Microscopic analysis of collective states in neutron-deficient doubly even xenon isotopes

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The low-lying collective states of xenon isotopes are studied by means of the boson expansion theory. The microscopic Hamiltonian is comprised of the self-consistent QQ interaction with higher-order (three- and four-body) terms, monopole- and quadrupole-pairing interactions in addition to the spherical limit of the Nilsson Hamiltonian. It is shown that the difficulty of weak strengths of the monopole-pairing interaction in this region, indicated by Rohoziński *et al.*, can be naturally remedied by including the quadrupole-pairing interactions.

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I. INTRODUCTION

Since Bohr, Mottelson, and Pines [1] introduced the concept of nuclear superconductivity, the pairing interactions have played crucial role in understanding various aspects of nuclear structure. Among them, the monopole-pairing interaction has been extensively studied since it seems to be the main component of the pairing interactions in nuclear physics.

Over 15 years ago, Rohoziński *et al.* [2] performed dynamic calculations of collective states for neutron-deficient doubly even Xe and Ba isotopes, making use of the collective Hamiltonian obtained fully microscopically. They found that in this region the pairing interaction strengths need to be reduced by 20% to reproduce roughly the general trends of the energy spectra. This is a characteristic problem of the effective interactions in this mass region. In their calculation, the monopole-pairing interaction was included but the quadrupole-pairing interaction was not. From our present point of view, however, the quadrupole-pairing interaction is expected to play important role in investigating nuclear quadrupole collective phenomena.

In the early 1970s, Bes and Broglia [3] introduced multipole-pairing interactions to explain the strong $L \neq 0$ (t,p) transitions around closed shell nuclei. Since then, the quadrupole-pairing interaction has found many applications, e.g., to low-lying quadrupole collective states in boson expansion theory [4,5], to the attenuation of the Coriolis interaction in an odd mass nucleus [6,7], to the shift in the band crossing frequency [8,9], etc.

Recently the origin of the multipole-pairing interactions has been clarified in terms of the local Galilean invariance of a nuclear system and, at the same time, the self-consistent strengths of such interactions are proposed [10]. So it is advisable to perform microscopic calculation of nuclear quadrupole collective motion in this region including the quadrupole-pairing interaction. The present paper is devoted for this purpose, and the strengths of the pairing interactions are investigated.

Numerical calculations are made by means of the boson expansion theory (BET), since it allows us to take into account higher-order terms neglected in the random phase approximation (RPA), and furthermore the adiabatic condition for particle motions can be avoided. Among various types of BET's, extensive numerical calculations have been performed with the formalism developed by Kishimoto and Tamura [4,5] (referred to as KT-1 and KT-2), and good agreements with experiments have been obtained in various region of the periodic table. The BET of Kishimoto and Tamura has been reformulated in a mathematically rigorous form as a normal-ordered linked-cluster expansion of the modified Marumori boson mapping [11] (referred to as KT-3). Boson operators in KT-3 are allowed to act upon the ideal-boson states. It is a very promising method for the description of anharmonicities in nuclear quadrupole collective motions, if the coupling to noncollective states is faithfully included in the calculation. The calculation in this paper is performed based on the BET formalism of Kishimoto and Tamura [4,5,11] with several refinements developed in our previous work [12,13].

II. MODEL HAMILTONIAN

In the microscopic analysis of nuclear structure, it is desirable to introduce reliable effective interactions between nucleons in the nucleus. The pairing + multipole interaction model has been extensively and successfully applied not only to low-lying collective states but also to giant resonances in spherical nuclei [14–17]. However, it has been shown that the same model failed to reproduce the prominent features of the observed data in deformed nuclei. The reason for it has been traced to the violation of the nuclear saturation and the self-consistency between the shape of an average potential and that of a density distribution (*nuclear self-consistency*) for the conventional multipole $(Q_\lambda Q_\lambda)$ interaction model, and an improved version of it, i.e., the doubly stretched multipole $(Q''_\lambda Q''_\lambda)$ interaction model, has been proposed [18,19]. The model has found many applications, e.g., to

giant resonances in deformed nuclei in RPA [18,19], to anharmonic γ vibrations in a self-consistent–collective-coordinate (SCC) method [20], to octupole modes built on superdeformed bands in the cranked shell model + RPA [21], and even to the octupole softness of the super- and hyperdeformed states [22,23].

Though the doubly stretched multipole interactions are noninvariant under spatial rotation when the equilibrium shape is not spherical, they have advantages that they are responsible for the fluctuation about deformed equilibrium. Furthermore, the doubly stretched multipole interactions simulate nicely some important effects of self-consistent and rotational-invariant higher-order (many-body) interactions which are induced so as to satisfy the *nuclear self-consistency* with higher-order accuracy especially when more than one mode is simultaneously excited in a system [19]. In this paper, to keep the rotational invariance of the model Hamiltonian, the conventional QQ interaction and additional quadrupole higher-order interactions are adopted instead of the doubly stretched $Q''Q''$ interaction, since, as shown in Ref. [19], effective two-body contracted part of the $\{QQ+$ quadrupole higher-order interactions$\}$ is almost equivalent to the $Q''Q''$ interaction in a deformed system.

The model Hamiltonian with which we start is given in fermion operators as

$$H = h_{\rm sp} + (H_{0\ \rm pair} - \lambda\hat{N}) + H_{2\ \rm pair} + V^{(2)} + V^{(3)} + V^{(4)}, \tag{1}$$

with

$$h_{\rm sp} = \sum_{jm} \varepsilon_j a^\dagger_{jm} a_{jm}, \tag{2}$$

$$H_{0\ \rm pair} = -\frac{G_0}{4}\hat{P}^\dagger_0 \hat{P}_0, \tag{3}$$

$$H_{2\ \rm pair} = -\frac{G_2}{2}(\hat{P}^\dagger_2 \cdot \hat{P}_2), \tag{4}$$

$$V^{(2)} = -\frac{\chi^{(2)}}{2}(\hat{Q}_2 \cdot \hat{Q}_2), \tag{5}$$

$$V^{(3)} = -\frac{\chi^{(3)}}{3!}[\sqrt{56\pi/5}(\hat{Q}_2\hat{Q}_2\hat{Q}_2) - 3\hat{R}_0(\hat{Q}_2 \cdot \hat{Q}_2)], \tag{6}$$

$$V^{(4)} = -\frac{\chi^{(4)}}{4!}\left[\frac{48\pi}{5}(\hat{Q}_2 \cdot \hat{Q}_2)^2 - 8\sqrt{56\pi/5}\hat{R}_0(\hat{Q}_2\hat{Q}_2\hat{Q}_2) + 12\hat{R}_0^2(\hat{Q}_2 \cdot \hat{Q}_2)\right], \tag{7}$$

$$\hat{N} = \sum_{jm} a^\dagger_{jm} a_{jm}, \tag{8}$$

$$\hat{P}^\dagger_0 = \sum_{jm} a^\dagger_{jm} a^\dagger_{\widetilde{jm}}, \tag{9}$$

$$\hat{P}^\dagger_{2\mu} = \sum_{jmj'm'} \langle jm|Q_{2\mu}|j'm'\rangle a^\dagger_{jm} a^\dagger_{\widetilde{j'm'}}, \tag{10}$$

$$\hat{Q}_{2\mu} = \sum_{jmj'm'} \langle jm|Q_{2\mu}|j'm'\rangle : a^\dagger_{jm} a_{j'm'} :, \tag{11}$$

$$\hat{R}_0 = \sum_{jmj'm'} \langle jm|r^2|j'm'\rangle : a^\dagger_{jm} a_{j'm'} :, \tag{12}$$

where the abbreviated notations are defined by

$$(Q_\lambda \cdot Q_\lambda) \equiv \sum_\mu Q_{\lambda\mu} Q_{\widetilde{\lambda\mu}}, \quad Q_{\widetilde{\lambda\mu}} \equiv (-1)^{\lambda-\mu} Q_{\lambda-\mu}, \tag{13}$$

$$(Q_\lambda Q_\lambda Q_\lambda) \equiv ([Q_\lambda Q_\lambda]^{(\lambda)} \cdot Q_\lambda). \tag{14}$$

In the above Hamiltonian, $H_{0\ \rm pair}$ and $H_{2\ \rm pair}$ are monopole- and quadrupole-pairing interactions, $V^{(2)}$ is the ordinary two-body QQ interaction, while $V^{(3)}$ and $V^{(4)}$ are the effective three- and four-body interactions introduced as the higher-order terms of the QQ interaction to recover the saturation and the self-consistency in higher-order accuracy [19,24,25].

In this paper we will expand our model Hamiltonian up to fourth order with respect to bosons, and in principle, we intend to take account of contributions from all the possible interactions up to the same order so as to include the effects of the kinematical anharmonicities originating from the fermion algebra and dynamical anharmonicities originating from the higher-order interactions on the same footing. This is the reason we adopt the quadrupole interaction up to the four-body interaction. In fact, the (QQQ) part of $V^{(3)}$ and $(QQ)^2$ part of $V^{(4)}$ are expected to contribute directly, i.e., without any recoupling of the angular momentum, to third- and fourth-order quadrupole collective boson Hamiltonians, respectively, while the $R(QQ)$ part of $V^{(3)}$ is expected to contribute to the fourth- order quadrupole collective Hamiltonian through the coupling between a two-quadrupole phonon 0^+ state and two-quasiparticle 0^+ states. Since $V^{(3)}$ and $V^{(4)}$ can be understood as higher-order terms of a Taylor expansion series of the $Q''Q''$ interaction with respect to the quadrupole deformation parameter, they are relatively small quantities compared to the QQ interaction. Therefore, in practical calculations, it is sufficient to include only a few of their lower-order boson expanded terms and their higher-order boson terms coming through the recoupling of the angular momentum are negligible except for a nucleus having very large quadrupole deformation. In fact the effect of $V^{(4)}$ is not so important in the present mass region because of a small coupling constant, though, as shown later, $V^{(3)}$ has visible contributions.

It should be noted that, because of the derivation, $V^{(2)}$, $V^{(3)}$, and $V^{(4)}$ are written in terms of the normal-ordered one-body operators $\hat{Q}_{2\mu}$ and $\hat{R}_0$, where the normal product $: \cdots :$ is defined with respect to quasiparticle operators. A detailed derivation and further discussions of such higher-order (many-body) interactions are given in Ref. [19]. Essentially the same type of many-body interactions has been independently derived by Marshalek [26], and the three-body interaction has been applied to the analysis of anharmonic γ vibrations in ^{168}Er by Matsuo and Matsuyanagi [27] and Stotts and Tamura [28], to the analysis of two-phonon states in Ru and Se isotopes by Aiba [29], and to the analysis of shape transition in

Sm isotopes by Yamada [30].

In the present calculation, single-particle base is constructed by using the spherical limit of the Nilsson Hamiltonian $h_{\rm sp}$ [31]. The model space is spanned by $2p_{1/2}$, $2p_{3/2}$, $1f_{5/2}$, $3s_{1/2}$, $2d_{3/2}$, $2d_{5/2}$, $1g_{7/2}$, $1g_{9/2}$, and $1h_{11/2}$ orbits for protons and $3s_{1/2}$, $2d_{3/2}$, $2d_{5/2}$, $1g_{7/2}$, $2f_{7/2}$, $1h_{9/2}$, $1h_{11/2}$, and $1i_{13/2}$ orbits for neutrons.

The strengths of the self-consistent QQ interaction and its higher-order terms are parametrized as

$$\chi^{(2)} = f_2\chi_2^{\rm self}, \quad \chi^{(3)} = f_3\chi_3^{\rm self}, \quad \chi^{(4)} = f_4\chi_4^{\rm self}, \qquad (15)$$

where $\chi_2^{\rm self}$, $\chi_3^{\rm self}$, and $\chi_4^{\rm self}$ are the self-consistent values of $\chi^{(2)}$, $\chi^{(3)}$, and $\chi^{(4)}$, respectively, which are derived in Ref. [19]. Including the extra factor of 2 due to the renormalization of quadrupole core polarization effect [32], they are given as

$$\begin{aligned}
\chi_2^{\rm self} &= 2\tfrac{4\pi}{5}\tfrac{m\omega_0^2}{A\langle r^2\rangle} \simeq 240A^{-5/3}\ [\mathrm{MeV}\,\nu^2],\\
\chi_3^{\rm self} &= \tfrac{\chi_2^{\rm self}}{A\langle r^2\rangle} \simeq 280A^{-3}\ [\mathrm{MeV}\,\nu^3],\\
\chi_4^{\rm self} &= \tfrac{\chi_3^{\rm self}}{A\langle r^2\rangle} \simeq 330A^{-13/3}\ [\mathrm{MeV}\,\nu^4],
\end{aligned} \qquad (16)$$

with

$$\nu \equiv \frac{m\omega_0}{\hbar} \simeq 1.0A^{-1/3}\ [\mathrm{fm}^{-2}], \qquad (17)$$

where the order estimations are performed by assuming the uniform density distribution for the nucleus. The dimensionless parameters f_2, f_3, and f_4 are introduced as adjustable parameters. To reduce the number of free parameters, they are set to $f_2 = f_3 = f_4$, and f_2 is varied slightly around the vicinity of the predicted value, i.e., unity.

For the strength of the quadrupole-pairing interaction, following the manner of Refs. [5,13], a similar parametrization is introduced as

$$G_2 = g_2\chi_2^{\rm self}, \qquad (18)$$

and g_2 is varied as another adjustable parameter. For simplicity, a single value of g_2 is used both for proton pairs and for neutron pairs.

For the monopole pairing, the interaction strengths $G_0(p)$ for protons and $G_0(n)$ for neutrons are fixed to fit the experimental gap energy through the gap equation. The gap parameters are derived from the experimental data of nuclear binding energy taken from the compilation by Wapstra and Audi [33]. Figure 1 shows the values of these quantities. For comparison, Rohoziński *et al.* [2] have used two different parameter sets for the strengths of the monopole-pairing interaction in their calculation. One is the standard pairing strengths (SPS's) given by

$$G_0(p) = 28.5/A\ [\mathrm{MeV}], \quad G_0(n) = 25.0/A\ [\mathrm{MeV}], \qquad (19)$$

and the other is the weak pairing strengths (WPS's) given by

$$G_0(p) = 22.8/A\ [\mathrm{MeV}], \quad G_0(n) = 20.0/A\ [\mathrm{MeV}]. \qquad (20)$$

The strengths of the monopole-pairing interaction adopted in the present calculation are in between the SPS's and the WPS's. Because of the derivation, however, they can be understood as the conventional standard values. In fact, these values are nearly compatible with the systematics [34]

$$G_0 = \frac{23.0}{A}\left(1 \pm 0.6\frac{N-Z}{A}\right)\ [\mathrm{MeV}], \qquad (21)$$

where the plus (minus) sign is for protons (neutrons).

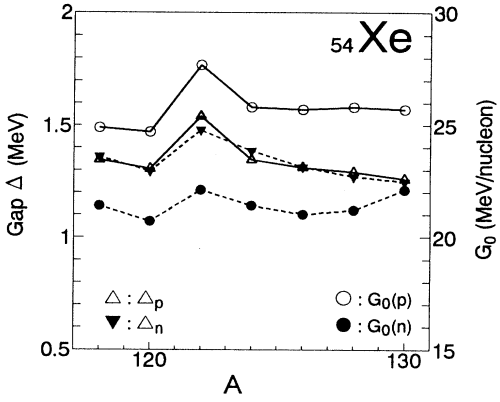

FIG. 1. The gap parameters (triangles) are shown in units of MeV, while the strengths of the monopole-pairing interaction (circles) are plotted in units of MeV/nucleon. Solid (dashed) lines connect the values for protons (neutrons).

Figure 2 shows the interaction strengths adopted in the present calculation to fit the experimental energy spectra. To reduce the number of free parameters, a single value of $f_2 = 0.93$ is used for all the isotopes. Thus g_2 is the only parameter which is varied freely to fit the experimental data.

Recently by using the concept of the local Galilean invariance as an important guiding principle, the origin of the multipole pairing interaction has been clarified and the self-consistent strength of the 2^λ-pole pairing interaction is proposed in Ref. [10] as

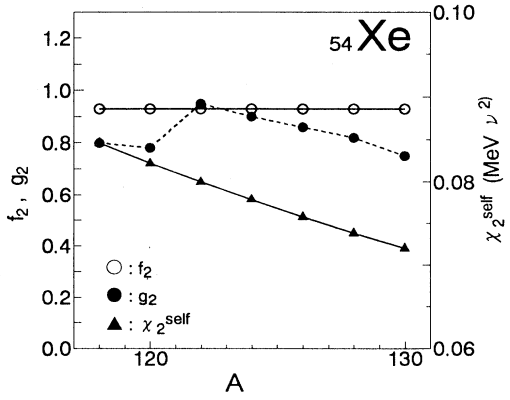

FIG. 2. The parameter for the strength of the QQ interaction f_2 and that of the quadrupole-pairing interaction g_2 adopted in the present calculation. As a reference, the theoretical value of the self-consistent strength $\chi_2^{\rm self}$ for the QQ interaction is plotted in units of MeV ν^2, where $\nu = M\omega_0/\hbar$.

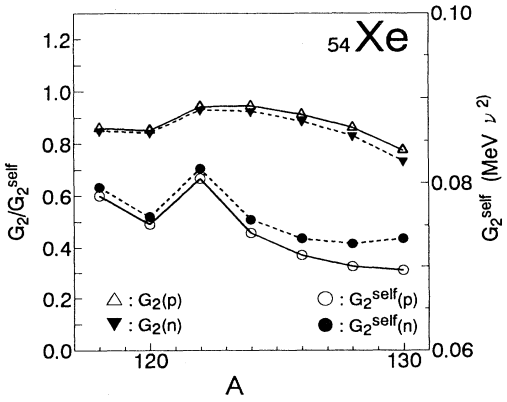

FIG. 3. The strength of the quadrupole-pairing interaction G_2 adopted in the present calculation is shown in units of $G_2^{\rm self}$. As a reference, the theoretical value of the self-consistent strength $G_2^{\rm self}$ is plotted in units of MeV ν^2, where $\nu = M\omega_0/\hbar$.

$$G_\lambda^{\rm self} = \left[\sum_{jj'} \frac{1}{4}\left(\frac{1}{E_j}+\frac{1}{E_{j'}}\right)\left|\frac{1}{\sqrt{2\lambda+1}}\langle j\|Q_\lambda\|j'\rangle\right|^2\right]^{-1}. \tag{22}$$

The comparison between the value of g_2 adopted in the present calculation and that of the theoretical self-consistent strength is given in Fig. 3. Here, a bump appears at $A = 122$ in the behavior of the absolute value of $G_2^{\rm self}$ both for protons and for neutrons. A similar bump can also be seen in Fig. 2 for the behavior of g_2 (i.e., G_2 plotted in units of $\chi_2^{\rm self}$). These bumps are due to the corresponding bump of the gap parameter of Fig. 1 which seem to be rather unnatural. However, as can be seen from Fig. 3, such an anomalous bump disappears when we plot G_2 in units of $G_2^{\rm self}$, and the value of G_2 is close to that of $G_2^{\rm self}$ both for protons and for neutrons. This means that even if there exists some unnatural behavior in the adopted parameters for the monopole-pairing interaction, the theoretical value of $G_2^{\rm self}$ can faithfully reflect such behavior and it can be a good reference value for the strength of the quadrupole-pairing interaction.

III. BOSON EXPANSION

We first define some of the basic quantities. The operator $d_{jm}^\dagger$ creates a quasiparticle in an orbit $|jm\rangle$, while d_{jm} annihilates it. The pair-creation operator and the so-called scattering operator are defined as

$$\begin{aligned} B_{j_1j_2\lambda\mu}^\dagger &= (-1)^{j_1-j_2-\lambda} B_{j_2j_1\lambda\mu}^\dagger \\ &= \sum_{m_1m_2} (j_1m_1j_2m_2|\lambda\mu) d_{j_1m_1}^\dagger d_{j_2m_2}^\dagger, \end{aligned} \tag{23}$$

$$\begin{aligned} C_{j_1j_2\lambda\mu}^\dagger &= (-1)^{j_1-j_2-\lambda} C_{j_2j_1\tilde{\lambda}\mu} \\ &= \sum_{m_1m_2} (j_1m_1\widetilde{j_2m_2}|\lambda\mu) d_{j_1m_1}^\dagger d_{j_2m_2} \equiv C_p^\dagger, \end{aligned} \tag{24}$$

where $p = \{j_1j_2\lambda\mu\}$ and

$$(j_1m_1\widetilde{j_2m_2}|\lambda\mu) = (-1)^{j_2-m_2}(j_1m_1j_2-m_2|\lambda\mu). \tag{25}$$

In the Tamm-Dancoff (TD) representation we use $B_a^\dagger$ defined by

$$B_a^\dagger \equiv B_{\alpha\lambda\mu}^\dagger = \sum_{j_1\le j_2} \psi_{j_1j_2\lambda}^{(\alpha)} D_{j_1j_2}^{-1} B_{j_1j_2\lambda\mu}^\dagger, \tag{26}$$

where $a = \{\alpha\lambda\mu\}$, α denoting a TD component for a fixed multipolarity λ, and $D_{j_1j_2} = (1+\delta_{j_1j_2})^{1/2}$. The basic commutation relations for these operators are given by

$$[B_a, B_b^\dagger] = \delta_{ab} - \sum_p P_{\tilde{a}b}^{(p)} C_p^\dagger, \quad [C_p^\dagger, B_a^\dagger] = \sum_b P_{\tilde{a}b}^{(p)} B_b^\dagger, \tag{27}$$

with

$$\begin{aligned} P_{\tilde{a}a'}^{(p)} &\equiv P_{\alpha\tilde{\lambda}\mu;\alpha'\lambda'\mu'}^{(j_1j_2kq)} = (\widetilde{\lambda\mu}\lambda'\mu'|kq)\hat{P}_{\alpha\lambda;\alpha'\lambda'}^{(j_1j_2k)}, \\ \hat{P}_{\alpha\lambda;\alpha'\lambda'}^{(j_1j_2k)} &\equiv \hat{\lambda}\hat{\lambda}' \sum_j \psi_{jj_2\lambda}^{(\alpha)} \psi_{j_1j\lambda'}^{(\alpha')} W(j_2j_1\lambda\lambda';kj) D_{jj_1} D_{jj_2}, \\ \hat{\lambda} &\equiv \sqrt{2\lambda+1}. \end{aligned} \tag{28}$$

We also introduce the ideal boson operators $A_a^\dagger$ and A_a which satisfy

$$[A_a, A_b^\dagger] = \delta_{ab}, \quad [A_a, A_b] = [A_a^\dagger, A_b^\dagger] = 0. \tag{29}$$

Orthonormal n boson states, which span the ideal boson space, are introduced as

$$|n:a) \equiv N(n:a)^{-1} A_{a_1}^\dagger A_{a_2}^\dagger \cdots A_{a_n}^\dagger |0), \tag{30}$$

where $N(n:a)$ is the boson normalization factor,

$$N(n:a)^2 \equiv (0|A_{a_n}\cdots A_{a_1} A_{a_1}^\dagger \cdots A_{a_n}^\dagger|0), \tag{31}$$

with the abbreviated notation $(n:a) \equiv (a_1, a_2, \ldots, a_n)$ with $a_1 \le a_2 \le \cdots \le a_n$. The corresponding n TD fermion-pair states

$$|n:a\rangle\rangle \equiv N(n:a)^{-1} B_{a_1}^\dagger B_{a_2}^\dagger \cdots B_{a_n}^\dagger |0\rangle \tag{32}$$

are not generally orthonormal and linearly independent. The fermion norm matrix is denoted as

$$\begin{aligned} \langle\langle n:a|m:b\rangle\rangle &\equiv \delta_{nm}(Z_n^2)_{a;b} \equiv \delta_{nm}(Z_n^2)_{a_1\cdots a_n;b_1\cdots b_n} \\ &\equiv \delta_{nm}(1_n - Y_n)_{a;b}, \end{aligned} \tag{33}$$

where the corresponding ideal boson norm matrix is defined by

$$(n:a|m:b) \equiv \delta_{nm}(1_n)_{a;b} \equiv \delta_{nm}(1_n)_{a_1\cdots a_n;b_1\cdots b_n}, \tag{34}$$

and Y_n is regarded as a matrix which measures the devi-

ation of the fermion norm from the corresponding ideal boson norm. For example, we have for $n = 2$ as

$$(1_2)_{a;b} = N(2:a)^{-1}N(2:b)^{-1}(\delta_{a_1b_1}\delta_{a_2b_2} + \delta_{a_1b_2}\delta_{a_2b_1}), \tag{35}$$

$$(Y_2)_{a;b} = N(2:a)^{-1}N(2:b)^{-1}Y(b_1a_1a_2b_2), \tag{36}$$

where

$$N(2:a) = N(a_1a_2) = \sqrt{1+\delta_{a_1a_2}}, \tag{37}$$

$$Y(a_1a_2a_3a_4) = \sum_p P^{(p)}_{a_1a_2}P^{(p)}_{a_3a_4}. \tag{38}$$

In order to construct orthonormalized fermion states, we have to assume that the inverse of Z_n, i.e., Z_n^{-1}, exists. One of the possible ways would be to divide the fermion space $\{|n:a)\rangle\rangle\}$ into two parts, T_F and $(1-T_F)$, i.e., the T_F space including the components that are retained, and $(1-T_F)$ excluding those. Then the orthonormalized fermion states can be obtained as

$$|n:t\rangle \equiv \sum_{t'}(Z_n^{-1})_{t;t'}|n:t'\rangle\rangle\ ; \tag{39}$$

here and in the following t, t', etc., indicate the components that belong to the T_F space while $\bar{t}$, $\bar{t}'$, etc., indicate those that belong to the $(1-T_F)$ space. To derive a physically meaningful boson mapping, the ideal boson space $\{|n:a)\}$ is also divided into two parts, T and $(1-T)$; i.e., the truncated space T for boson states is introduced as a replica of the T_F space for the fermion states. The one-to-one correspondence between the fermion state $|n:t\rangle$ and the boson state $|n:t)$ in the truncated space is obtained by using a mapping operator

$$U = \sum_{(n:t)}|n:t)\langle n:t| \tag{40}$$

as

$$|n:t) = U|n:t\rangle,\quad |n:t\rangle = U^\dagger|n:t). \tag{41}$$

At the same time, a boson image $(O_F)_B$ of a fermion operator O_F is defined by

$$(O_F)_B \equiv UO_FU^\dagger \tag{42}$$

so as to satisfy

$$\langle m:t|O_F|n:t'\rangle = (m:t|(O_F)_B|n:t') \tag{43}$$

in the truncated subspace.

The normal-ordered linked-cluster expansion of $(O_F)_B$ is obtained by expressing $|0)(0|$ and $(Z_n)_{t;t'}$ in an expansion form. For example, the basic operators are expressed as

$$\begin{aligned}(B^\dagger_{t_1})_B &= A^\dagger_{t_1} - \frac{1}{4}\sum_{t_2t_3t_4}Y(t_1t_2t_3t_4)A^\dagger_{t_2}A^\dagger_{t_3}A_{t_4} + O(\epsilon^2)\\ &= A^\dagger_{t_1} - \frac{1}{4}\sum_{t_2t_3t_4}N(t_2t_3)N(t_1t_4)(Y_2)_{t_2t_3;t_1t_4}\\ &\quad\times A^\dagger_{t_2}A^\dagger_{t_3}A_{t_4} + O(\epsilon^2),\end{aligned} \tag{44}$$

$$(C^\dagger_p)_B = \sum_{t_1t_2}P^{(p)}_{t_2t_1}A^\dagger_{t_1}A_{t_2} + \delta(C^\dagger_p)_B, \tag{45}$$

where ϵ denotes the expansion parameter such as $|Y_2|$. In Eq. (45), $\delta(C^\dagger_p)_B$ represents the fourth- and much higher-order terms, and it consists only of such terms, the so-called $\bar{t}$-sum terms, which contains only the TD components that are outside of the T space. The ϵ is very small, if we truncate the system to the collective TD component [11]. For the noncollective components, the corresponding ϵ is not always small. However, it is shown in KT-3 that if none of the noncollective components appears more than once (although the collective component may appear multiply), the theory remains still applicable, even if we retain only the lower-order terms of the expansion.

Since the above formalism of BET is based on the quasiparticle representation, it suffers from the spurious particle-number excitations associated with particle-number nonconservation. To remove such spurious modes, we use the prescription developed in Ref. [12]. Within the TD representation in our framework, a basic creation operator for the spurious mode can be expressed as

$$B^\dagger_{a=0} \equiv \sum_j \psi^{(\alpha=0)}_{jj\lambda=0}B^\dagger_{jj00}/\sqrt{2}\ , \tag{46}$$

with

$$\psi^{(\alpha=0)}_{jj\lambda=0} = \frac{w_j}{\sqrt{\sum_j(w_j)^2}},\quad w_j = \sqrt{2j+1}u_jv_j, \tag{47}$$

where u_j and v_j are the u and v factors in the BCS formalism. Then the spurious states $|n:\hat{a})\rangle\rangle$ in general are expressed as

$$\begin{aligned}|n:\hat{a})\rangle\rangle &= |a_1=0,a_2,a_3,\ldots,a_n)\rangle\rangle\\ &= N(n:\hat{a})^{-1}B^\dagger_0B^\dagger_{a_2}\ldots B^\dagger_{a_n}|0),\end{aligned} \tag{48}$$

which contains at least one spurious excitation mode $B^\dagger_0$. Here abbreviated notations are defined by

$$\begin{aligned}(n:\hat{a}) \equiv\ & (a_1=0,a_2,a_3,\ldots,a_n)\\ & \text{with}\ \ 0\le a_2\le a_3\le\cdots\le a_n,\end{aligned} \tag{49}$$

$$\begin{aligned}|n:\hat{a}) &= |a_1=0,a_2,a_3,\ldots,a_n)\\ &= N(n:\hat{a})^{-1}A^\dagger_0A^\dagger_{a_2}\cdots A^\dagger_{a_n}|0),\end{aligned} \tag{50}$$

$$\begin{aligned}N(n:\hat{a})^2 &\equiv N(a_1=0,a_2,a_3,\cdots,a_n)^2\\ &= (0|A_{a_n}\cdots A_{a_2}A_0A^\dagger_0A^\dagger_{a_2}\ldots A^\dagger_{a_n}|0).\end{aligned} \tag{51}$$

A projection operator onto the so-called *physical* quasiparticle subspace in the truncated space, which does not contain any spurious excitation mode, is introduced as

$$P_N \equiv T_N - \sum_{(n:\hat{t})} |n:\hat{t}\rangle\langle n:\hat{t}| \ , \tag{52}$$

with

$$|n:\hat{t}\rangle \equiv \sum_{\hat{t}'} (Z_n^{-1})_{\hat{t};\hat{t}'} |n:\hat{t}'\rangle\!\rangle, \tag{53}$$

where $|n:\hat{t}\rangle$ are orthonormalized spurious states belonging to the truncated subspace. Defining nonorthogonal *physical* quasiparticle state $|\overline{n:t}\rangle\!\rangle$ and corresponding norm matrices as

$$\begin{aligned} |\overline{n:t}\rangle\!\rangle \ &= \ |\overline{t_1,t_2,\ldots,t_n}\rangle\!\rangle \equiv P_N|n:t\rangle\!\rangle \\ &\text{with} \ \ t_1 \neq 0,\ldots,t_n \neq 0, \end{aligned} \tag{54}$$

$$\begin{aligned} (\overline{Z}_n^2)_{t;t'} &\equiv \langle\!\langle \overline{n:t}|\overline{n:t'}\rangle\!\rangle = \langle\!\langle n:t|P_N|n:t'\rangle\!\rangle \\ &\equiv (1_n - \overline{Y_n})_{t;t'}, \end{aligned} \tag{55}$$

then orthonormalized *physical* quasiparticle states are obtained as

$$|\overline{n:t}\rangle = \sum_{(n:t')} (\overline{Z}_n^{-1})_{t;t'} |\overline{n:t'}\rangle\!\rangle. \tag{56}$$

A boson mapping operator $\overline{U}$ for an improved BET, which maps orthonormal *physical* quasiparticle states $|\overline{n:t}\rangle\!\rangle$ onto ideal boson states $|n:t)$, is now given as

$$\overline{U} = \sum_{(n:t)} |n:t)\langle\overline{n:t}|, \tag{57}$$

while the mapping rules in the *physical* quasiparticle subspace are given by

$$|n:t) = \overline{U}|\overline{n:t}\rangle, \quad |\overline{n:t}\rangle = \overline{U}^\dagger |n:t), \tag{58}$$

$$(\overline{O_F})_B \equiv \overline{U} O_F \overline{U}^\dagger, \tag{59}$$

$$\langle\overline{m:t}|O_F|\overline{n:t'}\rangle = (m:t|(\overline{O_F})_B|n:t'). \tag{60}$$

As is clearly demonstrated above, the only difference between U and $\overline{U}$ is whether $|n:t\rangle$ or $|\overline{n:t}\rangle$ is mapped onto $|n:t)$; i.e., whether or not the truncated quasiparticle states contain spurious states. Exploiting the fact that the evaluation $\overline{Y_n}$ involves only the knowledge of Y_n, improved boson expansion coefficients are obtained with just a few modifications in the previous coefficients. For example, we obtain

$$\begin{aligned} (B_{t_1}^\dagger)_B = A_{t_1}^\dagger - \frac{1}{4} \sum_{t_2 t_3 t_4} N(t_2 t_3) N(t_1 t_4) (\overline{Y_2})_{t_2 t_3; t_1 t_4} \\ \times A_{t_2}^\dagger A_{t_3}^\dagger A_{t_4} + O(\epsilon^2), \end{aligned} \tag{61}$$

with

$$\overline{Y_2} = Y_2 + \Delta Y_2, \tag{62}$$

$$(\Delta Y_2)_{t_1 t_2; t_1' t_2'} = \sum_{tt'} (Y_2)_{t_1 t_2; 0t} [(1_2 - Y_2)^{-1}]_{0t;0t'} (Y_2)_{0t';t_1' t_2'}. \tag{63}$$

In practice, since there are two basic spurious modes, i.e., spurious excitation modes associated with a proton number and a neutron number, we apply the above procedure to both of them separately.

By use of Eqs. (45) and (61), we can bosonize the original fermion Hamiltonian of Eq. (1). The resulting Hamiltonian, up to fourth order, is given by

$$\begin{aligned} (H)_B = &\sum_{t_1 t_2} h_{11}(t_1 t_2)(A_{t_1}^\dagger \cdot A_{t_2}) + \sum_{t_1 \le t_2} h_{20}(t_1 t_2)\{(A_{t_1}^\dagger \cdot A_{t_2}^\dagger) + \text{H.c.}\} \\ &+ \sum_{t_1 \le t_2} \sum_{t_3} h_{21}(t_1 t_2 : t_3)\{([A_{t_1}^\dagger A_{t_2}^\dagger]^{(t_3)} \cdot A_{t_3}) + \text{H.c.}\} + \sum_{t_1 \le t_2 \le t_3} h_{30}(t_1 t_2 t_3)\{([A_{t_1}^\dagger A_{t_2}^\dagger]^{(t_3)} \cdot A_{t_3}^\dagger) + \text{H.c.}\} \\ &+ \sum_{t_1 \le t_2} \sum_{t_3 \le t_4} \sum_{I} h_{22}(t_1 t_2 : t_3 t_4; I)\{([A_{t_1}^\dagger A_{t_2}^\dagger]^{(I)} \cdot [A_{t_3} A_{t_4}]^{(I)})\} \\ &+ \sum_{t_1 \le t_2 \le t_3} \sum_{t_4} \sum_{I} h_{31}(t_1 t_2 t_3 : t_4; I)\{([A_{t_1}^\dagger A_{t_2}^\dagger]^{(I)} \cdot [A_{t_3}^\dagger A_{t_4}]^{(I)}) + \text{H.c.}\} \\ &+ \sum_{t_1 \le t_2 \le t_3 \le t_4} \sum_{I} h_{40}(t_1 t_2 t_3 t_4; I)\{([A_{t_1}^\dagger A_{t_2}^\dagger]^{(I)} \cdot [A_{t_3}^\dagger A_{t_4}^\dagger]^{(I)}) + \text{H.c.}\}, \end{aligned} \tag{64}$$

where the explicit expression of the coefficients h's are given in Ref. [12]. This Hamiltonian contains many terms: the purely collective parts in which all the boson labels in the summation are restricted only to the collective one, the coupling terms which contain collective as well as noncollective bosons, and finally the purely noncollective parts which contain noncollective branches only.

In order to include approximately the coupling effect between the collective and the noncollective modes, we use the perturbation theory for a quasidegenerate system [25,35–38]. The truncated boson space T is divided

into two parts, i.e., a P space and a Q space. Since we are interested in the quadrupole collective motions, the space spanned only by the coherent quadrupole TD mode, which we will denote as $t_{\rm coll} \equiv \{\theta 2\mu\}$, is referred to as the P space and the space containing all the other modes is referred to as the Q space. Then, along the line of the Feschbach formalism [39], we construct an effective boson Hamiltonian to be used in the P space as

$$H_{PP}^{\rm eff} \equiv H_{PP} + H'_{PP},$$
$$H'_{PP} \equiv H_{PQ}\frac{1}{E - H_{QQ}}H_{QP}\ , \tag{65}$$

with

$$H_{PP} \equiv P(H)_B P,\quad H_{QQ} \equiv Q(H)_B Q,$$
$$H_{PQ} \equiv H_{QP}^{\dagger} \equiv P(H)_B Q, \tag{66}$$

where E is a energy of the equation

$$(H_{PP}^{\rm eff} - E)P\Psi = 0. \tag{67}$$

Now we must construct the P space and the Q space more concretely. The above formalism is expressed in terms of the so-called A bosons, which correspond to the boson images of the original TD fermion operators in lowest order. However, it is more desirable to include the RPA-type correlations at the early stage of the calculation. For this purpose, we introduce new type of bosons, the so-called α bosons, for the collective branch defined by

$$A^{\dagger} = \psi\alpha^{\dagger} + \phi\tilde{\alpha},\quad \tilde{A} = \phi\alpha^{\dagger} + \psi\tilde{\alpha}, \tag{68}$$

with

$$\psi^2 - \phi^2 = 1. \tag{69}$$

Here and in the following, abbreviated notations such as

$$A^{\dagger} \equiv A_{t_{\rm coll}}^{\dagger} \equiv A_{\theta 2\mu}^{\dagger},\quad \tilde{A} \equiv A_{\theta\widetilde{2\mu}},$$
$$\alpha^{\dagger} \equiv \alpha_{2\mu}^{\dagger},\quad \tilde{\alpha} \equiv \alpha_{\widetilde{2\mu}} \tag{70}$$

are used for the quadrupole collective A bosons and α bosons. Because of Eq. (69), the coefficients ψ and ϕ can be expressed in terms of a single parameter z as

$$\psi = \frac{1}{2}(z + z^{-1}),\quad \phi = \frac{1}{2}(z - z^{-1}). \tag{71}$$

For the noncollective channels, the TD boson representation is fixed so as to diagonalize the h_{11} part of the boson Hamiltonian $(H)_B$ in the Q space. For practical purposes, we introduce the following two approximations: (i) The space truncation is performed so that the Q space is spanned by only those noncollective states in which none of the noncollective bosons excites more than once although the collective boson may excite multiply. (ii) the collective states are treated to have energies that are harmonic, i.e., $N\hbar\omega$ for collective N-phonon states, as far as they appear in the denominator of Eq. (65). These two approximations are the same as those adopted in KT-2 except for the restriction on the noncollective states. In the present calculations, all the possible noncollective boson states ($I^{\pi} = 0^{+}-4^{+}$) which can couple directly to the states with one or two collective phonons are included, while in KT-2 only the 2^{+} noncollective states were included as the Q-states.

By use of the technique developed in Refs. [5,12], which is based on the closure property for the intermediate states in the coupling Hamiltonian, the H'_{PP} term can be brought into an operator form so that the formal structure of it is just same as that of the purely collective Hamiltonian H_{PP}. The structure of the total Hamiltonian $H_{PP}^{\rm eff}$, which will be henceforth referred to as a collective Hamiltonian, is kept unchanged even when the coupling term H'_{PP} is taken into account. As a result, it can be expressed in a compact form as

$$H_{\rm coll} \equiv H_{PP}^{\rm eff} = U_0 + \sum_{mni} h_{mni}H_{mni}^{(\alpha)},$$
$$h_{mni} \equiv h_{mni}^{(\alpha)} + h_{mni}^{(\alpha)\prime}. \tag{72}$$

Here $H_{mni}^{(\alpha)}$ are operators of the form $(\alpha^{\dagger})^m(\tilde{\alpha})^n$ and the additional index i distinguishes terms that have the same m and n. They are expressed as

$$H_{11}^{(\alpha)} = (\alpha^{\dagger}\cdot\alpha),$$
$$H_{20}^{(\alpha)} = (\alpha^{\dagger}\cdot\alpha^{\dagger}) + (\alpha\cdot\alpha),$$
$$H_{21}^{(\alpha)} = ([\alpha^{\dagger}\alpha^{\dagger}]\cdot\alpha) + (\alpha^{\dagger}\cdot[\alpha\alpha]),$$
$$H_{30}^{(\alpha)} = ([\alpha^{\dagger}\alpha^{\dagger}]\cdot\alpha^{\dagger}) + (\alpha\cdot[\alpha\alpha]),$$
$$H_{22P}^{(\alpha)} = (\alpha^{\dagger}\cdot\alpha^{\dagger})(\alpha\cdot\alpha),$$
$$H_{22N}^{(\alpha)} = (\alpha^{\dagger}\cdot\alpha)\{(\alpha^{\dagger}\cdot\alpha) - 1\},$$
$$H_{22J}^{(\alpha)} = \hat{J}^2 - 6(\alpha^{\dagger}\cdot\alpha),$$
$$H_{31}^{(\alpha)} = (\alpha^{\dagger}\cdot\alpha^{\dagger})(\alpha^{\dagger}\cdot\alpha) + (\alpha^{\dagger}\cdot\alpha)(\alpha\cdot\alpha),$$
$$H_{40}^{(\alpha)} = (\alpha^{\dagger}\cdot\alpha^{\dagger})(\alpha^{\dagger}\cdot\alpha^{\dagger}) + (\alpha\cdot\alpha)(\alpha\cdot\alpha)\ , \tag{73}$$

with

$$\hat{J}^2 \equiv -10([\alpha^{\dagger}\alpha]^{(1)}\cdot[\alpha^{\dagger}\alpha]^{(1)}). \tag{74}$$

The coefficient $h_{mni}^{(\alpha)}$ comes entirely from the purely collective parts of Eq.(64), while $h_{mni}^{(\alpha)\prime}$ is the contribution from the noncollective couplings. The explicit expressions of these coefficients are given in Ref. [12].

The frequency of the harmonic motion, ω, and the parameter for the α transformation, z, are found by solving simultaneously the following equations:

$$h_{20}(z,\omega) = 0,$$
$$h_{11}(z,\omega) = \hbar\omega. \tag{75}$$

The first condition of Eq. (75) is equivalent to the so-called elimination of dangerous terms, which helps to make the P space in which the collective Hamiltonian is to be diagonalized much smaller than otherwise. The second condition of Eq. (75), which is a kind of the self-consistency condition, is nothing but a natural consequence of the perturbation theory for a quasidegenerate

system.

Now the collective Hamiltonian is expressed in terms entirely of the α bosons and the eigenenergies of the collective states can be obtained by diagonalizing it in the P space. To visualize the physical properties described by the collective Hamiltonian, the α bosons are transformed into momentum and conjugate coordinates defined by

$$\alpha^\dagger = \frac{1}{\sqrt{2}}(z^{-1}\beta_{2\mu} - iz\pi_{\widetilde{2\mu}}),$$
$$\alpha = \frac{1}{\sqrt{2}}(z^{-1}\beta_{\widetilde{2\mu}} + iz\pi_{2\mu}). \tag{76}$$

Performing this transformation, the collective Hamiltonian can be rewritten as

$$H_{\rm coll} = T(\beta_{2\mu}, \pi_{2\mu}) + V(\beta_{2\mu}) + c_0, \tag{77}$$

where the potential energy surface is given by

$$V(\beta_{2\mu}) \equiv V(\beta,\gamma) = c_2\xi^2\beta^2 + c_3\xi^3\beta^3\cos 3\gamma + c_4\xi^4\beta^4. \tag{78}$$

Here ξ is a suitable scaling parameter which relates our microscopic $\beta_{2\mu}$, which may be called $\beta_{2\mu}^{(\rm mic)}$, to the macroscopic $\beta_{2\mu}$ of the Bohr-Mottelson model. The latter, called $\beta_{2\mu}^{(\rm mac)}$, is related to the macroscopic quadrupole operator $\hat{Q}_{2\mu}^{(\rm mac)}$ as

$$\hat{Q}_{2\mu}^{(\rm mac)} = \frac{3}{4\pi}AR^2\beta_{2\mu}^{(\rm mac)}, \tag{79}$$

where A and R are the mass number and average radius of the nucleus. On the other hand, our $\beta_{2\mu}^{(\rm mic)}$ is related to the microscopic quadrupole operator $\hat{Q}_{2\mu}^{(\rm mic)}$ in lowest order as

$$\hat{Q}_{2\mu}^{(\rm mic)} = 2q_{10}(\alpha^\dagger + \tilde{\alpha}) = 2\sqrt{2}z^{-1}q_{10}\beta_{2\mu}^{(\rm mic)}, \tag{80}$$

where an extra factor of 2 comes from the fact that the effective charge of the mass quadrupole operator is assumed to be unity [32]. The q_{10} is the first-order coefficient of the bosonized quadrupole operator, the higher-order terms being much smaller. By setting

$$\hat{Q}_{2\mu}^{(\rm mic)} = \hat{Q}_{2\mu}^{(\rm mac)}, \tag{81}$$

we obtain

$$\beta_{2\mu}^{(\rm mic)} = \xi\beta_{2\mu}^{(\rm mac)}, \quad \xi = \frac{3AR^2z}{8\sqrt{2}\pi q_{10}}. \tag{82}$$

The coefficients c_2, c_3, and c_4 are given as

$$c_2 = \frac{1}{2}z^{-2}(h_{11} + 2h_{20} - 2h_{22P} - 6h_{22N} - 6h_{22J} - 7h_{31}),$$
$$c_3 = -\frac{1}{\sqrt{7}}z^{-3}(h_{21} + h_{30}),$$
$$c_4 = \frac{1}{4}z^{-4}(h_{22P} + h_{22N} + 2h_{31} + 2h_{40}). \tag{83}$$

It should be noted that, as discussed in KT-2, a strong correlation between the potential energy surface and the energy spectrum holds only under the condition that all the anharmonic terms in the kinetic energy $T(\beta_{2\mu}, \pi_{2\mu})$, i.e., those terms except the π^2 term, are sufficiently small. If such a condition is not met and a generalized collective mass depends crucially on coordinates, then to predict the energy spectrum from the shape of the potential energy surface only can be dangerous. In the boson expansion approach, the adiabatic assumption is not made and generally there appear terms that are in higher powers in π. Such terms give corrections to the theories based on the adiabatic assumption.

IV. RESULTS AND DISCUSSION

The effect of the quadrupole-pairing interaction on the low-lying collective states for ^{118}Xe is shown in Fig. 4. Here we see that the calculated level spacing is decreased as increasing the strength of the quadrupole-pairing interaction. The order of the two-phonon triplet states depends on the choice of g_2 in this case. In general, the order depends also on other force parameters such as f_2, f_3, f_4, etc., and is much sensitive to the choice of the single-particle levels. Therefore the difficulty in the order of the triplet states is not always so serious but sometimes can be remedied by the proper choice of the single-particle energies [40].

To visualize the effect of the higher-order interaction, the calculated level scheme and potential energy surface for ^{128}Xe are shown in Figs. 5 and 6. Though the higher-order interactions are included up to the four-body interaction for the case (b), we can verify from the order estimation of the interaction strengths given by Eq. (16) that the contributions from the four-body interaction are almost negligible for this region. Therefore the differences between (a) and (b) in these figures are mainly due to the contributions from the three-body interaction. In

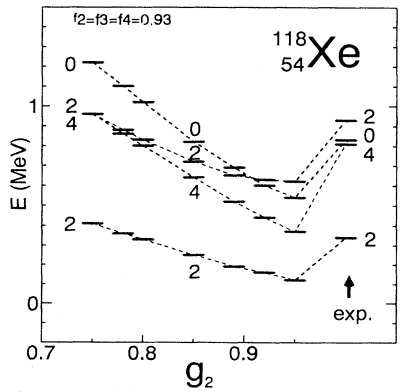

FIG. 4. Calculated energy levels of the first 2^+ and the two-phonon triplet states as a function of the quadrupole-pairing force parameter g_2 for ^{118}Xe. The strengths of the QQ interaction and its higher-order terms are fixed as $f_2 = f_3 = f_4 = 0.93$. For comparison, experimental levels are plotted in the right region.

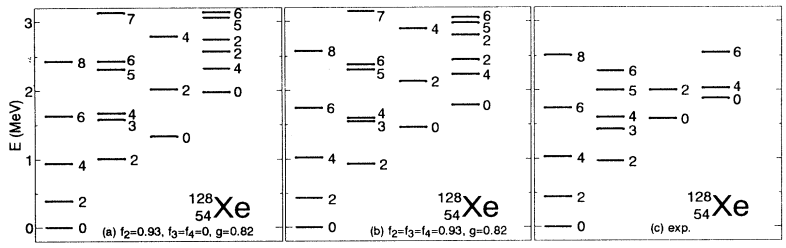

FIG. 5. Effect of the higher-order interactions on the spectrum of ^{128}Xe. The QQ interaction strengths are chosen to be $f_2 = 0.93, f_3 = f_4 = 0$ for (a) and $f_2 = f_3 = f_4 = 0.93$ for (b). In both cases, the strength of the quadrupole-pairing is fixed as $g_2 = 0.82$. All the resulting states with $E_x \leq 3$ MeV and $I \leq 8$ are listed. Especially the states in the ground band, quasi β band, and quasi γ band, are separately accumulated. Experimental levels are plotted in (c).

Fig. 5 we see that the excitation energies of the states in the quasi γ band are relatively lowered while those in the quasi β band are raised due to the three-body interaction. It should be noted here that if we consider the case of an axially symmetric deformed nucleus, the doubly stretched $Q''Q''$ interaction can be expanded as

$$-\frac{1}{2}\chi^{(2)}(\hat{Q}_2'' \cdot \hat{Q}_2'') = -\frac{1}{2}\sum_K \chi^{(2)}_{2K}\hat{Q}_{2K}\hat{Q}_{\widetilde{2K}} + \chi^{(2)}\frac{4}{3}\sqrt{5/16\pi}\varepsilon\hat{R}_0\hat{Q}_{20}, \quad (84)$$

with

$$\chi^{(2)}_{20} = (1 - \tfrac{4}{3}\varepsilon)\chi^{(2)},$$
$$\chi^{(2)}_{21} = (1 - \tfrac{2}{3}\varepsilon)\chi^{(2)},$$
$$\chi^{(2)}_{22} = (1 + \tfrac{4}{3}\varepsilon)\chi^{(2)}, \quad (85)$$

where ε is the quadrupole deformation parameter [19]. Therefore in such situation the $Q''Q''$ interaction is almost equivalent to the QQ interaction with different strengths for each K component. Since the effect of the three-body interaction can nicely be simulated by the doubly stretched $Q''Q''$ interaction, the above features of the excitation energy for the states in the quasi bands can be consistently understood by the results of the RPA calculations for a deformed nucleus [19] and by preforming order estimations of the interaction strengths of Eq. (85).

The calculated potential energy surfaces of the Xe isotopes studied in the present paper show γ-soft features. In fact, we see in Fig. 6 that the difference in energy between the two potential minima is rather small compared to the zero-point energy. These features are enhanced by the three-body interaction for ^{128}Xe. In this mass region, there is an interesting and important problem of the interplay of γ softness and triaxiality [41–44]. To obtain some perspective about this problem from the present type of analyses based on the BET, however, we have to know the γ dependence of the calculated potential energy surface up to at least the order of $(\beta^3 \cos 3\gamma)^2$. For this purpose, we have to expand the original fermion Hamiltonian up to at least sixth order in terms of the collective bosons. In the present calculation, however, the Hamiltonian is expanded only up to the fourth order and as a result the γ dependence of the potential surface is limited up to the order of $\beta^3 \cos 3\gamma$.

The theoretical spectra of Fig. 5(b) reproduce rather well the relative order of the experimental spectra, though the tendency to bunching of levels in the quasi γ band which is characteristic to γ instability is too prominent in the theoretical result. The same difficulty was observed in Refs. [5,13].

The systematics of the energy spectra for Xe isotopes obtained by the present calculation is shown in Fig. 7(a). The theoretical spectra reproduce very roughly the gen-

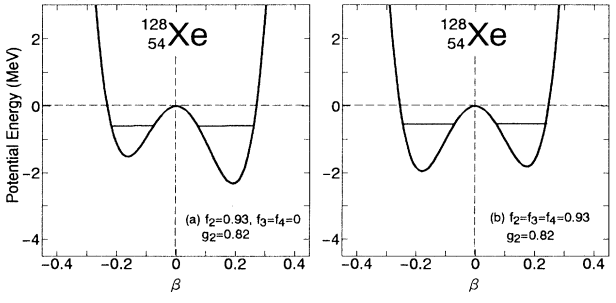

FIG. 6. Effect of the higher-order interactions on the potential energy surface of ^{128}Xe. Each potential is obtained by using the same parameter set as that of the corresponding spectrum in Fig. 5. The horizontal line indicates the ground state energy.

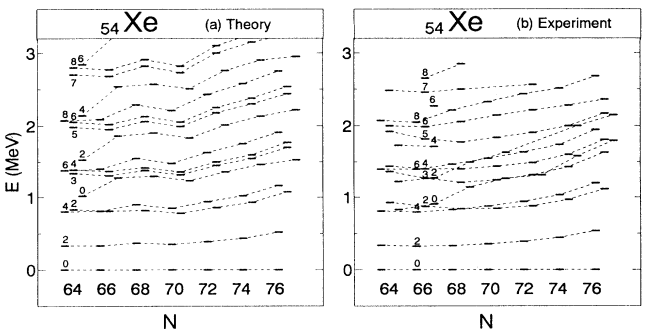

FIG. 7. Excitation energies for Xe isotopes: (a) theory, (b) experiment. Only the states in the ground band, quasi β band, and quasi γ band, are accumulated here.

eral trends of experimental spectra [Fig. 7(b)], though the characteristic degeneracy or bunching of pairs of states in the quasi γ band is too prominent in the theoretical result.

V. CONCLUSIONS

The low-lying quadrupole collective states of the neutron-deficient doubly even Xenon isotopes are studied by means of the BET. The higher-order terms of the QQ interaction are included to ensure the nuclear self-consistency, while the quadrupole-pairing interaction is included to ensure the local Galilean invariance.

In Ref. [2], it is indicated that the strengths of the monopole-pairing interaction need to be reduced by 20% to reproduce roughly the experimental data in this mass region, while in the present calculation where the quadrupole-pairing interaction is explicitly included in addition to the monopole-pairing interaction, such a conspicuous reduction is not necessary. The strengths of the monopole-pairing interaction adopted in the present calculation are in between SPS's of Eq. (19) and WPS's of Eq. (20), and at the same time those of the quadrupole-pairing interaction are close to the predicted values of the self-consistent strengths of Eq. (22).

The calculated potential energy surfaces in this mass region show γ-soft features. To get detailed information about the interplay of the γ softness and the triaxiality, however, further investigations based on a much higher-order boson expansion are advisable.

In summary, the present calculation gives useful information about schematic effective interactions in this mass region and provides an important step towards the microscopic description of low-lying quadrupole collective states in nuclei.

[1] A. Bohr, B.R. Mottelson, and D. Pines, Phys. Rev. **110**, 936 (1958).
[2] S.G. Rohoziński, J. Dobaczewski, B. Nerlo-Pomorska, K. Pomorski, and J. Srebrny, Nucl. Phys. **A292**, 66 (1977).
[3] D.R. Bes and B.A. Broglia, Phys. Rev. C **3**, 2349 (1971).
[4] T. Kishimoto and T. Tamura, Nucl. Phys. **A192**, 246 (1972).
[5] T. Kishimoto and T. Tamura, Nucl. Phys. **A270**, 317 (1976).
[6] I. Hamamoto, Nucl. Phys. **A232**, 445 (1974).
[7] K. Hara and S. Kusuno, Nucl. Phys. **A245**, 147 (1975).
[8] M. Wakai and A. Faessler, Nucl. Phys. **A295**, 86 (1978).
[9] K. Hara and S. Iwasaki, Nucl. Phys. **A348**, 200 (1980).
[10] H. Sakamoto and T. Kishimoto, Phys. Lett. B **245**, 321 (1990).
[11] T. Kishimoto and T. Tamura, Phys. Rev. C **27**, 341 (1983).
[12] H. Sakamoto and T. Kishimoto, Nucl. Phys. **A486**, 1 (1988).
[13] H. Sakamoto and T. Kishimoto, Nucl. Phys. **A528**, 73 (1991).
[14] L.S. Kisslinger and R.A. Sorensen, Mat. Phys. Medd. Dan. Vid. Selsk. **32**, No. 9 (1960); Rev. Mod. Phys. **35**, 853 (1963).
[15] M. Baranger and K. Kumar, Nucl. Phys. **A110**, 490 (1968); **A122**, 241 (1968).
[16] K. Kumar and M. Baranger, Nucl. Phys. **A110**, 529 (1968); **A122**, 273 (1968).
[17] T. Suzuki and C. Hinohara, Phys. Rev. C **9**, 1186 (1974).
[18] T. Kishimoto, J.M. Moss, D.H. Youngblood, J.D. Bronson, C.M. Rozsa, D.R. Brown, and A.D. Bacher, Phys. Rev. Lett. **35**, 552 (1975).
[19] H. Sakamoto and T. Kishimoto, Nucl. Phys. **A501**, 205 (1989); **A501**, 242 (1989).
[20] M. Matsuo and K. Matsuyanagi, Prog. Theor. Phys. **74**, 1227 (1985); **76**, 93 (1986).
[21] S. Mizutori, Y.R. Shimizu, and K. Matsuyanagi, Prog. Theor. Phys. **83**, 666 (1990); **85**, 559 (1991).
[22] W. Nazarewicz and J. Dobaczewski, Phys. Rev. Lett. **68**, 154 (1992).
[23] T. Nakatsukasa, S. Mizutori, and K. Matsuyanagi, Prog. Theor. Phys. **87**, 607 (1992); **89**, 847 (1991).
[24] T. Kishimoto, in *Proceedings of the International Conference on Highly Excited States in Nuclear Reaction* (RCNP, Osaka, 1980), p. 377.
[25] T. Kishimoto, T. Tamura, and T. Kammuri, Prog. Theor. Phys. Suppl. **74&75**, 170 (1983).
[26] E.R. Marshalek, Phys. Rev. Lett. **51**, 1534 (1983); Phys. Rev. C **29**, 640 (1984); Phys. Lett. B **244**, 1 (1990).
[27] M. Matsuo and K. Matsuyanagi, Prog. Theor. Phys. **78**, 591 (1987).
[28] S.A. Stotts and T. Tamura, Phys. Rev. C **40**, 2342 (1989).
[29] H. Aiba, Prog. Theor. Phys. **84**, 908 (1990).
[30] K. Yamada, Prog. Theor. Phys. **89**, 995 (1993).
[31] S.G. Nilsson, C.F. Tsang, A. Sobiczewski, Z. Szymański, S. Wycech, C. Gustafson, I.-L. Lamm, P. Möller, and B. Nilsson, Nucl. Phys. **A131**, 1 (1969).
[32] B.R. Mottelson, *Nikko Summer School Lectures* (NORDITA, Copenhagen, 1967), pub. 288.
[33] A.H. Wapstra and G. Audi, Nucl. Phys. **A432**, 1 (1985).
[34] J. Copnell, K. Kumar, S.J. Robinson, and C. Tenreito, J. Phys. G **18**, 1943 (1992).
[35] S. Okubo, Prog. Theor. Phys. **12**, 603 (1954).
[36] N. Fukuda, K. Sawada, and M. Taketani, Prog. Theor. Phys. **12**, 156 (1954).
[37] A. Messiah, *Quantum Mechanics* (North-Holland, Amsterdam, 1961), Vol. II, Chap. XVI.
[38] K. Suzuki and R. Okamoto, Prog. Theor. Phys. **71**, 1221 (1984).
[39] H. Feshbach, Ann. Phys. (N.Y.) **5**, 357 (1958); **19**, 287 (1962).
[40] T. Komatsubara, T. Hosoda, H. Sakamoto, T. Aoki, and K. Furuno, Nucl. Phys. **A496**, 605 (1989).
[41] R.F. Casten and P. von Brentano, Phys. Lett. **152**, 22 (1985).
[42] N.V. Zamfir and R.F. Casten, Phys. Lett. B **260**, 265 (1991).
[43] P.F. Mantica, Jr., B.E. Zimmerman, W.B. Walters, J. Rikovska, and N.J. Stone, Phys. Rev. C **45**, 1586 (1992).
[44] J. Yan, O. Vogel, P. von Brentano, and A. Gelberg, Phys. Rev. C **48**, 1046 (1993).