Continuum Tamm-Dancoff approximation calculations for the escape widths of the isobaric analog state and Gamow-Teller resonance in ^{208}Bi

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The proton escape widths of the isobaric analog state and the Gamow-Teller resonance in ^{208}Bi are calculated by solving the charge-exchange continuum Tamm-Dancoff equations. For the Gamow-Teller resonance the spreading contribution to the width is taken into account by the imaginary part of the optical potential. The calculated escape widths are in agreement with measurements obtained from a recent ^{208}Pb(^{3}He,tp)^{208}Bi experiment.

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It is well established that giant resonances (GR) are microscopically coherent superpositions of many correlated one-particle–one-hole (ph) configurations forming a highly collective state in the continuum. The two principal decay mechanisms of GR are the direct emission of particles from specific ph configurations and the spreading of the elementary ph configurations into more complicated n-particle–n-hole configurations. While the microscopic nature of the spreading is now fairly well established [1–4], the nature of the emission of nucleons by direct decay is an issue in nuclear theory that has yet to be satisfactorily resolved, due mainly to the lack of experimental data. The absence of experimental direct decay data is generally a reflection of the difficulty of performing coincidence experiments and also of subtracting the background component due to compound processes.

In 1980 Gaarde *et al.*, using the ^{208}Pb(^{3}He,tp)^{208}Bi charge exchange reaction with a beam energy $E(^3\mathrm{He})=81$ MeV measured the partial decay widths for both the isobaric analog state (IAS) and the Gamow-Teller resonance (GTR) [5]. The escape widths for the IAS obtained were consistent with previous experiments; however, the total measured escape width for the GTR was about the size of the total reported width of the resonance. Recently, this experiment has been repeated by Akimune *et al.* [6], with an $E(^3\mathrm{He})$ of 450 MeV. The main purpose of this paper is to compare these new experimental results with those obtained from charge exchange continuum Tamm-Dancoff approximation (CTDA) calculations [7,8].

Within the distorted-wave impulse approximation (DWIA), the singles (^{3}He,t) and the coincidence (^{3}He,tp) cross sections for the excitation of the GTR may be given, respectively, as [9]

$$\frac{d^2\sigma}{dEd\Omega t}=AS(E), \quad \frac{d^3\sigma}{dEd\Omega_t d\Omega_p}=\frac{1}{4\pi}A\Sigma_p S^{\uparrow}_{\mathrm{ph}}(E) \ , \tag{1}$$

where E is the excitation energy measured from the ground state of ^{208}Pb, $S(E)$ is the total GT strength function, and $S^{\uparrow}_{\mathrm{ph}}(E)$ is the partial escape component. $A(=[m_{\mathrm{He}}m_t/(2\pi\hbar^2)^2](p_b/p_a)|t_{\sigma\tau}|^2N_D)$ is a constant factor including the strength of the charge exchange transition t amplitude $t_{\sigma\tau}$ and the distortion factor N_D. Following Ref. [6], we define the escape width for a specific ph configuration as

$$\Gamma^{\uparrow}_{\mathrm{ph}}=\Gamma\,\frac{\int_{E_R-\Delta E}^{E_R+\Delta E}\int(d^3\sigma/dEd\Omega_t d\Omega_p)dEd\Omega}{\int_{E_R-\Delta E}^{E_R+\Delta E}(d^2\sigma/dEd\Omega_t)dE}
=\Gamma\,\frac{\int_{E_R-\Delta E}^{E_R+\Delta E}S^{\uparrow}_{\mathrm{ph}}(E)dE}{\int_{E_R-\Delta E}^{E_R+\Delta E}S(E)dE} \ , \tag{2}$$

where E_R is the resonance energy and ΔE defines the range of the integration. We choose $\Delta E=\Gamma/2$. The escape width for a specific hole state h is given by

$$\Gamma^{\uparrow}_{h}=\sum_p \Gamma^{\uparrow}_{\mathrm{ph}} \ . \tag{3}$$

Equation (2) shows that the escape widths are given by $S(E)$ and $S^{\uparrow}_{\mathrm{ph}}(E)$, which are evaluated by means of the continuum random phase approximation (CRPA) and/or CTDA method discussed in Refs. [7,8]. Since the method has already been given in detail in Refs. [7,8], we present here only a brief outline. We define the continuum wave function $|\Psi\rangle$ for the target excited by the Gamow-Teller transition operator $\hat{\rho}\equiv\sigma\tau$ as $|\Psi\rangle=\hat{G}|\rho\rangle$, where $|\rho\rangle=\hat{\rho}|\Phi\rangle$, $|\Phi\rangle$ being the ground state wave function, which we assume to be that of the pure shell model, $\hat{G}$ is the many-body Green's function (propagator) of the system and is given as $\hat{G}=[E-H+i\varepsilon]^{-1}$, where $H=H_h+H_p+V_{\mathrm{ph}}$, H_h, H_p, and V_{ph} being the hole-nucleus Hamiltonian, the particle Hamiltonian, and the residual ph interaction, respectively. H_p can be expressed as a sum of a kinetic energy operator and a complex energy-dependent optical potential. The imaginary part W_P in the potential describes the damping of p.

In Ref. [7], we introduced $|\Lambda\rangle$ as $|\Psi\rangle\equiv\hat{G}_0|\Lambda\rangle$, where $\hat{G}_0$ is the Green's function in the absence of V_{ph} (optical model Green's function). It is important to note here that we properly take into account the Pauli exclusion principle in

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$\hat{G}_0$ in the sense that particles are forbidden to propagate in occupied nucleon orbitals. Now $|\Lambda\rangle$ satisfies the integral equation

$$|\Lambda\rangle = |\rho\rangle + V_{\rm ph}\hat{G}_0|\Lambda\rangle. \tag{4}$$

The point of our method is to solve Eq. (4), after making the partial wave expansion,

$$|\Lambda\rangle = (1/r)\sum_{\rm ph} \lambda_{\rm ph}(r)|[y_p\phi_h]_{1m}\rangle \ , \tag{5}$$

by using the Lanczos method [10]. Quantities y_p and ϕ_h in the above expansion are the spin-angle wave function of the excited particle p and the hole-nucleus wave function, respectively. Once $\lambda_{\rm ph}(r)$ are obtained as solutions, it is easy to calculate $\psi_{\rm ph}(r)$, the radial wave functions for $|\Psi\rangle$. Using these $\lambda_{\rm ph}(r)$ and $\psi_{\rm ph}(r)$, and also the radial wave function $\rho_{\rm ph}(r)$ for $|\rho\rangle$, one can calculate $S_{\rm ph}^{\uparrow}$ as

$$S_{\rm ph}^{\uparrow} = S_{\rm ph} - S_{\rm ph}^{\downarrow} \ , \tag{6}$$

where the total and damping partial strength functions $S_{\rm ph}$ and $S_{\rm ph}^{\downarrow}$, respectively, are

$$S_{\rm ph} = \frac{1}{\pi}{\rm Im}[-\langle\rho_{\rm ph}|\psi_{\rm ph}\rangle], \quad S_{\rm ph}^{\downarrow} = \frac{1}{\pi}[\langle\psi_{\rm ph}| - W_p|\psi_{\rm ph}\rangle] \ . \tag{7}$$

The total strength function S is then given as $S=\Sigma_{\rm ph}S_{\rm ph}$.

Finally, we note that in this paper it is assumed that the total width Γ is expressed as $\Gamma = \Gamma^{\uparrow} + \Gamma^{\downarrow} = \Sigma_{\rm ph}\Gamma_{\rm ph}^{\uparrow} + \Sigma_{\rm ph}\Gamma_{\rm ph}^{\downarrow}$, where $\Gamma_{\rm ph}^{\downarrow}$ is defined in the same manner as is $\Gamma_{\rm ph}^{\uparrow}$ by replacing $S_{\rm ph}^{\uparrow}$ with $S_{\rm ph}^{\downarrow}$ in Eq. (2).

In terms of the formalism described above, we have performed calculations of the escape widths for proton emission from the GTR in ^{208}Bi. Though our main interests are in the GTR, calculations were also made for the IAS as a reference. Since the IAS escape widths are well established experimentally, they provide a constraint for the GTR calculations.

The theoretical parameters involved in the calculation are the residual ph interaction $V_{\rm ph}$ and the single particle potential for the excited particle p and the occupied hole states. $V_{\rm ph}$ is taken to be a simple zero-range interaction of the kind discussed in Ref. [9], which allows for the adjustment of the strength so that the calculated IAS and GTR resonance energies fit the experimental values. For the single particle potential for p and the hole state h, a Woods-Saxon potential is used with the diffuseness $a=0.65$ fm and the radius parameter $r_0=1.23$ or 1.20 fm. In the case where $r_0=1.20$ fm, the neutron hole state energies are adjusted to the experimental values. The above values of the parameters for the single particle potential are fixed such that the experimental emission widths of the IAS are reproduced as well as possible. For the GTR calculation, an imaginary potential is added to the above real potential, particularly for p, in order to take into account the damping (spreading). The parameters of the imaginary potential are taken from Ref. [11].

Figures 1 and 2 show $S(E)$ calculated with $r_0=1.23$ fm as a function of E for the IAS and GTR, respectively. For the IAS, a narrow resonance centered at approximately 18.83 MeV with a width of about 150 keV is obtained, which essentially exhausts the $N-Z$ sum rule. Spreading effects have not been included in this calculation, since, as is well known, the damping is specifically suppressed in this state. The solid line in Fig. 2 represents the calculated $S(E)$ for the GTR, while the dashed curve represents the Lorentzian function $S_L(E)=(\Gamma/2\pi)B[(E-E_R)^2+\Gamma^2/4]^{-1}$ with $\Gamma=4$ MeV, $E_R\approx 19.23$ MeV, and $B=82.9$. The width of the resonance was taken to be the width of the strength function at $S=\frac{1}{2}[S_{\rm min}+S(E_R)]$, where $S_{\rm min}$ is the value of the strength function between the small peak at 8.5 MeV and the main resonance. Note that the total width $\Gamma=4.0$ MeV of S_L agrees well with the experimental result of 3.80 MeV obtained by Akimune *et al.* [6]. This width mostly comes from the damping [6], which means that the damping predicted from the imaginary part of the optical potential can reproduce the observed width. Similar conclusions were previously drawn in Refs. [7,12] for the damping of the monopole and quadrupole GR states.

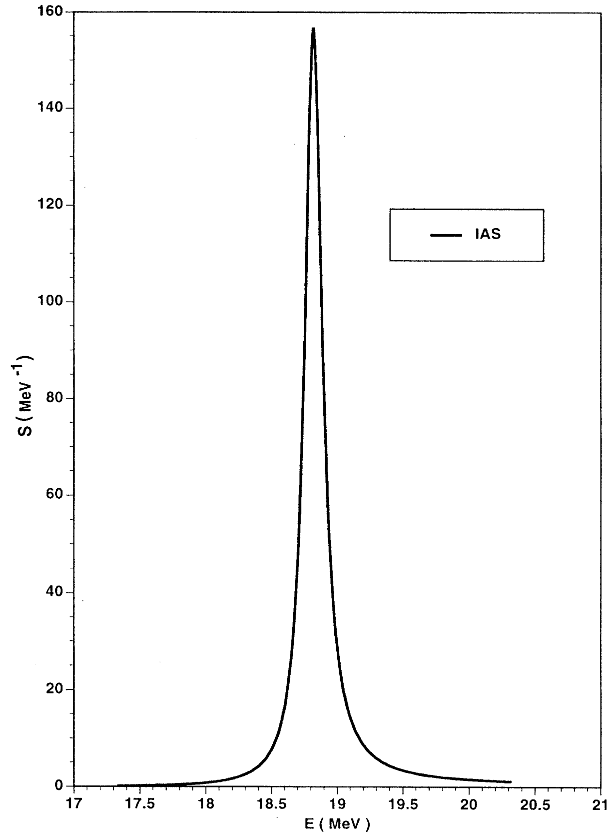

FIG. 1. Total strength function for the IAS in ^{208}Pb.

The value $B=82.9$ implies that the total integrated strength of S_L (82.9) is about 63% of the total sum rule limit of $\int_0^\infty S(E)dE=3(N-Z)$. The difference (37%) between $S(E)$ and $S_L(E)$ may be ascribed to the quenching of the resonance strength and may be compared to the experimental values of 40–50 % given in Refs. [1–4]. The predicted quenching is thus slightly smaller than the experimental value, which may be ascribed to the Δ-hole effect [4]. The peak at 6.8 MeV shown in Fig. 2 amounts to 15.6% of the sum rule, which may be compared with the experimental

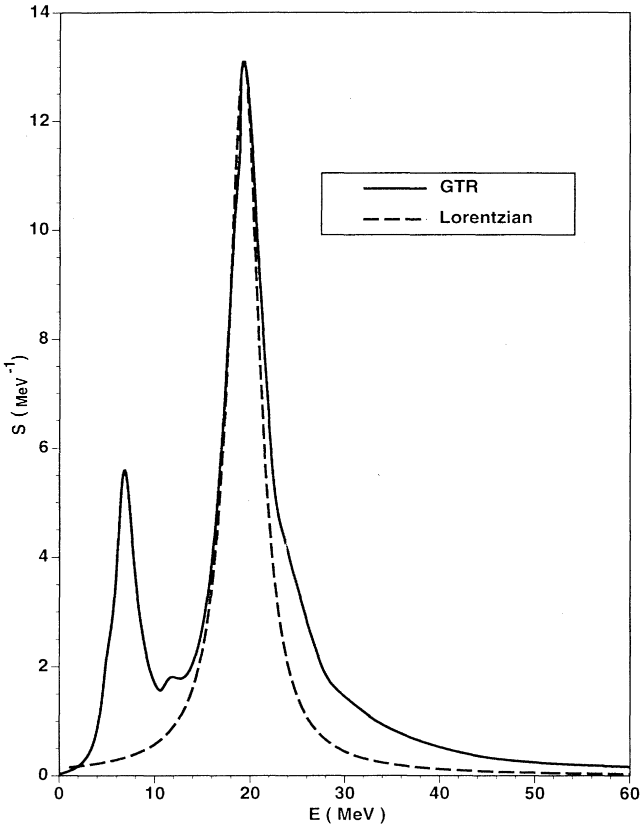

FIG. 2. Total strength function and corresponding Lorentzian function for the GTR state in ^{208}Pb.

strength of about 10% found by Akimune *et al.* [6] and Brockstedt *et al.* [13]. It is remarkable that the overall structure of $S(E)$ shown in Fig. 2 is very much similar to that obtained by Drozdz *et al.* [2] by taking into account explicitly the coupling to the 2p-2h states.

Table I summarizes the results of the calculated proton emission widths for the IAS and GTR. The results for $r_0 = 1.23$ fm are listed in column (a). In addition, the results of using $r_0 = 1.2$ fm and adjusting the neutron hole state energies to their experimental values are given in column (b). The results in column (d) for the GTR will be discussed later. For the IAS, except for the $f_{5/2}$ width, the calculated individual escape widths for both (a) and (b) agree nicely with the experimental values. There is almost perfect agreement with the experimental data for case (b) where the experimental neutron hole state energies are used. Note that calculations of $\Gamma_h^{\uparrow}$ have previously been made by Van Giai *et al.* [14] and also by Adachi and Yoshida [15]. These authors used a somewhat different (perturbative) approach for treating the continuum and predicted values fairly close to ours. The studies in Refs. [14,15], however, both predicted that $\Gamma_{p_{1/2}}^{\uparrow} > \Gamma_{p_{3/2}}^{\uparrow}$, which disagrees with experiment and our results. Most likely, we may ascribe this to their treatment of the continuum, specifically to the neglect of the coupling of the diagonal terms in the ph interaction to the continuum. This conclusion is based on the observation that the same result as theirs can be obtained if the diagonal coupling potential is neglected in our calculation. This indicates that it is important to treat the continuum coupling exactly in reproducing such a subtle feature as the relative magnitude between $\Gamma_{p_{1/2}}^{\uparrow}$ and $\Gamma_{p_{3/2}}^{\uparrow}$.

For the case of the GTR, the overall agreement of the calculated $\Gamma_h^{\uparrow}$ with the data is good, however, not as good as compared with that obtained for the IAS case; the predicted $\Gamma_h^{\uparrow}$ underestimate the experimental values. One possible reason for this may be attributed to our neglect of the distortion effects involved in the reaction cross sections. As is well known, the distortion reduces contributions from the nuclear interior to the cross section. This may then reduce the damping of the emitted particle p, which in turn increase the relative importance of the escape contribution.

In order to study this effect, we have repeated the calculations using a modified Gamow-Teller transition operator $\hat{\rho}' = f(r)\sigma\tau$, where $f(r)$ is a cutoff factor. It is assumed that

TABLE I. Theoretical and experimental partial proton and total escape widths in keV for the decay of the IAS in ^{208}Bi into neutron hole states in ^{207}Pb.

	IAS		Expt.	GTR			Expt.
$\Gamma_h^{\uparrow}$	*a*	*b*	*c*	*a*	*b*	*d*	*e*
$\Gamma_{p_{1/2}}^{\uparrow}$	63.4	53.9	51.9±1.6	42.5	31.7	78.2	58.4±11.2
$\Gamma_{f_{5/2}}^{\uparrow}$	11.2	24.9	26.4±2.0	16.7	22.9	15.6	incl. in $p_{3/2}$
$\Gamma_{p_{3/2}}^{\uparrow}$	74.1	61.7	64.7±3.4	44.4	43.3	69.4	101.5±15.6
$\Gamma_{i_{3/2}}^{\uparrow}$	0.2	0.13		3.0	2.6	2.9	8.3±9.2
$\Gamma_{f_{7/2}}^{\uparrow}$	1.2	4.3	4.2±0.6	2.9	7.0	2.8	15.6±7.4
$\Gamma_{h_{9/2}}^{\uparrow}$	0.002	0.04		0.2	0.5	0.2	
$\Sigma_h \Gamma_h^{\uparrow}$	150.1	145	147.2±4.3	109.7	113.4	169.1	184±49
Γ				4000	3600	4000	3750±250

[a]CTDA, $r_0 = 1.23$ fm.
[b]CTDA, $r_0 = 1.2$ fm, neutron hole state energies set to experimental values.
[c]Ref. [16], averaged results of Refs. [5,17,18].
[d]CTDA, modified GT transition operator is used.
[e]Ref. [6].

$f(r)=1-[1+e^{(4-R_c)/a}]^{-1}$ with $a=0.63$ fm and $R_c=5.92$ fm, which corresponds to the reduced radius parameter $r_c=1.0$ fm. The cutoff factor $f(r)$ eliminates the contribution from the nuclear interior and thus effectively simulates the distortion effect. The result of the calculations with this modified GT operator have demonstrated that the ratio $\Gamma_h^{\uparrow}/\Gamma$ increases with an increasing value of the cutoff radius R_c in $f(r)$, particularly for the GTR. In column (d) in Table I, we list the values of $\Gamma_h^{\uparrow}$ obtained with $R_c=5.92$ fm. As seen, the new calculated values are much closer to the experimental values. We note that similar calculations were also made for the $\Gamma_h^{\uparrow}$ values of the IAS, but the predicted values essentially remained unchanged. This insensitivity of the $\Gamma_h^{\uparrow}$ values of the IAS may be due to the absence of the damping in the IAS. With the modified Gamow-Teller operator, one can thus reproduce the data for both IAS and GTR simultaneously. Finally, we note that calculations of $\Gamma_h^{\uparrow}$ were made in Ref. [14]. The resultant $\Gamma_h^{\uparrow}$ are larger by about a factor of 2.5 as compared with the experimental values.

In conclusion we have performed CTDA calculations to obtain the partial escape widths for the GTR, as well as for the IAS, in ^{208}Bi. The CTDA treats the continuum effects in an exact manner, which is necessary to make reliable predictions of escape widths. Using a standard optical potential, two methods within the CTDA framework were employed to obtain the escape widths of the GTR. Since the IAS escape widths are well established experimentally, they provide a constraint for the GTR calculations. Using both methods the calculated escape widths for the IAS are in good agreement with the measured widths. For the GTR the calculated widths are in fair agreement with the measured escape widths obtained from a recent ^{208}Pb(^{3}He,tp)^{208}Bi experiment. This agreement improves if one takes into account the distortion effects involved in the excitation cross section.

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