Comparison of potential models with the *np* scattering data below 350 MeV

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As a follow-up on our earlier paper we calculate the χ^2 of various *NN* potential models with respect to the *np* scattering data. We find that only the most recent potential models give a reasonably good description of these data. Almost none of the potentials is truly an *NN* potential in the sense that it gives a good description of both *pp* and *np* scattering simultaneously.

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In a recent paper [1], we investigated the quality of a number of *NN* potential models with respect to the *pp* scattering data below 350 MeV. By comparing to the *pp* data we tested the isovector ($I=1$) partial waves of these models. In this Brief Report we will confront the models with the *np* data so that we can also test the isoscalar ($I=0$) partial waves.

In our previous comparison [1], we restricted ourselves to a comparison with respect to the *pp* data only, because of two reasons. First, at the time of our comparison we were still in the process of analyzing the *np* data, and so we did not have a complete and consistent *np* database. Second, our initial goal for comparing *NN* models was to bring attention to the fact that these models are often not *NN* models at all, but rather *pp* or *np* models; i.e., they were only fitted to one type (*pp* or *np*) of data. Especially in the last decade, it has been assumed by many that it is sufficient to fit the parameters of a potential model only to the *np* data. Simply adding the proper electromagnetic interaction is then assumed to provide the corresponding *pp* version of that potential, so that one in fact ends up with an *NN* potential. (This is much easier than fitting to the *pp* data directly, because in fitting the *np* data one only has to worry about the magnetic-moment corrections when calculating the observables.) For example, the full Bonn potential [2] was originally presented as an *NN* potential, although its parameters were in fact fitted only to the *np* data. In a later publication [3], the model had to be adjusted to make it also applicable to *pp* scattering. We wanted to show explicitly that in order to construct an *NN* potential model it is generally not good enough to only fit the *np* data.

One often argues incorrectly that the reason for this failure is only artificial, where the argument goes as follows. There are some very accurate *pp* differential cross sections [4] and *np* total cross sections [5] at low energies, which put very tight constraints on the *pp* and *np* $^1S_0$ scattering lengths. It is well known that the nuclear *pp* and *np* scattering lengths are different and that this difference cannot be explained by mass differences alone. It is then obvious that the choice of fitting a potential to either the *pp* or *np* scattering length will result in a totally incorrect *np* or *pp* scattering length, respectively, for that model. Very high χ^2 values on the *pp* data for models which are fitted to the *np* scattering length will then be the result.

A way to avoid this problem is to exclude these low-energy data and only include data above 5 MeV, say. Fitting to either the *pp* or *np* scattering length causes only minor differences in the quality of the description of the data at energies above 5 MeV. However, in doing this we found [1] that most of the potential models still gave a very poor description of the *pp* data. This indicates that the poor quality of these models is not only due to an incorrect scattering length, but has other sources as well.

Another argument that a high χ^2 on the *pp* data is only artificial is that one can slightly adjust one of the parameters of the potential model to ensure that it fits the *pp* $^1S_0$ scattering length. Because the scattering length is extremely sensitive to small changes in the potential, one expects that such an adjustment will be minimal and will hardly affect the other partial waves. This modification is then believed to improve the description of the low-energy data without affecting the description of the other data, resulting in an overall good description of the *pp* data. We investigated this [1] extensively using the Arg84 potential [6]. Instead of modifying one of the parameters, we completely replaced the *pp* $^1S_0$ phase shift of the Arg84 potential by the *pp* $^1S_0$ phase shift of the Nijmegen partial-wave analysis [7]. This corresponds to having an almost "perfect" $^1S_0$ phase shift; something which is very unlikely to be achievable by simply adjusting one of the potential parameters. Although this gives a considerable improvement, the resulting $\chi^2/N_{\rm data}$ is still rather high. The same trend is found when we apply this procedure to the coordinate-space version of the Bonn potential [2] or to the coordinate-space Bonn A and Bonn B potentials [8]. This indicates that the poor quality of these potential models with respect to the *pp* data is not only due to the fact that they have an incorrect $^1S_0$ phase shift. In fact, because of the mass difference between neutral and charged pions, the *pp* and *np* isovector phase shifts in the other partial waves are significantly different as well (see Table I of Ref. [9]). Therefore, the problem usually cannot be fixed by simply adjusting one of the parameters of the model.

Now that the Nijmegen partial-wave analysis of the *np* data below 350 MeV is finished [9], we have a complete and consistent *np* database, and we can complete our comparison

TABLE I. χ^2 on the np scattering data between 5 and 350 MeV for various NN potential models. A division is made showing the sub-χ^2 on the total cross sections, the differential cross sections, the analyzing powers, the spin-correlation parameters, the depolarizations, and the rotation parameters. The lower part shows the $\chi^2/N_{\rm data}$ for the 5–350 MeV and 0–350 MeV energy ranges, the latter both without (2510 data) and with (2514 data) the accurate total cross sections of Ref. [5].

	$N_{\rm data}$	HJ62	Reid68	TRS75	Paris80	Urb81	Arg84	BonnR	Bonn87	Nijm93
$\sigma_{\rm tot}$	225	2376	6599	389	1117	594	662	1624	895	554
$\sigma(\theta)$	1323	3379	5771	3273	2856	2726	2055	14908	3138	1554
A_y	738	2345	4217	3799	1275	1807	2019	3330	1762	1023
A_{yy}, A_{zz}	86	975	9456	1054	3899	1331	231	5822	1328	1499
D_t	43	56	133	87	108	81	51	296	90	52
A_t, R_t	43	60	127	176	202	145	82	348	89	100
all data	2458	9190	26303	8778	9457	6684	5100	26328	7301	4783
5–350 MeV	2458	3.7	10.7	3.6	3.8	2.7	2.1	10.7	3.0	1.9
0–350 MeV	2510	7.7	55.0	5.5	6.2	4.7	2.1	10.5	3.3[a]	1.9
	2514	4100	3740	2925	3300	3400	3.3	10.5	423[a]	1.9

[a]Value may be inaccurate because the SAID program possibly does not represent the low-energy Bonn phase shifts very well.

of NN potential models by confronting them with the np data. This allows us to investigate whether np potential models which give a very poor description of the pp data can in fact survive a quality test with respect to the np data. As we will see, some of them do. This once more strongly supports our statement that "a good fit to the np data does not automatically guarantee a good fit to the pp data" [1].

In this Brief Report we will confront a number of potential models with the np scattering data. This means that we have to calculate the phase shifts of the lower partial waves at all the energies at which experimental data are available, construct the scattering amplitude, and calculate the observables. Since we are only going up to $T_{\rm lab}=350$ MeV, it is sufficient to calculate the phase shifts due to the nuclear potential up to total angular momentum $J=6$. For the higher partial waves we can take the phase shifts as given by one-pion exchange. The np database we use is given in Ref. [9]. When we omit the low-energy data below $T_{\rm lab}=5$ MeV, the database will contain 2458 data between 5 and 350 MeV.

The potential models we consider are mainly the same as those of our pp comparison [1], namely, HJ62 [10], Reid68 [11], TRS75 [12], Paris80 [13], Urb81 [14], Arg84 [6], BonnR [2], Bonn87 [2], and Nijm93 [15]. The soft-core Nijmegen potential is here referred to as Nijm93. This is an updated version of the Nijm78 potential [16].

We made a change in notation with respect to the Bonn potentials, in that the coordinate-space Bonn potential is now denoted by BonnR; Bonn87 refers to the full Bonn np potential [2], and Bonn89 to the full Bonn pp potential [3]. Because we do not have the computer codes to calculate the phase shifts of the full Bonn potential ourselves, these phase shifts were obtained from the computer software SAID [17]. We take the phase shifts in steps of 1 MeV as generated by the SAID program, and then use linear interpolation to obtain the phase shifts at the experimental energies. Incidentally, this provides another reason for only using the np data above 5 MeV: The phase shifts below 10 MeV of a potential in the SAID program are represented by an effective-range parametrization which, especially for the S waves, is not good enough to represent the low-energy phase shifts to a high accuracy, and so the results from SAID for very low energies could be incorrect [18]. We checked for the other potentials that starting at 5 MeV, rather than at 10 MeV, does not make any difference for the conclusions we will draw below.

As a check that the way how we treat the full Bonn potential is justified, we applied the same procedure to the Nijmegen potential: We took the phase shifts of the Nijm93 potential in steps of 1 MeV as generated by the SAID program, interpolated to get the phase shifts at the experimental energies, calculated the total χ^2 with respect to the np data, and compared to the result we obtain when we calculate the Nijm93 phase shifts directly using our own computer code for the Nijm93 potential. The difference in χ^2 is only 0.4. However, if we include the data below 5 MeV as well, there is a large difference between the interpolated SAID and exact Nijmegen results. This means that for the present purpose we can use the 5–350 MeV phase shifts of the full Bonn potential as stored in the SAID program, but for calculations involving lower energies we have to make the proviso that the results could be inaccurate.

The results for the nine potential models are presented in Table I. Similarly to our comparison of potential models with respect to the pp scattering data [1], only a few of the models give a reasonable description of the np scattering data. The best ones are the Arg84 and Nijm93 models with $\chi^2/N_{\rm data}\approx 2$, followed by the Urb81 and Bonn87 models with a still reasonable $\chi^2/N_{\rm data}\approx 3$. To get some insight into where the high χ^2 for the other models comes from, we divide the total χ^2 in a set of sub-χ^2 for each different type of observable, as listed in Table I.

The relatively high $\chi^2(\sigma_{\rm tot})$ for most models is mainly due to one group of total cross sections [19], consisting of 70 data from 39 to 350 MeV. The statistical error on these data is rather small and the energy range is very large. So if the energy dependence (the shape) for the total cross sections as given by a potential model is different from what it is implied to be according to the experiment, the χ^2 rapidly increases. Indeed, the other total cross-section experiments cover much smaller energy ranges and the description of these data is generally much better with $\chi^2/N_{\rm data}$ ranging from 1.1 for the Arg84 potential to 3.7 for the BonnR poten-

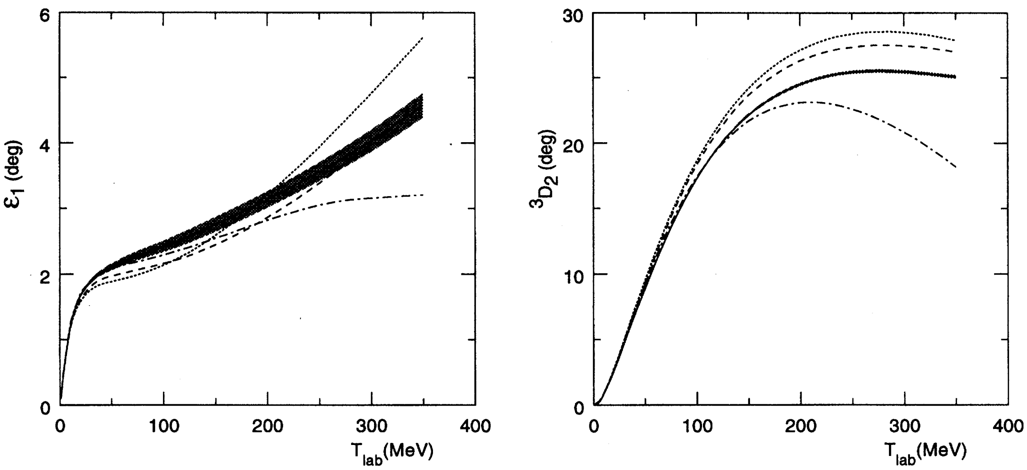

FIG. 1. ϵ_1 mixing parameter and $^3D_2$ phase shift of the Nijmegen partial-wave analysis with the statistical error (shaded band), the Paris potential (dotted line), the full Bonn potential (dash-dotted line), and the Nijmegen potential (dashed line).

tial. However, there still remains the fact that this group of 70 data can be described very well in the Nijmegen partial-wave analysis [9] with $\chi^2=56.8$.

High $\chi^2(\sigma(\theta))$ and $\chi^2(A_{yy},A_{zz})$ indicate an incorrect $^1P_1$ phase shift and ϵ_1 mixing parameter. For the HJ62, Reid68, and Urb81 models the $^1P_1$ phase shift becomes too negative at high energies, whereas for the BonnR model it does not become negative enough. At $T_{\rm lab}=300$ MeV the Nijmegen partial-wave analysis [9] gives $\delta(^1P_1) =-27.58(22)°$. Similarly, comparing to [9] $\epsilon_1=4.03(17)°$, the ϵ_1 mixing parameter is too high in the HJ62, Reid68, and TRS75 models, whereas it is too small in the Bonn models.

As an example of the differences between the phase shifts of the various potential models, we show in Fig. 1 the ϵ_1 mixing parameter and the $^3D_2$ phase shift. The solid curves are the values from the multiple-energy Nijmegen partial-wave analysis [9]. The shaded band represents the statistical error. Compared to the Nijmegen analysis, the Paris80 potential gives values which are too high, whereas the Bonn87 potential gives values which are too low. This is reflected in the relatively high χ^2 on the differential cross sections and spin-correlation parameters for these models.

In Table I we also give the $\chi^2/N_{\rm data}$ for the 5–350 MeV and 0–350 MeV energy ranges. For the latter energy range, we give the results with and without the very accurate low-energy cross sections [5]. This demonstrates the enormous changes that can occur when we include the low-energy data. The results involving low energies are different from what one would obtain for a potential model using the SAID [17] program. One reason is that in the SAID program the low-energy phase shifts are obtained from an effective-range interpolation which is not very accurate, whereas here we calculate the phase shifts by direct computation. A second reason is that the data of Ref. [5] are not present in the SAID database. Obviously, including these accurate low-energy data in a potential comparison can be misleading in that the large total χ^2 obscures the fact that a certain potential model can in fact describe the scattering data at energies above 5 MeV, say, reasonably well. Once again, this is the reason why we here focus on the 5–350 MeV results.

To summarize, we find that only the Arg84 and Nijm93 potentials give a good description of the np data ($\chi^2/N_{\rm data}\approx 2$), while the Urb81 and Bonn87 potentials can be qualified as reasonable ($\chi^2/N_{\rm data}\approx 3$). When we include the very accurate low-energy total cross sections, only the Nijm93 potential still has a good $\chi^2/N_{\rm data}$.

Before we can make statements about the general quality of NN potential models, we have to recapitulate the quality of these models with respect to the pp scattering data. Rather than referring to our previous publication [1], we here present the results for the pp data using the same procedure we use for the np data. So also for the pp data, we here give $\chi^2/N_{\rm data}$ results from a direct comparison to the pp data, rather than using the Nijmegen representation of the χ^2 hypersurface of the pp data. The results are given in Table II, where the 5–350 MeV entry should be compared with the 2–350 MeV entry in Table II of Ref. [1]. Note once more the enormous rise for some of the models (Arg84 and BonnR) when we include the low-energy (0–5 MeV) data. The quality with respect to the pp data for each model individually has already been discussed in our previous publication [1].

Comparing Tables I and II, we conclude that most potential models are not NN models at all, but give only a reasonable description of either the pp or the np scattering data (or

TABLE II. $\chi^2/N_{\rm data}$ on the pp scattering data for the 5–350 MeV and 0–350 MeV energy ranges.

	$N_{\rm data}$	HJ62	Reid68	TRS75	Paris80	Urb81	Arg84	BonnR	Bonn89	Nijm93
5–350 MeV	1590	9.7	2.5	3.3	2.2	5.9	6.9	12.4	1.8	1.9
0–350 MeV	1787	13.5	2.9	3.4	4.5	6.0	7615	1090	25.1[a]	1.8

[a]Value may be inaccurate because the SAID program possibly does not represent the low-energy Bonn phase shifts very well.

not even that). The only exception is the Nijm93 potential. The situation for the full Bonn potential is probably also still not unsatisfactory, although the description of the *np* data is considerably worse (but not too bad) than the description of the *pp* data. The TRS75 potential also shows a quality which is consistent for both *pp* and *np* scattering, although $\chi^2/N_{\rm data}=3.5$ on all data is not too good. All other potentials can be classified as either *pp* potentials (Reid68, Paris80), *np* potentials (Urb81, Arg84), or they do not give a satisfactory description of either *pp* or *np* scattering (HJ62, BonnR). Two other coordinate-space versions of the Bonn potentials, Bonn A and Bonn B [8], also belong to the last category. For the 5–350 MeV energy range, $\chi^2/N_{\rm data}$ for these two models is, for Bonn A, 9.4 on *pp* and 8.3 on *np* and, for Bonn B, 8.5 on *pp* and 8.9 on *np*. It is surprising to see the enormous difference in quality of the full Bonn potential on the one hand and all the Bonn coordinate-space versions on the other hand. The difference cannot be explained [2,8] by claiming that coordinate-space potentials are necessarily of inferior quality. The quality of the Nijmegen potential, which is a coordinate-space potential (but which also has an exactly equivalent momentum-space version), clearly contradicts this.

The results from Tables I and II are rather disappointing, considering the fact that all models were originally presented as being *NN* potentials. This demonstrates once again our point that in general one has to be careful when using these potential models in other calculations, like in few-nucleon scattering and bound-state calculations, *pp* bremsstrahlung, or nuclear matter calculations. In most cases these models cannot even describe both *pp* and *np* scattering with the same, satisfactory, quality.

We should mention that recently there have been constructed a number of new *NN* potentials which are truly *NN* potentials in the sense that they give an excellent description of both the *pp* and *np* data simultaneously. These are two Nijmegen potentials Nijm I and Nijm II [15], a regularized update of the old Reid68 potential [15], and an update of the old Arg84 potential [20]. All four models have the almost optimal $\chi^2/N_{\rm data}\approx 1$ on both *pp* and *np* data. Part of the success of these models is that they explicitly contain the one-pion-exchange potential with the proper neutral- and charged-pion masses. Another reason is of course that these models were explicitly fitted to both *pp* and *np* data simultaneously, giving the proper constraints on the isovector partial waves.

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