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Crystalline structure of the mixed confined-deconfined phase in neutron stars

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We calculate that the geometrical structure of the crystalline solid mixed phase of confined-deconfined hadronic matter in neutron stars—its form, size, spacing, and location in the star—is highly individualistic according to small changes in the stellar mass. This suggests a possible connection with the wide range of glitch behavior observed in different pulsars, since glitches are almost certainly associated with solid regions.

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Quark matter in the dense, high-pressure cores of neutron stars has been an intriguing subject of investigation since pioneering work in 1976 [1–3]. It was assumed, and is here also, that in cold matter the transition from quarks confined in hadrons to the deconfined phase of quark matter is first order. However, there has been a new development in our understanding of this transition that may eventually provide a link between the mixed phase of a compact star and observable pulsar phenomena. Although stars must be electrically neutral they do not need to be locally neutral, as was previously assumed. It is now realized that in the mixed confined-deconfined hadronic phase the electric charge on regions of hadronic and quark matter may not vanish in each region, but may simply annul each other over an appropriate one [4–8]. Nature will choose such a distribution of charges as minimizes the energy at each proportion of the phases in equilibrium, thus causing the pressure to vary with the proportion [4,5]. Imposing the constraint of identically vanishing charge may foreclose the possibility of finding a more energetically favorable distribution and renders the transition as a constant pressure one in which case the mixed phase could not occur in the monotonically varying pressure environment of a star.

More generally, a substance with n conserved charges (independent components) has $n-1$ degrees of freedom to rearrange charges between the phases in equilibrium so as to lower the energy. Consequently, the nature of a first order phase transition is fundamentally different between one-component substances such as water, and those with more than one [5]. This is the case with a neutron star, which, because of β equilibrium, is not composed solely of neutrons but also contains protons, electrons, probably hyperons, and, in the situation under study here, quarks. So there are two conserved charges, baryon and electric.

The internal force that favors a redistribution of charge is the isospin restoring force experienced by the confined phase [5,6]. It is embodied in the isospin symmetry energy in the empirical nuclear mass formula. Charge neutral neutron star matter in the pure phase is highly isospin asymmetric, being composed mostly of neutrons. However, in the mixed phase, hadronic regions can relieve the asymmetry to the degree allowed by overall charge neutrality by transferring charge to the quark phase in equilibrium with it. The bulk energy will be lowered by reducing the symmetry energy of neutron star matter at only a small cost in rearranging the quark Fermi surfaces. Thus the mixed phase region of the star will have positively charged regions of nuclear matter and negatively charged regions of quark matter arranged so as to minimize the sum of surface and Coulomb energies by forming a crystalline lattice. The dimensions involved will turn out to be on the nuclear scale.

An initial study of the lattice structure that was predicted in [4,5] was carried out by Heiselberg *et al.* [7], who investigated several important issues and suggested the possible relevance of the structure to pulsar glitches. Our purpose is to explore the radial extent and variation of the lattice structure in the environment of neutron star models to demonstrate the extreme sensitivity of the crystalline region on the stellar mass to which the great individuality [9,10] of the glitch behavior of different pulsars may be related in part. It seems plausible that all solid regions will be involved in the phenomenon of glitches—unpredictable small discontinuous decreases in pulsar rotational period, occurring on a time scale of days, months, or years, that are observed in many pulsars. A purely liquid or gaseous star is mechanically unable to generate sudden changes in angular momentum or moment of inertia by any conceivable mechanism.

Although the above physics responsible for structure in the confined-deconfined mixed phase is quite different from that causing a nuclear lattice in the stellar crust [11,12], the interplay of surface and Coulomb energies is identical. The problem is to find the particular geometry and size of the regions of rarer phase immersed in the other that minimize the sum of the above energies as a function of the proportion of phases in equilibrium. The pressure varies as a function of the proportion since the partition of charge is energetically optimized at each proportion [5]. And because the pressure monotonically decreases with radial distance from the center of the star, the geometry and its size and spacing will vary with the Schwarzschild radial coordinate. One may subdivide any locally inertial region of mixed phase into (Wigner-Seitz) cells containing one of the structures and a surrounding region of the other phase so that the total charge content of the cell is neutral. Adjacent cells therefore do not interact. We adapt the results of Ravenhall *et al.* [11], who treat three discrete geometric forms in such cells to write for the radius of the rare phase immersed in the other and the minimum of the sum of Coulomb and surface energies

$$\frac{1}{r^3}=\frac{4\pi[q_H(\chi)-q_Q(\chi)]^2e^2f_d(x)}{\sigma d}, \quad d=1,2,3, \quad (1)$$

$$\frac{E_C+E_S}{V}=6\pi x\left(\frac{\{\sigma d[q_H(\chi)-q_Q(\chi)]e\}^2f_d(x)}{16\pi^2}\right)^{1/3}, \quad (2)$$

where q_H and q_Q are the charge densities of hadronic and quark matter (in units of e) at volume proportion of quark matter $\chi=V_Q/V$ and σ is the surface interface energy. The ratio of droplet (rod, slab) volume to cell volume is denoted by $x=(r/R)^d$. When quark matter is the rare phase $x=\chi$ and otherwise $x=1-\chi$. In the case of drops or rods, r is their radius and R the half distance between centers, while for slabs r is the half thickness. The geometric forms are denoted by their dimensionality, $d=1$, 2, and 3 corresponding to slabs, rods, and drops, respectively.

The function $f_d(x)$ is given in all three cases by

$$f_d(x)=\frac{1}{d+2}\left[\frac{1}{(d-2)}(2-dx^{1-2/d})+x\right], \quad (3)$$

where the apparent singularity for $d=2$ has the correct limit for $d\rightarrow 2$, namely, $f_2(x)=(x-1-\ln x)/4$. We have supposed that the electrons are uniformly distributed throughout the mixed phase. In fact, we find that electrons are almost totally absent from the mixed phase as is generally the case when charge neutrality can be realized by baryon charge carrying particles [5].

Surface tension is a very difficult problem because it should be self-consistent with the two phases of matter, quark and hadronic, in equilibrium. Lattice gauge simulations have not provided an answer for cold isospin asymmetric matter. Gibbs studied the problem of surface energy, and as a gross approximation it is given by the difference in energy densities of the substances in contact times a length scale typical of the surface thickness [13], in this case of the order of the strong interaction range, $L=1$ fm. The surface interface energy should depend on the proportion of phases in phase equilibrium, just as everything else does,

$$\sigma=\text{const}\times[\epsilon_Q(\chi)-\epsilon_H(\chi)]\times L. \quad (4)$$

The constant should be chosen so that the structured phase lies below the unstructured one because of the additional degree of freedom that is exploited in the latter case. Heiselberg *et al.* [7] found a value of combined surface and Coulomb energy that satisfies this condition to be about 10 MeV. We choose the constant accordingly. It will be understood from the formulas written above that the structure size and the sum of surface and Coulomb energies scale with the surface energy coefficient as $\sigma^{1/3}$ independent of geometry. Therefore the location in the star where the geometry changes from one form to another is independent of σ.

The geometrical structure of the mixed phase occurs against the background of the bulk structure to good approximation. The energy and pressure are of course dominated by the bulk properties of matter. The equation of state of the confined hadronic phase is calculated as in Refs. [14,15]. The coupling constants of the theory are determined by the bulk nuclear properties, binding energy $B/A=-16.3$ MeV, saturation density $\rho_0=0.153$ fm^{-3}, symmetry energy coefficient $a_{\text{sym}}=32.5$ MeV, compression modulus $K=300$ MeV, and effective nucleon mass at saturation, $m^*/m=0.7$. The ratio of hyperon coupling to mesons as compared to nucleon couplings is chosen in accord with Ref. [15] to be $x_\sigma=0.6$, $x_\omega=0.653$. We compute the pure quark matter phase as in [5] using a bag constant $B^{1/4}=180$ MeV and the quark masses written there. The method of finding phase equilibrium in multicomponent systems is also described there.

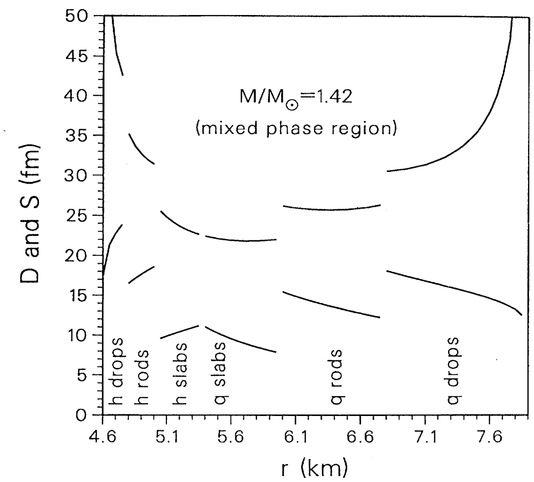

FIG. 1. Crystalline mixed region in a neutron star showing the diameter D (lower curves) and spacing S (upper curves) of indicated geometric structures as a function of radial Schwarzschild coordinate. Region internal to 4.6 km is pure quark matter. Notation "h drops" denotes hadronic drops immersed in quark matter, and so on. Stellar radius is 10.7 km.

To demonstrate the extreme dependence on stellar mass of the crystalline structure we compare two stars differing by only $0.02M_\odot$. In Fig. 1 we show the diameter of the geometrical objects, identify their forms, and plot the spacing between centers of the rare phase objects immersed in the other phase. The mass of the star is $M=1.42M_\odot$ and is at the mass limit. The pressure at the center and extending to 4.6 km is sufficiently high that this region is occupied by pure quark matter. The region between 4.6 and 7.8 km is in the crystalline confined-deconfined phase. Since the phase transition is completed before the center of the star is reached, the geometrical structures span the full range of forms. At the outer edge of the mixed phase region, quark drops of finite diameter are spaced, in the limit of $\chi=0$, at infinite distance. At greater depth and therefore proportion of quark matter the drops are more closely spaced and slightly larger in size. Deeper in the star, drops are no longer the favored configuration and merge to form rods of varying diameter and spacing. At still greater depth, the rods give way to slabs, then the roles of quark and hadronic matter interchange, and the forms are repeated in reverse order until at the inner edge of the mixed phase hadronic drops of finite size but spaced far apart are immersed in quark matter. In all cases the diameters

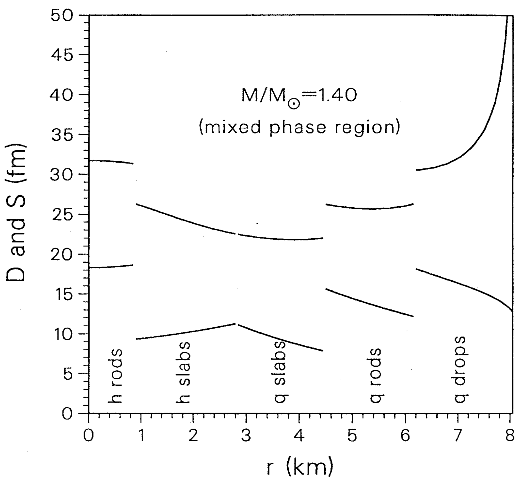

FIG. 2. Similar to Fig. 1 but for slightly less massive star. Mixed crystalline phase now extends to star's center. Radius is 12.3 km.

of the geometric forms lie between about 10 and 25 fm. Of course, were it not for the restriction to three discrete geometries, the discontinuities in the diameters and spacings shown in the figures would be interpolated by intermediate forms.

For lower-mass stars, the central density and pressure may be insufficient to fully convert quark matter to hadronic matter. This is the case for the neighboring mass star, $M=1.40M_\odot$, for which the geometries of the crystalline mixed phase are shown in Fig. 2. The crystalline phase extends to the center of the star and the central geometry consists of rods of hadronic matter immersed in quark matter. For a mere $0.02M_\odot$ change in mass the situation has changed radically from a star having a 4.6 km quark gas core, enclosed by a 3 km thick solid shell, to a star having no gas interior, but an 8 km solid sphere. For a star of mass 1.2 $M_\odot$ we find the quark rods occupying the region from the center to 2 km and quark drops from there to 7 km. For a 1 $M_\odot$ star, quark drops extend from the center to only 3.6 km. All of the above configurations are of course surrounded by neutron star liquid and finally a thin solid crust of nuclear forms embedded in an electron gas. Stars of somewhat lower mass than the above have no quark matter whatever.

The great sensitivity to stellar mass is due to the high compression in neutron stars so that the density is a rather flat function of radial coordinate except near the edge of the star, and a small change in central value and therefore mass has a large effect on where in the star a particular density occurs. Naturally, the details illustrated above pertain to the particular parameters chosen. The pattern of the results is, however, general.

Early work, by inadvertently closing off a degree of freedom, rendered the transition as one of constant pressure for all proportions in the mixed phase [1–3,16]. In such a case the mixed phase cannot exist in the gravitational field in a star. The pure quark phase was found to occur only at high density or pressure and therefore concentrated near the center. That is true here also. However, the mixed phase extends to a rather low density of only several times nuclear saturation density, as has been found by several authors when the star is allowed to exploit the degree of freedom opened when charge neutrality is imposed in the global rather than the local sense [4,5,7,8].

It is almost certain that solid regions in a pulsar will play a role in the period glitch phenomenon, whether associated with cracking of the solid as in star quakes, or with unpinning of superfluid vortices from solid regions. Glitch behavior is highly individualistic from one pulsar to another [10]. We tentatively suggest that this may be due to the extreme sensitivity on stellar mass of the radial extent of the solid crystal region and the forms that occupy it as well as the moments of inertia associated with different gaseous (quark), liquid, and solid regions. The interior solid region of the mixed phase and the crustal solid, separated by a nuclear liquid, offer interesting possibilities for interaction or stimulated response at the time of a glitch originating in one of them and in postglitch recovery.

The crystalline form cannot fit uniformly into the axial symmetry of a rotating star without there being lattice imperfections. As the star cools we envision that the crystalline structure will develop with relatively few imperfections but over time and successive glitches their number will grow, creating eventually a granular region. Thus we may also expect a long-term change in behavior with pulsar age.

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