Two-parameter formula for rotational spectra

Guo-Mo Zeng,[1,2,3] Wei Liu,[2] and En-Guang Zhao[1,3]
[1]*China Center of Advanced Science and Technology (CCAST, World Laboratory), P.O. Box 8730, Beijing 100080, China*
[2]*Department of Physics, Jilin University, Changchun, Jilin 130023, People's Republic of China*
[3]*Institute of Theoretical Physics, Academia Sinica, Beijing 100080, China*
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A new two-parameter formula for the rotational spectra of well-deformed nuclei is presented for the first time. The formula is deduced from experimental level systematics and alternatively from nuclear hydrodynamics. Comparisons with a great number of rotational spectra of even-even nuclei in rare-earth and actinide regions show that the formula can give a best fitting to the experimental data.

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I. INTRODUCTION

Recently abundant information about the high-spin levels of ground rotational bands (GRB's) in actinide nuclei arises from Coulomb excitation experiments with very heavy-ion beams. Because the moments of inertia of actinide nuclei are about twice as big as those of the rare-earth nuclei, the two-quasiparticle S bands do not compete with the GRB up to much higher spins than those in rare-earth nuclei. So, making use of the recent data, one can test further the applicability of various current formula for rotational spectra. Several authors did this work and concluded that the ab formulas

$$E(I)=a[\sqrt{1+bI(I+1)}-1] \tag{1}$$

are far better than other two-parameter formulas for describing the rotational bands [1]. However, in many cases the results predicted by ab formulas deviate obviously from the experimental data. It becomes therefore a motivation for the present work to seek a more accurate formula.

In this paper a new two-parameter formula is proposed for the first time and is used to analyze systematically the rotational spectra of well-deformed even-even nuclei both in rare-earth and actinide regions up to very high spin below the band crossing. We have carried out comparisons between the calculation results by using our formula and almost all experimental data for the ground-state rotational bands. An overall and excellent agreement is obtained especially for the actinide nuclei. We also compared our formula with Eq. (1) and the Harris two-parameter ω^2 expansion [2]

$$E=\alpha\omega^2+\beta\omega^4, \tag{2}$$

which is equivalent to the variable moment of inertia (VMI) model [3], and found that our formula is systematically better than both expressions. Thus in view of the results of Ref. [1] and references therein on the accuracy of ab and other expressions we can reasonably affirm that among all two-parameter formulas ours can give a best fitting to the observed rotational spectra.

In Sec. II we derive our formula in two ways. Comparisons of the new formula with experimental data and other formulas in detail are given in Sec. III. In Sec. IV we enumerate our major conclusions.

II. DERIVATION OF THE NEW FORMULA

The ab formula (1) was deduced by Holmberg and Lipas by investigating the curves of the moment of inertia $\mathscr{J}$ against E for a representative selection of nuclei according to the simplest expression of rotational spectra (see Fig. 2 in Ref. [4])

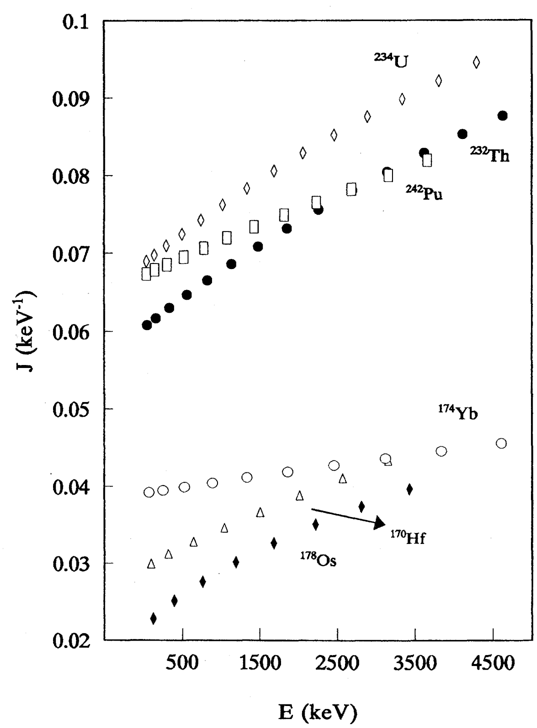

FIG. 1. Moment of inertia as a function of excitation energy according to Eq. (3) for some representative nuclei.

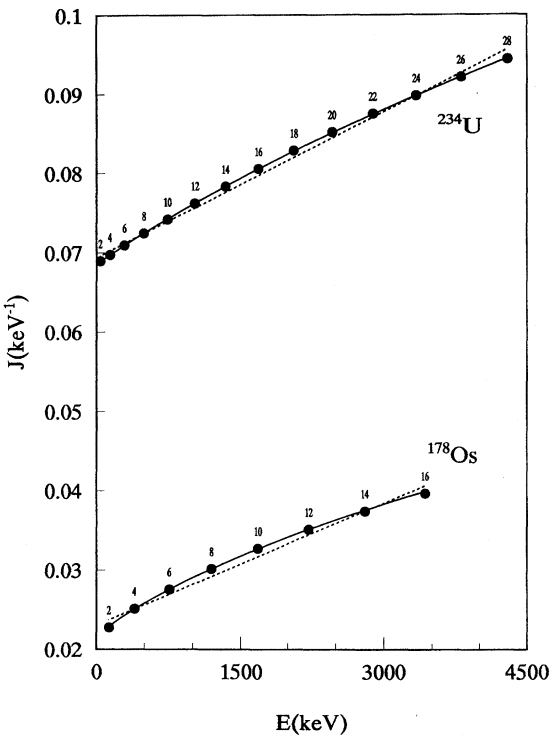

FIG. 2. $\mathscr{J}\sim E$ plots for ^{178}Os and ^{234}U with the solid circles denoting the data calculated from Eq. (3), solid line the pq fit, and dashed line the ab fit.

$$E(I)=\frac{\hbar^2}{2\mathscr{J}}I(I+1). \tag{3}$$

They assume that $\mathscr{J}$ varies approximately linearly with E, i.e.,

$$\mathscr{J}=c_1+c_2E. \tag{4}$$

Substituting Eq. (4) into Eq. (3) and solving the resulting quadratic equation yields the ab formula. In substance it takes into account the centrifugal stretching effect. An alternative derivation of Eq. (1) from the hydrodynamic model was also given by the above authors.

However, the linear relation between $\mathscr{J}$ and E is not obeyed well by many rotational spectra for low as well as very high spins. Some examples are presented in Fig. 1 and one can see that $\mathscr{J}$ does not increase with E as quickly as Eq. (4) described. In other words Eq. (4), and hence Eq. (1), overstresses the centrifugal stretching effect. So we take the starting point as giving a more accurate functional relation of $\mathscr{J}$ with E following Ref. [4]

Rewriting Eq. (1) (hereafter take $\hbar=1$), we get

$$E(I)=\frac{abI(I+1)}{\sqrt{1+bI(I+1)}+1}. \tag{5}$$

Comparing with Eq. (3), we have

$$\mathscr{J}=\frac{1}{2ab}[\sqrt{1+bI(I+1)}+1]. \tag{6}$$

By using Eq. (3) again and taking $\mathscr{J}$ in it as the zero approximation,

$$\mathscr{J}_0=\frac{1}{ab}, \tag{7}$$

Eq. (6) can be rewritten as

$$\mathscr{J}=\frac{1}{2ab}\left(\sqrt{1+\frac{2}{a}E}+1\right). \tag{8}$$

This is just the relation between $\mathscr{J}$ and E we obtained. Substituting it into Eq. (3) produces a cubic equation of E without a quadratic term,

$$E^3+a^2bI(I+1)E-\frac{a^3b^2}{2}[I(I+1)]^2=0. \tag{9}$$

This equation has only one real root for $b>0$, which can be written as

$$E(I)=a\left(\left\{\left(\frac{bI(I+1)}{2}\right)^2+\left[\left(\frac{bI(I+1)}{2}\right)^4+\left(\frac{bI(I+1)}{3}\right)^3\right]^{1/2}\right\}^{1/3}+\left\{\left(\frac{bI(I+1)}{2}\right)^2-\left[\left(\frac{bI(I+1)}{2}\right)^4+\left(\frac{bI(I+1)}{3}\right)^3\right]^{1/2}\right\}^{1/3}\right). \tag{10}$$

In order to distinguish between the above equation and ab formula [Eq. (1)], instead of a,b we take p,q denoting the two free parameters and let $x=qI(I+1)$; the above equation now reads

$$E(I)=p\left(\left\{\left(\frac{x}{2}\right)^2+\left[\left(\frac{x}{2}\right)^4+\left(\frac{x}{3}\right)^3\right]^{1/2}\right\}^{1/3}+\left\{\left(\frac{x}{2}\right)^2-\left[\left(\frac{x}{2}\right)^4+\left(\frac{x}{3}\right)^3\right]^{1/2}\right\}^{1/3}\right). \tag{11}$$

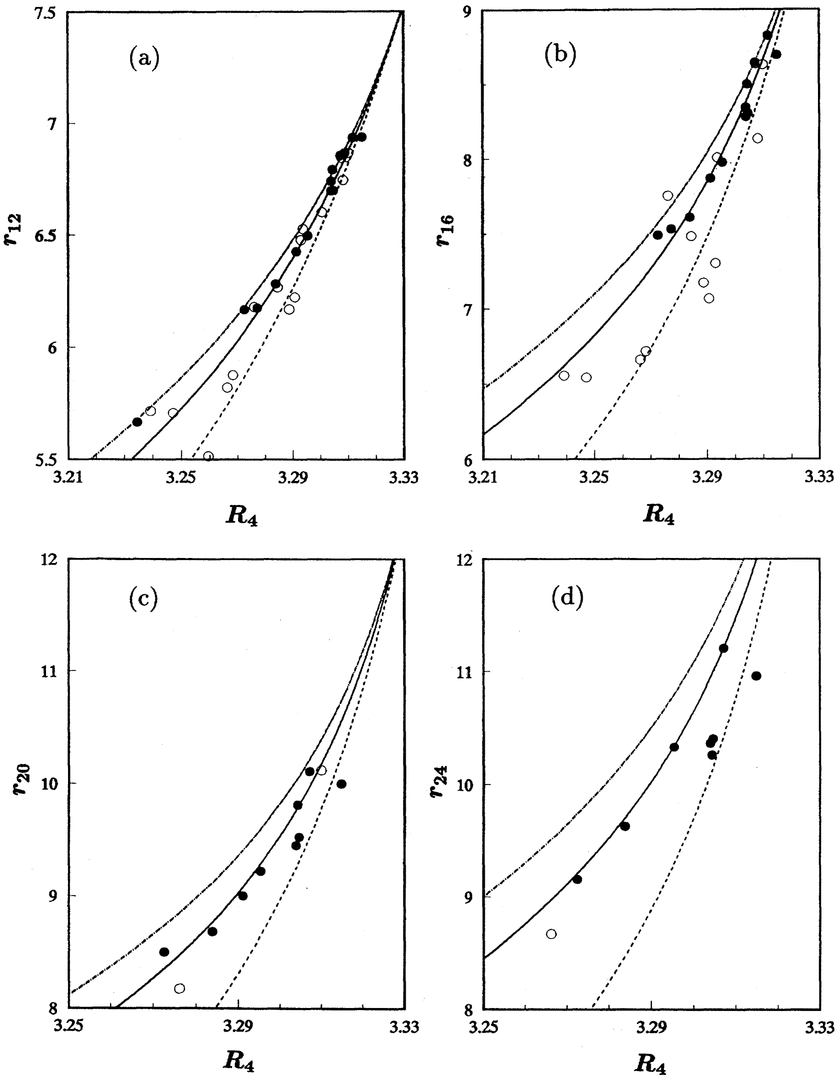

FIG. 3. The Mallmann plots of the pq, ab, and $\alpha\beta$ expressions, denoted by solid, dashed and dot-dashed lines respectively, and the experimental data denoted by solid circles for the actinide nuclei and open circles for the rare-earth nuclei. (a) $I=12$, (b) $I=16$, (c) $I=20$, (d) $I=24$.

This equation is just our new formula for rotational spectra; hereafter we call it the pq formula. It can be easily verified that the formula will reduce to a rigid rotor for small x.

We now turn to an alternative derivation of Eq. (11) from the hydrodynamic model. In this model the energy of a rotating nucleus is given by

$$E(\beta)=\frac{1}{2\mathscr{J}(\beta)}I(I+1)+\tfrac{1}{2}C(\beta-\beta_0)^2, \qquad (12)$$

where $\mathscr{J}=3B\beta^2$ from nuclear hydrodynamics, β_0 is the deformation parameter for a rigid nucleus, and β its stretched value. From the variational condition $\partial E/\partial\beta=0$, we can get a quadratic equation with which the dynamical equilibrium deformation must be satisfied:

$$(\beta-\beta_0)^2+\tfrac{1}{2}\beta_0(\beta-\beta_0)-\frac{E}{C}=0. \qquad (13)$$

TABLE I. Experimental and calculated energies (in keV) of ^{174}Yb and ^{232}Th with the parameters fixed by the experimental values of the 2^+ and 8^+ energy levels.

Nuclei	$I=2$	4	6	8	10	12	14	16	18	20	22	24	26	28	$\sigma\times10^3$
^{174}Yb Expt.	76.47	253.12	526.03	889.93	1336	1861	2457	3117	3836	4610					
pq	76.47	253.15	526.12	889.93	1338.31	1864.74	2462.89	3126.89	3851.44	4631.82					2.480
ab	76.47	253.19	526.23	889.93	1337.39	1861.06	2453.21	3106.28	3813.20	4567.46					3.688
^{232}Th Expt.	49.37	162.12	333.2	556.9	827.0	1137.1	1482.8	1858.6	2262.9	2691.5	3144.2	3619.6	4116.2	4631.8	
pq	49.37	162.15	333.37	556.90	826.62	1137.05	1483.52	1862.19	2269.89	2704.01	3162.38	3643.19	4144.90	4666.19	3.922
ab	49.37	162.26	333.66	556.90	824.72	1130.08	1466.64	1828.94	2212.47	2613.49	3028.99	3456.54	3894.17	4340.29	29.279

TABLE II. Experimental and calculated energies (in keV) of rare-earth nuclei with the parameters fixed by fitting the whole region of experimental values.

Nuclei		$I=2$	4	6	8	10	12	14	16	18	20	$\sigma\times10^3$
^{154}Sm	Expt.	81.98	266.79	543.74	902.65	1332.80	1825.7	2373	2967			
	pq	81.62	266.84	545.29	905.06	1335.07	1825.96	2370.21	2961.81			2.276
	ab	81.16	266.31	546.35	909.38	1342.76	1834.51	2374.08	2952.63			5.952
^{156}Gd	Expt.	88.97	288.18	584.71	965.09	1416.03	1924.41	2475.74				
	pq	88.84	288.67	585.55	964.64	1413.17	1921.07	2480.58				1.615
	ab	88.35	288.41	587.49	969.92	1420.06	1924.12	2470.71				3.937
^{162}Dy	Expt.	80.66	265.67	548.53	920.99	1374.91	1901.41	2492.4	3138.4	3831		
	pq	80.65	265.77	548.81	921.32	1374.39	1899.55	2489.31	3137.21	3837.78		0.847
	ab	80.38	265.32	548.97	923.19	1378.52	1905.13	2493.55	3135.05	3821.89		2.058
^{164}Dy	Expt.	73.39	242.23	501.32	843.67	1261.3	1745.8	2290.5				
	pq	73.43	242.29	501.21	843.13	1260.41	1745.65	2292.12				0.509
	ab	73.33	242.19	501.54	844.31	1262.21	1746.57	2288.92				0.627
^{164}Er	Expt.	91.40	299.44	614.4	1024.57	1518.0	2082.73	2702.5				
	pq	91.43	299.71	614.62	1023.92	1515.76	2079.78	2707.40				1.130
	ab	91.14	299.46	615.53	1026.84	1519.84	2081.43	2699.88				1.653
^{166}Er	Expt.	80.58	264.99	545.46	911.21	1349.64	1846.6	2389.4	2967.4			
	pq	80.98	265.35	543.84	905.46	1339.61	1837.10	2390.35	2993.21			5.393
	ab	80.77	265.34	545.29	909.46	1345.78	1842.61	2389.53	2977.67			2.199
^{166}Yb	Expt.	102.37	330.50	668.01	1098.25	1605.93	2175.94	2779.4	3490.0			
	pq	102.20	331.00	668.85	1097.82	1603.04	2173.10	2799.40	3475.39			3.185
	ab	101.60	330.90	671.96	1105.56	1613.04	2178.39	2788.73	3434.10			7.296
^{168}Yb	Expt.	87.73	286.55	585.25	970.02	1425.41	1935.9	2488.5	3073.0			
	pq	88.21	287.19	583.88	964.09	1415.29	1927.41	2492.59	3104.75			5.666
	ab	87.72	286.84	585.64	969.39	1423.09	1933.18	2488.30	3079.25			1.155
^{170}Yb	Expt.	84.26	277.45	573.54	963.67	1437.97	1983.78	2580.9				
	pq	84.39	277.89	573.31	961.43	1432.65	1977.99	2589.54				2.506
	ab	84.23	277.76	573.88	963.32	1435.51	1979.66	2585.52				1.325
^{172}Yb	Expt.	78.75	260.29	540.00	912.16	1370.11	1907.21	2518.4	3198.1			
	pq	78.64	260.18	540.29	913.01	1371.52	1908.86	2518.35	3193.86			0.924
	ab	78.53	260.02	540.42	913.93	1373.44	1911.22	2519.31	3189.96			1.936
^{174}Yb	Expt.	76.47	253.12	526.03	889.93	1336	1861	2457	3117	3836	4610	
	pq	76.51	253.19	525.96	889.17	1336.35	1860.84	2456.20	3116.51	3836.46	4611.33	0.368
	ab	76.33	252.83	525.83	890.00	1338.80	1865.02	2461.20	3119.99	3834.42	4598.06	1.571
^{176}Yb	Expt.	82.13	271.70	564.7	954.0	1431	1985	2602	3270			
	pq	82.46	272.39	564.36	951.11	1424.61	1976.90	2600.60	3289.13			3.594
	ab	82.30	272.13	564.49	952.26	1426.98	1979.59	2601.05	3282.78			2.262
^{168}Hf	Expt.	123.95	385.54	756.8	1213.5	1735.7	2305.8	2938.8	3622.9			
	pq	123.29	387.52	759.55	1214.41	1735.72	2312.65	2937.67	3605.26			3.558
	ab	120.38	385.54	766.47	1233.35	1761.93	2334.52	2938.80	3566.27			15.360
^{170}Hf	Expt.	100.80	321.99	642.9	1043.3	1505.5	2016.4	2567.2	3151.6			
	pq	101.12	322.65	641.53	1037.77	1496.93	2008.94	2566.59	3164.57			3.714
	ab	99.98	322.54	646.94	1050.43	1513.11	2019.55	2558.40	3121.48			5.978
^{172}Hf	Expt.	95.24	309.26	628.14	1037.30	1521.07	2064.44	2654.01	3277.17			
	pq	95.44	309.92	628.14	1033.98	1513.68	2056.45	2654.01	3300.02			3.657
	ab	94.79	309.41	630.24	1040.43	1523.21	2063.76	2649.88	3271.91			2.503
^{174}Hf	Expt.	90.99	297.38	608.26	1009.6	1485.9	2020.5	2597.5	3208.9			
	pq	91.50	298.20	606.98	1003.42	1474.62	2010.11	2601.67	3242.88			5.917
	ab	90.99	297.75	608.53	1008.44	1482.19	2015.82	2597.50	3217.62			1.659
^{176}Hf	Expt.	88.35	290.18	596.82	997.74	1481.07	2034.67	2646.6				
	pq	88.41	290.32	596.65	996.31	1478.28	2032.69	2651.20				1.218
	ab	88.20	290.18	597.53	998.97	1482.05	2034.50	2645.11				0.963
^{178}Hf	Expt.	93.18	306.62	632.18	1058.56	1571.0	2150.7	2777.6	3436.2			
	pq	93.85	307.67	630.96	1051.20	1556.21	2135.38	2779.90	3482.61			7.397
	ab	93.54	307.41	632.08	1054.88	1562.02	2140.13	2777.20	3462.92			4.341
^{182}W	Expt.	100.11	329.43	680.5	1144.4	1711.9	2372.5	3112.6				
	pq	99.91	329.58	681.52	1145.91	1712.23	2370.30	3110.92				1.137
	ab	99.75	329.39	681.94	1147.63	1715.03	2372.22	3107.59				2.112
^{178}Os	Expt.	131.65	398.1	761.3	1193.9	1681.9	2219.6	2804.5	3429.1			
	pq	130.91	398.98	762.52	1196.65	1687.00	2224.54	2803.05	3418.02			2.943
	ab	127.68	400.27	776.69	1221.73	1711.35	2230.42	2769.50	3322.63			20.434

The physical root of this equation is

$$\beta-\beta_0\equiv\Delta\beta=\frac{\beta_0}{4}\left[-1+\sqrt{1+\frac{16}{C\beta_0^2}E}\right], \quad (14)$$

and the moment of inertia now reads

$$\mathscr{J}=\tfrac{3}{2}B\beta_0^2\left[1+\sqrt{1+\frac{16}{C\beta_0^2}E}+O(\Delta\beta^2)\right]. \quad (15)$$

Dropping $O(\Delta\beta^2)$ from $\mathscr{J}$ and substituting it into Eq. (12) with its second term neglected we can obtain a cubic equation similar to Eq. (9) and its solution is of the same form as Eq. (11).

III. CALCULATIONS BY USING THE NEW FORMULA

A. $\mathscr{J}\sim E$ curves

First we rewrite Eq. (8) as

$$\mathscr{J}=c_1(\sqrt{1+c_2E}+1), \quad (16)$$

with c_1,c_2 regarded as two free parameters. Making use of this equation to fit almost all $\mathscr{J}\sim E$ curves of GRB's (below band crossing) for even-even nuclei with $R_4=E(4_1^+)/E(2_1^+)\geqslant 3.0$, we find that the agreement is very good for most nuclei. Two typical examples of ^{178}Os [18] and ^{234}U [27], which belong to rare-earth and actinide regions, respectively, are shown in Fig. 2. We see that the fits given by the pq formula are far better than those given by the ab formula.

B. Mallmann plots

Mallmann [5] pointed out that plots of $R_I\equiv(E_I-E_0)/(E_2-E_0)$ (or $r_I\equiv R_I-R_{I-2}$) vs R_4 of the even-even collective nuclei show a remarkably smooth trend. Reference [6] verified that for each two-parameter formula of rotational bands the Mallmann plot is unique and does not depend on the parameter values. The Mallmann plot can be compared directly with the experimental data of rotational spectra over the entire range of nuclei. Thus, the Mallmann plot can give a clear-cut picture of the relative validity of each formula for rotational spectra.

From Eq. (11) we have

$$R_I=\frac{\sqrt[3]{(x/2)^2+\sqrt{(x/2)^4+(x/3)^3}}+\sqrt[3]{(x/2)^2-\sqrt{(x/2)^4+(x/3)^3}}}{\sqrt[3]{(3q)^2+\sqrt{(3q)^4+(2q)^3}}+\sqrt[3]{(3q)^2-\sqrt{(3q)^4+(2q)^3}}}. \quad (17)$$

Taking q as a parametric variable, one can draw the Mallmann plot for the pq formula. As illustrative examples the Mallmann plots for $I=$ 12, 16, 20, and 24 are given in Fig. 3. For comparison, also given are the corresponding plots for the other two rotational formula with two parameters, i.e., the $\alpha\beta$ expression and ab expression according to Eq. (2) and Eq. (1), respectively.

All the experimental data now available [7–32] for the GRB's of even-even rare-earth and actinide nuclei (with band crossing angular momentum $I_c\geqslant 16$) are displayed in the figures. It can be seen that the Mallmann plots for the pq expression pass through the experimental points, while those for the $\alpha\beta$ expression lie higher and those for the ab expression lower than the data.

C. Energy spectra

In order to demonstrate further the accuracy and convergence of Eq. (11) for describing the experimental data, choosing ^{174}Yb [16] and ^{232}Th [26] as examples, we calculate the GRB's for both nuclei with the two parameters p and q fixed by the experimental values of the 2^+ and 8^+ energy levels. The results are listed in Table I and those obtained by Eq. (1) are also shown in the same table for comparison. It can be found that the agreements between the observed values and our results are very good up to very high spin while the calculations using Eq. (1) are worse than ours. The relative error σ in Table I is defined as

$$\sigma=\sqrt{\frac{1}{N}\sum_I\left[\frac{E_{\rm expt}(I)-E_{\rm calc}(I)}{E_{\rm expt}(I)}\right]^2}. \quad (18)$$

Using the pq expression we calculate all GRB's for the nuclei mentioned in the last subsection with the two parameters (p,q) fitted over the whole experimental bands by the least-squares procedure. The results are listed in Table II and Table III for rare-earth and actinide nuclei, respectively. The structures of the two tables are the same as Table I. From Table II we can see that the pq fits are well-behaved in all cases and better than ab fits for more than half of the nuclei. For ^{174}Yb, ^{168}Hf, and ^{178}Os the σ values from Eq. (11) are appreciably smaller than those from Eq. (1), whereas only for ^{168}Yb is the former obviously worse than the latter.

In Table III we have exciting results: Apart from ^{236}U, ^{238}U, and ^{248}Cm, the pq fits are apparently better than ab fits for all other actinide nuclei. The values of σ obtained by pq are only 1/4, even 1/10, as big as those obtained by ab. On the other hand, the $\mathscr{J}\sim E$ curves of the three exceptional nuclei mentioned above are nearly straight lines which can not be well described by Eq. (8)

IV. SUMMARY AND OUTLOOK

A new two-parameter formula with closed analytical form, Eq. (11), for the energy levels in the ground-state rotational bands of well-deformed nuclei is obtained from consideration of the accurate relation between the moment of inertia and experimental energy levels or, alternatively, from nuclear hydrodynamics. Analyses of $\mathscr{J}\sim E$ curves, Mallmann plots, and energy spectra reveal surprisingly good agreement between our predictions and observed data and show that among all two-parameter rotational formulas our

TABLE III. Experimental and calculated energies (in keV) of actinide nuclei with the parameters fixed by fitting the whole region of experimental values.

Nuclei		$I=2$	4	6	8	10	12	14	16	18	20	22	24	26	28	$\sigma\times10^3$
^{226}Ra	Expt.	67.67	211.54	416.6	669.4	960.0	1281	1629								
	pq	67.34	212.42	417.76	669.71	959.15	1279.98	1627.94								2.717
	ab	66.41	212.54	422.18	678.78	968.97	1283.04	1614.27								11.446
^{228}Th	Expt.	57.76	186.82	378.17	622.2	911.5	1238.7	1595.9								
	pq	57.64	187.02	378.72	622.90	911.21	1237.15	1595.78								1.235
	ab	57.29	186.82	380.02	626.39	915.61	1238.70	1588.32								5.036
^{230}Th	Expt.	53.20	174.10	356.6	594.1	879.7	1207.8	1572.9	1971.5	2397.8	2850	3325				
	pq	53.12	174.12	357.03	594.73	880.31	1207.78	1572.13	1969.33	2396.08	2849.72	3328.03				0.867
	ab	52.62	173.17	356.76	596.78	885.92	1216.92	1583.14	1978.74	2398.79	2839.18	3296.53				6.171
^{232}Th	Expt.	49.37	162.12	333.2	556.9	827.0	1137.1	1482.8	1858.6	2262.9	2691.5	3144.2	3619.6	4116.2	4631.8	
	pq	49.48	162.40	333.59	556.72	825.57	1134.58	1479.11	1855.31	2260.05	2690.75	3145.29	3621.89	4119.02	4635.40	1.461
	ab	48.85	161.03	332.56	557.96	830.99	1145.33	1494.98	1874.58	2279.44	2705.60	3149.70	3608.99	4081.16	4564.31	7.363
^{230}U	Expt.	51.72	169.5	347.0	578.3	856.5	1175.8	1531.7	1921.3							
	pq	51.67	169.40	347.44	578.91	857.13	1176.25	1531.42	1918.70							0.912
	ab	51.42	169.07	347.85	580.98	861.01	1180.68	1533.41	1913.52							4.039
^{232}U	Expt.	47.57	156.57	322.6	541.0	805.8	1111.5	1453.7	1828.1	2231.5	2659.7					
	pq	47.59	156.61	322.82	540.84	805.11	1110.51	1452.55	1827.47	2232.10	2663.80					0.741
	ab	47.40	156.34	323.13	542.65	809.03	1116.28	1458.65	1830.96	2228.65	2647.82					3.118
^{234}U	Expt.	43.50	143.35	296.07	497.04	741.2	1023.8	1340.8	1687.8	2063.0	2464.2	2889.7	3339	3808	4297	
	pq	43.53	143.41	296.02	496.71	740.61	1023.12	1340.18	1688.31	2064.57	2466.49	2892.00	3339.35	3807.04	4293.76	0.615
	ab	43.18	142.65	295.51	497.65	744.30	1030.38	1350.95	1701.41	2077.58	2475.83	2893.02	3326.44	3773.83	4233.24	6.821
^{236}U	Expt.	45.24	149.48	309.78	522.24	782.3	1085.3	1426.3	1800.9	2203.9	2631.7	3081.2	3550	4039	4549	
	pq	45.49	150.07	310.35	521.91	779.96	1079.89	1417.53	1789.21	2191.83	2622.71	3079.62	3560.61	4064.05	4588.51	4.839
	ab	45.15	149.30	309.71	522.52	783.08	1086.46	1427.69	1802.10	2205.36	2633.65	3083.58	3552.23	4037.10	4536.03	1.190
^{238}U	Expt.	44.91	148.41	307.21	517.8	775.7	1076.5	1415.3	1788.2	2190.7	2618.7	3067.2	3534.5	4017.3	4516.5	
	pq	45.12	148.86	307.88	517.80	773.88	1071.57	1406.71	1775.68	2175.40	2603.21	3056.88	3534.50	4034.43	4555.24	4.897
	ab	44.78	148.10	307.28	518.52	777.27	1078.65	1417.80	1790.06	2191.18	2617.35	3065.19	3531.82	4014.73	4511.77	1.450
^{236}Pu	Expt.	44.63	147.45	305.8	515.7	773.5	1074.3	1413.6	1786.0							
	pq	44.64	147.51	305.80	515.70	772.98	1073.40	1413.03	1788.30							0.632
	ab	44.57	147.41	305.91	516.35	774.28	1074.88	1413.36	1785.11							0.824
^{238}Pu	Expt.	44.08	145.98	303.4	513.4	772.8	1078.5	1427.2	1816.2	2240.5						
	pq	44.03	145.86	303.42	513.85	773.75	1079.57	1427.81	1815.15	2238.57						0.846
	ab	43.97	145.74	303.43	514.25	774.78	1081.18	1429.41	1815.43	2235.37						1.897
^{240}Pu	Expt.	42.82	141.69	294.32	497.52	747.8	1041.8	1375.6								
	pq	42.81	141.68	294.35	497.67	748.04	1041.76	1375.22								0.236
	ab	42.77	141.64	294.45	498.09	748.75	1042.26	1374.35								0.890
^{242}Pu	Expt.	44.54	147.3	306.4	518.1	778.7	1084.0	1431.3	1816.3	2235.6	2686.0	3163	3662			
	pq	44.58	147.52	306.40	517.91	778.23	1083.46	1429.84	1813.91	2232.56	2683.05	3163.00	3670.30			1.106
	ab	44.41	147.12	306.07	518.22	779.87	1086.90	1435.03	1820.06	2237.95	2684.96	3157.71	3653.16			1.870
^{244}Pu	Expt.	46.0	153.0	315.4	531.8	798.3	1110.7	1464.4	1855.4	2279.2	2730.7					
	pq	46.06	152.27	315.83	532.97	799.43	1110.93	1463.44	1853.33	2277.37	2732.78					1.904
	ab	45.93	152.00	315.74	533.56	801.16	1113.83	1466.81	1855.46	2275.50	2723.00					3.024
^{248}Cm	Expt.	43.38	143.8	298.8	506.0	761.9	1062.8	1404.3	1781.6	2189.7	2623.6	3079.4	3554.8	4049.8	4566.1	
	pq	43.70	144.52	299.95	506.52	760.34	1057.45	1394.06	1766.76	2172.49	2608.59	3072.73	3562.90	4077.36	4614.57	6.039
	ab	43.46	143.95	299.38	506.69	762.16	1061.66	1400.93	1775.80	2182.29	2616.72	3075.78	3556.50	4056.28	4572.81	1.936

new formula can give a best fitting to the experimental data. It is of great interest to exploit the microscopic basis of this formula.

The excellent agreements between our formula and experimental GRB's imply that the formula can be used to determine the adopted levels from different experiments and to predict high spin states to be measured.

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