Thermal effects on isoscalar giant resonance energies in hot nuclei

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The thermal effects on the energies of the isoscalar giant multipole resonances of hot nuclei are discussed and an approximate formula for the energy as a function of temperature is derived via a hydrodynamic theory. The energy difference between the isoscalar giant multipole resonance of a hot nucleus and its ground-state resonance depends on the competition between the volume expansion and the increase of the average kinetic energy per nucleon of hot nuclei, which lower and raise the resonance energy, respectively, and nearly counteract each other in magnitude. The variation of the isoscalar giant resonance energy with temperature is very small.

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I. INTRODUCTION

Over the past decade many aspects of giant resonances in hot nuclei have been studied. The experiments have found that the isovector giant dipole resonance (GDR) energies and strengths in hot nuclei are stable with respect to the corresponding ground-state resonance energies and strengths for nuclear temperatures $T \sim 1$–2 MeV and spins $I \leq 40\ \hbar$ [1,2]. Nevertheless, the width of a giant resonance increases rapidly with temperature [3–5], due to shape fluctuations. Meanwhile a number of theoretical investigations [1,6] have been performed as well and much progress has been made concerning the GDR. But the giant resonance in hot nuclei is such a sophisticated subject that many open problems still remain. Much effort has been put into studying its formation and decay, its energy, strength, and width dependences upon temperature and angular momentum, the relations between the giant resonances and other properties of nuclei, and so on. The study of giant resonances in hot nuclei has been one of the most interesting subjects in nuclear physics.

This paper is devoted to discussing the thermal effects on the energy variation of isoscalar giant multipole resonances in hot nuclei. A giant resonance is a collective mode of excitation in nuclei. The variation of the resonance energy with temperature sheds light on how the restoring force of the collective mode changes with temperature. Lipparini and Stringari [7] investigated the isoscalar giant monopole resonance and the GDR with the liquid drop model and derived a formula for the giant resonance energy as a function of temperature, predicting that the GDR energy would be reduced by 1 MeV when the temperature reached 4 MeV. Using a random-phase approximation (RPA), Garcias *et al.* [8] calculated the variation of the GDR energy for some nuclei for $T \geq 4$ MeV and obtained similar results. The most useful methods for theoretical studies of the giant resonances in hot nuclei are microscopic methods, e.g., Hartree-Fock (HF) + RPA [8], semiclassical methods, e.g., the Vlasov equation [9], and so on. These microscopic methods have mostly utilized numerical methods to obtain solutions. Just as in the experimental studies, most theoretical investigations of giant resonances in hot nuclei have paid attention to the properties of the GDR. In this paper the method of Nix and Sierk [10] has been adopted to study the energy variation of the isoscalar giant multipole resonance with temperature by using a hydrodynamic theory. The methods of statistical physics were employed to study the volume expansion of hot nuclei and the average kinetic energy per particle for a finite-temperature Fermi gas with interacting particles. Finally, the thermal effects that contribute to the energy variation of the giant resonances are analyzed and the temperature dependence of the energy of the isoscalar giant multipole resonances is discussed.

II. HYDRODYNAMIC METHOD

For the giant resonances in ground states many macroscopic models as well as microscopic models have been used to investigate their energies, widths, strengths, and other properties [11–13]. Bertsch [11] in his macroscopic model took account of the distortion of the Fermi surface and derived a formula for the energy of isoscalar giant quadrupole resonance as a function of nucleon number. Nix and Sierk extended this model with a hydrodynamic theory to isoscalar giant resonances of arbitrary multipole. Giant resonances are phenomena of the collective motion of nuclei and, hence, are suited to study by means of macroscopic theories. In isoscalar and isovector giant resonance neutrons and protons in nuclei move in and out of phase, respectively. A nucleus was treated by Nix and Sierk as an irrotational and incompressible fluid in which neutrons and protons move in phase with a small-amplitude, collective oscillation. For the isoscalar giant resonances in hot nuclei Nix and Sierk's method is still useful except that the parameters should be temperature dependent. Here the isoscalar giant resonances in hot nuclei were investigated by following Nix and Sierk's method. A formula was obtained for their energies, which is helpful to understand which factors affect their temperature dependences.

The foundation of Nix and Sierk's method is the Boltzmann equation,

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$$\frac{\partial f}{\partial t} + \vec{v} \cdot \nabla f - \frac{1}{m} \nabla U \cdot \nabla_v f = I(f), \tag{1}$$

where $f(\vec{r}, \vec{v}, t)$ is the distribution function for a particle at position $\vec{r}$ moving with velocity $\vec{v}$ at time t, $U(\vec{r})$ the mean single particle potential, $I(f)$ the collision term, and m the mass of the particle. Introducing the average velocity $\vec{u}(\vec{r}, t)$ and pressure tensor $P_{ij}(\vec{r}, t)$,

$$u_i \equiv \frac{1}{\rho} \int f v_i d^3 v, \tag{2}$$

$$P_{ij}(\vec{r}, t) = m \int (v_i - u_i)(v_j - u_j) f d^3 v = p_0 \delta_{ij} + \kappa_{ij}, \tag{3}$$

with nucleon number density $\rho = \int f(\vec{r}, t) d^3 v$, isotropic equilibrium pressure p_0, and pressure tensor deviation κ_{ij}. Considering the irrotation of the flow, i.e., $\nabla \times \vec{u} = 0$, and the incompression, i.e., $\nabla \cdot \vec{u} = 0$, and neglecting the collision term for small amplitudes of oscillation, one can simplify Eq. (1) as

$$\frac{\partial \kappa_{ij}}{\partial t} = -\frac{1}{3} m \rho_0 \langle v^2 \rangle \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right), \tag{4}$$

for a mean square velocity $\langle v^2 \rangle$ of a nucleon and equilibrium nucleon number density ρ_0. When $T = 0$, $\langle v^2 \rangle = \frac{3}{5} v_F^2$, where v_F is the Fermi velocity; but when $T > 0$, $\langle v^2 \rangle$ is larger and needs to be treated particularly. In deriving Eq. (4) the formula for the pressure tensor suggested by Bertsch [14] was used.

A nucleus also has a dissipative layer, but it is difficult to treat here. For simplicity this layer was taken to be a sharp cutoff surface. The nuclear surface for the nth multipole oscillations can be described as

$$R(\theta) = (R_0/\lambda)[1 + \alpha_n P_n(\cos(\theta)], \tag{5}$$

where $R_0 = r_0 A^{1/3}$ with r_0 the nuclear radius parameter and A the mass number, α_n a deformation parameter, and λ a factor for nuclear volume conservation while a nucleus is undergoing distortion. The parameter r_0 will increase with nuclear temperature and needs to be specifically analyzed.

The energy corresponding to the shape perturbation of a giant resonance from a spherical shape is

$$E = \int \left(\frac{\partial P_{ij}}{\partial x_j} + \rho \frac{\partial U}{\partial x_i} \right) dx_i d^3 r, \tag{6}$$

with the volume element displacement in the ith direction dx_i. The formula for the energy of the isoscalar giant multipole resonance in hot nuclei resulting from Nix and Sierk's method is

$$E_n = \hbar \sqrt{C_n / M_n}, \tag{7}$$

where C_n is the nuclear stiffness coefficient and M_n the inertia for the nth multipole oscillations. Expressed in terms of the basic constants of this problem these factors C_n and M_n are

$$C_n = \frac{8\pi(n-1)}{3n} m \rho_0 \langle v^2 \rangle R_0^3, \tag{8}$$

$$M_n = \frac{3}{n(2n+1)} A \, m \, R_0^2. \tag{9}$$

Equation (7) is the same in form as that for the ground-state nuclei, but the quantities r_0, ρ_0, and $\langle v^2 \rangle$ are temperature dependent. The average kinetic energy per nucleon in a nucleus is defined as $\tau(T) \equiv \frac{1}{2} m \langle v^2 \rangle$, with $\tau(0) = \frac{3}{5} \epsilon_F$ for Fermi energy ϵ_F. The nucleon number density at equilibrium is $\rho_0 = \frac{3}{4\pi r_0^3(T)}$. Combining Eqs. (7)–(9), one obtains the energy of the isoscalar giant multipole resonance in a hot nucleus as follows:

$$E_n(T) = E_n(0) \frac{r_0(0)}{r_0(T)} \sqrt{\frac{\tau(T)}{\tau(0)}}, \tag{10}$$

where $E_n(0)$ denotes the energy of the isoscalar giant resonance in a nucleus at the ground state.

It is clear that the most important ingredient of the energy of an isoscalar giant resonance in a hot nucleus is the resonance energy of the ground-state nucleus $E_n(0)$, which means the isoscalar giant resonance is primarily a surface effect. At the same time, the thermal effects lead to changes in resonance energy. In Eq. (10) the thermal effects on the energy of the isoscalar giant resonance are manifested in two ways: (1) the volume expansion and (2) the increase in the average kinetic energy per nucleon in the hot nucleus. The volume expansion and the increase in average kinetic energy per nucleon in a hot nucleus make the energy of the isoscalar giant resonance decrease and increase, respectively. The final explicit expression for the temperature dependence of the giant resonance energy is determined by the forms of $r_0(T)$ and $\tau(T)$, which are discussed in Secs. III and IV in detail.

III. VOLUME EXPANSION OF HOT NUCLEI

In general, the volume of a nucleus will expand when the temperature rises. Some authors [15,16] calculated the neutron and proton radii of some nuclei and the nuclear matter density with temperature with the help of the HF method. From their conclusions the nuclear radius parameter $r_0(T)$ and temperature T have the simple functional relationship

$$r_0(T) = r_0(1 + \eta T^2), \tag{11}$$

with r_0 the radius for the ground-state nucleus. It is necessary to determine the value of the radius parameter $r_0(T)$ at a definite temperature to fix the parameter η. The critical point (T_c, ρ_c) is a convenient temparature for this purpose in the equation of state for nuclear matter. Jaqaman, Mekjian and Zamick [17] used the finite-temperature HF theory with an effective nucleon-nucleon Skyrme interaction [Eq. (15)] to obtain an equation of state for nuclear matter, i.e.,

$$P = \rho T - a_0 \rho^2 + a_3(1 + \sigma) \rho^{2+\sigma}. \tag{12}$$

The parameters of the critical point of a finite nucleus deduced from the above equation are (when $\sigma = 1$) $\rho_c = 0.485 \rho_0$ (here ρ_0 is the nucleon density at absolute zero), and $T_c = 20.5$ MeV. This results in an expression for the parameter η, i.e., $(1+\eta T_c^2)^{-3} = 0.485$, from which one obtains $\eta = 6.5\times10^{-4}$ MeV^{-2}. Therefore the relation of the nuclear radius with temperature can be expressed as

$$r_0(T) = r_0(1+6.5\times10^{-4}T^2). \tag{13}$$

The increase of the nuclear radius with temperature given above is nearly equal to that suggested in Ref. [16] ($\eta = 4.2\times10^{-4}$ MeV^{-2}) and to that in Ref. [18] ($\eta = 7.0\times10^{-4}$ MeV^{-2}), but much smaller than that of Lipparini and Stringari [7].

IV. AVERAGE KINETIC ENERGY PER PARTICLE

The average kinetic energy per particle of a Fermi gas is $\frac{3}{5}\epsilon_F$ at absolute zero, but the expression becomes very complicated for temperature above absolute zero. Even for an ideal Fermi gas it is difficult to obtain an accurate analytic expression for the average kinetic energy per particle for temperature above absolute zero. But here an approximation will be taken for it. The particle occupation number for an energy level ε_q of a nuclear Fermi gas is determined by the Fermi distribution function

$$N_q(\varepsilon_q) = \frac{1}{1+\exp[(\varepsilon_q-\mu)/T]}, \tag{14}$$

where μ is the chemical potential of the nucleon and T the nuclear temperature with the same unit as energy. The energy per particle ε_q is composed of two parts, i.e., the kinetic energy $\frac{\hbar^2q^2}{2m}$ and a potential part that is more sophisticated. In the HF theory the potential part is calculated by using the Skyrme type interaction [19]

$$\begin{aligned} V = {} & -t_0(1+x_0P_\sigma)\delta(\vec r) + \frac{t_1}{2}[k^2\delta(\vec r)+\delta(\vec r)k^2] \\ & + t_2\vec k\delta(\vec r)\vec k + \frac{t_3}{6}(1+P_\sigma)\rho^\sigma\left[\frac{\vec r_1+\vec r_2}{2}\right]\delta(\vec r), \end{aligned} \tag{15}$$

where $\vec r = \vec r_1 - \vec r_2, \vec k = (\vec\bigtriangledown_1 - \vec\bigtriangledown_2)/2i$, and all t_i's and x_0 are parameters. The single particle energy for a finite-temperature Fermi gas according to the HF theory is given by

$$\varepsilon_q = \varepsilon_0 + \frac{\hbar^2q^2}{2m^*}, \tag{16}$$

where ε_0 is a constant independent of the wave vector q and m^* is the effective mass [20], which is expressed in the frame of the HF method with a Skyrme type interaction as

$$\frac{m^*}{m} = \left[1+\frac{m\rho}{8\hbar^2}(3t_1+5t_2)\right]^{-1}. \tag{17}$$

Inserting Eq. (16) into Eq. (14) and integrating it over all wave vectors, one gets the particle number N of the system and the average kinetic energy per particle τ, respectively, as

$$N = \frac{g\Omega}{(2\pi)^3}\int d^3qN_q = \frac{g\Omega}{\lambda^3}f_{3/2}(z), \tag{18}$$

$$\tau = \frac{g\Omega}{(2\pi)^3N}\int d^3q\frac{\hbar^2q^2}{2m}N_q = \frac{3}{2}\frac{m^*}{m}T\frac{g\Omega}{\lambda^3N}f_{5/2}f(z), \tag{19}$$

where $\Omega(T) = \frac{4\pi}{3}r_0^3(T)A$ is the nuclear volume, the partition function $z \equiv e^{(\mu-\varepsilon_0)/T}$, the thermal wavelength $\lambda \equiv (\frac{2\pi\hbar^2}{m^*T})^{1/2}$, $f_n(z) \equiv \frac{1}{\Gamma(n)}\int_0^\infty \frac{x^{n-1}dx}{1+z^{-1}e^x}$, and $g = 4$, the degeneracy factor of proton and neutron.

Because the limiting temperature for a nucleus is $T \sim 6$ MeV [21] and its thermal wavelength λ is much larger than the mean distance between nucleons, i.e., $(\lambda^3/g\Omega) \gg 1$, the hot nucleus can be treated as a Fermi gas of low temperature and high density. Expanding the functions $f_{3/2}(z)$ and $f_{5/2}(z)$ one obtains

$$f_{3/2}(z) = \frac{4}{3\sqrt\pi}(\ln z)^{3/2}\left[1+\frac{\pi^2}{8}(\ln z)^{-2}+\cdots\right], \tag{20}$$

$$f_{5/2}(z) = \frac{8}{15\sqrt\pi}(\ln z)^{5/2}\left[1+\frac{5\pi^2}{8}(\ln z)^{-2}+\cdots\right]. \tag{21}$$

Substituting Eq. (20) into Eq. (18), one obtains

$$T\ln z \simeq \tilde\epsilon_F\left[1-\frac{\pi^2}{12}\left(\frac{T}{\tilde\epsilon_F}\right)^2\right], \tag{22}$$

up to the second order in the nuclear temperature T with an effective Fermi energy $\tilde\epsilon_F \equiv \frac{m}{m^*}(\frac{\Omega(0)}{\Omega(T)})^{2/3}\epsilon_F$.

By combining Eqs. (17)–(21) and approximating them to the second order in the temperature, one obtains the average kinetic energy per particle as

$$\begin{aligned} \tau &= \frac{3}{2}\frac{m^*}{m}Tf_{5/2}(z)/f_{3/2}(z) \\ &= \frac{3}{5}\left(\frac{\Omega(0)}{\Omega(T)}\right)^{2/3}\epsilon_F\left[1+\frac{5\pi^2}{12}\left(\frac{T}{\tilde\epsilon_F}\right)^2\right]. \end{aligned} \tag{23}$$

Equations (22) and (23) are suitable for a Fermi gas of interacting particles. Using the expression for the nuclear radius [Eq. (13)] the average kinetic energy per nucleon can then also be rewritten to order T^2 as

$$\tau = \frac{3}{5}\epsilon_F\left(\frac{r_0(0)}{r_0(T)}\right)^2\left[1+\frac{5\pi^2}{12}\left(\frac{m^*}{m}\right)^2\left(\frac{T}{\epsilon_F}\right)^2\right]. \tag{24}$$

Although the kinetic energy per nucleon in hot nuclei is inversely proportional to the nuclear radius $r_0(T)$ it still increases smoothly with temperature.

V. THERMAL EFFECTS

Both the volume expansion and the increase in the average kinetic energy per nucleon of a hot nucleus arise from the increase in the nuclear temperature, but as pointed out in Sec. II these effects have contrary roles in the temperature dependence of the energy of an isoscalar giant multipole resonance. The competition between them will be examined and an approximate formula for the temperature dependence of the resonance energy will be derived.

Inserting Eqs. (11) and (24) into Eq. (10), one obtains the isoscalar giant multipole resonance energy,

$$E_n(T) = E_n(0)\left\{1 - \left[2\eta - \frac{5\pi^2}{24}\left(\frac{m^*}{m}\right)^2 \frac{1}{\epsilon_F^2}\right] T^2\right\}. \tag{25}$$

In the effective SKM* interaction $t_1 = 410.0$ and $t_2 = -135.0$ [20]. By taking $r_0 = 1.18$ fm and using Eq. (17), one finds $m^*/m = 0.806$. This value seems reasonable for hot nuclei. Inserting all the parameters' values into Eq. (25), an approximate formula for the energy of an isoscalar giant multipole resonance in a hot nucleus can be written as

$$E_n(T) = E_n(0)[1 - 2.0 \times 10^{-4} T^2]. \tag{26}$$

This formula shows that the energy of an isoscalar giant multipole resonance in a hot nucleus nearly does not change with temperature. When the temperature $T = 5$ MeV the energy of the isoscalar giant multipole resonance is reduced by no more than 0.1 MeV, which is smaller than the 1 MeV decrease of the GDR energy for a temperature of 4 MeV [7], and is also less than the decrease in the GDR energy obtained by using either the HF+RPA method or the Vlasov equation [8,9].

VI. CONCLUSION

In the above sections we have discussed the thermal effects on the giant resonances in hot nuclei. We conclude with a brief comment. There exist two factors that play contrary roles in determining the temperature dependence of the isoscalar giant multipole resonance energy in nuclei. The first one is the volume expansion which enlarges the nuclear inertia, and, hence, reduces the energy of the isoscalar giant resonance in a hot nucleus. The other one is the increase in the average kinetic energy per nucleon that increases the determinant of the pressure tensor, i.e., enhances the force against distortion of the Fermi surface, thereby increasing the resonance energy with temperature. Based on the theoretical calculations and analyses here, we conclude that the contributions of both the volume expansion and the increase in the average kinetic energy per nucleon to the giant resonance energy in hot nuclei nearly counteract each other. In other words, the energy of the isoscalar multipole giant resonance depends very weakly on the nuclear temperature.

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