Electromagnetic meson form factors in a covariant Salpeter model

C. R. Münz, J. Resag, B. C. Metsch, and H. R. Petry
Institut für Theoretische Kernphysik, Universität Bonn, Nußallee 14-16, 53115 Bonn, Germany
(Received 29 June 1994; revised manuscript received 13 February 1995)

We present a covariant formulation of the Salpeter equation for $q\bar{q}$ states in order to calculate bound-state transitions between mesons. The corresponding Bethe-Salpeter amplitudes are reconstructed from equal-time amplitudes which were obtained in a previous paper by solving the Salpeter equation for a confining plus an instanton-induced interaction. This method is applied to calculate electromagnetic form factors and decay widths of low-lying pseudoscalar and vector mesons, including predictions for CEBAF experiments. We also describe the momentum transfer dependence for the processes $\pi^0,\eta,\eta'\rightarrow\gamma\gamma^*$.

PACS number(s): 13.40.Gp, 14.40.Aq, 12.39.−x, 11.10.St

I. INTRODUCTION

In two previous papers [1,2] we presented a quark model for light mesons based on the Salpeter equation. We investigated a kernel that incorporates confinement and a residual instanton-induced quark interaction [3–5], which in this framework leads to the correct masses and flavor mixing of the π and η mesons. In general we obtained a satisfactory description of the mass spectrum of the low-lying pseudoscalar and vector mesons. We also calculated various decay observables such as the weak decay constants, the $\gamma\gamma$-decay width of the pseudoscalars, and the leptonic widths of vector mesons. A comparison with results of nonrelativistic models revealed the relevance of the relativistic treatment (including the correct normalization of the bound states) for the description of these observables, especially for the two-photon width of the pion.

All these transitions involve a nonhadronic final state and therefore could be calculated in the rest frame of the bound state where the amplitudes were determined. A relativistic quark model, however, should also be able to describe reactions with a mesonic final state. If we consider for example weak decays of heavy to light mesons or electromagnetic scattering with large momentum transfer, the outgoing meson recoils with relativistic velocity. The calculation of such transitions between mesonic states therefore involves a boost of at least one of the meson amplitudes. In the present contribution we will give a covariant formulation of the Salpeter equation [6], which enables us to treat this boost correctly and thus to investigate the region of large momentum transfer available for instance in CEBAF experiments in the near future [7–9].

A second important ingredient of any relativistic quark model is an adequate treatment of the off-shell properties of the quarks. Especially for mesonic states with large binding energy the negative-energy Dirac components become essential. If one considers form factors at high momentum transfer, the quarks are highly off shell. The Salpeter model in the form presented here allows for a consistent inclusion of these effects.

In Sec. II of this paper we repeat the covariant formulation of the Salpeter equation and the construction of a four-dimensional Bethe-Salpeter amplitude as well as its transformation properties. In Sec. III we present our ansatz to calculate the electromagnetic current in the Mandelstam formalism. Finally in Sec. IV we show our results for the electromagnetic $0^-\rightarrow0^-$ and $1^-\rightarrow0^-$ form factors, the corresponding decay widths, and the form factors for the processes $\pi^0,\eta,\eta'\rightarrow\gamma\gamma^*$.

II. COVARIANT FORMULATION OF THE SALPETER EQUATION AND RECONSTRUCTION OF THE BETHE-SALPETER AMPLITUDE

The Bethe-Salpeter equation for the amplitude

$$[\chi_P(x)]_{\alpha\beta}=\langle 0|T\Psi^1_\alpha(\eta_1 x)\bar{\Psi}^2_\beta(-\eta_2 x)|P\rangle, \tag{1}$$

reads in momentum space [10,2]:

$$\chi_P(p)=S^F_1(p_1)\int\frac{d^4p'}{(2\pi)^4}\,[-iK(P,p,p')\chi_P(p')]\times S^F_2(-p_2), \tag{2}$$

where $p_1=\eta_1P+p$, $p_2=\eta_2P-p$ denote the momenta of the quark and antiquark, respectively, P is the four-momentum of the bound state, and S^F and K are the Feynman propagators and the irreducible interaction kernel. Here η_1,η_2 are two arbitrary real numbers satisfying $\eta_1+\eta_2=1$.

In our model, both S_F and K are given by a phenomenological ansatz described as follows: We introduce the components of the relative momentum p parallel to the total momentum $p_{\|P}:=pP/\sqrt{P^2}$ and perpendicular to it $p_{\perp P}:=p-(pP/P^2)P$, and assume that the interaction kernel only depends on the components of p and p' perpendicular to P [6], i.e.,

$$K(P,p,p')=V(p_{\perp P},p'_{\perp P}). \tag{3}$$

Furthermore it is assumed that the propagators are given by their free form $S_i^F(p)=i/(\not{p}-m_i+i\epsilon)$, where m_i is an effective constituent-quark mass. From $d^4p=dp_{\|P}d^3p_{\perp P}$ one can integrate out $dp_{\|P}$ and $dp'_{\|P}$ in Eq. (2) by introducing a covariant Salpeter amplitude

$$\Phi_P(p_{\perp P}):=\int dp_{\|P}\chi_P(p_{\|P},p_{\perp P}), \tag{4}$$

which leads to

$$\Phi_P(p_{\perp P})=\int dp_{\|P}S_1^F(P/2+p)\int\frac{d^3p'_{\perp P}}{(2\pi)^4}[-iV(p_{\perp P},p'_{\perp P})\Phi_P(p'_{\perp P})]S_2^F(-P/2+p), \tag{5}$$

where $\int dp_{\|P}$ can in principle be calculated for any on-shell momentum P. Though all the quantities have been introduced covariantly, one has to choose a specific frame for solving the equation. Obviously the rest frame of the bound state is distinguished, where one has $p_{\perp P}:=(0,\vec{p})$. Hence in this frame the interaction is instantaneous in the original sense of Salpeter,

$$V(p_{\perp P},p'_{\perp P})|_{P=(M,\vec{0})}=V(\vec{p},\vec{p}\,'), \tag{6}$$

with the equal-time amplitude

$$\Phi(\vec{p}):=\int dp^0\chi_P(p^0,\vec{p})|_{P=(M,\vec{0})}=\int dp_{\|P}\chi_P(p_{\|P},p_{\perp P})|_{P=(M,\vec{0})}, \tag{7}$$

so that from Eq. (5) one arrives at the well-known Salpeter equation [11], i.e.,

$$\Phi(\vec{p})=\int\frac{d^3p'}{(2\pi)^3}\frac{\Lambda_1^-(\vec{p})\gamma^0[V(\vec{p},\vec{p}\,')\Phi(\vec{p}\,')]\gamma^0\Lambda_2^+(-\vec{p})}{M+\omega_1+\omega_2}-\int\frac{d^3p'}{(2\pi)^3}\frac{\Lambda_1^+(\vec{p})\gamma^0[V(\vec{p},\vec{p}\,')\Phi(\vec{p}\,')]\gamma^0\Lambda_2^-(-\vec{p})}{M-\omega_1-\omega_2}, \tag{8}$$

with the projectors $\Lambda_i^{\pm}=(\omega_i\pm H_i)/(2\omega_i)$, the Dirac Hamiltonian $H_i(\vec{p})=\gamma^0(\vec{\gamma}\vec{p}+m_i)$, and $\omega_i=(m_i^2+\vec{p}^{\,2})^{1/2}$.

The amplitudes Φ have been calculated by solving the Salpeter equation for a kernel including a confining plus a residual instanton-induced interaction. The parameters have been fixed to reproduce the masses of the pseudoscalar and vector mesons, the weak decay constant of the pion, and the leptonic width of the ρ meson. The results have been presented in [1], we use model V1 therein for the calculations presented below. The confinement interaction has been described by a timelike vector spin structure

$$\int d^3p'[V_C^V(\vec{p},\vec{p}\,')\Phi(\vec{p}\,')]=-\int d^3p'\mathscr{V}_C((\vec{p}-\vec{p}\,')^2)\gamma^0\Phi(\vec{p}\,')\gamma^0 \tag{9}$$

as a scalar confinement leads to a random phase approximation (RPA) instability of the Salpeter equation [12,13]. The scalar function $\mathscr{V}_C$ in coordinate space is given by a linearly rising potential $\mathscr{V}_C(r)=a_c+b_cr$, in analogy to nonrelativistic quark models; see, for instance, [14,5]. Note that $\gamma^0\otimes\gamma^0$ can be written as $\gamma_{\|P}\otimes\gamma_{\|P}$, so that the previous statements on formal covariance hold.

In order to reproduce the spectrum of the pseudoscalar mesons, we used an additional instanton-induced interaction given by 't Hooft [3–5,1]. It acts only on pseudoscalar and scalar mesons and has the form

$$\int d^3p'[V_T(\vec{p},\vec{p}\,')\Phi(\vec{p}\,')]=4G\int d^3p'V_{\text{reg}}(\vec{p}-\vec{p}\,')\{1\,\text{tr}[\Phi(\vec{p}\,')]+\gamma^5\text{tr}[\Phi(\vec{p}\,')\gamma^5]\}, \tag{10}$$

where G is a flavor matrix containing the coupling constants. Here summation over flavor has been suppressed and V_{reg} is a regularizing Gaussian function (see [1] for more details).

To arrive at a fully covariant calculation of transition matrix elements with energy-momentum conservation for both particles, the Bethe-Salpeter amplitude $\chi_P(p)$ depending on the relative four-momentum p has to be known. It can be reconstructed from the equal-time amplitude $\Phi(\vec{p})$ in the rest frame as follows. From the Bethe-Salpeter equation itself one finds that the amputated Bethe-Salpeter (BS) amplitude or meson-quark-antiquark vertex function $\Gamma_P(p):=[S_1^F(p_1)]^{-1}\chi_P(p)[S_2^F(-p_2)]^{-1}$ only depends on the variables of a three-dimensional subspace $p_{\perp P}$:

$$\Gamma_P(p)=\Gamma_P(p_{\perp P})=-i\int\frac{d^3p'_{\perp P}}{(2\pi)^4}[V(p_{\perp P},p'_{\perp P})\Phi_P(p'_{\perp P})]. \tag{11}$$

The vertex function is computed in the rest frame from the equal-time amplitude as

$$\Gamma_P(p_{\perp P})|_{P=(M,\vec{0})}=\Gamma(\vec{p})=-i\int\frac{d^3p'}{(2\pi)^4}[V(\vec{p},\vec{p}\,')\Phi(\vec{p}\,')]. \tag{12}$$

From the transformation law for the Dirac field operators $U_\Lambda\Psi(x)U_\Lambda^+=S_\Lambda^{-1}\Psi(\Lambda x)$ and the corresponding properties of the bound state $|P\rangle$ with mass M one derives that the Bethe-Salpeter amplitude behaves as

$$\chi_{\Lambda P}^{JM_J}(p)=\sum_{M_J'}S_\Lambda\chi_P^{JM_J'}(\Lambda^{-1}p)S_\Lambda^{-1}D_{M_JM_J'}^{J*}(u(\Lambda,P)), \tag{13}$$

under a Lorentz transformation Λ, where $u(\Lambda,P):=\Lambda_{\Lambda P}^{-1}\Lambda\Lambda_P$ is the corresponding Wigner rotation and the boost Λ_P is defined by $P=\Lambda_P(M,\vec{0})$.

For a pure boost Λ_P we can calculate the BS amplitude in any reference frame as

$$\chi_P(p)=S_{\Lambda_P}\chi_{(M,\vec{0})}(\Lambda_P^{-1}p)S_{\Lambda_P}^{-1}. \tag{14}$$

The covariant transformation law is justified from the covariant momentum dependence (3) of the interaction kernel. The kinematical boost (14) thus gives the solution of the Bethe-Salpeter equation for any on-shell momentum P of the bound state.

III. TRANSITION AMPLITUDES IN THE SALPETER FORMALISM

The general prescription for calculating any current matrix element between bound states has been given by Mandelstam [15], see, e.g., [16] for a comprehensive presentation. Consider for example the electromagnetic current operator: It may be calculated from the BS amplitudes and a kernel $K^{(\gamma)}$ as shown in Fig. 1. $K^{(\gamma)}$ denotes a kernel irreducible with respect to the incoming and outgoing quark-antiquark pairs, i.e., it includes all diagrams that cannot be split by just cutting the quark and the antiquark lines.

To lowest order the kernel shown in Fig. 2 reads explicitly

$$K_\mu^{(\gamma)}(P,q,p,p')=-e_1\gamma_\mu^{(1)}S_2^{F^{-1}}(-P/2+p)\delta(p'-p+q/2)-e_2\gamma_\mu^{(2)}S_1^{F^{-1}}(P/2+p)\delta(p'-p-q/2), \tag{15}$$

where p and p' denote the relative momenta of the incoming and outgoing $q\bar{q}$ pairs, e_1 and $-e_2$ are the charges of the quark and antiquark, and $q=P-P'$ is the momentum transfer of the photon. Without loss of generality we use $\eta_1=\eta_2=1/2$, as the result is independent of this choice.

For identical mesons in the initial and final state and vanishing photon momentum, this is consistent with the general normalization condition for bound states given by Cutkosky [17], which we already used in [1,2]. In that way the form factor is properly normalized.

For the electromagnetic current coupling, e.g., to the first quark we have explicitly

$$\langle P'|j_\mu^{(1)}(0)|P\rangle=-e_1\int\frac{d^4p}{(2\pi)^4}\,\mathrm{tr}\{\bar{\Gamma}_{P'}((p-q/2)_{\perp P'})S_1^F(P/2+p-q)\gamma_\mu S_1^F(P/2+p)\Gamma_P(p_{\perp P})S_2^F(-P/2+p)\} \tag{16}$$

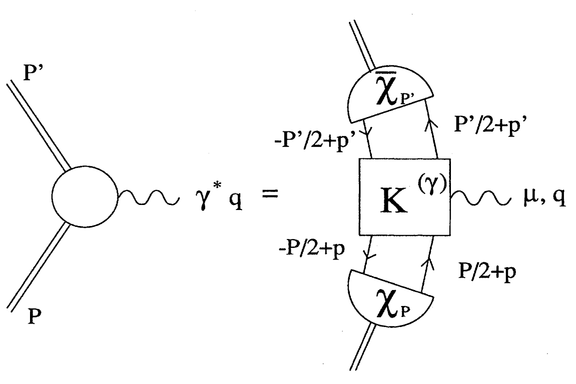

FIG. 1. The electromagnetic current in the Mandelstam formalism calculated from the Bethe-Salpeter amplitudes $\chi_P, \bar{\chi}_{P'}$ and the kernel $K^{(\gamma)}$. The relevant momenta are also defined.

in terms of the vertex functions given in the previous section. These amplitudes depend only on three components of the relative momentum variable. This is a direct consequence of our covariant Salpeter ansatz. In our model it is thus assumed that the dependence of the vertex function on the component parallel to the total momentum is negligible.

From the transformation properties given in Eq. (13) one can show that the matrix elements of the current *transform*

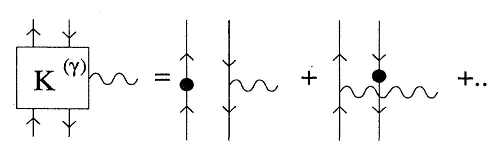

FIG. 2. Lowest order terms of the perturbative expansion of the kernel $K^{(\gamma)}$; the solid circle denotes an inverse quark propagator.

covariantly and thus can be evaluated in any reference frame. For instance the Lorentz transformation Λ of spinless mesons with momenta P and P' gives

$$\langle P'|j_\mu(x)|P\rangle = (\Lambda^{-1})_\mu^\nu \langle \Lambda P'|j_\nu(\Lambda x)|\Lambda P\rangle. \qquad (17)$$

We have chosen to perform the calculation in the rest frame of the incoming meson. According to Eq. (12) the vertex function of the initial meson depends only on the three-momentum $\vec{p}$ and not on p^0. In the case of elastic transitions with $Q^2 := -(P-P')^2 = 0$ we have $P' = P = (M,\vec{0})$, so that the outgoing vertex $\overline{\Gamma} = -\gamma_0 \Gamma^+ \gamma_0$ [2] is only a function of $\vec{p}$. Since the p^0 dependence is contained only in the one-particle propagators, the integral may be calculated analytically by contour integration according to the Feynman prescription. The remaining integration on $|\vec{p}|$ is done numerically.

For spacelike momentum transfer $Q^2 > 0$ or nonidentical mesons, the outgoing amplitude has to be boosted according to Eq. (14). This gives the vertex function an explicit dependence on the energy component p^0 of the relative momentum of the incoming quark-antiquark pair. There are, however, *no singularities* on the real p^0 axis. We thus cannot close the contour in the p^0 plane, as we do not know the analytic structure of Γ in the whole complex plane.

We therefore proceed as follows. The p^0 integral has the generic form

$$\int_{-\infty}^{+\infty} dp^0 \, \frac{f(p^0)}{\Pi_{i=1,6}(p^0 - p_i^0 \pm i\epsilon)}, \qquad (18)$$

where $f(p^0)$ is a regular function containing the spin term of the one-particle propagators and the vertex functions. The six poles correspond to the denominators of the three propagators. The integral can be decomposed into a contribution from the residues and a principal-value integral. We retain only the contribution from the poles because the principal-value integral should vanish in a consistent covariant field theory in order to yield a Hermitian current (in fact the principal-value term corresponds to the expectation value of an anti-Hermitian operator). Within our model the prescription is actually an additional assumption supplementing the Mandelstam formalism. We checked numerically (see Sec. IV) that the principal value indeed does not vanish.

Apparently this non-Hermitian contribution is induced by the wrong analytic structure of the vertex functions inherent to the neglect of retardation effects. Indeed, a calculation of the form factor for bound states of scalar massive particles with a massless scalar exchange *with retardation* (Wick-Cutkosky model [18]) along the same lines shows that the unphysical non-Hermitian part of the current vanishes [22].

For the numerical calculation of the spin part of the current we use an angular-spin decomposition of the Bethe-Salpeter amplitudes for particle-antiparticle states presented in [2]. The trace of the products of Dirac matrices is evaluated with standard algebraic software. The remaining integration on $|\vec{p}|$ and $\cos\Theta_p$ is performed numerically by a Gaussian integration routine with variable step size.

The radial part of the vertex functions as well as the Salpeter amplitudes has been expanded in a basis of 11 Laguerre functions. The results are found to be stable within a large range of the scale parameter of the basis.

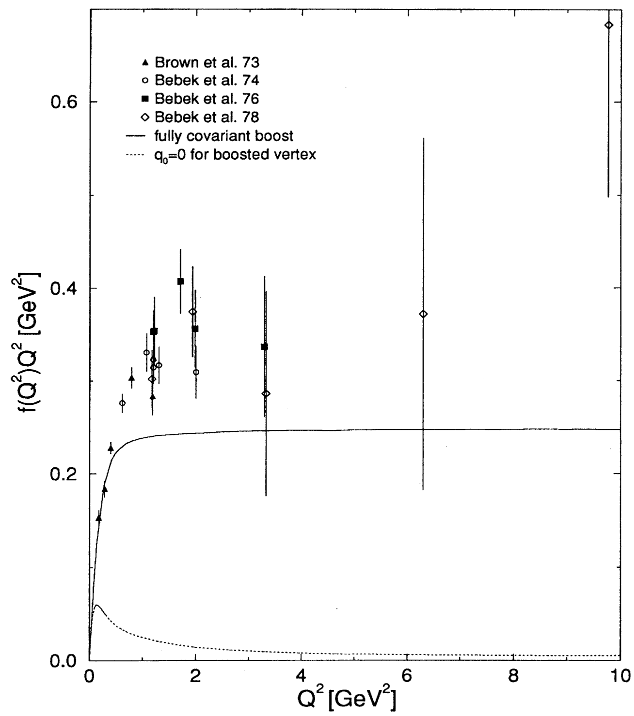

FIG. 3. The pion form factor at large momentum transfer compared to results from pion electroproduction; see [19] and the references given therein. The solid curve represents the calculation with the correct boost of the vertex function, the dashed line is obtained by neglecting the dependence on the time component of its relative momentum.

As our model includes confinement, we can also have mesons with mass M larger than the sum $m_1 + m_2$ of the constituents. The pinching singularities, which appear in general for such states at values of the relative momentum $\vec{p}$ where both the particle and the antiparticle are on their mass shell, are canceled by the zeros of the trace of the spin part, so that the integral remains well defined. In fact, the projection of the vertex function on positive energies $\Gamma_{\text{pos}}(\vec{p}) := \Lambda_1^+(\vec{p})\gamma^0\Gamma(\vec{p})\gamma^0\Lambda_2^-(-\vec{p})$ vanishes if both the quark and the antiquark are on shell. This means that the amplitude for the bound state decaying into a free quark and antiquark vanishes, so that confinement in this channel is guaranteed.

The present formulation of the Salpeter equation and the corresponding electromagnetic current in terms of the four-dimensional Bethe-Salpeter amplitude thus constitutes a covariant framework for the calculation of transitions between bound states which seems an interesting alternative to the existing calculations in the Salpeter model; see, for instance, [23,24]. There an effective current is introduced in terms of the three-dimensional equal-time amplitudes; four-momentum conservation is spoiled by integrating only in a three-dimensional subspace and therefore the formulation cannot be made covariant.

However, it should be noted at this point that our current given in Eq. (16) will in general not be conserved. Yet, the

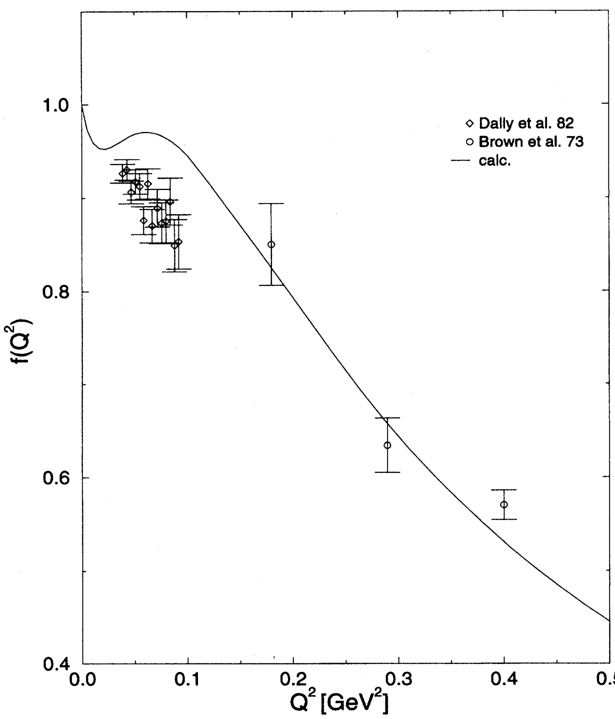

FIG. 4. The pion form factor at small momentum transfer [19,20]: Because of the large binding energy, the *shape* of the pion form factor becomes very sensitive to the form of the potential; this indicates the limits of the Salpeter approach for very deeply bound states.

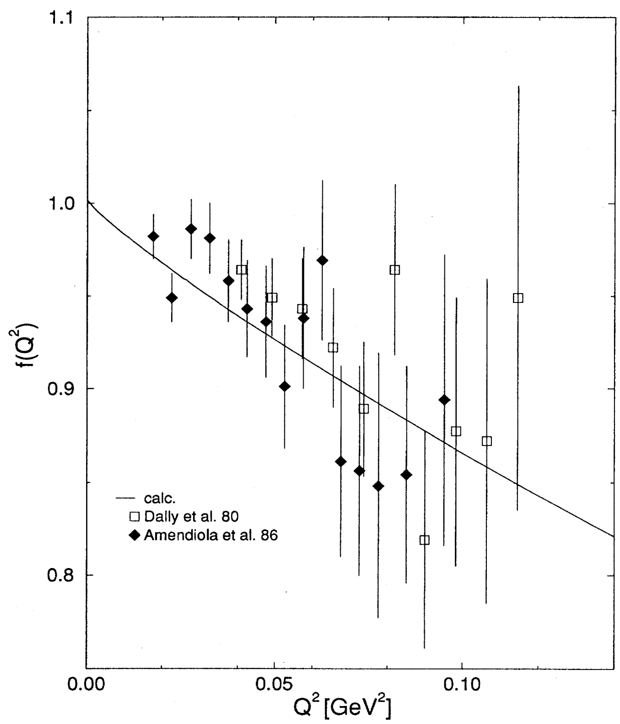

FIG. 5. The charged kaon form factor at small momentum transfer compared to data from electron scattering [26,27].

contraction of the current matrix element with the momentum transfer vanishes in the special case of a transition between identical pseudoscalar mesons or between a vector and a pseudoscalar state. This can be shown analytically from the formal covariance, time reversal, and parity, and is also fulfilled numerically in our model. Therefore the violation of current conservation does not affect the transitions that will be studied in the following.

IV. RESULTS AND DISCUSSION

As already mentioned in Sec. II, the parameters of the model were adjusted in a previous work [1] (model V1) to reproduce the mass spectrum and the decay observables with nonhadronic final states of the pseudoscalar and vector mesons. We refrained from readjusting them in order to illustrate the predictive power of our approach. The electromagnetic transitions calculated below thus have *no free parameters*, and give a further test of the Salpeter model for mesons as well as predictions for future experiments.

A. Form factors $M \to M'\gamma^*$

The electromagnetic form factor $f(Q^2)$ of pseudoscalar mesons is defined by

$$\langle P'|j_\mu(0)|P\rangle = ef(Q^2)(P'+P)_\mu, \tag{19}$$

with e the total electric charge of the meson. Consider first the pion which in a constituent-quark model is the most deeply bound state. In Fig. 3 we compare our results (solid line) for $Q^2 \cdot f(Q^2)$ with experimental data up to 10 GeV^2 [19]. The agreement is rather good even for high momentum transfer. The error bars for some of the data points are still very large, but an experiment planned at CEBAF [7] should improve the situation. Obviously $f(Q^2)$ behaves as $1/Q^2$ for large Q^2, which supports a statement by Isgur and Llewellyn Smith [21] that the form factor in this region should be explained by nonperturbative effects. We would like to mention similar calculations in the quasipotential formalism by Tiemeijer and Tjon [23] and in a separable ansatz including chiral symmetry breaking by Ito, Buck, and Gross [25]. Their calculations show a stronger falloff at higher momentum transfer. To analyze this effect we performed a calculation where the p^0 dependence of the outgoing vertex function is neglected (dashed line): The form factor is strongly suppressed at high Q^2. Our results at high momentum transfer thus depend importantly on the particular covariant treatment of the relative momentum dependence.

Yet, for small momentum transfer of the order of the quark mass the form factor of the pion is not strictly monotonic as one would expect, e.g., from a naive vector-dominance model, so that the determination of the charge radius seems ambiguous; see Fig. 4. We find that for states, where the binding energy is very large, as in the case of the pion, the *shape* of the form factor becomes sensitive towards the form of the interaction potential. By decreasing, e.g., the range of the instanton induced interaction, the maximum at 0.1 GeV^2 is less pronounced, but still present. We thus find that the Salpeter approach is limited for the calculation of the charge radius of very deeply bound states. The $1/Q^2$ behav-

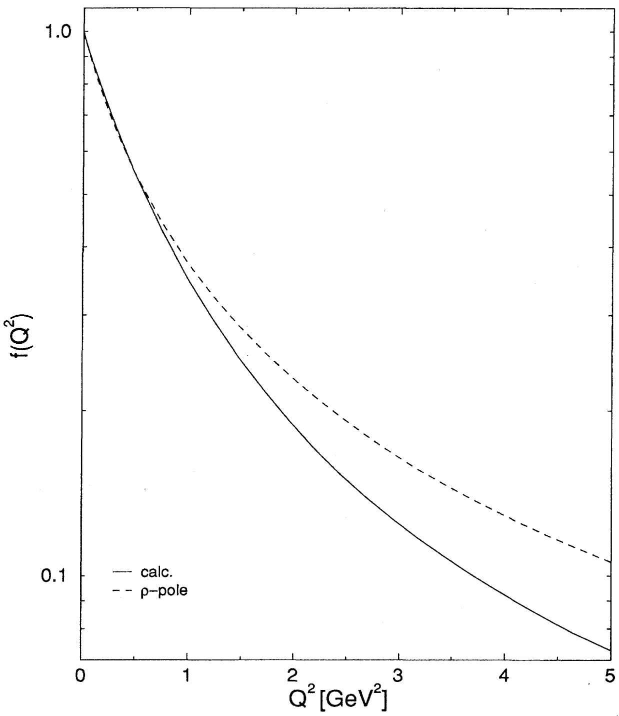

FIG. 6. The charged kaon form factor at large momentum transfer (solid line) compared to a ρ pole motivated by the VDM (dashed line).

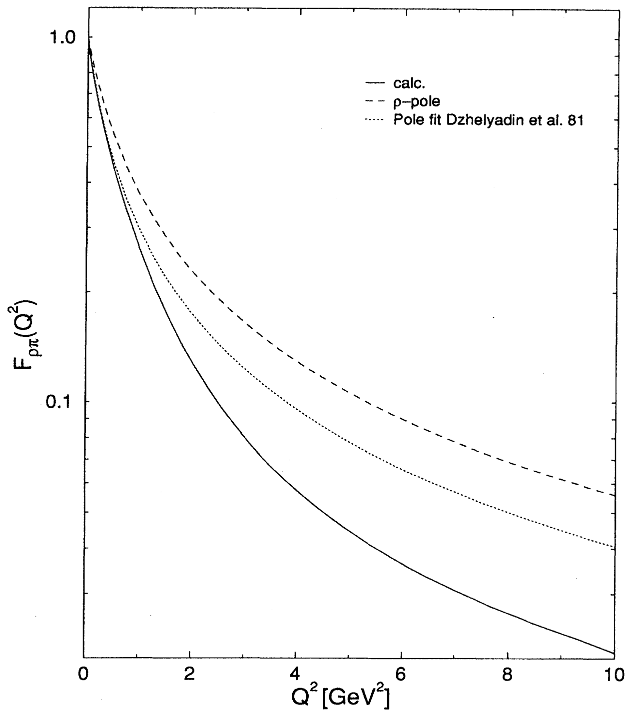

FIG. 8. The normalized $\rho\pi\gamma^*(\omega\pi\gamma^*)$ form factor at large momentum transfer.

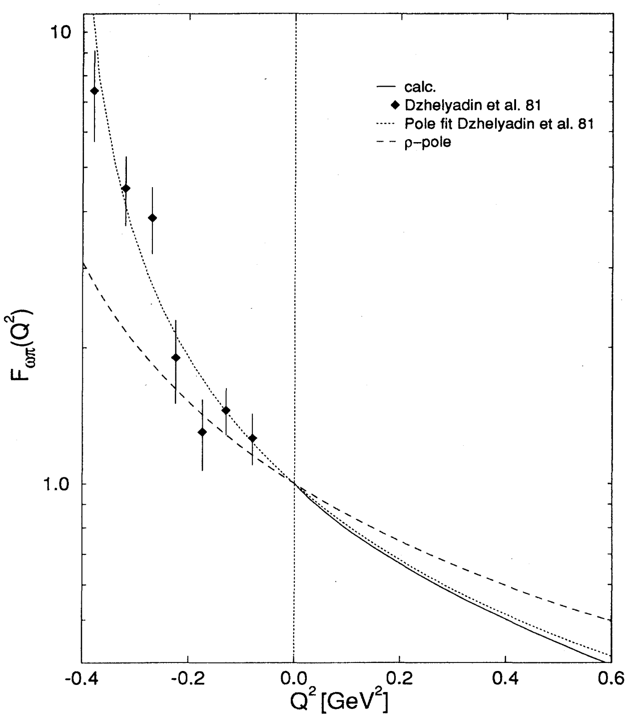

FIG. 7. Comparison of the normalized $\omega\pi\gamma^*$ form factor (solid line) in the spacelike region with an extrapolation of experimental data in the timelike region [29] (dotted line) and with a ρ-pole ansatz motivated by vector dominance (dashed line).

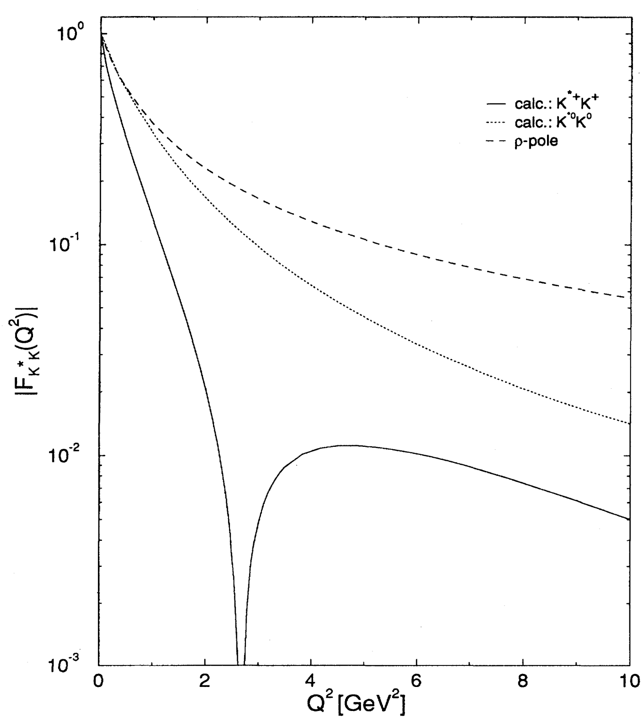

FIG. 9. The normalized neutral (dotted line) and charged (solid line) $K^*K\gamma^*$ form factors compared to VDM (dashed line).

TABLE I. Dependence of the K and K^* mass, weak decay constant f_K, charged kaon radius, decay widths, and zero of the form factor for $K^{*\pm}\to K^{\pm}\gamma$ on the ratio of the strange and nonstrange constituent-quark mass m_s/m_n (masses and decay constant given in MeV).

m_s/m_n	M_K	M_K^*	f_K	$\langle r^2_{K^\pm}\rangle$[fm]	$\Gamma_{K^{*\pm}\to K^\pm\gamma}$ [keV]	$\Gamma_{K^{*0}\to K^0\gamma}$ [keV]	Q_0^2 [GeV^2]
3.0	535	895	179	0.61	78	114	2.1
2.3	510	880	183	0.60	64	112	2.7
1.8	485	865	185	0.59	52	111	4.8
1.5	475	860	187	0.59	45	110	>10
1.0	465	855	189	0.57	27	109	–
Experiment	495	892	164	0.58 ± 0.04	50 ± 5	117 ± 10	

ior at high momentum transfer, on the other hand, is determined by the covariant boost of the vertex function and independent of the form of the interaction potential, so that we consider our results to be reliable.

One should also keep in mind that in our calculation the matrix element of the electromagnetic current contains an unphysical non-Hermitian term, which we dropped in a rather *ad hoc* manner. Its size is about 10% of the pion form factor at $Q^2=0.5$ GeV^2 and 30% at $Q^2=10$ GeV^2. As already mentioned, we consider this as an estimate of the accuracy of our approach.

In the case of the kaon we obtain a very good description of the form factor at small momentum transfer; see Fig. 5. We find an electromagnetic charge radius for the charged kaon of $\langle r^2_{K^\pm}\rangle^{1/2}_{\rm calc}=0.60$ fm as compared to $\langle r^2_{K^\pm}\rangle^{1/2}_{\rm expt}=0.58\pm0.04$ fm [26] or $\langle r^2_{K^\pm}\rangle^{1/2}_{\rm expt}=0.53\pm0.05$ fm [27] both from electron scattering data. For the neutral kaon we obtain $\langle r^2_{K^0}\rangle_{\rm calc}=-0.070$ fm^2 as compared to $\langle r^2_{K^0}\rangle_{\rm expt}=-(0.054\pm0.026)$ fm^2 [28]. The reliability of these results is not affected by an unnatural shape found for the pion form factor, as the binding energy of the kaon is much smaller, so that the Salpeter approach is justified.

In view of these agreements it is interesting to investigate the charged form factor in the region of high momentum transfer, which will be accessible in a CEBAF experiment [8]. Our prediction is plotted in Fig. 6 and compared to a vector dominance model (VDM). A deviation from a simple ρ-monopole ansatz $f(Q^2)=1/(1+Q^2/M_\rho^2)$ is predicted to appear for $Q^2>1$ GeV^2.

We studied the effects of the relativistic treatment by calculating only the contributions, where the quarks have positive energy. Apart from the noncovariance of the calculation this would correspond to using a reduced Salpeter equation. We find that the contribution of the negative energy states to the charged kaon form factor (and therefore to the normalization) is 25% for zero momentum transfer. It gives a radius of $\langle r^2{}_{K^\pm}\rangle^{1/2}_{\rm pos\ energy}=0.67$ fm, which is off the experimental error bars. Relativistic effects thus play an important role for light mesons even at small Q^2.

The form factor $f_{\rho\pi}(Q^2)$ for the process $\rho\to\pi\gamma^*$ (or $\omega\to\pi\gamma^*$) can be defined as

$$\langle\pi(P')|j_\mu(0)|\rho(P,\lambda)\rangle=e\frac{f_{\rho\pi}(Q^2)}{M_\rho}\,\varepsilon_{\mu\nu\sigma\tau}\epsilon^\nu(P,\lambda)P'^\sigma P^\tau, \tag{20}$$

where $\epsilon(P,\lambda)$ denotes the polarization vector of the ρ (or ω) meson with spin projection λ.

The transition $\omega\to\pi\gamma^*$ has been measured in the timelike region via the decay $\omega\to\pi^0\mu^+\mu^-$ [29], where the normalized quantity $F_{\omega\pi}(Q^2):=f_{\omega\pi}(Q^2)/f_{\omega\pi}(0)$ is fitted by a simple pole ansatz $F_{\omega\pi}(Q^2)=1/(1+Q^2/\Lambda^2_{\omega\pi})$ with $\Lambda^{\rm expt}_{\omega\pi}=(0.65\pm0.03)$ GeV. In Fig. 7 we compare the experimental results and the simple fit to our calculation in the spacelike region.[1] Our curve corresponds to a pole at $\Lambda^{\rm calc}_{\omega\pi}=0.63$ GeV. The extrapolation of the data to the spacelike region and our prediction thus agree excellently and differ from the vector dominance model which corresponds to a ρ pole at $\Lambda^{\rm VDM}_{\omega\pi}=0.77$ GeV, far off the experimental data. Thus we have found a process where a relativistic quark model is superior to the phenomenological vector dominance model even at small momentum transfer.

The $\rho\to\pi\gamma^*$ form factor, which in our model is degenerate with $\omega\to\pi\gamma^*$, is particularly important for calculating meson-exchange current corrections to the deuteron form factor. As there is also a direct experimental interest in this quantity [9,30], we plot our prediction for large momentum transfer in Fig. 8 and compare it to a simple ρ pole and to the pole fit of [29] for $\omega\to\pi^0\gamma^*$ discussed above.[2] At momentum transfer larger than 1 GeV^2, which is particularly relevant for relativistic effects in the deuteron, we find a deviation even from the simple pole fit. Ito *et al.* [25] obtained similar results for $\rho\to\pi\gamma^*$. These authors also discussed contributions beyond the impulse approximation including an interaction current [31]. They however neglect the confinement problem. Our absolute value of $f_{\rho\pi}(0)$ will be discussed in the context of the electromagnetic decay widths.

In the strange sector the form factors $K^{*\pm}\to K^\pm\gamma^*$ and $K^{*0}\to K^0\gamma^*$ are extremely interesting quantities, as the coupling of the current to the quark and the antiquark differ because of their unequal masses. Our results in Fig. 9 show a nearly VDM-like behavior for the neutral process at least at small momentum transfer. However, the picture is totally different for the charged kaons: The negative interference between the two currents leads to a zero in the form factor at $Q_0^2=2.7$ GeV^2, a region which is already highly relativistic. The effect may be understood qualitatively in a VDM-type

[1]A calculation of the quantity $F_{\omega\pi}(Q^2)$ becomes meaningless in the timelike region, as our model does not guarantee confinement in the $\rho\to\pi q\bar{q}$ channel, so that the graph diverges for $Q^2\to-(m_q+m_{\bar{q}})^2$.

[2]Assuming SU(2) flavor symmetry for $F_{\omega\pi}$ and $F_{\rho\pi}$ which of course might not be true experimentally.

TABLE II. Comparison of experimental and calculated electromagnetic meson decay widths.

Mesonic decay width [keV]	Experimental [32]	Calculated
$\Gamma(\rho^\pm \to \pi^\pm \gamma)$	68 ± 7	38
$\Gamma(\rho^0 \to \pi^0 \gamma)$	121 ± 31	38
$\Gamma(\omega \to \pi \gamma)$	717 ± 43	335
$\Gamma(K^{*\pm} \to K^\pm \gamma)$	50 ± 5	64
$\Gamma(K^{*0} \to K^0 \gamma)$	117 ± 10	112
$\Gamma(\rho \to \eta \gamma)$	58 ± 10	50
$\Gamma(\omega \to \eta \gamma)$	4.0 ± 1.7	5.6
$\Gamma(\phi \to \eta \gamma)$	56.9 ± 2.9	60
$\Gamma(\eta' \to \omega \gamma)$	5.9 ± 0.9	12.7
$\Gamma(\eta' \to \rho \gamma)$	59 ± 6	122
$\Gamma(\phi \to \eta' \gamma)$	< 1.8	0.18 ± 0.02

model, where the coupling to the quark and antiquark is assumed to be proportional to their respective magnetic moments and to a propagator of the corresponding vector meson, ρ, or Φ. The result, however, depends sensitively on the ratio of the mass of the strange (m_s) and nonstrange (m_n) constituent quark. We varied these masses keeping the sum of them as well as the other parameters fixed and obtained the dependence on the m_s/m_n ratio shown in Table I. Our original fit in [1] corresponds to $m_n = 170$ MeV and $m_s = 390$ MeV, i.e., a ratio $m_s/m_n = 2.3$.

Because of the good agreement with the experimental decay widths (see next section), we consider these calculations to be even more reliable than in the $\rho\pi\gamma$ case. We therefore encourage an experimental investigation of the strange form factors, e.g., at the CEBAF facility to provide empirical information on the ratio of the constituent-quark masses.

B. Decay width $M \to M'\gamma$ for the ground state mesons

There exist several measurements of an excited meson decaying to a state with lower mass by emission of a single real photon [32]. These processes provide a suitable test of the Bethe-Salpeter amplitudes especially for resonances where no detailed study of form factors is available. The results for the transitions between vector and pseudoscalar mesons are summarized in Table II. If the mass difference is large, the final meson is emitted with relativistic velocity, so that a covariant framework is essential.

In the semirelativistic ansatz of Godfrey and Isgur [33] the nonrelativistic decay formulas have been modified by terms $(m/E)^n$, where m is the quark mass and E its energy. These corrections involve new *ad hoc* parameters n. The relativistic framework presented here includes these effects

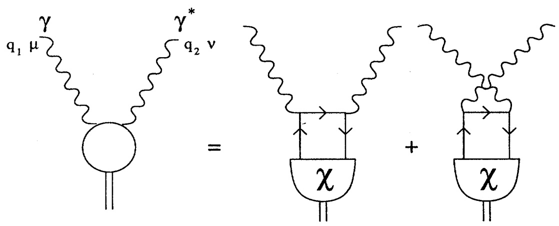

FIG. 10. Feynman graph for the decay into two photons.

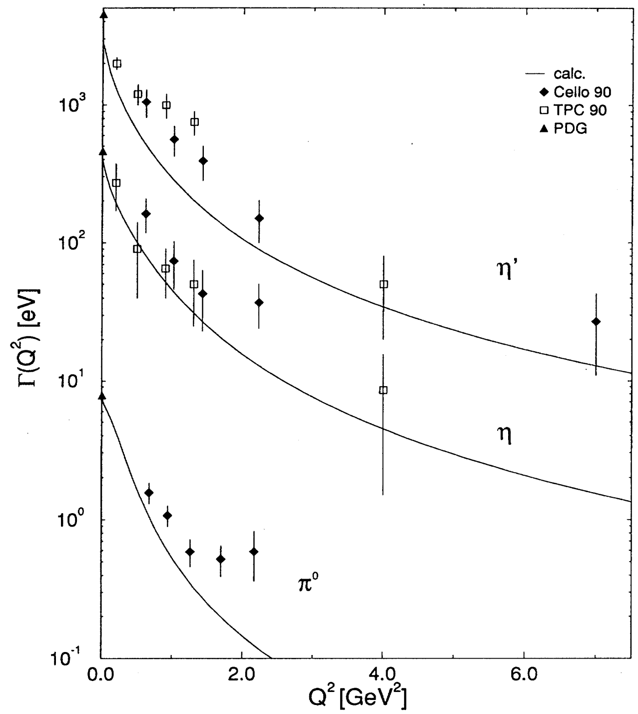

FIG. 11. The π^0, η, and $\eta' \to \gamma\gamma^*$ form factors compared to experimental data [34,35].

automatically, without free parameters.

The widths for decays with a pion in the final state are generally underestimated by a factor of 2.[3] Our results are consistent with those of Tiemeijer [24] in a similar equal-time formalism. This again indicates that the Salpeter formalism is not fully satisfactory in the case of very deeply bound states.

The rates $K^* \to K\gamma$ are in excellent agreement with measurements for both the neutral and the charged strange mesons. This, together with the good results of the form factor, indicates that the kaon is well understood in the Salpeter model. Again the width for the charged decay depends on the ratio of the constituent-quark masses (see Table I). A ratio of $m_s/m_n = 1.5-1.9$ is favored, whereas the neutral decay is almost insensitive to this quantity.

The electromagnetic decays involving a η (η') meson in the final (initial) state are a sensitive test of the $n\bar{n}$ and $s\bar{s}$ component of their Bethe-Salpeter amplitude and therefore of the interaction which induces flavor mixing. The decays $\rho, \omega \to \eta\gamma$ involve the $n\bar{n}$ component, $\Phi \to \eta\gamma$ the $s\bar{s}$ component. The agreement is excellent for these three rates, confirming the good description of the mixing coefficients for the η given in [1]. For the η' decaying into ρ or ω, our calculated rate is too large by a factor of around of 2, consistent with the fact that in our model the $n\bar{n}$ component of

[3]From SU(2)-isospin symmetry the decay width $\rho^0 \to \pi^0\gamma$ should be the same as for $\rho^\pm \to \pi^\pm\gamma$ and by a factor of 9 smaller than the width for $\omega \to \pi^0\gamma$, and therefore this experimental value has to be considered with care.

the η' is larger than the semiempirical value extracted from J/Ψ decay [1].

Our prediction for the decay width $\Phi\rightarrow\eta'\gamma$ includes the estimated error from the inaccuracy in the calculated meson masses which enters the transition matrix elements. As we underestimate the $s\bar{s}$ component of the η' only by a small amount [1], we expect our result for this experimental value to be quite accurate.

C. Form factors $M\rightarrow\gamma\gamma^*$

The structure of the Bethe-Salpeter amplitude for neutral pseudoscalar mesons may be tested by the production via a virtual and an (almost) real photon as done in $\gamma\gamma$ experiments at e^+e^- colliders [34,35]. To lowest order the process is given by the graphs shown in Fig. 10. If one of the photons is on shell, i.e., $q_1^2=0$, the amplitude can be parametrized as

$$T_{\mu\nu}(q_1,q_2)=\epsilon_{\mu\nu\alpha\beta}q_1^\alpha q_2^\beta f_{M\gamma}(Q_2^2). \tag{21}$$

As in the analysis of the experimental data given in [34,35], we define a width

$$\Gamma(Q^2)=\frac{M^3}{64\pi}f_{M\gamma}^2(Q^2), \tag{22}$$

which for $Q^2=0$ gives the actual decay width for a pseudoscalar into two real photons.

The calculated values of $\Gamma(Q^2)$ for the π^0, η, and η' are shown in Fig. 11 and compared to the experimental results [34,35]. The decay widths have already been published and are in good agreement with experimental data [1].

The width for the process $\pi^0\rightarrow\gamma\gamma^*$ depends sensitively on the quark mass, a result already found by Ito *et al.* [25]. Their optimal value of the nonstrange quark mass $m_n=250$ MeV is in rough agreement with our result of $m_n=170$ MeV, which had been adjusted to obtain the correct pion decay constant.[4] As in the case of the charged pion form factor we find a structure for $\Gamma_{\pi^0\rightarrow\gamma\gamma}(Q^2)$ for low momentum transfer, which, however, is much less pronounced. We thus again find that the Salpeter approach reaches its limits for states with extremely large binding energy. The width for this process is also somewhat underestimated at larger momenta.

The structure of the η form factor can be rather well described up to 4 GeV^2. This supports the conclusion based on other observables that the η meson is well understood. Although the η' width is too small by about 30%, the dependence on Q^2 is well reproduced up to 8 GeV^2.

V. SUMMARY AND CONCLUSION

The first part of this paper contains a covariant formulation of the Salpeter equation, in order to describe transitions between bound states with large momentum transfer. The key issue in this respect is the interrelation between the equal-time Salpeter amplitude and the meson-quark-antiquark vertex function. The former allows for the formulation of the Salpeter equation as a well defined eigenvalue problem for the meson masses and amplitudes; the latter allows for the reconstruction of the corresponding Bethe-Salpeter amplitude. With the formalism developed by Mandelstam one can use the vertex functions, which depend only on the coordinates of a three-dimensional—although covariant—subspace, to calculate mesonic transitions and decays. This procedure respects covariance and current conservation for the transitions studied. However, in order to obtain a Hermitian current we have to adopt the additional prescription to take only the residue contributions of the one-particle propagators, as otherwise the neglect of retardation effects would yield a anti-Hermitian principal-value integral.

Applying this ansatz to a relativistic quark model that includes confinement and an instanton-induced flavor mixing interaction, we investigated the electromagnetic properties of the light pseudoscalar and vector mesons for the scalar and isoscalar states. To demonstrate the predictive power of our approach, we used the Bethe-Salpeter amplitudes calculated in a previous work [1], and thus all the present results were obtained without any additional free parameter.

We find an excellent description of all observables measured so far, especially for the η and K meson, and also for the lowest vector mesons. The flavor mixing of the η, which can be measured in the decays $\rho,\omega,\Phi\rightarrow\eta\gamma$, is correctly reproduced. The kaon form factor at small momentum transfer as well as the $K^*\rightarrow K\gamma$ widths is in very good agreement with the data. Our quark model prediction in the spacelike region of the $\omega\rightarrow\pi\gamma^*$ form factor agrees with the phenomenological extrapolation from the data in the timelike region, but differs significantly from the standard vector dominance model. We present predictions for the processes $\rho\rightarrow\pi\gamma^*$ and the kaon form factor in the large Q^2 regime which will be measured at CEBAF in the near future. We showed that the transition form factor $K^{*\pm}\rightarrow K^\pm\gamma^*$ represents an interesting observable, as its form depends strongly on the ratio of strange and nonstrange quark mass. Because of the negative interference of the quark and the antiquark currents, one obtains a zero in the amplitude, which we predict at $Q_0^2\approx 2.7$ GeV^2. From the decay width into a real photon we find $m_s/m_n=1.9-1.5$.

The description of the $1/Q^2$ behavior of the charged pion form factor in the region of high momentum transfer are sensitive to the formally covariant boost and our treatment of the time component of the relative momentum. Further work, however, is needed to assess the reliability of our model for evaluating the form factor. At present we are therefore studying electromagnetic transitions in a similar framework on the basis of the Wick-Cutkosky model [22].

In our opinion, we reach the limits of the Salpeter ansatz in the case of the pion, due to its strong binding. We find that the shape of the charged and neutral form factor on the scale of the quark mass becomes sensitive to the interaction. In our model there are structures which are not seen in experimental data. We could not find a kernel that includes confinement and describes the form factors in this region. This strong dependence is only present for states with extremely large binding energy and therefore does not, e.g., affect the kaon properties.

[4] A larger quark mass would give a smaller width at zero momentum transfer, but also a smaller slope.

The results show that a relativistic treatment of constituent quarks in the framework of the Salpeter model for mesons, including a relativistic normalization and a covariant boosting of the amplitudes is able to describe quantitatively various properties of the ground state pseudoscalar and vector mesons. In view of this success we shall devote future work to apply the formalism to a detailed study of the complete meson spectrum using the one-gluon-exchange potential for heavy systems.

ACKNOWLEDGMENTS

We appreciate the help of S. Hainzl, G. Zöller, and M. Fuchs in numerical calculations as well as encouraging remarks by K. Althoff. One of the authors (C.R.M.) would like to thank the CEBAF theory group and J. Piekarewicz for hospitality and interesting and fruitful discussions. This work was partially supported by the Bundesministerium für Forschung und Technologie.

[1] C. R. Münz, J. Resag, B. C. Metsch, and H. R. Petry, Nucl. Phys. **A578**, 418 (1994).
[2] J. Resag, C. R. Münz, B. C. Metsch, and H. R. Petry, Nucl. Phys. **A578**, 397 (1994).
[3] G. 't Hooft, Phys. Rev. D **14**, 3432 (1976).
[4] M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, Nucl. Phys. **B163**, 46 (1980).
[5] W. H. Blask, U. Bohn, M. G. Huber, B. C. Metsch, and H. R. Petry, Z. Phys. A **337**, 327 (1990).
[6] S. J. Wallace and V. B. Mandelzweig, Nucl. Phys. **A503**, 673 (1989).
[7] CEBAF experiment 93-021, The Charged Pion Form Factor, D. Mack, spokesman.
[8] CEBAF experiment 93-018, Kaon Electroproduction, O. K. Baker, spokesman.
[9] CEBAF experiment 93-012, Electroproduction of Light Quark Mesons, M. Kossov, spokesman.
[10] E. E. Salpeter and H. A. Bethe, Phys. Rev. **84**, 132 (1951).
[11] E. E. Salpeter, Phys. Rev. **87**, 328 (1952).
[12] A. Archvadze, M. Chachkhunashvili, and T. Kopaleishvili, Few-Body Syst. **14**, 53 (1993).
[13] J. Parramore and J. Piekarewicz, Nucl. Phys. **A585**, 705 (1995).
[14] E. Eichten *et al.*, Phys. Rev. Lett. **34**, 369 (1975).
[15] S. Mandelstam, Proc. R. Soc. London **233**, 248 (1955).
[16] D. Lurie, *Particles and Fields* (Interscience Publishers, New York, 1968).
[17] R. E. Cutkosky and M. Leon, Phys. Rev. **135**, B1445 (1964).
[18] G. C. Wick, Phys. Rev. **96**, 1124 (1954); R. E. Cutkosky, *ibid.* **96**, 1135 (1954).
[19] C. J. Bebek *et al.*, Phys. Rev. D **17**, 1693 (1978), and references therein.
[20] E. B. Dally *et al.*, Phys. Rev. Lett. **48**, 375 (1982).
[21] N. Isgur and C. H. Llewellyn Smith, Phys. Rev. Lett. **52**, 1080 (1984).
[22] B. C. Metsch (unpublished).
[23] P. C. Tiemeijer and J. A. Tjon, Phys. Lett. B **277**, 38 (1992).
[24] P. C. Tiemeijer, Ph.D. thesis, University of Utrecht, 1993.
[25] H. Ito, W. W. Buck, and F. Gross, Phys. Rev. C **45**, 1918 (1992); Phys. Lett. B **287**, 23 (1992).
[26] S. R. Amendiola *et al.*, Phys. Lett. B **178**, 435 (1986).
[27] E. B. Dally *et al.*, Phys. Rev. Lett. **45**, 232 (1980).
[28] W. R. Molzon *et al.*, Phys. Rev. Lett. **41**, 1213 (1978).
[29] R. I. Dzhelyadin *et al.*, Phys. Lett. **102B**, 296 (1981).
[30] O. K. Baker (private communication).
[31] F. Gross and D. O. Riska, Phys. Rev. C **36**, 1928 (1987).
[32] Particle Data Group, K. Hikasa *et al.*, Phys. Rev. D **45**, S1 (1992).
[33] S. Godfrey and N. Isgur, Phys. Rev. D **32**, 189 (1985).
[34] TPC-Two-Gamma Collaboration, H. Aihara *et al.*, Phys. Rev. Lett. **64**, 172 (1990).
[35] CELLO Collaboration, H.-J. Behrend *et al.*, Z. Phys. C **49**, 401 (1991).