Response of mica to weakly interacting massive particles

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We calculate spin-dependent cross sections for the scattering from mica of hypothetical weakly interacting dark-matter particles such as neutralinos. The most abundant odd-A isotopes in mica, ^{27}Al and ^{39}K, require different shell-model treatments. The calculated cross sections will allow the interpretation of ongoing experiments that look for tracks due to the interaction of dark-matter particles with nuclei in ancient mica.

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I. INTRODUCTION

The nature of the dark matter in our galaxy and elsewhere has become increasingly puzzling [1,2]. Although it is too early to make definitive statements, ongoing experiments seem to imply that there are not enough macroscopic objects in the galactic halo [3] to account for the gravitational attraction felt by luminous objects near the edge of the disk. One alternative is dark matter in the form of elementary particles. For several years now, weakly interacting massive particles (WIMP's) that arise in supersymmetric extensions of the standard model have been attractive candidates [4]. A variety of experiments to detect WIMP's are either already operating or in the planning/prototype stage.

One promising project [5] uses ancient mica as a detector. The idea is that over long periods of time a countable number of WIMP's should have collided with underground mica and left recognizable tracks. At present, the experiment is not sensitive enough to test the WIMP hypothesis fully, but by increasing the amount of mica examined, experimenters hope to reach the required level of sensitivity within a few years. Then, confirming or rejecting the WIMP hypothesis will require a knowledge of their cross sections for scattering from the various elements in mica. In this paper we calculate these cross sections so that the experiments can be properly interpreted. Our results are general enough to be applied to any heavy Majorana particle; we explicitly calculate the "spin-dependent" response of mica. While "spin independent" cross sections for supersymmetric neutralinos [6–9], perhaps the most plausibly motivated WIMP's, are usually larger than the spin-dependent cross sections in most of the nuclei in mica [10], there are still regions of supersymmetric parameter space for which this is not so. Furthermore, WIMP's with no scalar interactions, such as heavy Majorana neutrinos, have not yet been completely ruled out. A good calculation of the spin-dependent response is therefore warranted. The spin independent scattering can be easily calculated following, for example, the work of Ref. [11].

The composition of mica is 58% ^{16}O, 16% Si (mostly ^{28}Si), 12% ^{27}Al, 5% K (mostly ^{39}K), and 9% H. Since the scattering from even-even nuclei is entirely spin independent — and therefore easily calculable — we need to concentrate here only on ^{27}Al and ^{39}K. (No nuclear-structure calculation is required for H.) The nucleus ^{27}Al is also one of the active ingredients in a very high-resolution ($\Delta E_{\rm FWHM}\approx 200$ eV) and low-threshold ($E_{\rm th}\approx 500$ eV) sapphire-crystal (Al_2O_3) detector that is currently under development [12]. The low threshold of this experiment makes it ideal for detecting lighter WIMP's.

Though both aluminum and potassium nuclei are nominally classified to be in the sd shell, ^{39}K is at the upper edge of the shell and its wave functions contain significant admixtures of one- and two- particle-hole states involving higher shells. Most of the correlations in ^{27}Al, by contrast, are produced by purely sd-shell configurations. The techniques we apply in the two cases are therefore quite different. After a general discussion of spin-dependent scattering in Sec. II, we will examine the two odd-A nuclei independently in Secs. III and IV. In Sec. V we briefly discuss the implications of these calculations for mica-based detection.

II. SPIN-DEPENDENT SCATTERING

At low momentum transfer, the tree-level coupling of neutralinos to quarks (excluding the exchange of Higgs bosons) yields the elastic scattering amplitude from a nucleus $|N\rangle$:

$$M = A\langle N|a_p\mathbf{S}_p + a_n\mathbf{S}_n|N\rangle\cdot\mathbf{s}_\chi \ , \qquad (1)$$

where $\mathbf{S}_p$ and $\mathbf{S}_n$ are the total proton and neutron spins in the nucleus, $\mathbf{s}_\chi$ is the spin of the neutralino, a_p and a_n are coupling constants that depend on the composition of the neutralino (in terms of the four or more neutral fermionic partners of the gauge and Higgs bosons) and on the distribution of nucleon spin among the different types of quarks, and the quantity A depends on other physics at the supersymmetry scale [6]. Though these specifics apply only to neutralinos, the spin-dependent scattering amplitudes for other Majorana WIMP's will have the same general form, Eq. (1). The spin-dependent cross section is a product of the square of the amplitude and a phase-space factor.

When the momentum transferred to the nucleus by the WIMP, q, becomes comparable to the inverse radius of the nucleus, structure functions (form factors) resembling those associated with elastic magnetic electron or neutrino scattering modify Eq. (1). The more general expression for the spin-dependent cross section is [11]

$$\frac{d\sigma}{dq^2}=\frac{8G_F^2}{(2J+1)v^2}S_A(q), \tag{2}$$

where G_F is the Fermi weak four-point effective coupling constant and v the neutralino velocity. The subscript A stands for "axial." The function $S_A(q)$ can always be written in terms of three fundamental structure functions

$$S_A(q)=a_0^2S_{00}(q)+a_1^2S_{11}(q)+a_0a_1S_{01}(q). \tag{3}$$

The labels 0 and 1 refer to isospin; the constant a_0 is given by a_p+a_n and a_1 by a_p-a_n. The three structure functions contain expectation values of operators of the form $j_L(qr)\times[Y_L\sigma]^{L\pm1}$ (L even), which depend on spatial coordinates as well as nucleon spins. Details appear in Ref. [11]. Without rewriting all the expressions here, we describe the calculation of $S_A(q)$ in ^{27}Al and ^{39}K.

III. ALUMINUM

The nucleus ^{27}Al lies in the middle of the sd shell. To obtain the ground-state wave function we diagonalize Wildenthal's "universal" sd interaction [13] in the full $0\hbar\omega$ space. This interaction has been meticulously developed and tested over many years and together with effective operators accurately reproduces most low-energy observables in sd-shell nuclei [14]. We carry out our calculations with the Lanczos m-scheme shell-model code CRUNCHER [15] and its auxiliary codes; the m-scheme basis for ^{27}Al contains 80 115 Slater determinants. A very similar calculation for ^{29}Si is reported in [16], where more details appear.

The agreement of the calculated spectrum with that measured in ^{27}Al is excellent, reflecting the effort that went into constructing the interaction. More relevant is the accuracy of the calculated magnetic moment, which derives from the same spin operators that determine the WIMP structure functions at $q^2=0$. The measured magnetic moment is $\mu_{\rm expt}=+3.6415\mu_N$. Using free-particle g factors and our wave functions, we obtain a value $\mu_{\rm calc}=+3.584\mu_N$, which agrees well with experiment. By contrast, the single-particle value is $\mu_{\rm SP}=+4.793\mu_N$.

Although Ref. [14] advocates the use of effective operators in the sd shell, we obtain a calculated magnetic moment in ^{27}Al closer to experiment with the free-nucleon spin and orbital-angular-momentum operators (the same was true in ^{29}Si). Unfortunately, we overpredict the Gamow-Teller matrix element by a factor of about 1.25, a problem that the effective operators would remedy. Thus we have conflicting evidence on the use of effective operators. In the absence of a better prescription, we choose to use the free-nucleon g factors. The consequence for WIMP scattering is that we do not quench our calculated cross sections at all. In ^{39}K, where our calculation provides more information, we make a similar decision.

TABLE I. The value of (the z projection of) the nuclear spin and orbital-angular-momentum matrix elements for ^{27}Al.

	$\langle\mathbf{S}_p\rangle$	$\langle\mathbf{S}_n\rangle$	$\langle\mathbf{L}_p\rangle$	$\langle\mathbf{L}_n\rangle$
ISPSM	0.5	0	2.0	0
OGM	0.25	0	2.25	0
EOGM ($g_A/g_V=1.0$)	0.333	0.043		
EOGM ($g_A/g_V=1.25$)	0.304	0.073		
Shell model	0.3430	0.0296	1.7812	0.3461

Reference [17] develops two schemes, based on the "odd group model" (OGM) and an extension of it (EOGM) for extracting values of neutron and proton spins — the quantities that determine spin-dependent WIMP cross sections at $q^2=0$ — from magnetic moments and β-decay lifetimes. These values for ^{27}Al appear in Table I alongside the spins and orbital angular momenta from our calculation. The most surprising result is the large value for $\langle\mathbf{L}_n\rangle$ obtained in the shell-model calculation. On comparing the numbers in Table I, we see that the single-particle model (ISPSM) and OGM disagree with the shell model; the large value of the neutron's orbital angular momentum is the likely explanation of the OGM's failure to reproduce the spin angular momenta. By contrast, the EOGM results with $g_A/g_V=1$ (a quenching of the Gamow-Teller matrix element) are very close to our shell-model values. This agreement with the most sophisticated phenomenological analysis is heartening. However, since spin-dependent neutralino scattering involves the axial nuclear current, the failure of any analysis to reproduce β-decay lifetimes accurately without *ad hoc* quenching — a problem not mentioned in any previous work — is troubling. As a result our aluminum calculation has an uncertainty from this source of roughly 30%. The situation is better in the potassium calculation since the core-polarization corrections, to be discussed in the next section, explain much of the β-decay quenching.

As described above, when heavy particles transfer momentum comparable to the inverse nuclear radius, structure functions modify the simple form Eq. (1) for the scattering amplitude. In the shell model, the expectation value of any one-body operator is easy to calculate, so the evaluation of structure functions poses no additional problem. We did, however, improve upon an approximation in previous works, namely, the use of harmonic oscillator single-particle wave functions; the true nuclear mean-field potential is closer to a Woods-Saxon than a harmonic oscillator form. For comparison we calculate the structure functions with both types of wave functions. We use a length parameter of $b=1.73$ fm for

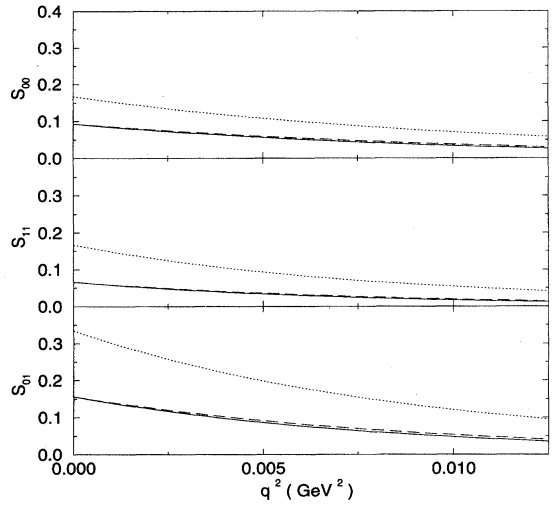

FIG. 1. The three ^{27}Al structure functions S_{00} (top), S_{11} (middle), and S_{01} (bottom) as a function of momentum transfer squared. The dotted lines are the single-particle functions, the dashed lines the full functions with harmonic oscillator single-particle wave functions, and the solid lines the full functions with Woods-Saxon single-particle wave functions.

oscillator functions and the standard parameters of Ref. [18] for the Woods-Saxon potential. Figure 1 shows the three structure functions in both cases as a function of q^2 up to the maximum possible momentum transfer for a WIMP moving at 700 km/sec (more than twice the mean WIMP velocity). The Woods-Saxon wave functions make only small changes in the results.

For the momentum transfers in the figure, the Woods-Saxon structure functions are given to high accuracy by the third-order polynomials

$$S_{00}=0.092\,951\,6-0.472\,059\,y+1.059\,96\,y^2-1.011\,48\,y^3,$$

$$S_{11}=0.065\,723\,2-0.449\,840\,y+1.350\,41\,y^2-1.685\,08\,y^3,$$

$$S_{01}=0.156\,330\,0-0.935\,958\,y+2.457\,79\,y^2-2.726\,21\,y^3, \tag{4}$$

where $y=(bq/2)^2$. As noted above, these three functions allow the calculation of spin-dependent cross sections for any heavy Majorana particle.

IV. POTASSIUM

For the nucleus ^{39}K, a shell-model diagonalization is difficult to perform because of the severe truncations required to the active model space. Since this nucleus is so near the boundary between the *sd* and *pf* shells, excitations of particles into higher shells can have significant effects that are often not well simulated by effective operators. Despite the difficulties, our first attempt to treat this nucleus was within the framework of the shell model, with the Hamiltonian and method advocated in Ref. [19]. Computational complexity, however, kept us from allowing more than two particles out of the *sd* shell into the *fp* shell. After diagonalization, the $2p$-$3h$ excitations we did include made up approximately 40% of the ground-state wave function. While the excitations changed the orbital angular momentum significantly, they had almost no effect on the spin, and our magnetic moment disagreed badly with experiment, indicating that further correlations are indeed important. Any further complexity makes use of the shell model difficult, however. We turn instead to an alternative scheme, based on perturbation theory, that was successfully implemented a few years ago in calculations of several spin-dependent observables in closed-shell-plus- (or minus-) one nuclei [20]. Since this method has never before been applied to WIMP structure functions, we describe it in more detail than we did our *sd* shell-model results.

We begin by dividing the Hamiltonian into a one-body term and a residual interaction: $H=H_0+\mathscr{V}$, where $H_0=T+U$, the sum of kinetic energy and one-body potential energy operators, and $\mathscr{V}=V-U$, the difference between the two- and one-body potential energy operators. The eigenfunctions of H_0, which we take to be a harmonic oscillator, form the basis of the calculation. The unperturbed ground state of ^{39}K, a single-proton hole in the $d_{3/2}$ orbital, gives the single-particle (Schmidt) magnetic moment: $-0.676\mu_N$. Corrections come from the perturbative expansion in the residual interaction. Very generally, to first order in a closed-shell-plus- (or minus-) one nucleus the matrix element of a one-body operator, F, is given by

$$\langle\psi_b|F|\psi_a\rangle=\langle b|F|a\rangle+\sum_{\alpha\neq a,b}\frac{\langle b|F|\alpha\rangle\langle\alpha|\mathscr{V}|a\rangle}{E_a-E_\alpha}+\sum_{\alpha\neq a,b}\frac{\langle b|\mathscr{V}|\alpha\rangle\langle\alpha|F|a\rangle}{E_b-E_\alpha}. \tag{5}$$

Here a and b are single-hole valence states and α an infinite set of single-hole or two-hole one-particle ($2h$-$1p$) states constructed from the eigenfunctions of H_0; in our work $a=b=0d_{3/2}$. The energy denominators also come from H_0; they are integral multiples of $2\hbar\omega$, where $\hbar\omega$ is the characteristic oscillator energy. We choose $\hbar\omega=45A^{-1/3}-25A^{-2/3}=11.1$ MeV, corresponding to $b=1.934$ fm, for $A=39$. We explicitly evaluate the intermediate-state summations for denominators equal to $2\hbar\omega$, $4\hbar\omega$, $6\hbar\omega$, and $8\hbar\omega$, and then extrapolate the results geometrically.

The two-body matrix elements of $\mathscr{V}$ in Eq. (5) contain the one-body potential through the $-U$ components, known as Hartree-Fock insertions [21]. If the unperturbed Hamiltonian H_0 were chosen to be the Hartree-Fock Hamiltonian that minimized the energy of a single Slater determinant characterizing the closed-shell core, then the Hartree-Fock insertions would have no effect. Reference [20] shows that even with an oscillator Hamiltonian the Hartree-Fock insertions do not affect magnetic moments and their effects ought to be small for the operators discussed here. We therefore drop the Hartree-Fock insertions altogether. Furthermore, all first-order corrections to magnetic moments, and to the operators used here in the $q=0$ limit, also vanish because the spin and

orbital-angular-momentum operators cannot create (or annihilate) a particle-hole state at LS closed shells (selection rules require the particle and hole states to have the same orbital structure). At nonzero q, the first-order corrections to $S_A(q)$, while not identically zero, are still small. It is essential, therefore, to consider perturbation theory through to second order, and we add the following additional terms to Eq. (5):

$$\sum_{\alpha,\beta\neq a,b}\langle b|F|\alpha\rangle\frac{\langle\alpha|V|\beta\rangle\langle\beta|V|a\rangle}{(E_a-E_\alpha)(E_a-E_\beta)}-\sum_{\alpha\neq a,b}\langle b|F|\alpha\rangle\frac{\langle\alpha|V|a\rangle\langle a|V|a\rangle}{(E_a-E_\alpha)^2}+\sum_{\alpha,\beta\neq a,b}\frac{\langle b|V|\beta\rangle\langle\beta|V|\alpha\rangle}{(E_b-E_\beta)(E_b-E_\alpha)}\langle\alpha|F|a\rangle$$
$$-\sum_{\alpha\neq a,b}\frac{\langle b|V|b\rangle\langle b|V|\alpha\rangle}{(E_b-E_\alpha)^2}\langle\alpha|F|a\rangle+\sum_{\beta,\gamma\neq a,b}\frac{\langle b|V|\beta\rangle}{(E_b-E_\beta)}\langle\beta|F|\gamma\rangle\frac{\langle\gamma|V|a\rangle}{(E_a-E_\gamma)}-\frac{1}{2}\sum_{\beta\neq a,b}\frac{\langle b|V|\beta\rangle\langle\beta|V|b\rangle}{(E_b-E_\beta)^2}\langle b|F|a\rangle$$
$$-\frac{1}{2}\sum_{\beta\neq a,b}\langle b|F|a\rangle\frac{\langle a|V|\beta\rangle\langle\beta|V|a\rangle}{(E_a-E_\beta)^2}. \tag{6}$$

Here, as before, a and b are single-hole valence states, α an infinite set of $2h$-$1p$ states, and β,γ an infinite set of $2h$-$1p$ and $3h$-$2p$ states. Again the selection rules on $\langle b|F|\alpha\rangle$ and $\langle\alpha|F|a\rangle$ forbid the first four terms of Eq. (6) from contributing at $q=0$, and only allow small contributions at higher q. The fifth, sixth, and seventh terms, however, each contain intermediate-state summations over $2h$-$1p$ and $3h$-$2p$ states that are not constrained by selection rules and converge only slowly with increasing energy denominators. (The last two terms renormalize the single-hole matrix element $\langle b|F|a\rangle$.) These three terms are sometimes called the number-conserving set [22] because if the one-body operator were replaced by the number operator the terms would contribute nothing. This is apparent when the last three terms in Eq. (6) are rewritten in the following equivalent form (with $a=b$):

$$\left\langle a\left|V\frac{Q}{e}\left[F,\frac{Q}{e}V\right]\right|a\right\rangle, \tag{7}$$

where the operator Q/e is $\Sigma_\beta|\beta\rangle\langle\beta|/(E_a-E_\beta)$. If the one-body operator F were to commute with both the energy denominator e and the residual interaction V, as does the number operator, the second-order correction to the diagonal matrix element would vanish. But spin-dependent one-body operators do not commute with spin-dependent residual interactions, resulting, for example, in second-order corrections to magnetic moments from tensor forces [23].

Here we try two different residual interactions. The first (which we call I), used in Ref. [20] for magnetic-moment calculations, is based on the one-boson-exchange potential of the Bonn type [24], but limited to the four or five important meson exchanges. For use in finite nuclei, this interaction should be converted to a G matrix; here we approximate the procedure crudely by introducing a short-range correlation function. The resulting interaction has a weak tensor-force component typical of Bonn potentials. Our second interaction [25] (called II here) is a full G matrix constructed from the Paris potential [26] and parametrized in terms of sums over Yukawa functions of various ranges and strengths. Interaction II has a strong tensor force, producing effects described below.

Our results for the orbital angular momenta and spins in ^{39}K appear in Table II. As was the case earlier, the spins from both forces agree well with those obtained in the phenomenological EOGM. To test our results against data, we perform a comprehensive calculation of magnetic moments and Gamow-Teller matrix elements, including (under the heading MEC) meson-exchange currents, isobar currents, and other relativistic effects (see [20]) in addition to the core polarization described above. Table III presents the results for both isoscalar and isovector quantities. (The magnetic moment of ^{39}K is the sum of the isoscalar and isovector moments.) The MEC correction is small for isoscalar magnetic moments and β decay, but is significant for isovector electromagnetic operators. There is little difference between the two interactions for isoscalar operators and the results are

TABLE II. The value of (the z projection of) the nuclear spin and orbital-angular-momentum matrix elements for ^{39}K.

	$\langle\mathbf{S}_p\rangle$	$\langle\mathbf{S}_n\rangle$	$\langle\mathbf{L}_p\rangle$	$\langle\mathbf{L}_n\rangle$
ISPSM	-0.3	0	1.8	0
OGM		0		0
EOGM ($g_A/g_V=1.0$)	-0.196	0.055		
EOGM ($g_A/g_V=1.25$)	-0.171	0.030		
Force I	-0.197	0.051	1.339	0.308
Force II	-0.184	0.054	1.068	0.564

TABLE III. Corrections to the single-particle (SP) values of the isoscalar (μ_S) and isovector (μ_V) magnetic moments and diagonal Gamow-Teller β-decay matrix element [M(GT)], for the mass $A=39$ system from a Bonn-based, weak-tensor force, I, and a Paris-based, strong-tensor force, II. The corrections are due primarily to second-order core-polarization (CP) and meson-exchange currents (MEC). See text for further explanation.

	Force	SP	CP	MEC	Sum	Expt.
μ_S	I	0.636	0.066	0.004	0.706	0.707
	II	0.636	0.064	0.008	0.708	0.707
μ_V	I	−0.512	−0.138	0.364	−0.286	−0.315
	II	−0.512	−0.378	0.369	−0.521	−0.315
M(GT)	I	−0.976	0.176	−0.009	−0.809	−0.647
	II	−0.976	0.205	−0.016	−0.787	−0.647

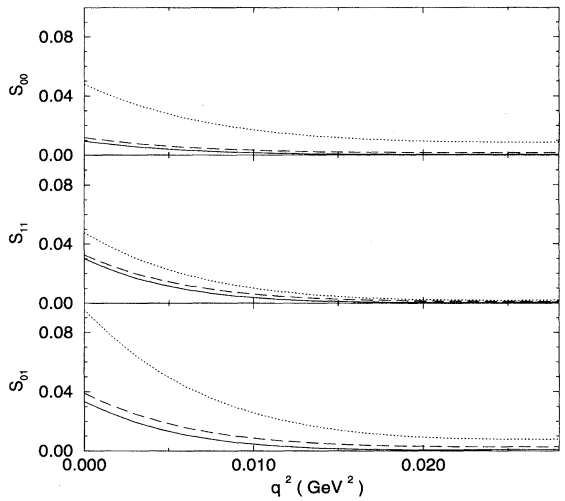

FIG. 2. The three ^{39}K structure functions S_{00} (top), S_{11} (middle), and S_{01} (bottom) as a function of momentum transfer squared. The dotted lines are the single-particle functions, the dashed lines the full functions obtained from the modified Bonn interaction, and the solid lines the full functions obtained from the Paris-based G matrix (see text).

in good agreement with experimental values. For the isovector operators, interaction II produces larger corrections and poorer agreement with the experimental isovector magnetic moment. But, although neither interaction reproduces the Gamow-Teller β-decay matrix element well, interaction II does better, and we recommend it for that reason. Compared with ^{27}Al, much smaller *ad hoc* quenching is required to reproduce the measured lifetime; the correlations induced by the core-polarization produce about two-thirds of the required reduction themselves.

Turning now to the WIMP structure functions in Fig. 2, we see that all three are strongly quenched from their single-particle values. The strongest quenching is in $S_{00}(q)$, which is reduced to 25% and 20% of the single-particle value for the two residual interactions considered. Our preferred choice, the solid line (corresponding to interaction II), is the lowest in all three diagrams. The solid lines are accurately reproduced by the following fourth-order polynomials in $y=(bq/2)^2$:

$$S_{00}=0.009\,499\,9-0.061\,971\,8\,y+0.162\,844\,y^2 -0.194\,282\,y^3+0.089\,105\,4\,y^4,$$

$$S_{11}=0.029\,812\,7-0.217\,636\,0\,y+0.623\,646\,y^2 -0.814\,418\,y^3+0.405\,027\,0\,y^4,$$

$$S_{01}=0.033\,204\,4-0.231\,943\,0\,y+0.638\,528\,y^2 -0.798\,523\,y^3+0.380\,975\,0\,y^4. \quad (8)$$

We repeat that these structure functions completely determine spin-dependent cross sections for any heavy Majorana particle.

V. DISCUSSION

What are the prospects for mica as a WIMP detector? At present the limits that can be set on WIMP cross sections are about 10^{-32} cm^2, [5] several orders of magnitude higher than expected cross sections for neutralinos, which very rarely exceed 10^{-35} cm^2 in light nuclei [2,27]. But by analyzing more mica the experiments may reach the necessary level of sensitivity within a few years. Because the efficiency with which each element in mica can be detected when it recoils is different, we cannot easily present an overall cross section for a representative neutralino, as is the common practice in the literature. But we note that the calculated efficiencies [28] begin to increase at a recoil energy of about 20 keV, corresponding to a q^2 of about 10^{-3} GeV^2 in Al and about 1.6×10^{-3}GeV^2 in K. If the 20 keV were a detection threshold, then as the curves in Figs. 1 and 2 show the number of events below threshold would be a small fraction of the total.

Whatever the precise characteristics of the detector, the expected number of events can now be calculated by folding the efficiencies together with the cross sections presented here and with the flux of WIMP's, which, unfortunately, is still poorly constrained. But continuing work with the mica, with complementary detectors, and with telescopes should make it possible to rule in or out most hypothetical WIMP's within the next decade.

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[1] V. Trimble, Annu. Rev. Astron. Astrophys. **25**, 425 (1989).
[2] J.R. Primack, B. Sadoulet, and D. Seckel, Annu. Rev. Nucl. Part. Sci. **38**, 751 (1988).
[3] See, e.g., C. Alcock *et al.*, Nature **365**, 623 (1993).
[4] M.S. Turner, Phys. Scr. **T36**, 167 (1991); J. Ellis, *ibid.* **T36**, 142 (1991).
[5] D.P. Snowden-Ifft, E.S. Freeman, and P.B. Price, Phys. Rev. Lett. **74**, 4133 (1995).

[6] K. Griest, Phys. Rev. D **38**, 2357 (1988); **62**, 666 (1989).
[7] M. Srednicki and R. Watkins, Phys. Lett. B **225**, 140 (1989).
[8] G.F. Giudice and E. Roulet, Nucl. Phys. **B316**, 429 (1989).
[9] M. Drees and M.M. Nojiri, Phys. Rev. D **48**, 3483 (1993).
[10] G. Jungman (private communication).
[11] J. Engel, Phys. Lett. B **264**, 114 (1991); J. Engel, S. Pittel, and P. Vogel, J. Int. Mod. Phys. E **1**, 1 (1992).
[12] S. Cooper *et al.*, Max-Planck-Institut für Physik Report No. MPI-PhE/93-29, 1993; P. Colling *et al.*, Max-Planck-Institut für Physik Report No. MPI-PhE/94-13, 1994.
[13] B.H. Wildenthal, Prog. Part. Nucl. Phys. **11**, 5 (1984).
[14] B.A. Brown and B.H. Wildenthal, Phys. Rev. C **28**, 2397 (1983); Nucl. Phys. **A474**, 290 (1987).
[15] D.A. Resler and S.M. Grimes, Comput. Phys. **2**, 65 (1988).
[16] M.T. Ressell *et al.*, Phys. Rev. D **48**, 5519 (1993).
[17] J. Engel and P. Vogel, Phys. Rev. D **40**, 3132 (1989).
[18] T.W. Donnelly and J.D. Walecka, Nucl. Phys. **A201**, 81 (1973).
[19] E.K. Warburton, J.A. Becker, and B.A. Brown, Phys. Rev. C **41**, 1147 (1990).
[20] I.S. Towner, Phys. Rep. **155**, 263 (1987).
[21] P.J. Ellis and H.A. Mavromatis, Nucl. Phys. **A175**, 309 (1971).
[22] P.J. Ellis and S. Siegel, Phys. Lett. **34B**, 177 (1971).
[23] K. Shimizu, M. Ichimura, and A. Arima, Nucl. Phys. **A226**, 282 (1974).
[24] R. Machleidt, K. Holinde, and Ch. Elster, Phys. Rep. **149**, 1 (1987); R. Machleidt, Adv. Nucl. Phys. **19**, 189 (1989).
[25] A. Hosaka, K.-I. Kubo, and H. Toki, Nucl. Phys. **A444**, 76 (1985).
[26] M. Lacombe *et al.*, Phys. Rev. C **21**, 861 (1980).
[27] G. Jungman, M. Kamionkowski, and K. Griest, Phys. Rep. (to be published).
[28] D.P. Snowden-Ifft (private communication).