Three-flavor quark matter in the chiral color dielectric model

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We investigate the properties of quark matter at finite density and temperature using the nonlinear chiral extension of the color dielectric model. Assuming that the square of the meson fields develops a nonzero vacuum expectation value, the thermodynamic potential for interacting three flavor matter has been calculated. It is found that $\langle K^2 \rangle$ and $\langle \eta^2 \rangle$ remain zero in the medium whereas $\langle \pi^2 \rangle$ changes in the medium. As a result, u and d quark masses decrease monotonically as the temperature and density of the quark matter is increased. In the present model, the deconfinement density and temperature are found to be lower compared to lattice results. We also study the behavior of pressure and energy density above critical temperature.

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I. INTRODUCTION

The quark structure of hadrons suggested the possibility of a transition from nuclear to quark phase at high density and/or temperature. This indicates that there can be a phase transition, usually termed as deconfinement transition, inside neutron stars at very high densities so that neutron stars may now consist of quark cores. On the other hand, the heavy ion collision experiments can also produce quark matter. In fact the increasing availability of high energy beams gives a new dimension to the quest for the quark gluon plasma. But the scenario gets complicated because of chiral symmetry. At low density or temperature chiral symmetry is broken and eventually at some high density or temperature it is supposed to be restored. So with an increase in density and/or temperature there can be two transitions, one is the deconfined transition where nucleons dissolve into quarks and the other is the chiral transition where chiral symmetry is restored.

QCD is the theory of strong interaction. So one would like to understand the properties of matter at extreme conditions and the possible phase transitions using QCD. At present the only method of such an investigation is the lattice calculation. The best established results of lattice calculations [1] show that although the energy density attains an ideal gas value very quickly above deconfined transition temperature T_c, it takes a much larger value of T for the pressure to attain the same behavior. This is an indication of nonperturbative behavior which is poorly understood. The character of the phase transition depends sensitively on the number of light quark flavors. Although there are large uncertainties, the current status is that for two-flavor light quarks, the transition is second order, while for three or more flavors it appears to be first order. But then the lattice calculation has its own complication, besides the problem of the large computer time needed. Until today, the lattice calculation has been limited to systems with zero chemical potentials. It is unclear whether strange quarks should be treated as light or heavy. Also one should note that in the real world since the light quark masses are not strictly zero, the second order phase transition for light quark flavors may turn into a rapid crossover.

One can investigate the matter at extreme conditions, without so much complication, by using QCD motivated models, which have been devloped and successfully applied to the study of the properties of baryons. One can get a simple physical understanding of the different characteristics of matter. There is one more advantage of using such models. In the lattice, it is difficult to represent the chiral symmetry of the continuum. On the other hand, in models one can implement chiral symmetry exactly. Many such investigations have already been done [2–6]. Recently, there are fresh attempts to understand the chiral symmetry transition using the linear σ model [7,8]. In Ref. [7] the authors have used the SU(3) $\times$ SU(3) linear σ model Lagrangian in terms of the 3×3 matrix field M. In fact this Lagrangian corresponds to the action with M as order parameter. They find that for realistic meson masses the chiral transition becomes a crossover phenomena, i.e., the phase transition behaves smoothly in all thermodynamic quantities. With a decrease in pion mass, the order of a transition changes from crossover to first order and with a decrease in σ mass, it changes from crossover to second order. Also at the transition point only the SU(2)$\times$SU(2) part of the chiral symmetry is restored. Some authors have studied the temperature dependence of chiral condensate from an interacting pion gas, by using the fact that the condensate is related to the derivative of the free energy density with respect to the bare quark mass [9]. They find a critical temperature $T_c \sim \sqrt{2} f_\pi$.

In our earlier calculation [5], we used the nonlinear chiral color dielectric model (CCDM) to study the properties of two flavor quark matter at finite temperature and density. There we have shown that with our ansatz of nonzero $F_\pi = \langle \vec{\pi}^2 \rangle / f_\pi^2$ and vanishing $\langle \vec{\pi} \rangle$ chiral symmetry is restored gradually at very high temperature $T \sim (2-3) T_c$, T_c being the deconfinement temperature, which is around 100–120 MeV. This model has been applied further to study the properties of stars [10,11]. In the present paper we have extended our ansatz to study three flavor matter at finite density and temperature. Now our ansatz implies a nonzero $F_\phi = \langle \phi^2 \rangle / f_\pi^2$, where ϕ corresponds to the meson fields, i.e., pion (π), kaon (K), or eta (η) [10–12]. We find that in the medium F_K and F_η are zero, whereas F_π attains a nonzero value for the range of density and temperature we have considered. This implies that strange quark mass remains constant in the medium whereas u and d quark masses decrease with density. So the chiral transition occurs only in the two flavor sector in our model. Also, the meson excitations play almost no role in determining the properties of quark matter.

The paper is organized as follows. In Sec. II we define the CCDM and give a brief outline of our ansatz, most of the detail being left for the Appendix. In Sec. III, we present the results of our calculation, and finally our conclusion is given in Sec. IV.

II. THE MODEL

The color dielectric model (CDM) is based on the idea of Nelson and Patkos [13]. In this model one generates the confinement of quarks and gluons dynamically through the interaction of these fields with scalar field or color dielectric field χ. In the present work we have used the chiral extension [10,11,14] (CCDM) of this model to calculate quark matter equation of state. The Lagrangian density of CCDM is given by

$$L(x)=\bar{\psi}(x)(i\gamma^{\mu}\partial_{\mu}-\{m_0+[m/\chi(x)]U_5\} + \tfrac{1}{2}g\gamma_{\mu}\lambda_a A^a_{\mu}(x))\psi+(f_{\pi}^2/4)\mathrm{Tr}(\partial_{\mu}U\partial^{\mu}U^{\dagger}) - \tfrac{1}{2}m_{\phi}^2\phi^2(x)-\tfrac{1}{4}\chi^4(x)[F^a_{\mu\nu}(x)]^2 + \tfrac{1}{2}\sigma_v^2[\partial_{\mu}\chi(x)]^2-U(\chi), \tag{1}$$

where $U=e^{i\lambda_a\phi^a/f_{\pi}}$ and $U_5=e^{i\lambda_a\phi^a\gamma_5/f_{\pi}}$, $\psi(x)$, $A_{\mu}(x)$, and $\chi(x)$ and $\phi(x)$ are quark, gluon, scalar (color dielectric) and meson fields, respectively. m and m_{ϕ} are the masses of quark and meson, f_{π} is the pion decay constant, $F_{\mu\nu}(x)$ is the usual color electromagnetic field tensor, g is the color coupling constant, and λ_a are the Gell-Mann matrices. The flavor symmetry breaking is incorporated in the Lagrangian through the quark mass term $\{m_0+[m/\chi(x)]U_5\}$, where $m_0=0$ for u and d quarks. So masses of u, d, and s quarks are m, m, and m_0+m, respectively. So for a system with broken flavor symmetry, i.e., flavor asymmetric matter, strange quark mass will be different from u and d quark masses. The meson matrix then consists of a singlet η, triplet of π, and quadruplet of K. One can also describe a flavor symmetric matter using the above Lagrangian. For such systems, $m_0=0$ and all the quark masses become same. The meson matrix Φ then becomes a eight component field. The self-interaction $U(\chi)$ of the scalar field is assumed to be of the form

$$U(\chi)=\alpha B\chi^2(x)[1-2(1-2/\alpha)\chi(x)+(1-3/\alpha)\chi^2(x)], \tag{2}$$

so that $U(\chi)$ has an absolute minimum at $\chi=0$ and a secondary minimum at $\chi=1$. The interaction of the scalar field with quark and gluon fields is such that quarks and gluons cannot exist in the region where $\chi=0$. In the limit of vanishing meson mass, the Lagrangian of Eq. (1) is invariant under chiral transformations of quark and meson fields.

There are two different approaches used when chiral models are applied for the study of hadrons. One is cloudy bag model [15], which is a perturbative method. The another one is hedgehog approach [16], which is nonperturbative but is not applicable for infinite matter. In our calculation, in an attempt to go beyond the perturbative approach of cloudy bag model, we assume that because of nonvanishing quark and antiquark densities the square of the expectation value of meson fields develops a nonzero value, i.e., $\langle\phi^2\rangle\neq 0$. On the other hand we assume that expectation value of the meson field vanishes in the medium . For an infinite system of quarks we can assume that $\langle\phi^2\rangle$ is independent of space and time. The meson excitations are then defined in terms of the fluctuations about $\langle\phi^2\rangle$, so that $\phi^2=\langle\phi^2\rangle+\phi'^2$. Defining $F_{\phi}=\langle\phi^2\rangle/f_{\pi}^2$, the CDM Lagrangian can be rewritten in terms of F_{ϕ}'s and meson excitations ϕ'. In our calculation the scalar field χ and F_{ϕ} have been calculated in the mean field approximation and quark-gluon, gluon-gluon, and quark-meson excitations are treated perturbatively. Thus, keeping the terms upto two meson exchange interaction we get

$$L(x)=\bar{\psi}(x)(i\gamma^{\mu}\partial_{\mu}-\{m_0+[m/\chi(x)]f_1(F_{\phi})\}+\tfrac{1}{2}g\gamma_{\mu}\lambda_a A^a_{\mu}(x))\psi-(1/4)\chi^4(x)[F^a_{\mu\nu}(x)]^2-U(\chi)+\tfrac{1}{2}\sigma_v^2[\partial_{\mu}\chi(x)]^2 - \tfrac{1}{2}m_{\phi}^2 f_{\pi}^2 F_{\phi}+\tfrac{1}{2}f_2(F_{\phi})[\partial_{\mu}\phi'(x)]^2-\tfrac{1}{2}m_{\phi}^2\phi'^2(x)-\frac{im}{\chi}\bar{\psi}(x)\gamma_5 f_{I1}(F_{\phi})\psi(x)+\frac{m}{2f_{\pi}^2\chi}\bar{\psi}(x)f_{I2}(F_{\phi})\psi(x). \tag{3}$$

The last two terms are the interactions of meson excitations with quarks. The f_1 and f_2 are the 3×3 diagonal matrices, each term of which is a polynomial in F_{ϕ}'s, i.e., F_{π}, F_K, F_{η}, and their cross terms. f_{I1} and f_{I2} are interaction matrices for one meson and two meson exchanges, respectively. These are also 3×3 matrices but are not diagonal. In the Appendix we have given the method of evaluating the f_1, f_2, f_{I1}, and f_{I2} and also the expressions of their nonzero contributions for three flavor asymmetric quark matter, symmetric matter, and two flavor quark matter. For three flavor (both flavor symmetric and asymmetric) the above polynomials are not summable, but are fast convergent and terms up to third order gives reasonably good convergence. On the other hand, these polynomials are simpler for two flavor matter and can be summed to a compact form as given in Appendix and Ref. [5]. In our calculations scalar field χ and F_{ϕ} have been evaluated in the mean field approximation and quark-gluon and quark-meson interactions are treated perturbatively. In this approximation, χ is independent of space and time for uniform quark matter. In an actual calculation, the mean field values of χ and F_{ϕ} are determined by minimizing the thermodynamical potential Ω. Here, to have a qualitative feeling of the different F_{ϕ}'s, let us consider the equation of motion for F_{ϕ} at zero temperature and without interactions:

$$\left[i\gamma^{\mu}\partial_{\mu}-\left(m_0+\frac{m}{\chi}f_1(F_{\phi})\right)\right]\psi(x)=0, \tag{4}$$

$$U'(\chi)=\frac{m}{\chi^2}f_1(F_\phi)\frac{\gamma}{2\pi^2}\int_0^{k_f}k^2dkm^*/(k^2+m^{*2})^{1/2}, \quad (5)$$

$$f_2(F_\phi)\partial^\mu\partial_\mu\phi'(x)+m_\phi^2\phi'(x)=0, \quad (6)$$

$$m_\phi^2f_\pi^2=\frac{m}{\chi}f_1'(F_\phi)\frac{\gamma}{2\pi^2}\int_0^{k_f}k^2dkm^*/(k^2+m^{*2})^{1/2}, \quad (7)$$

where $m^*=(m/\chi)[f_1(F_\phi)]_{11,22}$ for u, d quarks and $m^*=m_0+(m/\chi)[f_1(F_\phi)]_{33}$ for s quarks. k_F is the Fermi momentum and γ is the spin-color degeneracy factor for quarks. The equation of motion for quarks [Eq. (4)] shows that m^* can be identified with the effective quark masses in the presence of χ and F. Also the equation of meson fluctuations show that for nonzero F_ϕ the energy momentum relation for meson fluctuation is $E_\phi(k)=\sqrt{k^2+m_\phi^{*2}}$, where $m_\phi^*=m_\phi/\sqrt{f_2(F_\phi)}$. Thus the effective pion mass increases with F_ϕ and hence density in the medium. The mean field values of χ and F_ϕ can be evaluated by solving Eqs. (4)–(7) self-consistently.

The characteristics of F_ϕ can be understood from Eq. (7). The physical values of F_ϕ should be greater than zero. Equation (7) yields a positive value of F_ϕ only if

$$\frac{\gamma}{2\pi^2}\int_0^{k_f}k^2dkm^*/(k^2+m^{*2})^{1/2}\geq(m_\phi f_\pi\chi/m)^2$$

that is, if k_f (or the quark density) is larger than a certain value. Below this density, F_ϕ remains zero and, therefore, Eq. (7) becomes redundant. Now since $m_\pi<m_K<m_\eta$, the respective Fermi momentum k_F(for π)$<k_F$(for K)$<$ k_F(for η), i.e., as quark density increases, F_π will appear first. The F_K and F_η will appear much later as the masses of K and η are much higher than the pion mass. Also, F_ϕ increases with quark density. This means that effective quark masses will decrease with density and may vanish for a certain value of F_ϕ.

Using the modified Lagrangian we calculate the thermodynamic potential Ω up to second order in quark-gluon coupling and quark-meson couplings [4,5]. The thermodynamic quantities are obtained from Ω using standard methods [4,5]. The mean field values of χ and F_ϕ are obtained by minimizing Ω with respect to χ and F_ϕ:

$$(\partial\Omega/\partial\chi)_{T,\mu}=0, \quad (\partial\Omega/\partial F_\phi)_{T,\mu}=0. \quad (8)$$

III. RESULTS AND DISCUSSION

The thermodynamic properties of the quark matter are calculated for a number of parameter sets of chiral CDM (B, m, α, and g_s) which reproduce baryon masses. Earlier calculations show that these parameters are not determined uniquely by the fitting procedure [14]. In particular it has been found that good fits to baryon masses are obtained for 0.6 GeV$\leq m_{GB}\leq$3 GeV, $m_q(u,d)\leq$125 MeV, $m_q(s)\sim$300 MeV, and $B^{1/4}\leq$ 150 MeV, where $m_{GB}=\sqrt{2B\alpha/\sigma_v^2}$. On the other hand, if one insist for good fits of charge radius and magnetic moments as well, then the fits are better for lower values of m_{GB}, $m_q(u,d)$, and $B^{1/4}$. But, lower bag pressure $B^{1/4}$ gives a lower value for transition density and tempera-

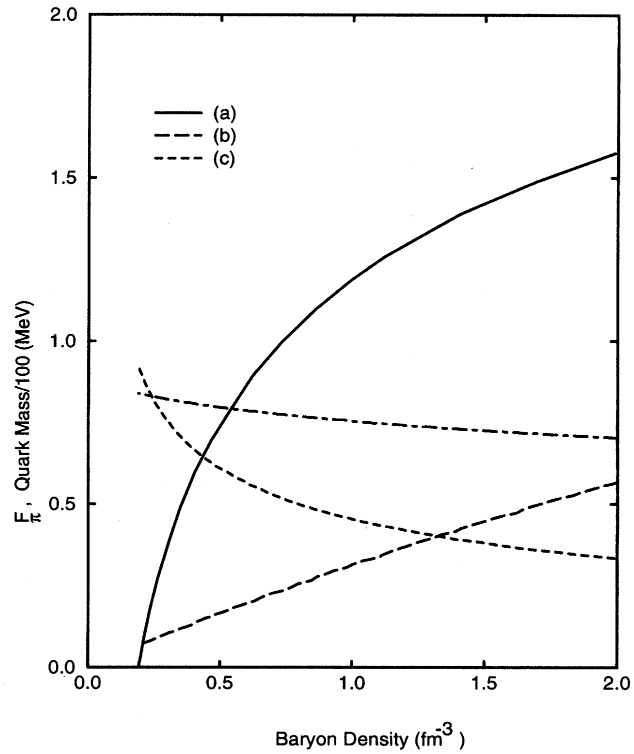

FIG. 1. Baryon density vs F_π and quark mass m_q (u and d); (a) and (b) correspond to F_π and m_q for the flavor asymmetric case; (c) and (d) correspond to F_π and m_q for the flavor symmetric case, respectively.

ture. In fact in Ref. [11] we found that for such parameter sets one even gets a stable quark matter phase at nuclear matter densities. So we have used parameter sets for which one gets reasonable behavior and we find that the results for these parameter sets are qualitatively similar. Therefore we have considered a representative parameter set ($B^{1/4}$= 152 MeV, $m_{u,d}$= 92 MeV, m_s=295 MeV, α=36, and g=1.002) for a detailed discussion. Since the analytic derivatives of the thermodynamic potential with respect to χ and F_ϕ's are not easy to obtain, we have numerically minimized $\Omega(T,\mu)$ as function of χ and F_ϕ. We find that the mean field values of χ remain close to 1 for both two and three flavor matter, whereas for three flavor quark matter F_K and F_η remain zero in the medium for the range of density as well as temperature considered in our calculation. F_π is zero for small values of T and μ to about 2 or more for T=400 MeV and μ= 500 MeV. From the expressions given in the Appendix it can be found that u and d quarks couple with all the F_ϕ's, i.e., F_π, F_K, and F_η. On the other hand, the s quark couples with only K and η. So $F_K=F_\eta=0$ implies that strange sector of the SU(3)×SU(3) decouples and strange quark mass remains constant in the medium. m_u and m_d become small at large values of baryon density n_B and temperature T. So, in our model, chiral symmetry restoration occurs for SU(2)× SU(2) sector only. The variation of F_π with n_B and T are shown in Fig. 1 (curves a and b) and Fig. 2 (curves a and b), respectively. Corresponding curves for flavor symmetric matter are shown by the curves c and d. We find that varia-

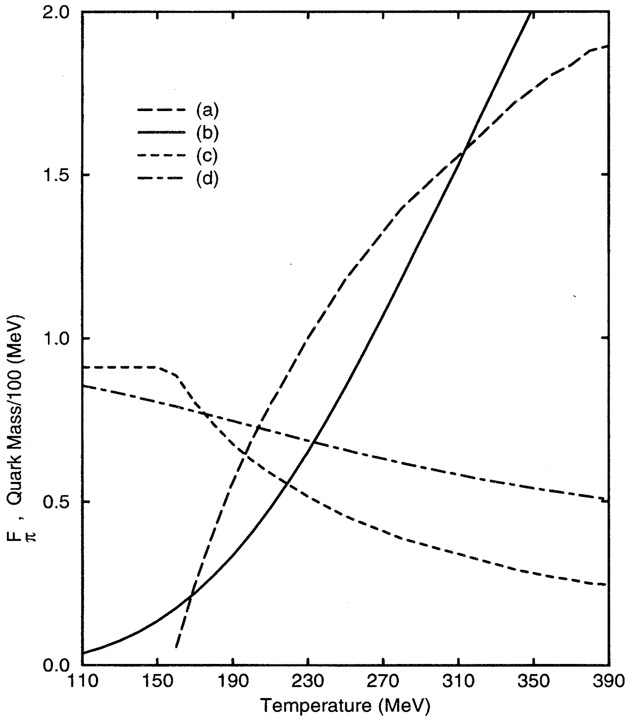

FIG. 2. Temperature vs F_π and quark mass m_q (u and d) (a) and (b) correspond to F_π and m_q for the flavor asymmetric case; (c) and (d) correspond to F_π and m_q for the flavor symmetric case, respectively.

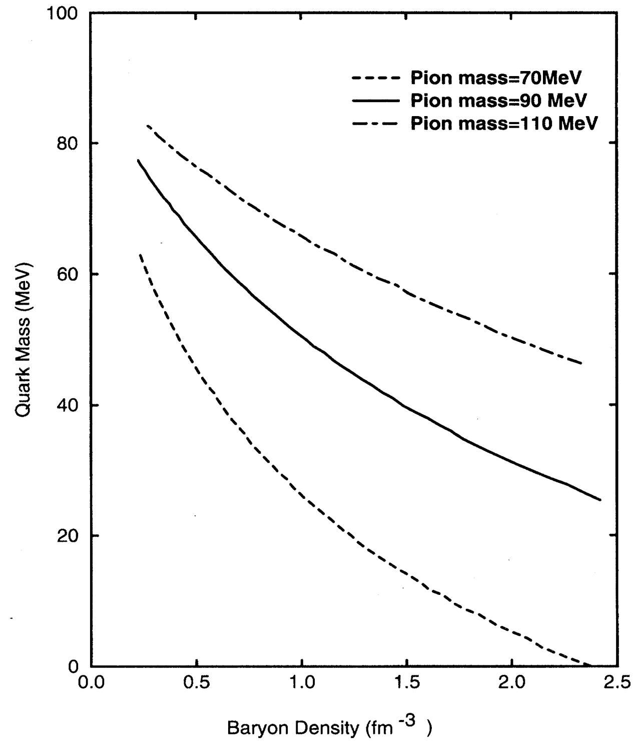

FIG. 3. Baryon density vs m_q for different pion mass m_π, (a) for $m_\pi = 110$ MeV, (b) for $m_\pi = 90$ MeV, and (c) for $m_\pi = 70$ MeV.

tion of quark mass with n_B and T is slower for flavor symmetric matter.

The equation of motion for F_π [Eq. (7)] shows that for lower values of m_π the density at which F_π is nonzero is lowered. It is also obvious that one cannot lower m_π arbitrarily as below a certain value of m_π, there is no solution. In Fig. 3, we have plotted quark mass with n_B for different m_π. We find that for lower m_π, quark mass approaches zero faster, i.e., the chiral restorations occur at lower density for lower pion mass. Similar behavior is obtained for finite T and zero μ as well. But the nature of the chiral transition, in our calculation, does not change with m_π and the chiral restoration phase is reached smoothly. This behavior is different compared to Ref. [7], where authors find that the nature of the transition changes from crossover to first order with decreasing pion mass. In our model, the effective pion mass increases with increase in density and temperature. We find that initially the pion mass increases to about 150 MeV and then remains constant. As discussed earlier, as the chiral behavior of the $SU(3)\times SU(3)$ is described by $SU(2)\times SU(2)$ part; the similar behavior is observed for two flavor matter as well [5].

Figure 4 shows the variation of pressure with density for different values of temperature. At zero temperature the pressure is negative for $n_B < 0.35$ fm^3. This density defines the boundary of the quark phase [5]. The density at which the pressure vanishes decreases with increasing temperature and above a certain critical temperature ($\sim 110-120$ MeV in our model) the pressure is always positive. This temperature can be defined as the critical temperature for the quark-gluon plasma (QGP) phase transition in our model. We find that critical temperature for two flavor matter (~ 115 MeV) is higher than the three flavor matter (~ 110 MeV). For flavor symmetric case the transition temperature is lowest (~ 105 MeV). But, as evident from the values quoted, the T_c is not much sensitive to the different composition of the quark matter system. The best evidence from the lattice QCD calculations show that for the two dynamical flavor case, the transition temperature is around 150 MeV and the transition density is around 4–8 times the nuclear matter density. For three flavor matter there are no convincing calculations, but one can assume it to be not much less than 150 MeV, keeping in mind the large value of m_s compared to u and d quarks. So, in our model, the critical temperature for deconfined transition is lower than the lattice QCD results.

For an ideal gas of quarks and gluons, $E/T^4 = \pi^2/30(\gamma_g + \frac{7}{4}\gamma_q = 15.63$ for three flavor matter and $P = \frac{1}{3}E$. Lattice calculations [1] show that the energy density increases above the T_c and quickly attains the ideal gas value. On the other hand, pressure needs a larger temperature ($T \sim T_c$) to reach the ideal gas value. This is one indication that nonperturbative effects are still present, in the QGP, at least just above the critical temperature. The values of E/T^4 and $3P/T^4$ for $\mu = 0$, from our model calculation, are given in Fig. 5. The figure shows that at high temperature E/T^4 approaches a value close to 15 and, as the temperature is lowered, there is a small increase in E/T^4. We do not show

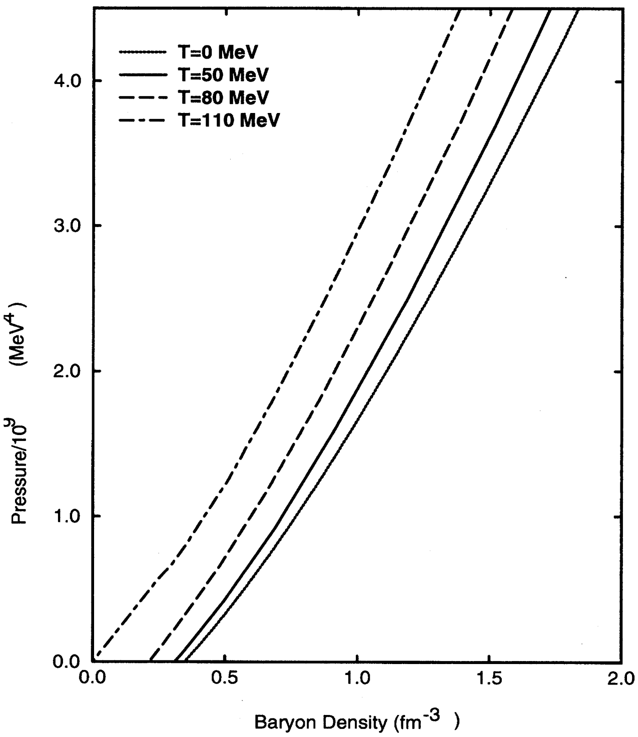

FIG. 4. Baryon density vs pressure for different temperatures, (a) $T=0$ MeV, (b) $T=50$ MeV, (c) $T=80$ MeV, and (d) $T=110$ MeV.

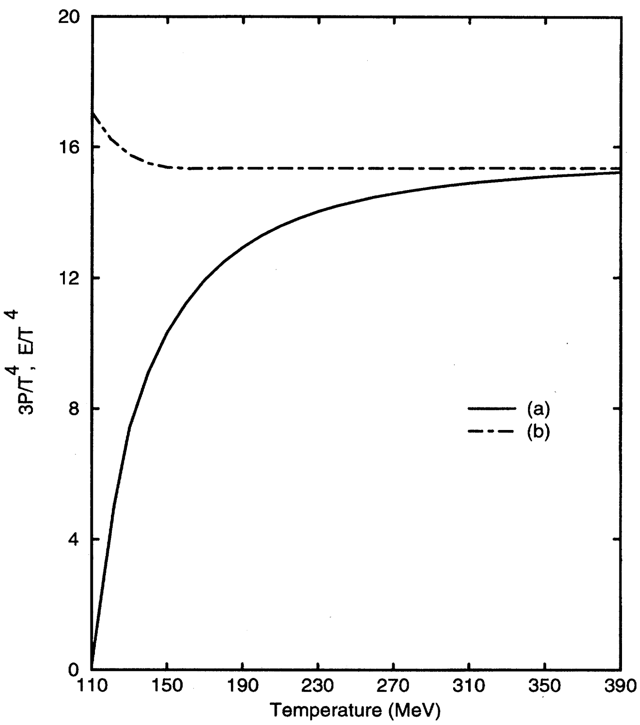

FIG. 5. Pressure and energy variation with temperature above critical temperature for zero chemical potential, (a) $3P/T^4$ and (b) E/T^4.

the results below T_c as the calculation is not applicable at these temperatures. Figure 5 also shows that P/T^4 is a monotonically increasing function of temperature. We have also fitted E/T^4 and P/T^4 with functions of the form $f(T)=\Sigma a_i/T^i$ with i running from 0 to 4. We find that a good fit to E/T^4 is obtained with $a_0=14.38$, $a_1=644.15$, $a_2=-1.15\times10^5$, $a_3=0$, and $a_4=9.22\times10^8$. For P/T^4, a good fit is obtained with $a_0=5.68$, $a_1=-292.12$, $a_2=27753.10$, $a_3=0$, and $a_4=-7.699\times10^8$. The corresponding fits for two flavor matter is similar to the one obtained in Ref. [5]. Because of the constant background of the scalar field χ and F, we expect that the energy density and pressure should have a temperature independent term.

IV. CONCLUSION

The nonlinear version of the chiral color dielectric model is used to study the properties of quark matter in the present work. We have assumed that due to nonzero quark and antiquark densities, the square of the meson fields (pion, kaon, and eta) develops a vacuum expectation value. In the matter, the $\langle K^2\rangle$ and $\langle \eta^2\rangle$ remain zero throughout the range of densities and temperatures considered. On the other hand, $\langle \pi^2\rangle$ has a nonzero value in the medium. As a result, the s quark mass remains constant in the medium, whereas effective u and d quark masses decrease continuously as the density and temperature of the quark matter is increased. If we interpret this as a restoration of chiral symmetry, our calculation implies that chiral symmetry is restored smoothly in the chiral color dielectric model. Also, the chiral restoration occurs in SU(2)×SU(2) part only. There is a small increase in the pion mass with temperature and density in our model. Furthermore, if the pion mass is decreased, the quark mass drops faster, i.e., the chiral restoration occurs for lower density and temperatures. Here one thing should be noted that quark-meson interactions play almost no role in determining the quark matter characteristics as their contribution to Ω is much less compared to pion background field and the quark-gluon interaction contribution [5].

The pressure of the quark matter becomes negative at lower densities and temperatures, which implies that a confined phase is reached when the temperature or density of the quark matter is lowered. We find that the critical temperature does not vary much for different composition of the quark matter. The values of the critical temperature and density at which deconfinement transition take place are smaller than the values obtained in lattice calculations [1]. The behavior of the pressure and density calculated for $\mu=0$ is quantitatively similar to that obtained in lattice calculations.

APPENDIX

Here we will describe the method of calculating the Lagrangian (3) in terms of the $\langle\phi^2\rangle/f_\pi^2=F_\phi$ and the meson excitations ϕ'. The meson octet Φ is

$$\lambda_a \phi^a = \Phi = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & \frac{-\pi_0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & \frac{-2\eta}{\sqrt{6}} \end{pmatrix}, \tag{A1}$$

where λ_a are the Gellmann matrices. The chiral symmetry operator and meson kinetic energy operator are $U_5 = e^{i\gamma_5 \Phi/f_\pi}$ and $U = e^{i\Phi/f_\pi}$. The general procedure is to expand the exponential and contract the different fields following the different rules as prescribed below:

$$\pi^+\pi^- = \pi^-\pi^+ = \pi^{02} = \langle \pi^2 \rangle / j_\pi = F_\pi f_\pi^2 / j_\pi, \tag{A2}$$

$$K^+K^- = K^-K^+ = K^0\bar{K}^0 = \bar{K}^0 K^0 = \langle K^2 \rangle / j_K = F_K f_\pi^2 / j_K, \tag{A3}$$

$$\eta^2 = \langle \eta^2 \rangle / j_\eta = F_\eta f_\pi^2 / j_\eta, \tag{A4}$$

where in a flavor asymmetric matter the factors due to isospin recoupling are $j_\pi = 3$ for pions, $j_K = 4$ for kaons and $j_\eta = 1$ for eta. For flavor symmetric matter $j_\pi = j_K = j_\eta = 8$.

In general for an operator $O(\phi(x))$, for any positive integer n, we can write

$$O(\phi(x)) = \langle O(\phi(x)) \rangle + \sum_n [O(\phi(x))]_n, \tag{A5}$$

where $[O(\phi(x))]_n$ implies that there are n meson fields $[\phi'(x)]$ in the expression. For our present calculation, we need terms up to $n=2$. Let us now evaluate the expansion for different combinations of meson fields:

$$\begin{aligned} \langle (\pi^+\pi^-)^n \rangle &= n \langle \pi^+\pi^- \rangle \langle (\pi^+\pi^-)^{n-1} \rangle \\ &= n(n-1) \langle \pi^+\pi^- \rangle^2 \langle (\pi^+\pi^-)^{n-2} \rangle \\ &= \frac{n!}{j_\pi^n} \langle \pi^2 \rangle^n \end{aligned} \tag{A6}$$

where the factor n implies that there are n ways to contract one $(\pi^+\pi^-)$ out of n $(\pi^+\pi^-)$ fields: Once first contraction is done, then second one has $n-1$ possibilities. Hence we get the factor $n(n-1)$. Similarly one can get the form for other fields:

$$\begin{aligned} \langle (\pi^0)^{2n} \rangle &= (2n-1) \langle \pi^{02} \rangle \langle (\pi^0)^{2n-2} \rangle \\ &= (2n-1)(2n-3) \langle \pi^0 \rangle^2 \langle (\pi^0)^{2n-4} \rangle \end{aligned} \tag{A7}$$

$$= \frac{(2n-1)!!}{j_\pi^n} \langle \pi^2 \rangle^n, \tag{A8}$$

$$\langle (\eta)^{2n} \rangle = (2n-1)!! j_\eta^n \langle \eta \rangle^n, \tag{A9}$$

$$\langle (K^+K^-)^n \rangle = \frac{n!}{j_K^n} \langle K^2 \rangle^n, \tag{A10}$$

$$\langle (K^0\bar{K}^0)^n \rangle = \frac{n!}{j_K^n} \langle K^2 \rangle^n. \tag{A11}$$

The two meson interaction term can be evaluated following the above procedure:

$$[(\pi^+\pi^-)^n]_2 = \frac{n^2(n-1)!}{j_\pi^{n-1}} \langle \pi^2 \rangle^{n-1} \pi^+\pi^-. \tag{A12}$$

The n^2 factor shows that there are $n \times n$ ways to collect a $\pi^+\pi^-$ pair from n pairs. The remaining $n-1$ pairs can be contracted in $(n-1)!$ ways. For other meson fields we get

$$[(\pi^0)^{2n}]_2 = \frac{n(2n-1)(2n-3)!!}{j_\pi^{n-1}} \langle \pi^2 \rangle^{n-1} \pi^{02}, \tag{A13}$$

$$[\eta^{2n}]_2 = \frac{n(2n-1)(2n-3)!!}{j_\eta^{n-1}} \langle \eta^2 \rangle^{n-1} \eta^2, \tag{A14}$$

$$[(K^+K^-)^n]_2 = \frac{n^2(n-1)!}{j_K^{n-1}} \langle K^2 \rangle^{n-1} K^+K^-, \tag{A15}$$

$$[(K^0\bar{K}^0)^n]_2 = \frac{n^2(n-1)!}{j_K^{n-1}} \langle K^2 \rangle^{n-1} K^0\bar{K}^0. \tag{A16}$$

For odd m, $[\phi^{2n}]_m$ vanishes, since $\langle \phi \rangle = 0$. Also for odd powers, $\langle \phi\phi^2 \rangle = 0$. The one meson exchange terms are calculated from the terms containing odd powers of the meson concerned. For any positive integer n we can write

$$\begin{aligned} [(\pi^+)^n(\pi^-)^{n+1}]_1 &= (n+1)\pi^- \langle \pi^+\pi^- \rangle^n \\ &= \frac{(n+1)n!}{j_\pi^n} \langle \pi^2 \rangle^n \pi^-, \end{aligned} \tag{A17}$$

where one π^- can be picked up from the $n+1$ fields in $n+1$ ways and rest of the n pairs of $\pi^+\pi^-$ can be contracted in $n!$ ways. Similarly,

$$[(\pi^+)^{n+1}(\pi^-)^n]_1 = \frac{(n+1)n!}{j_\pi^n} \langle \pi^2 \rangle^n \pi^+, \tag{A18}$$

$$[(\pi^0)^{2n-1}]_1 = \frac{(2n-1)(2n-3)!!}{j_\pi^{n-1}} \langle \pi^2 \rangle^{n-1} \pi^0, \tag{A19}$$

$$[\eta^{2n-1}]_1 = \frac{(2n-1)(2n-3)!!}{j_\eta^{n-1}} \langle \eta^2 \rangle^{n-1} \eta, \tag{A20}$$

$$[(K^+)^n(K^-)^{n+1}]_1 = \frac{(n+1)n!}{j_K^n} \langle K^2 \rangle^n K^-, \tag{A21}$$

$$[(K^+)^{n+1}(K^-)^n]_1 = \frac{(n+1)n!}{j_K^n} \langle K^2 \rangle^n K^+, \tag{A22}$$

$$[(K^0)^n(\bar{K}^0)^{n+1}]_1=\frac{(n+1)n!}{j_K^n}\langle K^2\rangle^n\bar{K}^0, \tag{A23}$$

$$[(K^0)^{n+1}(\bar{K}^0)^n]_1=\frac{(n+1)n!}{j_K^n}\langle K^2\rangle^n K^0. \tag{A24}$$

Case a: Three flavor asymmetric matter. So starting from U_5, considering only the even powers we get a 3×3 diagonal matrix $[f_1(F_\phi)]_{ii}=\langle U_5\rangle$, with fully contracted terms, which contributes to the quark mass terms.

$$\begin{aligned}[f_1(F_\phi)]_{1\,1}=[F_1(F_\phi)]_{2\,2}=&1-\frac{F_\pi}{2!}+\frac{5F_\pi^2}{34!}-\frac{35F_\pi^3}{96!}\cdots-\frac{F_k}{22!}+\frac{3F_k^2}{44!}-\frac{3F_k^3}{26!}\cdots-\frac{F_\eta}{32!}+\frac{F_\eta^2}{34!}-\frac{5F_\eta^3}{96!}\cdots\\&+F_\eta\left(\frac{2F_\pi}{4!}-\frac{25F_\pi^2}{36!}+\cdots\right)-5F_\eta^2\frac{F_\pi}{5!}\cdots+F_k\left(\frac{3F_\pi}{24!}-\frac{25F_\pi^2}{66!}+\cdots\right)-7F_k^2\frac{F_\pi}{26!}\cdots\\&+F_\eta\left(\frac{F_k}{24!}-\frac{3F_K^2}{26!}\cdots\right)-3F_\eta^2\frac{F_k}{26!}\cdots+O(F_\phi^4)\end{aligned} \tag{A25}$$

$$\begin{aligned}[f_1(F_\phi)]_{3\,3}=&1-\frac{F_k}{22!}+\frac{3F_k^2}{24!}-\frac{3F_k^3}{6!}\cdots-\frac{4F_\eta}{32!}+\frac{16F_\eta^2}{34!}-\frac{320F_\eta^3}{96!}\cdots+F_\eta\left(\frac{2F_\pi}{4!}-\frac{25F_\pi^2}{36!}\right)\cdots-F_\eta^2\frac{5F_\pi}{6!}\cdots\\&+F_k\left(\frac{F_\pi}{4!}-\frac{5F_{pi}^2}{36!}\right)\cdots-F_k^2\frac{7F_\pi}{26!}\cdots+F_\eta\left(\frac{3F_k}{4!}-\frac{15F_k^2}{26!}\right)\cdots-19F_\eta^2\frac{F_k}{6!}-\frac{2F_\pi F_k F_\eta}{6!}\cdots+O(F_\phi^4).\end{aligned} \tag{A26}$$

As stated in the text, we find that $F_k=F_\eta=0$ for the range of density ($\sim15-20$ times normal nuclear matter density) and temperature ($\sim3-4$ times the critical temperature) considered. This implies that in the above expression, the terms with only F_π will contribute. The resultant terms can be summed up to give the new masses of u, d and s quarks. So, for flavor asymmetric quark matter the nonzero corrections to quark mass (u and d) terms are

$$[f_1(F_\pi)]_{11}=[f_1(F_\pi)]_{2\,2}=(1-F_\pi)e^{-F_\pi/6}. \tag{A27}$$

Correction to one meson exchange interaction are obtained from the odd powers in the expansion of U_5:

$$\begin{aligned}[f_{I1}(F_\pi)]_{11}=&\left(1-\frac{5F_\pi}{33!}+\frac{35F_\pi^2}{95!}\cdots\right)\frac{\pi^0}{f_\pi}\\&+\left(\frac{9F_\pi}{33!}+\frac{75F_\pi^2}{95!}\cdots\right)\frac{\eta}{\sqrt{3}},\end{aligned}$$

$$\begin{aligned}[f_{I1}(F_\pi)]_{22}=&-\left(1-\frac{5F_\pi}{33!}+\frac{35F_\pi^2}{95!}\cdots\right)\frac{\pi^0}{f_\pi}\\&+\left(\frac{9F_\pi}{33!}+\frac{75F_\pi^2}{95!}\cdots\right)\frac{\eta}{\sqrt{3}},\end{aligned}$$

$$[f_{I1}(F_\pi)]_{12}=\left(1-\frac{5F_\pi}{33!}+\frac{35F_\pi^2}{95!}\cdots\right)\frac{\sqrt{2}\pi^+}{f_\pi},$$

$$[f_{I1}(F_\pi)]_{21}=\left(1-\frac{5F_\pi}{33!}+\frac{35F_\pi^2}{95!}\cdots\right)\frac{\sqrt{2}\pi^-}{f_\pi},$$

$$[f_{I1}(F_\pi)]_{13}=\left(1-\frac{F_\pi}{3!}+\frac{5F_\pi^2}{35!}\cdots\right)\frac{\sqrt{2}k^+}{f_\pi},$$

$$[f_{I1}(F_\pi)]_{31}=\left(1-\frac{F_\pi}{3!}+\frac{5F_\pi^2}{35!}\cdots\right)\frac{\sqrt{2}k^-}{f_\pi},$$

$$[f_{I1}(F_\pi)]_{23}=\left(1-\frac{F_\pi}{3!}+\frac{5F_\pi^2}{35!}\cdots\right)\frac{\sqrt{2}k^0}{f_\pi},$$

$$[f_{I1}(F_\pi)]_{32}=\left(1-\frac{F_\pi}{3!}+\frac{5F_\pi^2}{35!}\cdots\right)\frac{\sqrt{2}\bar{k}^0}{f_\pi},$$

$$[f_{I1}(F_\pi)]_{33}=0. \tag{A28}$$

The correction to two meson exchange terms come from the even terms of U_5:

$$\begin{aligned}(f_{I2})_{11}=&\left(1-\frac{5F_\pi}{18}+\frac{7F_\pi^2}{216}\right)\pi^+\pi^-\\&+\left(1-\frac{5F_\pi}{18}+\frac{7F_\pi^2}{216}\right)\pi^{02}\\&+\left(1-\frac{7F\pi}{36}+\frac{11F_\pi^2}{648}\right)K^+K^-\\&+\left(1-\frac{F_\pi}{12}+\frac{F_\pi^2}{81}\right)\eta^2,\end{aligned}$$

$$(f_{I2})_{22}=\left(1-\frac{5F_\pi}{18}+\frac{7F_\pi^2}{216}\right)\pi^+\pi^- +\left(1-\frac{5F_\pi}{18}+\frac{7F_\pi^2}{216}\right)\pi^{02}+\left(1-\frac{7F\pi}{36}+\frac{11F_\pi^2}{648}\right)K^0\bar{K}^0+\left(1-\frac{F_\pi}{12}+\frac{F_\pi^2}{81}\right)\eta^2,$$

$$(f_{I2})_{33}=\left(1-\frac{2F_\pi}{4!}+\frac{10F_\pi^2}{36!}\right)K^+K^- +\left(1-\frac{2F_\pi}{4!}+\frac{10F_\pi^2}{36!}\right)K^0\bar{K}^0. \quad (A29)$$

The correction to meson masses comes through the correction to kinetic energy term.

$$f_{2\pi}=1-\frac{F_\pi}{9}+\frac{4F_\pi^2}{273!}\cdots,$$

$$f_{2k}=1-\frac{7F_\pi}{12}+\frac{11F_\pi^2}{108}\cdots,$$

$$f_{2\eta}=0. \quad (A30)$$

The $f_{2\eta}=0$ implies that there is no change in the mass of η meson in the medium.

Case b: Three flavor symmetric matter. Three flavor symmetric matter consist of a single eight component meson field ϕ. Also all the quark masses become equal. The corrections are obtained by putting $j_\pi=j_K=j_\eta=8$. The correction to mass term is given by

$$f_1(F_\phi)=1-\frac{2F_\phi}{32!}+\frac{5F_\phi^2}{64!}-\frac{95f_\phi^3}{726!}\cdots . \quad (A31)$$

The one meson exchange term is given by

$$f_{I1}(F_\phi)=\left(1-\frac{10F_\phi}{83!}+\frac{380F_\phi^2}{38^25!}\cdots\right)\frac{\Phi}{f_\pi}. \quad (A32)$$

The two meson exchange term is

$$f_{I2}=\left(1-\frac{313F_\phi}{576}+\frac{4859F_\phi}{103680}\right)\phi^2. \quad (A33)$$

The correction to mass term is given by

$$f_2(F_\phi)=1-\frac{F_\phi}{8}+\frac{463}{1620}\frac{F_\phi^2}{8^2}\cdots . \quad (A34)$$

Case c: Two flavor matter. In case of two flavor matter, only pions are present in the medium. So that the meson matrix is now a 2×2 matrix. The contraction rules are as given for pions in case a. The results are same as the one obtaind for the pions in case a. All the corrections are now summable and take the forms as given below:

$$(f_1)_{11}=(f_1)_{22}=(1-F_\pi)e^{-F_\pi/6}, \quad (A35)$$

$$(f_2)_{11}=(f_2)_{22}=\frac{1}{3}\left(2+\frac{1.5}{F_\pi}(1-e^{-2F_\pi/3})\right), \quad (A36)$$

$$(f_{I1})_{11}=\left(1-\frac{F_\pi}{9}\right)e^{-F_\pi/6}\pi^0=f_{I2}\ , \quad (A37)$$

where Eqs. (A35), (A36), and (A37) are the corrections to quark mass term, pion mass term, and quark-pion interaction term. The other elements for interactions can be obtained similarly.

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