Longitudinal response functions of ³He and ³H by Lorentz kernel transformations

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The longitudinal response functions of three-body nuclei are obtained from the inversion of their integral transforms with a Lorentz kernel. The full dynamics of initial and final states is treated within this method. Results are in excellent agreement with those of previous quite different calculations. A comparison with experimental data is also presented.

PACS number(s): 21.45.+v, 25.30.Fj

I. INTRODUCTION

Inclusive electron scattering on nuclei leads to nuclear continuum states. Already for three nucleons the boundary conditions of those states are quite complex [1], not to speak of a larger number of particles. For three nucleons the continuum has been mastered [2] and already applications to inelastic electron scattering on three-nucleon ground states have appeared [3,4]. Nevertheless a method which relies on bound state techniques, avoiding asymptotic boundary conditions of oscillatory nature or related singularity structures in momentum space treatments, would be welcome.

Two attempts in this direction have been done to calculate inclusive scattering response functions. One relies on a Stieltjes transform [5], the other on a Laplace transform [6]. Both are still plagued by the ill posed nature of the corresponding inverse transformations. Nevertheless they have been already successfully applied [6,7] to inclusive scattering on the α particle and ³He, thereby treating the nuclear dynamics in its full complexity. The transforms of both the theoretical and experimental responses have been compared to each other. This provided very important information through the insight that in the cases studied final state interactions were absolutely mandatory. Though that useful information was obtained without transforming into the energy space the direct view onto the measured responses should be the aim.

In Ref. [8] an integral transform with a Lorentz kernel was proposed and tested in a soluble case, i.e., the deuteron.

The particular shape of the kernel which does not spread the information about the response function, but averages it in a more or less narrow range, has allowed the inversion with great accuracy. Encouraged by that work we have now performed a realistic study of inclusive electron scattering on the $3N$ bound states. Realistic forces are employed and the predictions are compared both to results obtained with a more direct method and to data.

The theoretical formulation will be given in Sec. II and our results in Sec. III. We finish with a brief outlook.

II. THEORETICAL FORMULATION

We start by reviewing briefly the initial steps of the integral transform with a Lorentz kernel as introduced in [8]. Let

$$R(\omega) = \sum_{f \neq 0} |\langle f|\hat{O}|0\rangle|^2 \delta(\omega + E_0 - E_f) \tag{1}$$

be an inelastic response function, where $|0\rangle$ and $|f\rangle$ are eigenstates to the underlying Hamiltonian H with energies E_0 and E_f, respectively. Then the transformation with a Lorentz kernel is defined by

$$\Phi(\sigma_R, \sigma_I) = \int_{\omega_{\text{th}}}^{\infty} d\omega \, \frac{R(\omega)}{\sigma_I^2 + (\omega - \sigma_R)^2}, \tag{2}$$

where ω_{th} corresponds to the lowest inelastic threshold. Inserting (1) into (2) yields

$$\Phi(\sigma_R, \sigma_I) = \sum_{f \neq 0} |\langle f|\hat{O}|0\rangle|^2 \frac{1}{\sigma_I^2 + (E_f - E_0 - \sigma_R)^2} = \langle 0|\hat{O}^\dagger \frac{1}{H - E_0 - \sigma_R - i\sigma_I} \frac{1}{H - E_0 - \sigma_R + i\sigma_I} \hat{O}|0\rangle - \frac{|\langle 0|\hat{O}|0\rangle|^2}{\sigma_I^2 + \sigma_R^2}. \tag{3}$$

The transform $\Phi(\sigma_R, \sigma_I)$ is the sum of an elastic contribution and a norm $\langle \Psi_0|\Psi_0\rangle$, with $|\Psi_0\rangle$ defined by

$$|\Psi_0\rangle = \frac{1}{H - E_0 - \sigma_R + i\sigma_I} \hat{O}|0\rangle. \tag{4}$$

For $\sigma_I \neq 0$ the asymptotic behavior of $|\Psi_0\rangle$ is exponentially damped like for a bound state. Obviously $|\Psi_0\rangle$ obeys the inhomogeneous equation

$$(H - E_0 - \sigma_R + i\sigma_I)|\Psi_0\rangle = \hat{O}|0\rangle. \tag{5}$$

<u>52</u> 1778

Let us now regard the case of three nucleons and use the Faddeev scheme. For

$$H = H_0 + \sum_i V_i \tag{6}$$

(V_i being the pair interactions in usual odd man out notation) and assuming a single particle operator

$$\hat{O} = \sum_i \hat{O}_i , \tag{7}$$

we can rewrite Eq. (4) as

$$|\Psi_0\rangle = \frac{1}{E_0 + \sigma_R - i\sigma_I - H_0} \sum_i V_i |\Psi_0\rangle$$

$$- \frac{1}{E_0 + \sigma_R - i\sigma_I - H_0} \sum_i \hat{O}_i |0\rangle . \tag{8}$$

This defines Faddeev components as

$$|\Psi_0\rangle \equiv \sum_i |\psi_i\rangle . \tag{9}$$

Since the particles are identical only one Faddeev component is needed and the other two result by the sum of a cyclical and anticyclical permutation P. Thus

$$|\Psi_0\rangle = (1+P)|\psi\rangle , \tag{10}$$

with

$$|\psi\rangle \equiv \frac{1}{E_0 + \sigma_R - i\sigma_I - H_0} V_1 |\Psi_0\rangle$$

$$- \frac{1}{E_0 + \sigma_R - i\sigma_I - H_0} \hat{O}_1 |0\rangle . \tag{11}$$

Replacing $|\Psi_0\rangle$ by (10) and performing the standard steps one gets

$$|\psi\rangle = G_0 t_1 P |\psi\rangle - (G_0 + G_0 t_1 G_0) \hat{O}_1 |0\rangle , \tag{12}$$

with the free propagator

$$G_0 \equiv \frac{1}{E_0 + \sigma_R - i\sigma_I - H_0} \tag{13}$$

and the NN t operator defined by

$$t_1 = V_1 + V_1 G_0 t_1 . \tag{14}$$

Equation (12) is an inhomogeneous Faddeev equation, evaluated at the complex energy $E_0 + \sigma_R - i\sigma_I$. Once the Faddeev amplitude ψ has been determined we get

$$\Phi(\sigma_R, \sigma_I) = 3\langle\psi|(1+P)|\psi\rangle - \frac{|\langle 0|\hat{O}|0\rangle|^2}{\sigma_I^2 + \sigma_R^2} , \tag{15}$$

where the factor 3 results from the antisymmetry of $|\Psi_0\rangle$.

As an application we now regard inelastic electron scattering on ^{3}He and ^{3}H and concentrate on the longitudinal

response function $R(\omega, |\vec{Q}|)$ at fixed momentum transfer $|\vec{Q}|$, driven by the density operator $\hat{O} \equiv \rho$. We solve the inhomogeneous Faddeev equation by our standard tools in momentum space [1,9]. In the partial wave decomposition we keep the NN force only in the dominant states $^1S_0$ and $^3S_1 - ^3D_1$. However, we include states of total $3N$ angular momenta up to $15/2^\pm$. We have checked that higher angular momenta give negligible contributions. The resulting set of two-dimensional integral equations are solved precisely with a typical accuracy of about 1%. We use the simplest single nucleon operator without relativistic corrections as described in Refs. [3,4]

$$\rho = \sum_{i=1}^{Z} F_p e^{i\vec{Q}\cdot\vec{r}_i} + \sum_{i=1}^{N} F_n e^{i\vec{Q}\cdot\vec{r}_i} , \tag{16}$$

where the Gari-Krümpelmann [10] parametrization has been used for F_p and F_n representing the proton and neutron form factors.

The explicit form of the partial wave representation of ρ applied on the $3N$ bound state $|0\rangle$ in momentum space is given in [11]. Since ρ breaks isospin we include states of total isospin $T = 1/2$ and $3/2$. The Coulomb force between the two protons is neglected. For the $3N$ bound state we use consistently just the five channel solutions of the corresponding Faddeev equations.
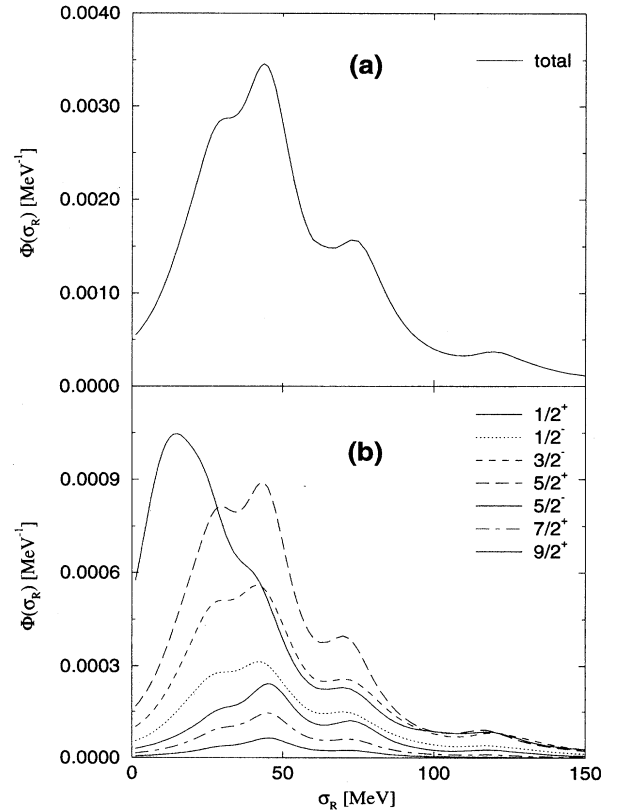

FIG. 1. The Lorentz transform of the longitudinal response function of ^{3}He at $|\vec{Q}| = 300$ MeV/c ($\sigma_I = 10$ MeV); (a) the total sum of all channels contributions; (b) single channel contributions.
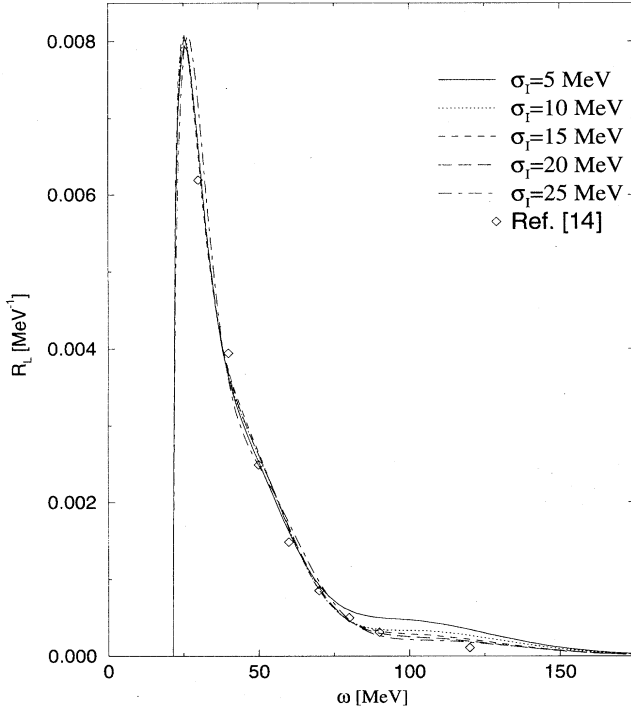
FIG. 2. The monopole contribution to the longitudinal response function of ^{3}He at $|\vec{Q}| = 300$ MeV/c, for different values of σ_I. The results are compared with those obtained in Ref. [14] (diamonds).

III. RESULTS AND DISCUSSION

In Ref. [11] we have shown that the longitudinal response function has little sensitivity to the choice of the NN force. Therefore here we restrict ourselves to the Reid [12] and Bonn-B [13] NN forces.

The inhomogeneous Faddeev equation has to be solved for each total $3N$ angular momentum J and the two parities separately. $\Phi(\sigma_R, \sigma_I)$ is then obtained from Eq. (3) summing up all partial amplitudes.

In Fig. 1(a) we show $\Phi(\sigma_R, \sigma_I)$ for $\sigma_I = 10$ MeV as a function of σ_R and for $|\vec{Q}| = 300$ MeV/c. In this example the Reid potential has been used. One can notice how the form of the transform resembles that of $R(\omega, |\vec{Q}|)$, with the typical quasielastic peak around $|\vec{Q}|^2/2M \approx 50$ MeV. Of course this is expected because of the bell shaped nature of the kernel.

Since $R(\omega, |\vec{Q}|)$ builds up additively out of the different J^π contributions, one could generate them separately out of the corresponding Φ amplitudes. In Fig. 1(b) some of these amplitudes are shown for J^π ranging from $1/2^+$ to $9/2^+$. One sees that the shapes of the transforms are very similar in all cases but in the channel $J^\pi = 1/2^+$. This corresponds to the monopole part of the operator and it is directly connected to the three nucleon ground state. So it is wise to add first all J^π contributions up to $15/2^\pm$ except for $1/2^+$ and then invert that sum and the $1/2^+$ piece separately. This same procedure was applied successfully in Ref. [8]. The second term in (15) is present only in the state $1/2^+$. In this case the cancellation between the two terms may be delicate. We notice that the
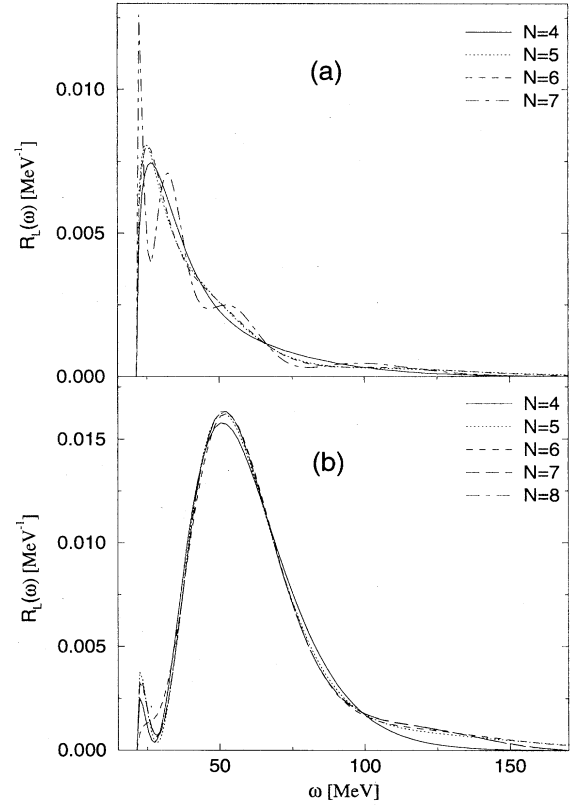

FIG. 3. The longitudinal response function of ^{3}He at $|\vec{Q}| = 300$ MeV: dependence on the number of basis functions; (a) monopole contribution; (b) sum of all other amplitudes.

second term can be replaced identically by $-9|\langle 0|\psi\rangle|^2$. This is the way we actually determine Φ for $J = 1/2^+$, which is superior since the Faddeev component $|\psi\rangle$ acts like a projector on five channels for the ground state. If we did not make this replacement the full bound state $|0\rangle$ with an infinite number of channels should have been taken into account.

The inversion follows the way proposed in [8]. We expand the structure function in a set of functions $\chi_n(\omega)$

$$R(\omega, |\vec{Q}|) = \sum_{n=1}^{N} a_n \chi_n(\omega),\qquad(17)$$

with

$$\chi_n(\omega) = \omega^{n-1/2} e^{-\omega/E_0}.\qquad(18)$$

This leads to

$$\Phi(\sigma_R, \sigma_I) = \sum_{n}^{N} a_n \xi_n(\sigma_R, \sigma_I),\qquad(19)$$

with

$$\xi_n(\sigma_R, \sigma_I) = \int_{\omega_{th}}^{\infty} d\omega \frac{\chi_n(\omega)}{\sigma_I^2 + (\omega - \sigma_R)^2}.\qquad(20)$$

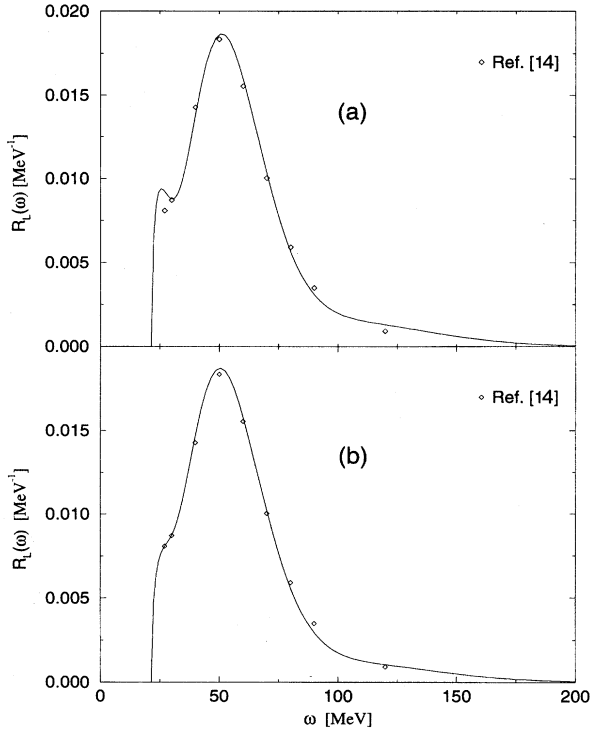FIG. 4. The longitudinal response function of ^{3}He at $|\vec{Q}| = 300$ MeV; (a) sum of the monopole contribution [$N=6$, $E_0=10.25$ MeV in Fig. 3(a)] to the total contribution of the other channels [$N=6$, $E_0=11$ MeV in Fig. 3(b)]; (b) the result with the cut at low energy (see text). The results (Reid potential) are compared with those obtained in Ref. [14] (diamonds).

For fixed σ_I the ($N+1$) parameters a_n and E_0 are optimized to represent $\Phi(\sigma_R, \sigma_I)$ for I values of σ_R with $I \gg N$. We have calculated $\Phi(\sigma_R, \sigma_I)$ for σ_I ranging from 5 to 25 MeV in steps of 5 MeV and for σ_R from 1.25 to 200 MeV in $I = 100$ points.

We have found that the functions resulting from this inversion procedure are independent on the values of σ_I. An example can be seen in Fig. 2, where the monopole contribution to $R(\omega, |\vec{Q}|)$ of ^{3}He is shown for different values of σ_I. Therefore in the following we will present results for the fixed value of $\sigma_I = 10$ MeV.

In Fig. 3 the typical quality of the convergence in N of the results is shown, both for the monopole part and for the rest of the amplitudes considered. The range of variation of the curves from $N=4$ to $N=7$ in the former case and from $N=4$ to $N=8$ for the latter gives an estimate of the magnitude of the error in our results. Higher values of N give rise either to a sudden transition to unphysical oscillations, as is shown in Fig. 3(a) for the case of $N=7$ (which is of course rejected), or to response functions which are not positive in the whole range considered. The natural limit in the number of basis functions is clearly given by the accuracy of the transform.

Before proceeding to show the total result, Fig. 3(b) deserves further comment. All curves present some kind of almost stable oscillations at low energy. However, their
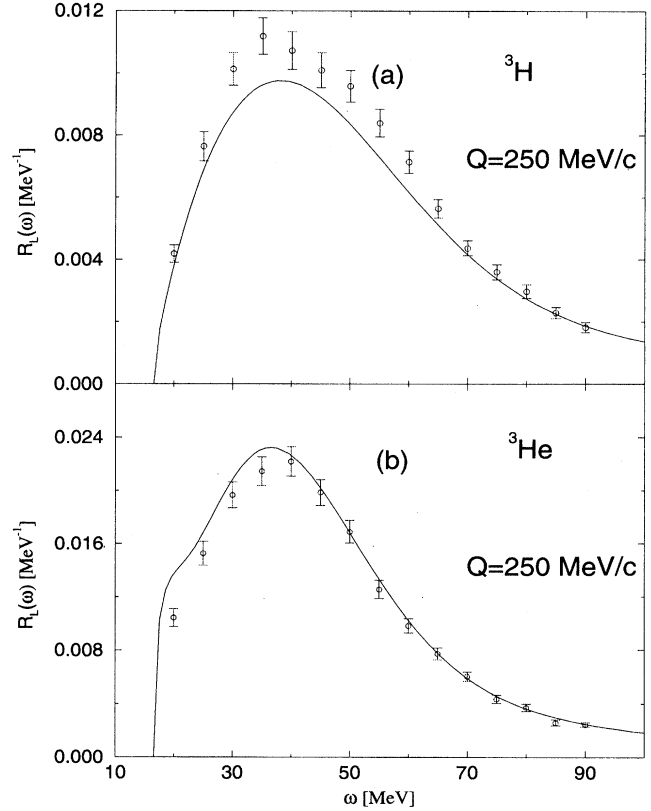

FIG. 5. The longitudinal response function of ^{3}H (a) and ^{3}He (b) at $|\vec{Q}| = 250$ MeV/c (Bonn-B potential). Experimental data are taken from Ref. [16].
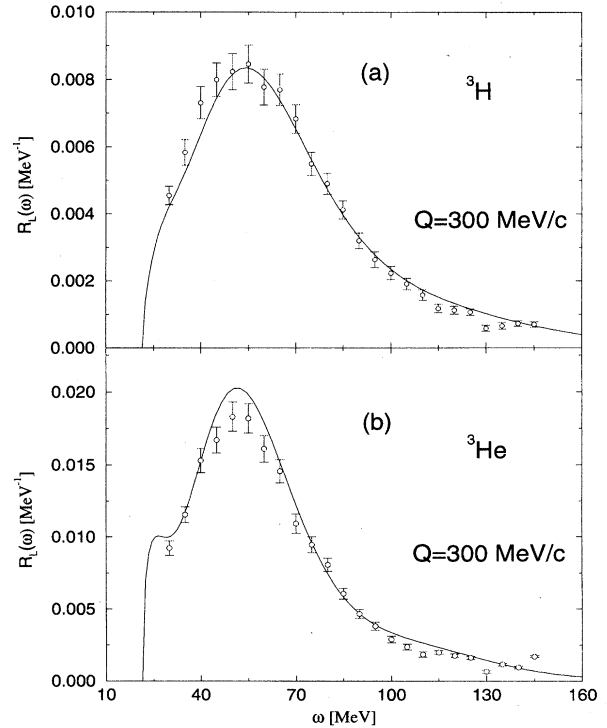

FIG. 6. The same as Fig. 5 at $|\vec{Q}| = 300$ MeV/c.

"wavelength" seems to be smaller than the resolution σ_I of the kernel. So they are presumably to be considered unphysical since, in addition, no such structures are seen in the transform (see Fig. 1).

The sum of the two contributions to the response function is shown in Fig. 4(a). The curve corresponding to $N=6$ has been chosen for the reasons mentioned above. Nevertheless a sort of shoulder shows up at low energy. In the same figure and also in Fig. 2 the present results are compared with those obtained in a totally different way in Ref. [14], i.e., using directly the continuum. The agreement is almost perfect but for the first and maybe the last point, i.e., just in the regions where the inversion seems to be affected by larger uncertainties.

As to the low energy part of the spectrum one could try to improve the situation using the results on the Stieltjes transform [5,15] or applying the following procedure. Since the low energy behavior of the result in Fig. 3(b) is rather uncertain and since, as it was also shown in Ref. [8], the monopole contribution will dominate the response up to about 15–20 MeV above threshold, one could obtain the total response matching the pure monopole part at lower energies and the total sum at higher ones (we choose in this case $N=7$) at some point between 15 and 20 MeV above threshold. The result of this procedure is shown in Fig. 4(b) where a nice agreement is obtained also for the first point of the direct calculation.

In Figs. 5 and 6 our predictions obtained with the Bonn-B potential for both ^{3}He and ^{3}H are compared to experimental data for $|\vec{Q}| = 250$ MeV/c and $|\vec{Q}| = 300$ MeV/c. One can see an overall satisfactory agreement, except in the peak of ^{3}H at $|\vec{Q}| = 250$ MeV/c and of ^{3}He at $|\vec{Q}| = 300$ MeV/c.

In conclusion we have shown that obtaining the response function of the three body systems from the inversion of a Lorentz kernel integral transform is a feasible task. The results are in excellent agreement with those obtained with the direct method and in rather good accord with experimental data. While the results do contain the full dynamics of both initial and final states the lengthy calculation of the continuum is avoided.

The present method has been applied within a completely nonrelativistic framework and the good agreement with the experimental data suggests that at these values of $|\vec{Q}|$ relativistic corrections are of minor importance. Results at larger momentum transfer could also be obtained in the same way, provided that higher J^π components are included in the solution of Eq. (5). One could then explore at which values of $|\vec{Q}|$ relativistic effects would begin to become significant. Work along these lines is in progress.

ACKNOWLEDGMENTS

[1] See, for instance, W. Glöckle, *The Quantum Mechanical Few-Body Problem* (Springer, Berlin, 1983).

[2] H. Witała, Th. Cornelius, and W. Glöckle, Few-Body Syst. **3**, 123 (1988).

[3] S. Ishikawa, H. Kamada, W. Glöckle, and J. Golak, Nuovo Cimento A **107**, 305 (1994); J. Golak, H. Kamada, H. Witała, and W. Glöckle, Phys. Rev. C **51**, 1638 (1995).

[4] S. Ishikawa, H. Kamada, W. Glöckle, J. Golak, and H. Witała, Phys. Lett. B **339**, 293 (1994).

[5] V. D. Efros, Yad. Fiz. **41**, 1498 (1985) [Sov. J. Nucl. Phys. **41**, 949 (1985)]; V. D. Efros, W. Leidemann, and G. Orlandini, Few-Body Syst. **14**, 151 (1993).

[6] J. Carlson and R. Schiavilla, Phys. Rev. Lett. **68**, 3682 (1992); Few-Body Syst. Suppl. **7**, 349 (1994).

[7] V. Yu. Dobretsov, V. D. Efros, and Bin Shao, Yad. Fiz. **58**, 1601 (1995) [Phys. At. Nucl. **58** (9) (1995)].

[8] V. D. Efros, W. Leidemann, and G. Orlandini, Phys. Lett. B **338**, 130 (1994).

[9] W. Glöckle, in *Models and Methods in Few Body Systems,* edited by L. S. Ferreira, A. C. Fonseca, and L. Streit, Lecture Notes in Physics Vol. 273 (Spinger-Verlag, Berlin, 1987), p. 3; W. Glöckle, in *Computational Nuclear Physics 1, Nuclear Structure*, edited by K. Langanke, J. A. Maruhn, and S. E. Koonin (Springer-Verlag, Berlin, 1991), p. 152.

[10] M. Gari and W. Krümpelmann, Phys. Lett. B **173**, 10 (1986).

[11] J. Golak, H. Witała, H. Kamada, D. Hüber, S. Ishikawa, and W. Glöckle, Phys. Rev. C **52**, 1216 (1995).

[12] R. V. Reid, Ann. Phys. (N.Y.) **50**, 411 (1968).

[13] R. Machleidt, Adv. Nucl. Phys. **19**, 189 (1989).

[14] J. Golak (private communication).

[15] S. Martinelli, thesis, University of Trento, 1995.

[16] K. Dow *et al.*, Phys. Rev. Lett. **61**, 1706 (1988).