Are octupole vibrations harmonic?

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(Received 2 March 1995)

The anharmonicity of octupole vibrations is deduced over a wide range of Z and A through the examination of summed $B(E3)\uparrow$ values for all 3^- states in a given nuclide versus the centroid energy for those states. Data obtained with the (p,p'), (d,d'), and (α,α') reactions are used, as opposed to an earlier study based on Coulomb excitation and lifetime data. Results of the new analysis support earlier observations of significant anharmonicity, but leave open the possibility of greater harmonicity than was previously supposed. Fits to the experimental data are discussed within the framework of the hydrodynamical model and possible mechanisms for the observed behavior are suggested.

PACS number(s): 21.10.Re, 23.20.Js, 21.60.Ev

I. INTRODUCTION

Low-lying vibrational excitations of the nucleus have been studied almost since the very beginning of nuclear spectroscopy, and a great deal of information has been accumulated. The systematic behavior of quadrupole transition probabilities $B(E2)\uparrow$ from the ground states to the lowest 2_1^+ states in even-even nuclei shows smooth behavior as a function of Z, A and excitation energy [1]. This behavior can be reproduced using the simple harmonic-vibrational model, even though individual nuclei display strong anharmonic effects. However, the empirical situation for transition probabilities from the ground states to the low-energy octupole states (LEOS) appears to be more complicated than in the quadrupole case.

In this work, we investigate the systematic behavior of octupole transition probabilities $B(E3)\uparrow$ from the ground states of even-even nuclei to the LEOS. In addition to examining the systematic behavior of quadrupole transition probabilities, the authors of Ref. [1] also examined octupole transition probabilities to the lowest 3^- states. They applied an empirical criterion to show that while the quadrupole mode is almost harmonic, the octupole mode has strong anharmonicity. Reference [1] restricted attention to $B(E3)$ values derived from Coulomb excitation and lifetime data. Here we consider data obtained with inelastic scattering measurements. It is well known that in many nuclei the LEOS is fragmented among a number of 3^- states. Here we take into account all of the observed fragments by considering the octupole strength for all observed 3^- states in the low-energy regime. This expanded data set gives a revised estimate of the anharmonicity of the octupole mode. In addition, a schematic model is described, in terms of which such an anharmonicity may be understood.

II. THEORETICAL BASIS OF THE ANALYSIS

We recapitulate the description of the quadrupole and octupole vibrations put forth by Bohr and Mottelson [2]. Because we are interested in describing only rough systematics, this theoretical framework is deliberately presented here in its most general form.

Multipole excitations (phonons) with angular momentum λ and projection μ are described by collective coordinates $\alpha_{\lambda\mu}$ and conjugate momenta $\pi_{\lambda\mu}$. To introduce a physical scale for transition probabilities requires knowing how the phonons are connected to the original nucleon variables. The collective transition operator for electric multipole excitations is usually taken to be proportional to the collective coordinate,

$$(E\lambda)_\mu = Q_\lambda \alpha_{\lambda\mu}, \tag{1}$$

where the amplitude Q_λ can be determined from microscopic theory or phenomenological model. Equation (1) defines the scale, which leads in the harmonic approximation to the reduced transition probability for the single-phonon excitation

$$B(E\lambda)\uparrow \equiv B(E\lambda;0\rightarrow\lambda) = \frac{2\lambda+1}{2D_\lambda\omega_\lambda} Q_\lambda^2. \tag{2}$$

Here ω_λ is the phonon frequency and D_λ is the inertial parameter for the mode under consideration. The inverse dependence on the transition energy is therefore a characteristic feature of any model with harmonic vibrations and "natural" transition operators as defined in Eq. (1).

The simplest and best-known example is the hydrodynamical model, in which the excitations correspond to small-

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amplitude shape vibrations of an incompressible liquid drop. It gives the collective multipole moment, as used in Eq. (1),

$$Q_\lambda = \frac{3}{4\pi} ZeR^\lambda, \tag{3}$$

where Z is the nuclear charge and R is the average (spherical) nuclear radius ($\propto A^{1/3}$). If the liquid motion is irrotational, the hydrodynamical mass parameter is, for a nucleus with mass number A,

$$D_\lambda^{\mathrm{irr}} = \frac{3}{4\pi\lambda} mAR^2, \tag{4}$$

where m is the nucleon mass. Combining these expressions, the model then predicts global behavior for transition probabilities as follows:

$$B(E\lambda)\uparrow \propto Z^2 A^{(2\lambda-5)/3} \omega_\lambda^{-1}. \tag{5}$$

The simplest general form for the dependence of $B(E3)\uparrow$ on A, Z, and 3^- state energy E_{3^-} for harmonic octupole vibrations is therefore straightforward— $B(E3)\uparrow$ will be inversely proportional to E_{3^-}:

$$B(E3)\uparrow \propto \frac{Z^2 R^6}{D_3} \frac{1}{E_{3^-}}. \tag{6}$$

Assuming $R \propto A^{1/3}$ and replacing the mass parameter D_3 for octupole vibrational flow with its irrotational value $D_3^{\mathrm{irr}} \propto AR^2$, we obtain

$$B(E3)\uparrow \propto Z^2 A^{1/3} (E_{3^-})^{-1}. \tag{7}$$

We are therefore interested in determining whether this expression for harmonic behavior reproduces the data. To this end, we perform a fit of the function

$$B(E3)\uparrow Z^{-2} A^{-1/3} = K E_{3^-}^{\eta}, \tag{8}$$

where K and η are determined by the fit. Reference [1] used the same procedure. Purely harmonic behavior results if $\eta = -1$. Deviations from this value indicate the degree of anharmonicity in octupole vibrations, and the scatter of data points around the line defined by Eq. (8) indicates the degree of noncollectivity of the octupole mode.

In this work, we have examined the available data on octupole states in nuclei with $A \geq 60$. We consider the entire LEOS, including all of the known fragments, in nuclei which have two or more known 3^- states with known $B(E3)\uparrow$ values. We compare the summed $B(E3)\uparrow$ to the *centroid* energy for all 3^- states within a given nucleus

$$E_{\mathrm{cent}} = \frac{\Sigma B(E3)\uparrow \times E}{\Sigma B(E3)\uparrow} \tag{9}$$

and obtain the constants K and η given in Eq. (8).

III. DETAILS OF THE ANALYSIS

It is well known that observed transition strengths in nuclei are dependent upon the reaction energy and type of probe [3]. This dependence arises from differences in how various probes interact with protons in the nucleus as opposed to neutrons. Because protons and neutrons may have differing motions within the nucleus, the observed deformation length $\delta = \beta R$ depends upon the ratio of the external-field interaction strengths b_n/b_p. We refer the reader to Ref. [3] for a more complete treatment of these differences, and merely note here that they are significant enough to warrant separate consideration of trends in scattering data depending on the probe used.

We have separated our data set into two subsets: (a) (p,p') reactions at $E_p < 50$ MeV for which $b_n/b_p \simeq 3$, and (b) (d,d') and (α,α') reactions for which $b_n/b_p = 1$. An extensive literature search of primary sources from 1967 through 1993 was conducted, and measured E and $B(E3)\uparrow$ data were tabulated for all nuclei with at least two definitely identified 3^- states of energy less than or equal to (20 MeV $A^{-1/3}$)—a cutoff chosen to isolate possible contributions from the low-energy octupole resonance (LEOR) found at or near (31 MeV $A^{-1/3}$) [4]. Data from reactions below the Coulomb barrier were not included in this analysis, because there are comparatively little data of good quality on higher-lying 3^- states from these reactions and Coulomb excitation measures different properties than inelastic scattering reactions above this barrier. A complete list of references used in the (p,p') compilation [5–12] and in the (d,d') and (α,α') compilation [8,11–33] is given at the end.

For inelastic (p,p') scattering, a total of 138 states in 24 nuclei were tabulated, and for inelastic (d,d') and (α,α') scattering, 238 states in 56 nuclei. The data sets were fitted to Eq. (8) by means of a least-squares-fit routine identical to that used in Ref. [1]. The $B(E3)\uparrow$ values themselves were either taken directly from the references, or calculated from the deformation parameter β (or the deformation length $\delta = \beta R$). The expression for this calculation is [34]

$$B(E3)\uparrow = (3ZR^3\beta/4\pi)^2, \tag{10}$$

where R is the nuclear radius defined by $R = 1.2A^{1/3}$. The deformation parameters are usually extracted using the deformed optical-model potential. It has recently been suggested by Beene *et al* [35]. that a more realistic analysis should use a folding model.

The fits were performed with individual points weighted by either $1/\sigma$ or $1/\sigma^2$, where σ is the uncertainty in $\Sigma B(E3)\uparrow$. The $1/\sigma$ weighting matches that used in Raman *et al.* [1], where it was chosen as a means of accounting for the often quite optimistic uncertainties quoted in the primary references. We preferred to use the canonical $1/\sigma^2$ weighting, and to account for optimistic (or nonexistent) uncertainty estimates by adjusting the uncertainties themselves. This was done by using a global estimate of $\sigma(\beta^2) = 0.15\beta^2$ for each state. As will be seen below, choice of weighting had a measurable but not dominant effect on the fits. Figure 1 illustrates the fits for the two data sets with $1/\sigma^2$ weighting. Table I summarizes our results, specifically the exponents for the

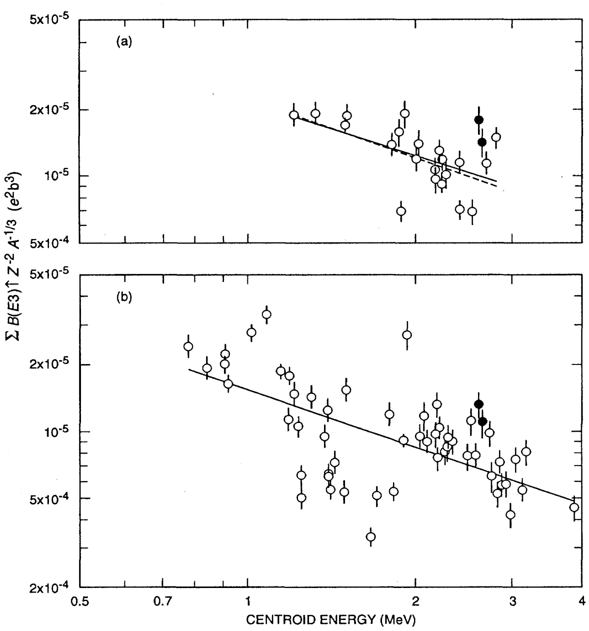

FIG. 1. Fits of summed transition strength vs centroid energy for the (a) proton and (b) deuteron and alpha data sets. The solid lines show results of least-squares fitting with $1/\sigma^2$ weighting (see Table I). The dashed line in (a) results when the two solid data points for ^{206,208}Pb are omitted in the fit. The corresponding dashed line in (b) is nearly indistinguishable from the solid line and, therefore, is not shown. The fitted K and η values given in Table I are strongly anticorrelated. Varying only the K values within the uncertainty ranges given in Table I accounts for $\sim$ 75% of the data points in (a) and $\sim$ 48% in (b).

fits with Z and A weightings as shown in Eq. (8), where $\eta = -1.00$ for harmonic behavior.

According to Ohm *et al.* [36], $B(E3)\uparrow$ values for the first 3^- states of a small subset of nuclei near closed shells do show an $\eta = -1$ dependence. However, this conclusion has been questioned by Horen *et al.* [37] after a remeasurement of the lifetime of the first 3^- state in ^{96}Zr and a refitting of the $B(E3)\uparrow$ data for 13 nuclei.

IV. RESULTS AND DISCUSSION

The proton and deuteron/alpha data sets give fits which are consistent with one another. The spread between values with the two types of uncertainty weightings is always much less than the uncertainty of the final numbers, indicating that the fitting process itself does not contribute strongly to the final results.

After the initial fit of the (p,p') data set with all data points included, the points for the Pb isotopes were seen to lie farther from the fit from most others. One possible explanation for this discrepancy is that in Pb the energy of the LEOS 3^- centroid is much closer to that of the low-energy octupole resonance (LEOR) than in any other element. The difference between the LEOS and LEOR energies is approximately 2.6 MeV for both ^{206}Pb and ^{208}Pb, whereas differences for other nuclei in the set are all 3.5 MeV or greater. This proximity could lead to greater mixing between the LEOR and the LEOS, causing the Pb points to skew the data set. Fits without the Pb data were therefore performed to examine whether a significantly different result would be obtained. The fitted η values for the data sets without Pb (see Table I) are of some interest, as they are still within one σ of the results for the full data set, but are also within one σ of -1.0. From this result, we infer that octupole transitions are generally not harmonic, but that subsets of nuclei may undergo harmonic octupole vibrations. This inference allows for the possibility of a few instances of observable two-octupole-phonon multiplets in the chart of the nuclides, and lends some encouragement to those searching for such states. However, more high-resolution scattering data are needed to improve the reliability of this analysis.

It is worth noting that while the values of η deduced from the (p,p') data set are identical to those deduced from the $(d,d'),(\alpha,\alpha')$ data set, the values of K from the two data sets are somewhat different. If the LEOS is a purely isoscalar excitation, then the $B(E3)\uparrow$ values measured with different probes should be equal, and it would be expected that the values of K are equal as well. The difference in the K values determined here warrants further investigation.

V. THEORETICAL PREDICTIONS FOR THE FIT PARAMETERS

To make microscopic predictions for the average global behavior of transition probabilities, we may use the schematic microscopic model with pairing and multipole-multipole forces as the main components of the residual interaction. The coupling constant of multipole interaction is given [2] by the self-consistency condition between the den-

TABLE I. Summary of fits from the current study.

Multipolarity	Probe	Weighting	$K(\times 10^{-5})$	η
$E3$	p	$1/\sigma$	2.12 ± 0.42	-0.75 ± 0.26
		$1/\sigma^2$	2.13 ± 0.41	-0.79 ± 0.26
$E3$, no Pb	p	$1/\sigma$	2.31 ± 0.44	-0.91 ± 0.26
		$1/\sigma^2$	2.27 ± 0.43	-0.90 ± 0.25
$E3$	d,α	$1/\sigma$	1.51 ± 0.15	-0.78 ± 0.15
		$1/\sigma^2$	1.55 ± 0.15	-0.85 ± 0.15
$E3$, no Pb	d,α	$1/\sigma$	1.52 ± 0.15	-0.82 ± 0.15
		$1/\sigma^2$	1.55 ± 0.15	-0.89 ± 0.15
$E3$ from Ref. [1]	various	$1/\sigma$	1.8 ± 0.3	-0.72 ± 0.13 (vibrational)
		$1/\sigma$	0.59 ± 0.05	-0.55 ± 0.15 (rotational)

sity and the field acting on the particles. In such an approach, low-lying collective vibrations appear as coherent soundlike two-quasiparticle excitations rather than pure surface hydrodynamical waves. Pairing is very important in this description, because the phonon frequencies as a rule fall within the energy gap $2\bar{E}$ for Cooper pair breaking, $\tau \equiv \omega_\lambda/2\bar{E} < 1$. The adiabaticity of vibrations allows us to discuss both the soft quadrupole ($\tau \ll 1$) mode and the octupole mode, whose adiabaticity is less pronounced.

The overall systematics [1] for quadrupole transitions, $\lambda = 2$, confirms the ω_2^{-1} dependence, but clearly prefers $\sim A^{-2/3}$ scaling, whereas Eq. (5) gives $\sim A^{-1/3}$ (and $\sim A^{1/3}$ in the octupole case). It is also well known that the residual interactions and shell effects make nonmagic nuclei much softer than in the hydrodynamical model. Liquid-drop calculation of the restoring force C_2 leads to quadrupole frequencies ω_2 which are typically several times higher than empirical ones (hydrodynamical frequencies are qualitatively correct only in near-magic nuclei).

For the quadrupole case, nonmagic spherical nuclei are apparently at the limit of the phase transition to static deformation. Therefore we can make estimates of $B(E\lambda)\uparrow$ in the adiabatic random-phase approximation (RPA) [38] near the critical point where the restoring force vanishes. In fact, in this region, strong anharmonic effects, mostly due to the quartic anharmonicity [38,39], stabilize the nucleus. However, the general structure of the excitation spectrum is still similar to that of the harmonic vibrator as a result of the prevailing $\mathcal{O}(5)$ symmetry. Qualitative estimates [38] give the following simple expression for the global behavior of the transition probabilities:

$$B(E\lambda)\uparrow = \frac{2\lambda+1}{2} e^2 \left(\frac{Z}{A}\right)^2 \Omega q_\lambda^2 \frac{2\bar{E}}{\omega_\lambda}. \tag{11}$$

Here, $q_\lambda \simeq \langle r^{2\lambda}\rangle \xi^2$ is the typical reduced matrix element for a single-particle transition ($\xi \simeq 1/2$ is an average reduction factor connected with the pairing effects), and $2\bar{E} \simeq \epsilon_F/\sqrt{A}$ is the two-quasiparticle excitation energy. The factor $\Omega \simeq \sqrt{A}$ characterizes the degree of collectivity of the phonon mode (the number of quasiparticles contributing to the phonon formation). In the quadrupole case, using $\langle r^4\rangle = 0.95A^{4/3}$ fm^4, we obtain (ω_2 is given in keV)

$$B(E2)\uparrow \simeq 8.5\xi^2 Z^2 A^{-2/3} \omega_2^{-1} e^2\mathrm{b}^2. \tag{12}$$

We can directly compare this with the empirical fit of Raman *et al.* [1],

$$B(E2)\uparrow = (2.6 \pm 0.6) Z^2 A^{-2/3} \omega_2^{-1} e^2\mathrm{b}^2. \tag{13}$$

Although the numerical agreement may be coincidental, the general trend is reproduced in a natural way. Because the zero-point vibrational amplitude in soft spherical nuclei is of the same order of magnitude as the static quadrupole deformation in permanently deformed (rotational) nuclei, we expect the same estimate to be valid for rotational excitations as well. This expectation is confirmed in Ref. [1].

Low-lying octupole excitations appear less regularly and do not have energies as low as those of quadrupole modes. Nevertheless, there are 3^- levels with energies lower than $2\bar{E}$ which display typical signatures of collective nature. The liquid-drop model allows octupole shape vibrations among other isoscalar modes of natural parity. The reduced transition probabilities in the hydrodynamical model are predicted, according to Eqs. (4)–(6), to satisfy

$$B(E3)\uparrow Z^{-2}A^{-1/3} \approx 2.2\times 10^{-4}[\omega_3(\mathrm{MeV})]^{-1} e^2\mathrm{b}^3. \tag{14}$$

The value of K thus obtained is larger than that seen in the fits by an order of magnitude. The inverse energy dependence shown here is stronger than that seen in the fits; we will discuss the discrepancy below.

The adiabatic microscopic model used earlier in the quadrupole case gives a different A dependence for the octupole mode. Expressed in a form similar to Eq. (14), it reads

$$B(E3)\uparrow Z^{-2}A^{-1/3} \approx 15\times 10^{-5}(\Omega A^{-5/6}) \times \xi^2[\omega_3(\mathrm{MeV})]^{-1} e^2\mathrm{b}^3. \tag{15}$$

The collectivity factor Ω should be weaker here than in the quadrupole case ($\sim A^{1/2}$). Because the elementary octupole excitation involves an intruder single-particle level with parity opposite to that of the rest of the subshells in the same major shell, we expect $\Omega \sim A^{1/3}$. Then the numerical coefficient in Eq. (15) becomes $15\times 10^{-5}A^{-1/2}\xi^2 \simeq 0.4\times 10^{-5}$ for medium mass nuclei, which is too small when compared to the fitted value of K. In addition, the approximation of extreme adiabaticity, $\tau \ll 1$, used in the derivation of Eq. (11) for the octupole case, is unrealistic. As can be seen from the RPA solution, the transition amplitude for larger values of τ acquires a factor $(1-\tau^2)^{-1}$, which can compensate the coherence factor ξ^2 and bring the microscopic vibrational estimate [see Eq. (2)] closer to the average K seen in the fits. However, it cannot explain the deviation of the effective exponent η in the energy dependence towards the values $\eta < 1$. There are, however, other considerations that may account for this.

Another change in the global behavior can be expected, resulting from the anharmonicity of vibrations. As we have already mentioned, the strong anharmonic effects in the quadrupole case still preserve the general oscillator-like structure of the 2^+ excitations; the situation might well be different for octupole excitations. Intrinsic anharmonicity of the octupole mode should be weak, because the negative parity does not permit mixing of one- and two-phonon states. Admixture of the two possible three-phonon multiplet members 3^- is small because of the relatively high ω_3. However, the interaction of octupole excitations with the soft quadrupole mode can be quite important.

The simplest anharmonic coupling which conserves the number of octupole phonons and therefore does not need any mixing with high-lying states leads to the phenomenological Hamiltonian

$$H=H_2+H_3^{(0)}+x\sum_{\mu}[(f^\dagger f)_{2\tilde{\mu}}d_\mu+d_\mu^\dagger(f^\dagger f)_{2\mu}], \quad (16)$$

where H_2 stands for the quadrupole part, including all anharmonic corrections, and $H_3^{(0)}$ is the purely harmonic octupole vibrator $H_3^{(0)}=\Sigma_\mu \omega_3 f_\mu^\dagger f_\mu$. We introduce the notations f_μ and d_μ for the octupole and quadrupole boson operators, respectively.

The model Hamiltonian, Eq. (16), gives an expression of the following type for the energy E_3 of the lowest octupole excitations:

$$E_3=\omega_3-a\frac{x^2}{\omega_2}, \quad (17)$$

where the numerical factor a can depend on the details of the quadrupole anharmonicity. The approximate self-consistent solution for the low-lying eigenstates of the Hamiltonian of Eq. (16) which leads to Eq. (17) can be derived as follows.

The quadrupole excitations can be modeled [38,39] by an effective vibrator with the low frequency ω_2. This means that the effective equations of motion for the quadrupole degree of freedom interacting with the octupole mode are

$$i\dot{d}_\mu=\omega_2 d_\mu+x(f^\dagger f)_{2\mu}. \quad (18)$$

Thus, the presence of the octupole excitation induces the effective deformation $\langle d_\mu\rangle$ of the soft quadrupole field,

$$\langle d_\mu\rangle=-\frac{x}{\omega_2}\langle (f^\dagger f)_{2\mu}\rangle. \quad (19)$$

Since the octupole field is time reversal invariant and the coupling constant can be taken as real, we have $\langle d_\mu\rangle=\langle d_{\tilde{\mu}}^\dagger\rangle$.

The nonzero quadrupole deformation (19), in turn, renormalizes the octupole excitation energy as seen from the equation of motion for the octupole mode,

$$i\dot{f}_\mu=\omega_3 f_\mu+x\sum_{\mu'\nu}\langle 3\tilde{\mu}\ \ 3\mu'|2\tilde{\nu}\rangle f_{\mu'}\langle d_\nu\rangle. \quad (20)$$

The set of equations (19) and (20) can be easily solved for the axially symmetric case when the octupole motion singles out the symmetry axis in the isotropic system. Then we can look for the solution proportional to the corresponding spherical function,

$$f_\mu=\sqrt{\frac{4\pi}{7}}Y_{3\mu}(\mathbf{n}), \quad (21)$$

where the normalization factor is chosen for the single-phonon octupole state 3_1^-, $\Sigma_\mu f_\mu^\dagger f_\mu=1$. The induced quadrupole deformation has the same symmetry axis,

$$\langle d_\mu\rangle=Y_{2\mu}(\mathbf{n})\,\delta_2, \quad (22)$$

and its magnitude δ_2 is to be found from (19) and (21). Simple algebra with spherical functions gives

$$\delta_2=-4\sqrt{\frac{\pi}{105}}\frac{x}{\omega_2}. \quad (23)$$

The same ansatz (21) satisfies the equation of motion in Eq. (20) for the octupole mode with energy E_3, which takes the form of Eq. (17) with the numerical factor $a=8/21$.

The coupling constant x in the model Hamiltonian can be calculated microscopically in the same way as quadrupole anharmonic terms [40]; it depends smoothly on the octupole frequency ω_3. Then the main modification of the global expression [Eq. (15)] would be the substitution of ω_3, which was identified in the harmonic approximation with the excitation energy E_3, by $E_3+(\text{const})/\omega_2$. If this is the case, we may expect an anticorrelation of the transition probability $B(E3)$ with the quadrupole frequency ω_2.

In addition, we note that octupole vibrations are more surfacelike than quadrupole ones. In the limit of surface waves on a liquid drop, the normal frequencies are scaled as $\omega\propto A^{-1/2}$. If the collective strength can grow as $\Omega\propto(\text{area})\propto A^{2/3}$, then we have from Eq. (15), $B(E3)\propto\omega^{-2/3}$. So in this limiting case, $\eta=-2/3$, and we could argue that the physical situation lies somewhere in the range $-1<\eta<-2/3$. Either or both of these effects can lead to deviations of η from the harmonic value, in particular placing it in the range seen in the various fits.

VI. CONCLUSIONS

To summarize, we have demonstrated that octupole vibrations are generally not harmonic via an analysis of (p,p'), (d,d'), and (α,α') data on 3^- states. We have compared the summed transition strengths $B(E3)\uparrow$ of all known 3^- states in each nucleus against the centroid of the state energies, and fitted a functional dependence that serves as a measure of anharmonicity of octupole vibrations in nuclei. Our results confirm the earlier results of Raman *et al.*, which used first 3^- states in a larger sampling of nuclei. Trial fits with the Pb nuclei (in which the LEOS may be mixed with the LEOR) omitted imply that the overall harmonicity of these octupole vibrations may be greater than initially anticipated. Our results for η do not correspond to the simple harmonic prediction of the hydrodynamical model, but can be explained by invoking interaction with the quadrupole mode and the surfacelike nature of octupole vibrations.

These results are particularly relevant because a new generation of radiocative ion beam facilities will make possible the measurement of $B(E3)$ values in many unstable isotopes via scattering reactions in inverse kinematics. We anticipate that the availability of such reactions will add significantly to our knowledge base in this area.

ACKNOWLEDGMENTS

This work was supported in part by the National Science Foundation under Grants PHY-9210082 and PHY-9403666, in part by the State of Florida, and in part by the US DOE under Contract DE-AC05-84OR21400 with Martin Marietta Energy Systems, Inc.

[1] S. Raman, C. W. Nestor, Jr., S. Kahane, and K. H. Bhatt, Phys. Rev. C **43**, 556 (1991).
[2] A. Bohr and B. M. Mottelson, *Nuclear Structure Vol. II* (Benjamin, Reading, 1975), p. 405.
[3] A. M. Bernstein, V. R. Brown, and V. A. Madsen, Comments Nucl. Part. Phys. **11**, 203 (1983).
[4] M. W. Kirson, Phys. Lett. **108B**, 237 (1982).
[5] W. T. Wagner, G. M. Crawley, G. R. Hammerstein, and H. McManus, Phys. Rev. C **12**, 757 (1975).
[6] J. E. Finck, G. M. Crawley, J. A. Nolen, Jr., and R. T. Kouzes, Nucl. Phys. **A407**, 163 (1983).
[7] P. D. Cottle, K. A. Stuckey, and K. W. Kemper, Phys. Rev. C **38**, 2843 (1988).
[8] M. Pignanelli, N. Blasi, S. Micheletti, R. de Leo, M. A. Hofstee, J. M. Schippers, S. Y. van der Werf, and M. N. Harakeh, Nucl. Phys. **A519**, 567 (1990).
[9] A. Hogenbirk, H. P. Blok, and M. N. Harakeh, Nucl. Phys. **A524**, 251 (1991).
[10] E. Müller-Zanotti, R. Hertenberger, H. Kader, D. Hofer, G. Graw, Gh. Cata-Danil, G. Lazzari, and P. F. Bortignon, Phys. Rev. C **47**, 2524 (1993).
[11] D. Hofer, M. Bisenberger, R. Hertenberger, H. Kader, H. J. Maier, E. Müller-Zanotti, P. Schiemenz, G. Graw, P. Maier-Komor, G. Molnár, W. Unkelbach, J. Hebenstreit, H. Ohm, D. Paul, P. von Rossen, and M. Fujiwara, Nucl. Phys. **A551**, 173 (1993).
[12] M. Pignanelli, N. Blasi, J. A. Bordewijk, R. de Leo, M. N. Harakeh, M. A. Hofstee, S. Micheletti, R. Perrino, V. Yu. Ponomarev, V. G. Soloviev, A. V. Sushkov, and S. Y. van der Werf, Nucl. Phys. **A559**, 1 (1993).
[13] R. Bloch, B. Elbek and P. O. Tjǿm, Nucl. Phys. **A91**, 576 (1967).
[14] T. Grotdal, K. Nybø, T. Thorsteinsen, and B. Elbek, Nucl. Phys. **A110**, 385 (1968).
[15] P. O. Tjǿm and B. Elbek, Nucl. Phys. **A107**, 385 (1968).
[16] E. Veje, B. Elbek, B. Herskind, and M. C. Olesen, Nucl. Phys. **A109**, 489 (1968).
[17] N. Alpert, J. Alster, and E. J. Martens, Phys. Rev. C **2**, 974 (1970).
[18] N. Alpert, J. Alster, E. J. Martens, and W. Pickles, Phys. Rev. C **4**, 1230 (1971).
[19] F. T. Baker and R. Tickle, Phys. Rev. C **5**, 182 (1972).
[20] Th. W. Elze and J. R. Huizenga, Nucl. Phys. **A187**, 545 (1972).
[21] J. S. Boyno, J. R. Huizenga, Th. W. Elze, and C. E. Bemis, Jr., Nucl. Phys. **A209**, 125 (1973).
[22] S. J. Burger and G. Heymann, Nucl. Phys. **A243**, 461 (1975).
[23] R. C. Thompson, J. R. Huizenga, and Th. W. Elze, Phys. Rev. C **12**, 1227 (1975).
[24] S. W. Yates, A. M. Friedman, and I. Ahmad, Phys. Rev. C **12**, 795 (1975).
[25] H. J. Gils, H. Rebel, J. Buschmann, H. Klewe-Nebenius, G. P. Nowicki, and W. Nowatzke, Z. Phys. A **279**, 55 (1976).
[26] J. Rekstad and P. O. Tjǿm, J. Phys. G **3**, 411 (1977).
[27] H. P. Morsch, C. Sükösd, M. Rogge, P. Turek, H. Machner, and C. Mayer-Böricke, Phys. Rev. C **22**, 489 (1980).
[28] I. Kumabe, M. Matoba, M. Inoue, S. Matsuki, and E. Takasaki, Bull. Inst. Chem. Res. Kyoto Univ. **60**, 106 (1982).
[29] I. M. Govil, H. W. Fulbright, D. Cline, E. Wesolowski, B. Kotlinski, A. Backlin, and K. Gridnev, Phys. Rev. C **33**, 793 (1986).
[30] M. Lahanas, D. Rychel, P. Singh, R. Gyufko, D. Kolbert, B. van Krüchten, E. Madadakis, and C. A. Wiedner, Nucl. Phys. **A455**, 399 (1986).
[31] P. Singh, D. Rychel, R. Gyufko, B. van Krüchten, M. Lahanas, and C. A. Wiedner, Nucl. Phys. **A458**, 1 (1986).
[32] I. M. Govil, H. W. Fulbright, and D. Cline, Phys. Rev. C **36**, 1442 (1987).
[33] B. Schürmann, D. Rychel, B. van Krüchten, J. Speer, and C. A. Wiedner, Nucl. Phys. **A475**, 361 (1987).
[34] A. M. Bernstein, in *Advances in Nuclear Physics*, edited by M. Baranger and E. Vogt (Plenum, New York, 1969), Vol. 3, p. 411.
[35] J. R. Beene, D. J. Horen, and G. R. Satchler, Phys. Lett. B **344**, 67 (1995).
[36] H. Ohm, M. Liang, G. Molnár, S. Raman, K. Sistemich, and W. Unkelbach, Phys. Lett. B **241**, 472 (1990).
[37] D. J. Horen, R. L. Auble, G. R. Satchler, J. R. Beene, I. Y. Lee, C. Y. Wu, D. Cline, M. Devlin, R. Ibbotson, and M. W. Simon, Phys. Rev. C **48**, R2131 (1993).
[38] V. G. Zelevinsky, Int. J. Mod. Phys. E **2**, 273 (1993).
[39] O. K. Vorov and V. G. Zelevinsky, Nucl. Phys. **A439**, 207 (1985).
[40] V. G. Zelevinsky, in *Perspectives for the Interacting Boson Model on the Occasion of its 20th Anniversary*, edited by R. F. Casten, A. Vitturi, A. B. Balantekin, B. R. Barrett, J. N. Ginocchio, G. Maino, and T. Otsuka (World Scientific, Singapore, 1994), p. 307.