$$\frac{\partial}{\partial r_\nu}P+\rho\frac{\partial}{\partial r_\nu}V=\rho\frac{\partial}{\partial r_\nu}\frac{\delta\epsilon}{\delta\rho}\approx\rho_{\rm eq}\frac{\partial}{\partial r_\nu}\left[\left(\frac{\delta^2\epsilon}{\delta\rho^2}\right)_{\rm eq}\delta\rho\right], \tag{18}$$

where ϵ is the energy density of particles

$$\epsilon=\epsilon_{\rm kin}+\epsilon_{\rm pot},$$

and $\epsilon_{\rm pot}$ is the potential energy density which is related to the mean field V by

$$V=\delta\epsilon_{\rm pot}/\delta\rho. \tag{19}$$

Note that $(\delta\epsilon/\delta\rho)_{\rm eq}$ is the chemical potential which does not depend on the space coordinate $\vec{r}$ for the equilibrium state of the nucleus. We have used this fact when deducing Eq. (18).

The density variation $\delta\rho$ in (18) can be represented by a displacement field. It follows from (8), (9), (15), and (16) that

$$\delta\rho=\rho-\rho_{\rm eq}=-{\rm div}(\rho_{\rm eq}\vec{\chi}). \tag{20}$$

In order to obtain an expression for the quantity $Q_{\alpha\beta}$ in (7), we use the procedure of Refs. [11,12,18] for calculating the energy integral containing the linearized memory-dependent collision term $\delta St(\vec{p},t)$. It should be pointed out that in this method the equilibrium component $f_{\rm eq}$ of the Wigner distribution function is taken as the Fermi distribution $f_{\rm eq}=1/[1+\exp\{(\epsilon_{\rm eq}-\mu)/T\}]$ depending on the quasiparticle energy $\epsilon_{\rm eq}$ in equilibrium, the temperature T, and the chemical potential μ. The dynamical component of the distribution function has the form $\delta f(\vec{r},\vec{p},t)=-(\partial f_{\rm eq}/\partial\epsilon_{\rm eq})\,\psi(\vec{r},\hat{p},t)$, where ψ depends only on the direction of the momentum. As a result, we find for (7)

$$Q_{\alpha\beta}=-P'_{\alpha\beta}/\tau_2(\omega_0,T). \tag{21}$$

The quantity τ_2 is the relaxation time in the case of a quadrupole deformation of the Fermi surface. It is dependent on the real part of ω ($\omega_0\equiv{\rm Re}\ \omega$) and the temperature T. In the case where $T,\hbar\omega_0\ll\mu$ we have [11,12]

$$1/\tau_2(\omega_0,T)=\{1+C_\omega[\hbar\omega_0/(2\pi T)]^2\}/\tilde{\tau}_2(T). \tag{22}$$

The frequency dependence of τ_2 is due to the retardation effects in the collision integral. Note that we deal with a collision integral related to real transitions. By definition, an expression for this collision integral should contain a δ function associated with the conservation of energy. It can be seen that the expression will contain two δ functions and these δ functions will include the shifts $\pm\hbar{\rm Re}\ \omega$ in the arguments, if the retardation effect is taken into account [17,18]. Therefore the real part of ω only enters in (21) and (22).

The dependence on temperature T in Eq. (22) results from the T dependence of the equilibrium distribution function near the Fermi momentum. The quantity $\tilde{\tau}_2(T)$ is the thermal relaxation time [21,22]. The factor C_ω defines the magnitude of the partial relaxation time τ_2 in the quantum region $\hbar\omega_0\gg T$. The values C_ω and $\tilde{\tau}_2(T)$ will be discussed later (see Sec. IV).

Taking into account (11), (12) and (21) we solve Eq. (6) with respect to $P'_{\alpha\beta}$ and obtain

$$P'_{\alpha\beta}=-\eta_\omega\tilde{\Lambda}_{\alpha\beta}-P_{eq}\Lambda_{\alpha\beta}(\omega\tau_2)^2/[1+(\omega\tau_2)^2], \tag{23}$$

where

$$\tilde{\Lambda}_{\alpha\beta}=\frac{\partial}{\partial r_\beta}u_\alpha+\frac{\partial}{\partial r_\alpha}u_\beta-\frac{2}{3}\frac{\partial u_\nu}{\partial r_\nu}\delta_{\alpha\beta},$$

$$\Lambda_{\alpha\beta}=\frac{\partial}{\partial r_\beta}\chi_\alpha+\frac{\partial}{\partial r_\alpha}\chi_\beta-\frac{2}{3}\frac{\partial\chi_\nu}{\partial r_\nu}\delta_{\alpha\beta}.$$

The quantity η_ω in (23) is given by

$$\eta_\omega=(P_{\rm eq}/\omega)(\omega\tau_2)/[1+(\omega\tau_2)^2]. \tag{24}$$

This quantity determines the time irreversible contribution to the pressure tensor (23) and can be considered as the viscosity coefficient due to the relaxation occurring on the distorted Fermi surface. Expression (24) is valid independently of the nucleon's collision rate. The viscosity goes to zero in both the rare and frequent collision regimes.

Finally, using (18), (19), and (23) we find from (17) the equation for the normal mode of vibrations of the displacement field

$$\begin{aligned}\omega^2\rho_{\rm eq}\chi_\nu+(\rho_{\rm eq}/m)\frac{\partial}{\partial r_\nu}\kappa\frac{\partial}{\partial r_\mu}\rho_{\rm eq}\chi_\mu\\+\frac{(\omega\tau_2)^2}{1+(\omega\tau_2)^2}\frac{\partial}{\partial r_\mu}(P_{\rm eq}/m)\Lambda_{\nu\mu}\\=-i\omega\frac{\partial}{\partial r_\mu}(\eta_\omega\Lambda_{\nu\mu}/m).\end{aligned} \tag{25}$$

This equation has the form of a Fourier transform of the Navier-Stokes equation for the displacement field. The quantity κ in (25) is the local incompressibility of the nuclear liquid taken at equilibrium,

$$\kappa=(\delta^2\epsilon/\delta\rho^2)_{\rm eq}. \tag{26}$$

The last term on the left side of (25) describes the influence of the dynamic distortion of the Fermi surface.

III. THE DAMPING OF COLLECTIVE EXCITATIONS

The local equation (25) is rather complicated to solve in a general case. For a qualitative analysis of the dependence of the GMR collisional widths on temperature we will consider the nucleus as a homogeneous system with a sharp surface. We will assume that all equilibrium quantities $\rho_{\rm eq}, P_{\rm eq}$ etc. do not depend on the position $\vec{r}$ in the nuclear interior $r\leq R_{\rm eq}$ ($R_{\rm eq}$ is the equilibrium radius of the nucleus). Note that this simple model satisfactorily describes the main characteristics of the GMR if we introduce some boundary conditions on the nuclear surface [11,12,20,23,24].

Calculating the divergence from the left and right parts of (25), we find an equation for the density vibration inside the nucleus. Taking into account Eq. (19), we obtain

$$\omega^2\delta\rho+(K'_\omega/9m)\nabla^2\delta\rho=-i\omega(4\eta_\omega/3m\rho_{eq})\nabla^2\delta\rho. \quad (27)$$

Here

$$K'_\omega=K+8(\epsilon_{\rm kin}/\rho)_{\rm eq}(\omega\tau_2)^2/[1+(\omega\tau_2)^2], \quad (28)$$

and $K\equiv 9\kappa\rho_{\rm eq}$ is the adiabatic compressibility modulus.

We will look for the solution of Eq. (27) inside the nucleus for a giant resonance of multipolarity L. Using the form

$$\delta\rho=\text{const}\times j_L(kr)Y_{LM}(\hat{r}), \quad (29)$$

we find that k and ω are connected by the dispersion relation

$$\omega^2=(K'_\omega/9m)k^2+i\omega(4\eta_\omega/3m\rho_{\rm eq})k^2. \quad (30)$$

The solution of this equation

$$\omega=\omega_0+i\Gamma/2\hbar \quad (31)$$

defines the energy of the giant resonance $\hbar\omega_0$ and its width Γ. Note that this definition of the width is in agreement with the response function method in Fermi liquids in the case of small damped collective vibrations ($\omega_0\gg$ Im ω) [14,25,26]. When $\hbar\omega_0\gg\Gamma$ we find from (30) (k is real)

$$\Gamma=\frac{4}{3}\frac{\hbar\omega_0^2 m}{S^2 v_F^2\rho_{\rm eq}}\eta_{\omega_0}. \quad (32)$$

Here $S\equiv\omega_0/v_F k$ is the velocity of sound in units of the Fermi velocity v_F,

$$S^2=K'_{\omega_0}/(9mv_F^2)\equiv S_r^2+(S_f^2-S_r^2)/[1+(\omega_0\tau_2)^2], \quad (33)$$

where $S_r\equiv S(\omega_0\tau_2\gg 1)$ and $S_f\equiv S(\omega_0\tau_2\ll 1)$ are the velocities of the zero and first sounds, in units of v_F, respectively,

$$S_r^2=[K+8(\epsilon_{\rm kin}/\rho)_{\rm eq}]/9mv_F^2, \quad (34)$$

$$S_f^2=K/9mv_F^2. \quad (35)$$

Using the expressions (24), (34), and (35) we obtain

$$\Gamma=2B(S)\ \hbar\omega_0\ \omega_0\tau_2/[1+(\omega_0\tau_2)^2]\ , \quad (36)$$

where

$$B(S)^{-1}\equiv 15S^2/2 =B(S_r)^{-1}+[B(S_f)^{-1}-B(S_r)^{-1}]/[1+(\omega_0\tau_2)^2]. \quad (37)$$

Expression (36) for Γ can then be written in the form

$$\Gamma=2\tilde{c}B(S_r)\ \hbar\omega_0\ \frac{\omega_0\tau_2}{1+\tilde{c}(\omega_0\tau_2)^2},\qquad \tilde{c}\equiv B(S_f)/B(S_r), \quad (38)$$

which is the general expression for the width applicable for different values for the ratio ω_0/ω_c. In particular, it is valid in the transition region from the zero sound regime to the first sound regime. From (38) and (22) [$\tau_2\equiv\tau_2(\omega_0,T)$] we can see that the width of the GMR is determined by the temperature and the frequency ω_0 of the normal mode.

Note that in the theory of Fermi liquids one usually studies not the damping coefficient $\gamma\equiv\Gamma/2\hbar$ but, rather, the amplitude attenuation coefficient α [16,27]. This coefficient is found as the imaginary part of the wave number k (ω is real): $k=k_0-i\alpha$. Using (24) and (32) we obtain ($k_0\gg\alpha$)

$$\alpha=\frac{2}{3}\frac{\omega^2 m}{S^3 v_F^3\rho_{\rm eq}}\ \eta_\omega=\frac{2\omega}{S^3 v_F^3\rho_{\rm eq}}\frac{\omega\tau_2}{1+(\omega\tau_2)^2}.$$

The general form of this relation agrees with the form of the interpolation formula assumed in [16] [Eq. (1.186)].

IV. GENERALIZATION AND DISCUSSION

Above we have taken into account only the dynamical quadrupole deformations of the Fermi surface. In Refs. [11,12] expressions were obtained for the GMR widths in the two limiting cases of rare ($\omega_0\tau\gg 1$) and frequent ($\omega_0\tau\ll 1$) collisions, but all multipolarities of the distortion of the Fermi surface have been taken into account. They have the form

$$\Gamma(\omega_0\tau\gg 1)\equiv\Gamma_z=2R(S_r)\frac{\hbar}{\tau}\equiv 2\frac{R(S_r)}{d_2}\hbar\omega_0\frac{1}{\omega_0\tau_2}, \quad (39)$$

$$\Gamma(\omega_0\tau\ll 1)\equiv\Gamma_f=2\frac{B(S_f)}{d_2}\hbar\omega_0\tau\equiv 2B(S_f)\hbar\omega_0\tau_2, \quad (40)$$

if the effective nucleon mass $m^*\equiv m(1+F_1/3)$ is approximately equal to m.

The function $B(S_f)$ in (40) is defined by (37). The function $R(S_r)$ in (39) depends both on the magnitude of the distortion of the Fermi surface in the absence of the collision integral and on the scattering probability of quasiparticles. It has the form [11,12]

$$R(S)=S\sum_{\ell\geq 2}d_\ell(2\ell+1)Q_\ell^2(S)/\{2SF_0Q_1(S)[Q_1(S)F_0-1] -Q_1'(S)\}, \quad (41)$$

where $Q_\ell(S)$ are the Legendre functions of the second kind defining the magnitude of the distortion of the Fermi surface in the collisionless regime. This function (41) was first derived in [27–29] though in a somewhat different form. The quantity d_ℓ is equal to

$$d_\ell=\langle W\Phi_\ell\rangle/\langle W\rangle, \quad (42)$$

where W is the probability of scattering of nucleons near the Fermi surface. The function $\Phi_\ell\equiv 1+P_\ell(\hat{p}_2\hat{p}_1)-P_\ell(\hat{p}_3\hat{p}_1)-P_\ell(\hat{p}_4\hat{p}_1)$ defines the angular constraints for scattering of nucleons within the distorted layers of the Fermi surface with multipolarity ℓ (P_ℓ is a Legendre polynomial). The symbol $\langle\cdots\rangle$ denotes the averaging over angles of the relative momentum of the colliding particles.

The function $R(S)$ is proportional to $B(S)$ for large values of S,

$$R(S)=d_2B(S)=2d_2/15S^2, \qquad S\gg 1. \tag{43}$$

If the magnitude of S tends to 1, $R(S)$ also approaches 1

$$R(S)\approx 1, \qquad S\to 1+0. \tag{44}$$

The quantity τ is the relaxation time for the collective motion:

$$\tau(\omega,T)\equiv\tau_2(\omega,T)d_2\equiv\tilde{\tau}(T)/\{1+C_\omega(\hbar\omega/2\pi T)^2\}, \tag{45}$$

where $\tilde{\tau}(T)\equiv\tau(\omega=0,\ T)\equiv\tilde{\tau}_2(T)d_2$ and

$$1/\tilde{\tau}(T)=T^2g(m^*/\hbar^2)^3\langle W\rangle/24\pi^2. \tag{46}$$

The velocity S_r of zero sound is the solution of the dispersion equation

$$Q_1(S_r)=1/[F_0+S_r^2F_1/(1+F_1/3)]. \tag{47}$$

The speed of the first sound is given by

$$S_f^2=(1+F_0)(1+F_1/3)/3. \tag{48}$$

For simplicity here and in (41) only the Landau parameters F_0, and F_1 of the interaction between the nucleons were taken into account.

We now compare the relation (38) with (39) and (40) for the width. In the first sound regime expression (38) coincides with (40). This feature is due to the dominating role of the quadrupole deformation of the Fermi surface when the collisions of nucleons are frequent.

In the regime of rare collisions the contributions of distortions with different multipolarities are governed by the variation of the mean field, i.e., they depend on the value of the zero sound speed S_r. In the case of $S_r\gg 1$, which corresponds to strong repulsive interaction $F_0\gg 1$ [30], the Fermi surface distortion is restricted to the multipolarities $\ell\leq 2$ and expression (38) coincides with (39). This is evident from Eq. (43). For $S_r\approx 1$, an essential contribution to the width is given by the scattering of quasiparticles in the layers of the Fermi surface with multipolarities $\ell>2$. As a result, the magnitude of Γ calculated by (38) is $2d_2/15$ times smaller than that obtained from (39).

Using the exact expression (38) for the width in the case of quadrupole deformations of the Fermi surface as a basis, we propose a refined formula that describes the transition from the hydrodynamic regime to the zero sound regime taking into account all multipolarities of the distortion of the Fermi surface. In a general case the function $R(S_r)$ defines the contributions to the width of the scattering of quasiparticles in the layers of the Fermi surface with different multipolarities in the regime of rare collisions. Therefore we replace $B(S_r)$ by $R(S_r)/d_2$ in order to correct equation (38) in the zero sound regime with $S_r\approx 1$ [this substitution does not change (38) in the case $S_r\gg 1$ in view of (43)]. In this way we have from (38) the following expression for Γ:

$$\Gamma=2c\,\frac{R(S_r)}{d_2}\,\hbar\omega_0\,\frac{\omega_0\tau_2}{1+c\omega_0\tau_2^2}$$

$$\equiv 2q\,R(S_r)\,\hbar\omega_0\,\frac{\omega_0\tau}{1+q(\omega_0\tau)^2}, \tag{49}$$

where the cutoff factors c and q are equal to

$$c\equiv B(S_f)\,d_2/R(S_r), \quad q\equiv c/d_2^2. \tag{50}$$

This expression has the correct behavior in the limiting cases of $\omega_0\tau_2\gg 1$ and $\omega_0\tau_2\ll 1$ irrespective of the value of S_r and coincides with (38) when the Fermi surface distortion is restricted to the multipolarities $\ell\leq 2$.

To a good approximation, the condition $S_r\approx 1$ is fulfilled for the model of the nucleus in which nuclear-matter solutions of the Landau-Vlasov equation are combined with boundary conditions [11,12]. In nuclear matter, the relation $S_r\approx 1$ is also valid both for isovector and isoscalar vibrations when well-defined collective modes are present [26,30-31]. In this case of $S_r\approx 1$, we have from (49) the following expression for the collisional width:

$$\Gamma=2q\,\hbar\omega_0\,\frac{\omega_0\tau}{1+q(\omega_0\tau)^2}\approx 2.5c\,\hbar\omega_0\,\frac{\omega_0\tau_2}{1+c(\omega_0\tau_2)^2}, \tag{51}$$

where

$$q\approx 1/6S_f^2\approx 1/2(1+F_0)(1+F_1/3), \quad c\approx 16q/25. \tag{52}$$

To derive (51) and (52) from (49) and (50), Eq. (44) and (47) and the value $d_2=4/5$ were used. This value of d_2 is exact if the collision probability w is isotropic. Equation (51) represents a simple formula linking the expressions for the CD width in two physically different regimes of the rare and frequent collisions. Below we will use these relations for the calculation of Γ.

Let us compare the expression (51) for the collisional width with the one that corresponds to the width in the relaxation rate approximation [8–10,13,29], in which

$$\Gamma\equiv\Gamma_0=2\hbar/\tau=2\hbar\omega_0/\omega_0\tau. \tag{53}$$

As it is evident from (39) and (44) or (49) and (51), strictly speaking, the formula (53) is valid for describing collisional damping only in the regime of rare collisions. The distinction between (51) and (53) is due to the self-consistent mean field. The width Γ_0 increases as the quantity $\omega_0\tau$ decreases. In contrast to Γ_0, the width Γ [Eq. (51)] approaches zero when $\omega_0\tau$ tends to zero. Note that the factor of 2 in (53) results from the relation (31) between Γ and Im ω. It accounts for the fact that the quantity τ [Eq. (45)] is the relaxation time for a motion of the collective coordinate rather than the collective energy.

Crucial quantities in the calculations of the collisional width Γ Eq. (49) are the thermal relaxation time $\tilde{\tau}(T)$ and the cutoff factor q. The value of q is determined by the velocity of the zero sound, i.e., the Landau parameters F_l of the interaction between nucleons. The thermal relaxation time $\tilde{\tau}(T)$ leads to a nuclear viscosity in the regime of frequent collisions [15,22,32–34]. Most estimates of the ther-

TABLE I. Values of parameter α extracted from the corresponding references.

Ref.	[22]	[32]	[10]	[35]	[33]
α MeV	4.9	2.4	5.2	3.2	19.3

mal relaxation time $\tilde{\tau}(T)$ in nuclear matter are based on the expression (46). For example, this value was calculated in Refs. [10–12,22,32,33,35] with different assumptions about the collision probability W. Using Eq. (4.6) we introduce the parameter α by the relation

$$\tilde{\tau}_2(T)/\hbar = \alpha T^{-2} \quad [T,\alpha \text{ in MeV}]. \tag{54}$$

In Table I we give a list of the parameters α which were derived using different approaches [10,22,32,33,35]. The differences between the estimates obtained in Ref. [33] and Refs. [22,32] are rather large. They result from different values of the nucleon-nucleon cross section in a medium and in free space [36-38]. The variation in these estimates of α is probably due to this fact. Reference [33] gives apparently an overestimate for the value of $\tilde{\tau}$. For example, in [8] the value of the nucleon-nucleon cross section in a medium was assumed to be smaller by only a factor of 2 than the value in free space σ_{free}. Using the value of $\sigma_{\text{free}}=40$ mb, [22] one finds $\alpha=9.8$.

The magnitude of the factor C_ω in (22) and (45) was originally obtained by Landau [39] in the calculation of the absorption coefficient γ in a Fermi liquid. It was shown that in the zero sound regime the variation with temperature and frequency of the coefficient γ and consequently of the damping width, which is proportional to γ, is similar to that given by (39) $[R(S_r)=1]$ and (45) with $C_\omega=1$. This result in the regime of rare collisions agrees rather well with those obtained by the variety of microscopic approaches to damping in Fermi systems in the case of small amplitude deviation from a steady state [29,40–42].

The factor C_ω may also be calculated in a direct way within the framework of a kinetic theory if the exact expression for the collision integral is known. In the general case, the total dynamic part of this collision integral consists of three terms,

$$\delta St = \delta St_f + \delta St_\epsilon + \delta St_w.$$

The first term δSt_f is connected with the variation of the distribution function δf. The second one δSt_ϵ is governed by a variation of the mean field. The third term δSt_w results from the screening effect for the free two-body scattering in a hot Fermi system due to high frequency collective vibrations. In Refs. [11,12,18], the quantities C_ω associated with δSt_f and δSt_ϵ were calculated to be $C_\omega \equiv C_\omega^{(f)}=3$ and $C_\omega \equiv C_\omega^{(\epsilon)}=1$, respectively (see also [10]). These results are in disagreement with the value of 1 obtained in Ref. [13] for the factor $C_\omega \equiv C_\omega^{(f\epsilon)}$ for the combined variation $\delta St_f + \delta St_\epsilon$. Note that when compared to δSt_f the quantity δSt_ϵ contains a complementary effective interaction amplitude arising from a variation of the self-consistent field. In our opinion this indicates that the value $C_\omega=1$ can be obtained only by incorporating into the collision integral the contribution of δSt_w. At present the term δSt_w is rather poorly studied in kinetic theory. Because of this, we will use below the value $C_\omega=1$, which corresponds to Landau's prescription [15].

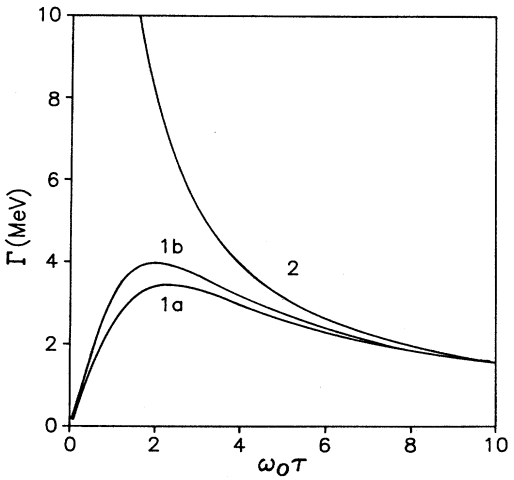

FIG. 1. The collisional width as a function of $\omega_0\tau$. The curves 1a and 1b are obtained using (51) and (52): For curve $1a$ we use $q=0.192$ and for $1b$ we use $q=0.254$; curve 2 is the relaxation rate approximation Γ_0, Eq. (53).

Strictly speaking, the preceding discussion is concerned with isoscalar vibrations. However some of the relations deduced for the isoscalar case remain the same for the isovector case by changing the meaning of certain quantities and other relations are valid as a good approximation. For example, the general form of Eq. (27) for the volume density vibrations will be correct for isovector GMR if the isoscalar density variation and compressibility modulus are replaced by the corresponding isovector quantities [20,43]. Note that we took into account volume oscillations $\delta\rho$ of the form (29), as in the hydrodynamic Steinwedel-Jensen model [44]. Because of this, the dispersion relation (30) and the general form (38) of the collisional width can be valid only in heavy nuclei. As can be seen from the results of Ref. [10], the thermal relaxation times for isovector and isoscalar modes are in agreement in the case of quadrupole distortion of the momentum distribution. For isovector vibrations the condition $S_r \approx 1$ is also fulfilled both in the models of a nucleus as a homogeneous system with a sharp surface and in nuclear matter [11,12,26,30,31]. All these facts make it possible to use the expressions (51) and (52) with α from Table I to estimate the isovector collisional widths in hot heavy nuclei.

Figure 1 shows the dependence of the collisional widths on $\omega_0\tau$. The curves $1a$ and $1b$ are found by employing Eqs. (51) and (52), respectively, in the case of the GDR. We used the values of the isovector amplitudes $F_0=1.6$ and $F_1=0$, [45] i.e., $q=0.192$ ($S_f=0.931$) to obtain curve $1a$. Curve $1b$ was obtained using $q=0.254$. This value corresponds to the velocity of the first sound $S_f=0.81$, which was evaluated in [26,30] for an isovector sound in nuclear matter. We also used $\hbar\omega_0=15.6$ MeV (this value corresponds to the energy of the GDR in ^{112}Sn). Curve 2 corresponds to Γ_0 of Eq. (53).

The width Γ is peaked at the value of $\omega_0\tau=q^{-1/2}$. It is equal to $\Gamma_{\max}=\hbar\omega_0 q^{1/2}$ at the maximum. The value of Γ_0 at

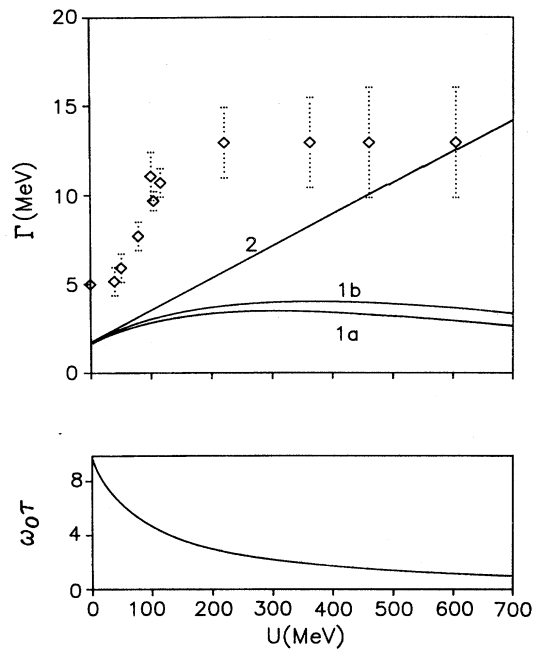

FIG. 2. The excitation energy dependence of width and $\omega_0\tau$ for the giant dipole resonance in the Sn region is shown for the ^{112}Sn nucleus. The experimental data were taken from [1]. The notations of the curves in the top part of the figure are the same as in Fig. 1.

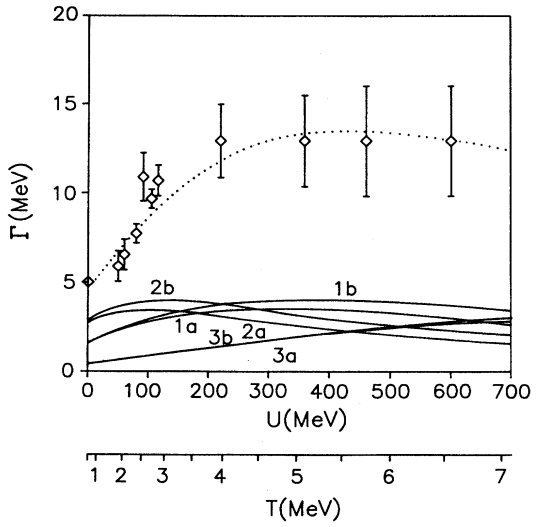

FIG. 3. The collisional widths of the GDR resonance in hot ^{112}Sn for different estimates of the thermal relaxation time. All curves are obtained using Eqs. (51) and (52): For curves $1a-3a$ we use $q=0.192$, for curves $1b-3b$ we use $q=0.254$; we use also $\alpha=4.9$ for $1a$ and $1b$ curves, $\alpha=2.4$ for $2a$ and $2b$ curves, and $\alpha=19.3$ for $3a$ and $3b$ curves. The dotted curve presents the fit of the experimental data using Eq. (51) with $q=0.78$ and $\alpha=3.25$.

the peak $\omega_0\tau=q^{-1/2}$ is twice as large as that of Γ. There are no sudden transitions from zero sound mode to first sound mode.

In Fig. 2 we show the collisional width of the giant dipole resonance and the magnitude of $\omega_0\tau$ in the Sn region as functions of excitation energy, using as an example the ^{112}Sn nucleus. The experimental data were taken from Fig. 6 of Ref. [1]. Considering the experimental data we assume that the energy $\hbar\omega_0$ of the giant dipole resonance is independent of temperature. We used the relation between the temperature and excitation energy U deduced in the Fermi gas model, i.e., $U=aT^2$, where a is the level density parameter. We took the value $a=A/8$ where A is the atomic mass number. As is seen from Fig. 2, Eq. (51) (curves $1a$ and $1b$) leads to a smoother behavior of the collisional width with increasing excitation energy as compared with the prediction of the relaxation rate approximation (the zero sound model, curve 2).

In Fig. 3 the collisional widths of the GDR resonance in hot ^{112}Sn for different estimates of the thermal relaxation time are shown. All curves are found by means of (51) and (52). The curves $1a-3a$ are calculated with $q=0.192$ and curves $1b-3b$ with $q=0.254$. We used the following values of the parameter α governing the thermal relaxation time $\tilde{\tau}_2(T)$: $\alpha=4.9$ for curves $1a$ and $1b$, $\alpha=2.4$ for curves $2a$ and $2b$, and $\alpha=19.3$ for curves $3a$ and $3b$ [see Eq. (54) and Table I].

As can be seen from this figure, for the realistic values of q calculated by Eq. (52) with F_0 and F_1 from [26,30,45], the magnitude of the contribution of collisional damping to the GDR width does not exceed 50% of the experimental values in the temperature range $T\approx 2.5-7$ MeV, irrespective of the estimates for the relaxation time. This result is similar to the result of the microscopic calculations [6]. The collisional contribution to the GDR width depends rather strongly on the relaxation time at low temperatures. It rises as the relaxation time $\tau(\omega,T)$ decreases.

The dotted curve in Fig. 3 is the result of a phenomenological fit to the experimental data using Eq. (51). The values $\alpha=3.25$ and $q=0.78$ were deduced from this fit. We have to point out the limitation of such a type of fit because the collision damping presented by Eq. (51) gives only a partial contribution to the total width of the GDR. As noted in the Introduction, a few additional mechanisms of spreading should be taken into account in order to reproduce the total widths of giant resonances. This fact is reflected in the fitted value of $q=0.78$ which is significantly larger than the realistic estimates of $q=0.2-0.3$. However, the phenomenological fit may be useful for calculating the values of the total width in the range of temperatures where experimental data are absent.

Before closing we would like to add some concluding remarks.

(1) Expressions (36) and (38) and thus (49) and (51) for Γ are derived from formula (30) under the assumption that the condition $\hbar\omega_0 \gg \Gamma$ is fulfilled. Therefore these expressions are not valid for an overdamped collective motion.

(2) The frequency ω_0 may depend on the temperature since $S\equiv S(\tau_2)$ and the relaxation time τ_2 is a function of T. For example, the velocity S may shift towards lower values with increasing temperature, as can be seen from (33). Therefore the frequency could have the same behavior when k is independent of the excitation energy. This variation of frequency with temperature was observed in Ref. [46].

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[1] J. J. Gaardhoje, Annu. Rev. Nucl. Part. Sci. **42**, 483 (1992).
[2] G. Enders *et al.*, Phys. Rev. Lett. **69**, 249 (1992).
[3] K. A. Snover, Nucl. Phys. **A553**, 153c (1993).
[4] A. Bracco *et al.*, Nucl. Phys. **A569**, 51c (1994).
[5] W. E. Ormand, F. Camera, A. Bracco, A. Maj, P. F. Bortignon, B. Million, and R. A. Broglia, Phys. Rev. Lett. **69**, 2905 (1992).
[6] F. V. De Blasio, W. Cassing, M. Tohyama, P. F. Bortignon, and R. A. Broglia, Phys. Rev. Lett. **68**, 1663 (1992).
[7] P. G. Reinhard, H. L. Yadov, and C. Toepffer, Nucl. Phys. **A458**, 301 (1986).
[8] Cai Yanhuang and M. Di Toro, Phys. Rev. C **39**, 105 (1989).
[9] A. Smerzi, A. Bonasera, and M. Di Toro, Phys. Rev. C **44**, 1713 (1991).
[10] S. Ayik and D. Boiley, Phys. Lett. B **276**, 263 (1992).
[11] V. M. Kolomietz, A. G. Magner, and V. A. Plujko, Yad. Fiz. **56**, 110 (1993) [Phys. At. Nucl. **56**, 209 (1993)]; Nucl. Phys. **A545**, 99c (1992).
[12] V. M. Kolomietz, A. G. Magner, and V. A. Plujko, Z. Phys. A **345**, 131 (1993).
[13] S. Ayik and D. Boiley, Phys. Lett. B **284**, 482E (1992).
[14] R. W. Hasse, Nuovo Cimento A **87**, 109 (1985).
[15] A. A. Abrikosov and I. M. Khalatnikov, Rep. Prog. Phys. **22**, 329 (1959).
[16] D. Pines and P. Nozieres, *The Theory of Quantum Liquids* (Benjamin, New York, 1966), Vol. 1.
[17] V. M. Kolomietz, A. G. Magner, and V. A. Plujko, Yad. Fiz. **55**, 2061 (1992) [Sov. J. Nucl. Phys. **55**, 1043 (1992)] .
[18] V. M. Kolomietz and V. A. Plujko, Yad. Fiz. **57**, 992 (1994) [Phys. At. Nucl. **57**, 931 (1994)].
[19] V. M. Kolomietz and H. H. K. Tang, Phys. Scr. **24**, 321 (1981).
[20] V. M. Kolomietz, *Local Density Approach for Atomic and Nuclear Physics* (Naukova Dumka, Kiev, 1990) (in Russian).
[21] G. A. Brooker and J. Sykes, Ann. Phys. (N.Y.) **61**, 387 (1970).
[22] G. Bertsch, Z. Phys. A **289**, 103 (1978).
[23] G. Eckart, G. Holzwarth, and J. P. Da Providencia, Nucl. Phys. **A364**, 1 (1981).
[24] G. Holzwarth and G. Eckart, Nucl. Phys. A **396**, 171c (1983).
[25] A. I. Akhiezer, I. A. Akhiezer, and I. Ya. Pomeranchuk, Zh. Eksp. Teor. Fiz. **41**, 478 (1961) [Sov. Phys. JETP **14**, 343 (1962)]; Nucl. Phys. **40**, 139 (1963).
[26] D. Gogny and R. Padjen, Nucl. Phys. **A293**, 365 (1977).
[27] G. Baym and C. Pethick, in *The Physics of Liquid and Solid Helium*, edited by K. H. Bennemann and J. B. Ketterson (Wiley, New York, 1978), Vol. 29, p. 1; *Landau Fermi-liquid Theory* (John Wiley and Sons, New York, 1991).
[28] C. J. Pethick, Phys. Rev. **185**, 384 (1969).
[29] K. Ando, A. Ikeda, and G. Holzwarth, Z. Phys. A **310**, 223 (1983).
[30] R. Padjen, Can. J. Phys. **57**, 99 (1979).
[31] B. I. Barz, Yu. L. Bolotin, and M. I. Kazarinov, Yad. Fiz. **27**, 1167 (1978) [Sov. J. Nucl. Phys. **27**, 619 (1978)].
[32] H. S. Köhler, Nucl. Phys. **A378**, 159 (1982).
[33] G. Wegmann, Phys. Lett. **50B**, 327 (1974).
[34] V. M. Kolomietz, A. G. Magner, and V. A. Plujko, Z. Phys. A **345**, 137 (1993).
[35] R. Alkofer, H. Hofmann, and P. J. Siemens, Nucl. Phys. **A476**, 213 (1988).
[36] A. Lejeune, P. Grange, M. Martzolff, and J. Cugnon, Nucl. Phys. **A453**, 189 (1986).
[37] G. Lingxiao, Z. Yizhong, and W. Nörenberg, Nucl. Phys. **A459**, 77 (1986).
[38] K. Niita, W. Cassing, and U. Mosel, Nucl. Phys. **A504**, 391 (1989).
[39] L. D. Landau, Zh. Eksp. Teor. Fiz. **32**, 59 (1957) [Sov. Phys. JETP **5**, 101 (1957)].
[40] G. M. Eliashberg, Sov. Phys. JETP **14**, 886 (1962).
[41] S. Adachi and P. Schuck, Nucl. Phys. **A496**, 485 (1989).
[42] M. Tohyama, Z. Phys. **A335**, 413 (1990).
[43] M. Di Toro, U. Lombardo, and G. Russo, Nuovo Cimento A **87**, 174 (1985).
[44] P. Ring and P. Schuck, *The Nuclear Many-Body Problem* (Springer, New York, 1980).
[45] V. A. Khodel and E. E. Saperstein, Nucl. Phys. **A348**, 261 (1980).
[46] A.G. Magner, V.M. Kolomietz, H. Hofmann, and S. Shlomo, Phys. Rev. C **51**, 2457 (1995).