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Chiral prediction for the πN S-wave scattering length a^- to order $\mathscr{O}(M_\pi^4)$

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We evaluate the S-wave pion-nucleon scattering length a^- in the framework of heavy baryon chiral perturbation theory up-to-and-including terms of order M_π^4. We show that the order M_π^4 piece of the isovector amplitude at threshold, $T^-_{\rm thr}$, vanishes exactly. We predict for the isovector scattering length, $0.088M_{\pi^+}^{-1} \leqslant a^- \leqslant 0.096M_{\pi^+}^{-1}$.

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Chiral symmetry severely constrains the interactions of the strongly interacting particles at low energies, first formulated in terms of current algebra (CA). One of the most splendid successes of CA was Weinberg's prediction for the S-wave pion-nucleon scattering lengths [1]

$$a^+_{\rm CA}=0, \quad a^-_{\rm CA}=\frac{M_\pi}{8\pi F_\pi^2}\frac{1}{1+M_\pi/m_p}=0.079M_\pi^{-1}, \quad (1)$$

with $M_\pi=139.57$ MeV the charged pion mass, $m_p=938.27$ MeV the proton mass, $F_\pi=92.4\pm0.3$ MeV the pion decay constant, and the superscripts $+/-$ refer to the isoscalar and isovector πN amplitude, respectively. The Karlsruhe-Helsinki phase shift analysis of πN scattering [2] leads to $a^-=(0.092\pm0.002)M_\pi^{-1}$ and $a^+=(-0.008\pm0.004)M_\pi^{-1}$, impressively close to the CA prediction, Eq. (1). The Karlsruhe–Helsinki analysis is based on dispersion relations thus requiring a solid database as input. However, over the last few years there has been some controversy about the low-energy πN data which has not yet been settled. Consequently, the uncertainties in $a^\pm$ are presumably larger and even the sign of a^+ could be positive. A more direct way to get a handle on these zero momentum (i.e., threshold) quantities is the measurement of the strong interaction shift (ϵ_{1S}) and the decay width (Γ_{1S}) in pionic atoms. The PSI-ETH group has recently presented first results of their impressive measurements in pionic deuterium and pionic hydrogen [3–5] (for earlier work in the field see, e.g., [6–8]). The S-wave scattering lengths are related to ϵ_{1S} and Γ_{1S} via the Deser-type formulas [9,10],

$$\epsilon_{1S}=-C(a^++a^-)(1+\delta_\epsilon),$$

$$\Gamma_{1S}=4\,qC(1+1/P)[a^-(1+\delta_\Gamma)]^2, \quad (2)$$

with q the momentum of the π^0, P the Panofsky ratio, and the constant $C=4E_{1S}/r_B$ is proportional to the point-Coulomb binding energy E_{1S} of the system in the $1S$ state divided by the pion Bohr radius r_B. δ_Γ and δ_ϵ describe electromagnetic corrections to the scattering lengths; see [10]. The consequent analysis of the data leads to [5] $a^-=(0.096\pm0.007)M_\pi^{-1}$ and $a^+=(-0.0077\pm0.0071)M_\pi^{-1}$. If one combines the pionic hydrogen shift measurement with the one from the pionic deuterium, one has $a^-=(0.086\pm0.002)M_\pi^{-1}$ and $a^+=(0.002\pm0.001)M_\pi^{-1}$. The largest uncertainty comes from the width measurement of pionic hydrogen. Both determinations are consistent within one standard deviation. We conclude that a^- is larger than the CA value and that a^+ is consistent with zero.

Heavy baryon chiral perturbation [11] allows to systematically calculate the corrections to the CA predictions, Eq. (1), for the S-wave scattering lengths. This topic was already addressed in Ref. [12]. In this paper, we want to further sharpen the prediction for the isovector scattering length. In particular, we will calculate all contributions which are of order M_π^4 and furthermore give a realistic estimate of the uncertainties for the one-loop result (which was not done in [12]). The starting point is the effective pion-nucleon Lagrangian

$$\mathscr{L}_{\pi N}=\mathscr{L}^{(1)}_{\pi N}+\mathscr{L}^{(2)}_{\pi N}+\mathscr{L}^{(3)}_{\pi N}+\mathscr{L}^{(4)}_{\pi N}+\mathscr{L}^{(2)}_{\pi\pi}, \quad (3)$$

where the superscript (i) refers to the number of derivatives or meson mass insertions. The structure of $\mathscr{L}_{\pi N}$ is discussed in detail in the review [13] and a pedagogical introduction can be found in [14]. The S-wave scattering lengths can be calculated from the on-shell forward πN scattering amplitude $T^{ba}=T^+(\omega)\delta^{ba}+i\epsilon^{bac}\tau^cT^-(\omega)$ with $\omega=p\cdot q/m_p$ the pion laboratory energy. Evaluation at threshold $\omega=M_\pi$ gives them as $a^\pm=T^\pm_{\rm thr}/[4\pi(1+M_\pi/m_p)]$. The one-loop result for $T^-_{\rm thr}$ to order M_π^4 reads

$$T^-_{\rm thr}=\frac{M_\pi}{2F_\pi^2}+\frac{g^2_{\pi N}M_\pi^3}{8m_p^4}+\frac{M_\pi^3}{16\pi^2F_\pi^4}\left(1-2\ln\frac{M_\pi}{\lambda}\right)+B^r(\lambda)M_\pi^3 + T^{-,4}_{\rm thr}+\mathscr{O}(M_\pi^5), \quad (4)$$

where the chiral corrections at order M_π^3 come from the expansion of the nucleon pseudovector Born term, the loop

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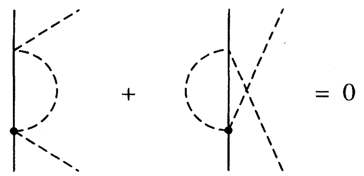

FIG. 1. Cancellation of diagrams at order M_π^4 due to the crossing property, Eq. (6). Solid and dashed lines denote nucleons and pions, respectively. The heavy dot denotes an insertion from $\mathcal{L}_{\pi N}^{(2)}$.

corrections, and the counterterms. $T_{\rm thr}^{-,4}$ denotes the order M_π^4 contribution to the isovector forward πN amplitude at threshold to be discussed below. We note that the contribution from the Born term is just 1% of $a_{\rm CA}^-$. To determine the counterterm contribution, one needs to pin down the value of $B^r(\lambda)$ which is a particular combination of some low-energy constants from $\mathcal{L}_{\pi N}^{(3)}$ [12,13]. In fact, these low-energy constants are not known and have been estimated in [12] via resonance exchange,

$$B^r(\lambda)=B^{\rm res}=\frac{g_{\pi N}^2}{2m_p^2m_\Delta^2}\left(Z-\frac{1}{2}\right)^2+\frac{Rg_{\pi N}^2}{8m_p^2(m_p+m_{N*})^2}. \tag{5}$$

In Eq. (5), we have exhibited on the corresponding $\Delta(1232)$ and the $N^*(1440)$ contributions. The range of the various resonance parameters like R and Z is discussed in [12]. This procedure induces a spurious dependence on the scale of dimensional renormalization (λ) which enters through the divergences from the one loop contribution. In [12], λ was fixed at the mass of the $\Delta(1232)$ since this resonance is most important in the estimation of the contact term $B^r(\lambda)$. However, in the framework of resonance saturation this scale should be varied between $M_\eta=550$ MeV and $m_{N*}=1.44$ GeV. Were we able to fix $B^r(\lambda)$ from some data, the prediction for a^- would, of course, be λ independent.

We now turn to the calculation of the order M_π^4 corrections. First, contact terms from $\mathcal{L}_{\pi N}^{(4)}$ cannot contribute to $T_{\rm thr}^-$ at $\mathcal{O}(M_\pi^4)$ because of the crossing property of the isovector forward πN amplitude,

$$T^-(\omega)=-T^-(-\omega), \tag{6}$$

with ω the pion laboratory energy. Thus we are left with one-loop graphs with exactly one insertion from $\mathcal{L}_{\pi N}^{(2)}$. Many of these diagrams cancel with their crossed partners due to the crossing property, Eq. (6), and specific properties of the loop functions entering, as exemplified in Fig. 1. In particular, the terms with one insertion proportional to $c_{1,2,3,4}$ from $\mathcal{L}_{\pi N}^{(2)}$ vanish because

$$J_0(\omega)+J_0(-\omega)=-\frac{1}{4\pi}\sqrt{M_\pi^2-\omega^2}=0, \tag{7}$$

at threshold, $\omega=M_\pi$ (the pertinent loop functions are given in Appendix B of [13]). However, this does not hold for the

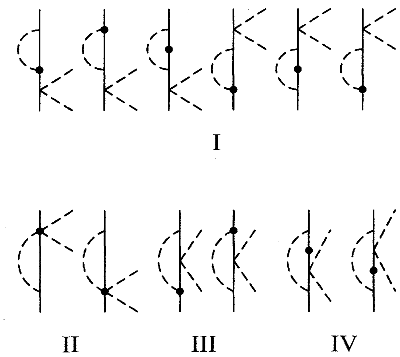

FIG. 2. Classes of nonvanishing diagrams at order M_π^4. For notations, see Fig. 1.

13 diagrams shown in Fig. 2. These remaining graphs can be grouped into five classes as indicated in Fig. 2. Straightforward calculation gives

$$T_{\rm thr}^{-,4}=\frac{g_A^2M_\pi^4}{128\pi m_pF_\pi^4}(9+8-2+5-20)=0, \tag{8}$$

where we have exhibited the contributions from the various classes separately. This result is reminiscent of the nonrenormalization of the isovector charge of the nucleon. Consequently, the next chiral corrections appear at order M_π^5, from two-loop graphs, one-loop graphs with one insertion from $\mathcal{L}_{\pi N}^{(3)}$, counterterms, and so on. They are suppressed by four powers of $M_\pi/(4\pi F_\pi)$ compared to the leading order term, Eq. (1) and are thus expected to be small, even though the exact numerical coefficients can be of order 10 [as indicated from the $\mathcal{O}(M_\pi^3)$ correction to the CA result].

Varying the various resonance parameters which enter the estimate of $B^{\rm res}$ and the scale λ within their bounds, we find as a conservative estimate for the one-loop prediction of a^-,

$$0.088M_{\pi^+}^{-1}\leq a^-\leq 0.096M_{\pi^+}^{-1}, \tag{9}$$

which is consistent with the various empirical values discussed before and 10–20 % larger than the CA prediction [1]. As already stressed in [12], it is the chiral loop correction at order M_π^3 which closes the gap between the lowest order (CA) prediction and the empirical value. An indication of the size of the next corrections can be obtained by writing the one-loop result as $a^-=a_{\rm CA}^-(1+\delta_1)\simeq a_{\rm CA}^-\exp(\delta_1)$. The next correction follows to be $\delta_1^2/2$ which is of the order of 1–2 % of $a_{\rm CA}^-$. We stress that this should only be considered indicative. The uncertainty in the prediction, Eq. (9), could be further decreased if one could determine $B^r(\lambda)$ from some other process.

To summarize, we have sharpened the chiral perturbation theory prediction for the isovector πN S-wave scattering length a^- by showing that the term of order M_π^4 in $T_{\rm thr}^-$ is identical to zero. Varying all uncertainties entering the one-loop expression for a^- [12], we arrived at the band, Eq. (9), which is consistent with the existing data. Concerning the isoscalar scattering length a^+, no firm prediction can be given at present as discussed in [12] since there are large cancellations which make the result for a^+ very sensitive to some not very accurately known counterterms.

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