Intermittency and multifractality of medium-energy particles in proton-nucleus interactions at 800 GeV

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We investigate the intermittent and multifractal behaviors of the multiplicity fluctuations of medium energy particles in 800 GeV proton interactions with emulsion nuclei. For our analysis, we have used ordinary scaled factorial moments (SFM) denoted as F_q, modified SFM, (F'_q) and modified multifractal moments, (G'_q) with suppressed statistical contributions. For a close comparison between the F'_q and G'_q moments, they have both been expanded in a set of basic functions each of which has a characteristic fractal behavior as a function of the resolution in phase space. The experimental values of anomalous fractal dimensions d_q and Renyi dimensions D_q are determined for $q=2-5$. The latter are corrected for statistical contamination in the multiplicity fluctuations. The observed variation of d_q and $D_q^{\rm dyn}$ with q supports a self-similar random cascading mechanism for the particle production process. It is found that $D_q^{\rm dyn}+d_q=1$.

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I. INTRODUCTION

Recently, the dynamics of multiparticle production has been probed from the interesting investigations of nonstatistical multiplicity fluctuations in narrow regions of phase space (intermittency) [1–4]. An attractive formalism proposed by Bialas and Peschanski using scaled factorial moments (SFM's) suggested a self-similar behavior of the particle production process [5]. Evidence for intermittency has been reported in a variety of processes like e^+e^- [6], hadron-hadron [7], hadron-nucleus [8], and nucleus-nucleus [9] collisions. Apart from the intermittent behavior, the multifractal nature of particle density fluctuations has also been confirmed in various collision processes with different targets and varying primary energies. Some authors [10] have attributed the observed multiplicity fluctuations to the existence of short-range correlations among the secondary particles. This approach, while accounting for the nonstatistical multiplicity fluctuations, still falls far short of explaining the power-law scaling in various reactions. The latter reflects a multifractal or self-similar property of the underlying dynamics. Hwa and co-worker [11,12] proposed a set of multifractal moments G_q in order to investigate the observed large density fluctuations. These G_q moments are, however, dominated by statistical fluctuations when the event multiplicity is low. In order to overcome this limitation, Hwa and Pan [13] and Derado *et al.* [14] suggested a modified form of these multifractal moments, denoted here as G'_q, which suppress statistical noise. These authors have also established a relation between the fractal and intermittency indices [13–16]. In these investigations, the analysis has been performed on the fast shower particles ($\beta\geqslant 0.7$). It would be interesting to investigate the intermittency in terms of multifractals for medium-energy particles ($0.7>\beta\geqslant 0.3$), for this will provide another window to understand the dynamics of particle production process.

Recently [17], we have found evidence of multifractal behavior in target-fragmented medium-energy protons in proton-AgBr interactions at 800 GeV, which is presently the highest energy for fixed targets. The present work has been done at the same energy in proton interactions with all nuclei in emulsion. Here attention will be focused on the analysis of intermittency in terms of multifractals for medium-energy particles and to obtain a relation between the two. Following Hwa and Pan [13], we analyze the modified SFM's, denoted here by F'_q, and expand them in a set of basic functions $B_{q,k}$, each of which exhibits a characteristic fractal behavior as the size of the phase space is reduced. This is similar to the expansion of a scattering amplitude in terms of a set of Regge poles, each of which exhibits characteristic Regge-type asymptotic behavior. We determine the value of the trajectory function $\lambda(q)$ which specifies the fractal behavior $M^{\lambda(q)}$ with the number of bins M tending to infinity. Here $\lambda(q)$ is analogous to the Regge trajectory $\alpha(t)$ which characterizes the power-law behavior $s^{\alpha(t)}$ with s tending to infinity. In order to study the relation between the fractal and intermittency indices, we perform a multifractal analysis of the present interactions using the noise-suppressed G'_q moments which are also expanded in the same set of basic functions. Our analysis exhibits a close analogy between intermittency and multifractality for medium-energy particles.

II. EXPERIMENTAL DETAILS

A stack of 40 Ilford G5 emulsion pellicles of dimension $10\times 8\times 0.06$ cm^3 was exposed to a proton beam of energy 800 GeV at Fermilab. The beam flux was 8.7×10^4 particles/cm^2. The scanning of interactions was done with high-resolution microscopes at 40× resolution by the area-scanning method. Using the double-scan data, the scanning efficiency was calculated for each observer and the overall efficiency was found to be greater than 90%. In order to

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make sure that the interactions are due to the beam only, all the interactions were followed back. The interactions lying within 25 μm each from the air and the glass surface have not been considered for measurement purposes. Following the above criteria, 3500 events were scanned.

All the charged secondaries of these events were classified according to the nuclear emulsion terminology [18]. Secondary tracks having $I \leq 1.4 I_0$, $\beta(v/c) \geq 0.7$ were designated as shower tracks, those having $10 I_0 \geq I > 1.4 I_0$, $0.7 > \beta \geq 0.3$ were designated as grey tracks, and black tracks have $I > 10 I_0$, $\beta < 0.3$, where I and I_0 are the ionizations of secondary and primary tracks, respectively. The values of N_s, N_g, and N_b denote the multiplicities of shower, grey, and black tracks, respectively. The shower tracks are due to fast particles produced in elementary nucleon-nucleon collisions. The heavy tracks (N_h) include grey and black tracks. The former are mostly due to medium-energy (30–400 MeV) knock-on nucleons, whereas the latter are produced by low-mass target fragments resulting from evaporation of the nucleus.

For measuring the emission angle (θ) of a charged particle, the coordinates (x,y,z) at the vertex of the interaction and at two points each on the primary beam track and the grey track were measured under the 100× oil immersion objective.

Let (x_{b1}, y_{b1}, z_{b1}) and (x_{b2}, y_{b2}, z_{b2}) be the coordinates of two points on the primary beam track and (x_{g1}, y_{g1}, z_{g1}) and (x_{g2}, y_{g2}, z_{g2}) be the coordinates of two points on the grey track. The emission angle (θ) of the grey track is given by

$$\theta = \cos^{-1}\left[\frac{x_b x_g + y_b y_g + z_b z_g}{l_b l_g}\right] , \quad (1)$$

where

$$x_b = x_{b1} - x_{b2} ,$$

$$y_b = y_{b1} - y_{b2} ,$$

$$z_b = (z_{b1} - z_{b2}) \times \text{shrinkage factor} ,$$

$$x_g = x_{g2} - x_{g1} ,$$

$$y_g = y_{g2} - y_{g1} ,$$

$$z_g = (z_{g2} - z_{g1}) \times \text{shrinkage factor}$$

$$l_b = (x_b^2 + y_b^2 + z_b^2)^{1/2} ,$$

$$l_g = (x_g^2 + y_g^2 + z_g^2)^{1/2} .$$

The values of x, y, and z coordinates are measured in micrometers (μm). The measurement error in θ was found to be $\sim 8 \times 10^{-4}$ rad ($\sim 0.05°$). In the following we shall omit the subscript g in N_g for convenience, and from now on we shall discuss the grey track distribution only. The total number of events with at least one grey track was found to be 1544, and the average multiplicity of grey tracks in the event sample was 3.37.

The usual longitudinal distribution variable pseudorapidity is not suitable for the study of these particles as they are heavier than pions and also less energetic. As followed in earlier works [17,19], we choose the cosine of the emission angle ($\cos\theta$) as the basic distribution variable in our one-dimensional analysis. The parameter $\cos\theta$ will, hereinafter, be referred to as W.

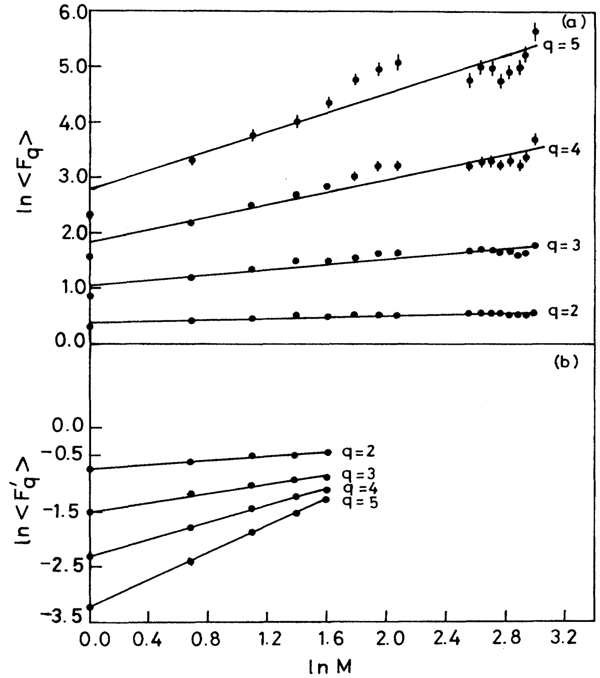

FIG. 1. (a) $\ln\langle F_q \rangle$, (b) $\ln\langle F'_q \rangle$ as a function of $\ln M$. Lines represent the least-squares fits to the data points.

III. DATA ANALYSIS, RESULTS, AND DISCUSSION

To study multiplicity fluctuations of the medium-energy particles produced in high-energy hadron-nucleus collisions, we have worked in the one-dimensional space of W. The total W space $[-1, +1]$ was divided into M equal parts, each having a bin size of $\delta W = 2/M$. In the phenomenological investigation of intermittency in multiparticle processes, the quantities of interest are the scaled factorial moments (SFM's). They are traditionally defined as

$$\langle F_q \rangle = \frac{1}{M} \sum_{j=1}^{M} \left\langle \frac{n_j(n_j - 1) \cdots (n_j - q + 1)}{(\langle N \rangle / M)^q} \right\rangle . \quad (2)$$

Here $j = 1, 2, \ldots, M$ labels a bin, n_j is the multiplicity of bin j, N is the total multiplicity in M bins for a single event, and $\langle N \rangle$ is the result for the total multiplicity obtained after averaging over the ensemble of interactions. To probe any evidence of intermittency in the production mechanism of grey tracks, the $\langle F_q \rangle$ moments were studied as a function of the bin size, as shown in Fig. 1. It is observed from Fig. 1(a) that (a) $\ln\langle F_q(\delta W) \rangle$ is positive for all q and δW and (b) $\ln\langle F_q(\delta W) \rangle$ increases approximately linearly with $-\ln(\delta W)$ for $0.1 \leq \delta W \leq 2.0$, i.e.,

$$\ln\langle F_q(\delta W) \rangle = -\Phi_q \ln\langle \delta W \rangle + C_q , \quad (3)$$

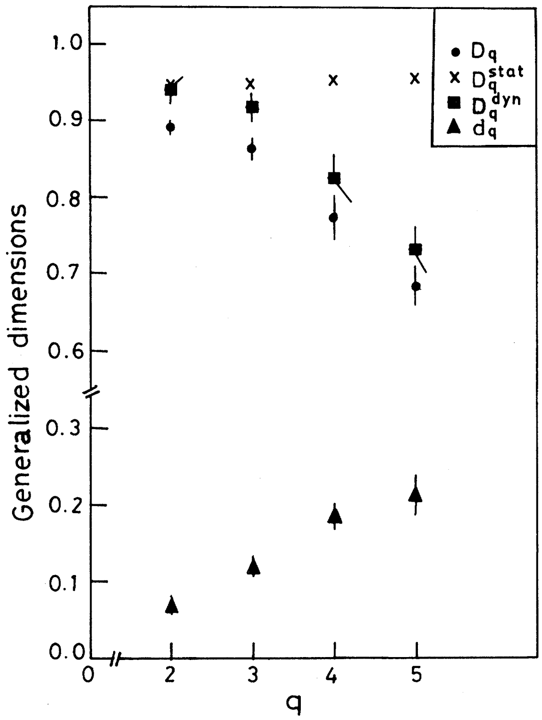

FIG. 2. Generalized dimensions D_q, $D_q^{\rm stat}$, $D_q^{\rm dyn}$, and the anomalous fractal dimension d_q as a function of order q.

where Φ_q and C_q are constants. The observation of such a scale invariant behavior encourages us to understand this characteristic in terms of the theory of multifractals [20], which assumes a self-similar cascading type of scenario in particle production. In this approach, after the collision between initial particles, the system is assumed to first decay into, say, P pieces. After a proper time interval, each of these pieces further dissociate into P pieces and this fragmentation process continues until the total energy has dissipated. The multifractal behavior of the interaction is illustrated from the observed variation of the anomalous fractal dimension $d_q = \Phi_q / q - 1$ as a function of q, as shown in Fig. 2. The values of intermittency index Φ_q are given in Table I for q in the range 2–5.

To facilitate comparison with a proper multifractal description, Hwa and Pan [13] and Derado *et al.* [14] have proposed the following definition of SFM's:

$$\langle F'_q \rangle = \frac{1}{M} \sum_{j=1}^{M} \left\langle \frac{n_j(n_j-1)\cdots(n_j-q+1)}{N^q} \right\rangle . \tag{4}$$

Thus the $\langle F'_q \rangle$ are defined with $\langle N \rangle$ in (2) replaced by the event multiplicity N in the W space $[-1,+1]$. With this change in normalization, the new definition of SFM's is closer to the original proposal of Bialas and Peschanski [5], namely,

$$F_q^{\rm BP} = M^{q-1} \sum_{j=1}^{M} \frac{n_j(n_j-1)\cdots(n_j-q+1)}{N(N-1)\cdots(N-q+1)} . \tag{5}$$

The double-logarithm plot of the modified SFM as a function of M is shown in Fig. 1(b). The intermittency strength parameters which specify the power-law behavior of F'_q, i.e., $a_q = \delta \ln \langle F_q \rangle / \delta \ln M$, were determined from slopes of the least-squares fittings in the linear region of Fig. 1(b), which extends for M only in the range of 1–5. The values of these indices are shown in Table I, and their dependence on q is shown in Fig. 8, below. Although the normalizations of $\langle F_q \rangle$ and $\langle F'_q \rangle$ moments are quite different, the values of the slope parameters determined from their power-law behaviors are not significantly different. From the definition of F_q (F'_q) moments [Eqs. (2) and (4)], it is clear that the deviation of the slope parameters Φ_q (a_q) from zero is a measure of nonstatistical fluctuations of dynamical origin. A slope parameter equal to zero would imply no power law and hence no dynamics. It is clear that following different approaches we should arrive at the final underlying dynamics. Hence it would be natural to expect the similarity in the values of Φ_q and a_q.

The multifractal description of the multiplicity fluctuations was investigated using an alternative to the factorial moments. These multifractal moments provide a direct measure of generalized dimensions of multiplicity fluctuations in particle physics. To suppress the contributions from statistical noise, multifractal moments are defined as [13,14]

$$\langle G'_q \rangle = \left\langle \sum_{j=1}^{M} (n_j/N)^q \Theta(n_j - q) \right\rangle , \tag{6}$$

where $\Theta(n_j - q)$ is the step function that cuts off at $n_j = q$, i.e., $\Theta(n_j - q) = 0$ for $n_j < q$ and $\Theta(n_j - q) = 1$ for $n_j \geq q$. The introduction of the cut in the definition of G'_q moments allows one to make a closer comparison between the F'_q and G'_q moments. This definition allows q to be a real integer, positive or negative. The low multiplicity of the present interactions puts a serious constraint on our ability to obtain any information based on negative q values. We obtained an increasing dependence of $\ln \langle G'_q \rangle$ on $\ln M$ only for two to three values of $\ln M$. This was considered to be statistically insufficient, and hence negative q moments were not consid-

TABLE I. Values of slope parameters a_q, Φ_q, $\tau_q^{\rm dyn}$, $\lambda(q,0)$, and the dynamical Renyi dimensions $D_q^{\rm dyn}$ as calculated from (i) $\tau_q^{\rm dyn}/q-1$ and (ii)$-\lambda(q,0)/q-1$. The errors (in parentheses) are standard.

q	a_q	Φ_q	$\tau_q^{\rm dyn}$	$\lambda(q,0)$	$\tau_q^{\rm dyn}/q-1$	$-\lambda(q,0)/q-1$
2	0.167 (0.028)	0.068 (0.010)	0.941 (0.015)	−0.889 (0.023)	0.941 (0.015)	0.941 (0.026)
3	0.398 (0.028)	0.236 (0.027)	1.832 (0.036)	−1.737 (0.100)	0.916 (0.018)	0.921 (0.054)
4	0.764 (0.025)	0.560 (0.049)	2.466 (0.093)	−2.322 (0.200)	0.822 (0.031)	0.833 (0.071)
5	1.209 (0.001)	0.848 (0.094)	2.916 (0.119)	−2.719 (0.381)	0.729 (0.029)	0.725 (0.099)

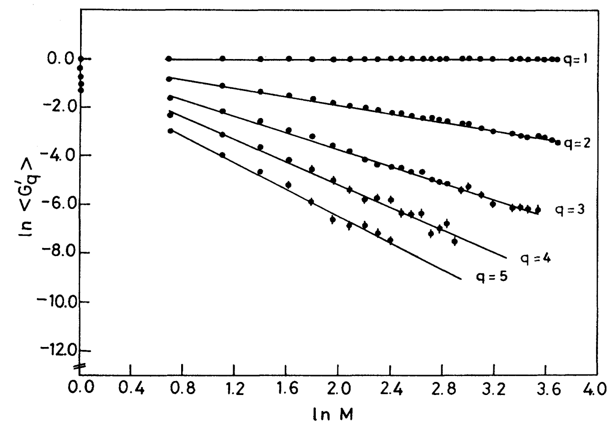

FIG. 3. $\ln\langle G'_q\rangle$ as a function of $\ln M$ for $q=1-5$. Straight lines represent the least-squares fits to the data points.

ered for further analysis. On the other hand, we obtained 28 values of $\ln M$ for positive q values.

Figure 3 shows a plot of $\langle G'_q\rangle$ as a function of M for positive q. It is evident that $\langle G'_q\rangle$ exhibits power-law behavior at large M as given by

$$\langle G'_q\rangle \propto M^{-\tau_q} \quad . \tag{7}$$

The values of mass exponents τ_q in the power-law behavior have been determined from the slopes of $\ln\langle G'_q\rangle$ vs $\ln M$ plots. The generalized dimensions $D_q=\tau_q/(q-1)$ were obtained from τ_q and the variation with q is shown in Fig. 2. In order to determine the degree of statistical contribution in the multiplicity fluctuations, we have adopted the procedure proposed by Chiu *et al.* [21]. For determining $\langle G'_q\rangle_{\text{stat}}$, the tracks in each event were distributed randomly; G'_q was evaluated for corresponding randomized events and averaged with respect to the experimental distribution. It is found in our data that $\langle G'_q\rangle_{\text{stat}}$ exhibits power-law behavior at large M with exponents τ_q^{stat} close to $q-1$. The variation of $\tau_q^{\text{stat}}/(q-1)=D_q^{\text{stat}}$ as a function of q is also shown in Fig. 2. The dynamical contributions to D_q^{stat} were obtained from

$$D_q^{\text{dyn}}=D_q-D_q^{\text{stat}}+1 \quad . \tag{8}$$

Table I lists the values of τ_q^{dyn} and D_q^{dyn}. The behavior of D_q^{dyn} vs q is also shown in Fig. 2. The multifractal structure of the self-similarity problem is evident from the decrease of D_q^{dyn} with increase in q. Figure 2 clearly illustrates the complementary nature of the intermittency and multifractal approaches. As can be seen, the anomalous fractal dimension d_q and the Renyi dimension D_q^{dyn} add up to unity.

We have also studied in the present interactions a simple fractal interpretation for the power-law dependence of $\langle F'_q\rangle$ and $\langle G'_q\rangle$ on M. First, we have considered the unnormalized weights $\bar{Q}_n(M,N)$ that a bin has n out of N particles in an event. Define

$$\bar{Q}_n(M,N)=\langle Q_n(M,N)\rangle_N \quad . \tag{9}$$

Here $Q_n(M,N)$ is the frequency of occurrence of multiplicity n over M bins in an event of multiplicity N, i.e., $\sum_{n=0}^{N} Q_n(M,N)=M$ and $\sum_{n=1}^{N} nQ_n(M,N)=N$. Furthermore, the angular brackets $\langle\cdots\rangle_N$ denote an average over all events with fixed multiplicity N. In terms of $Q_n(M,N)$, a summation over all bins in W is equivalent to that over the elements of the subset S_n containing n particles followed by summation over all n. Thus the F'_q and the G'_q moments can be expanded to give

$$F'_q=M^{q-1}\sum_{n=q}^{N}\frac{Q_n(M,N)N^{-q}n!}{(n-q)!} \tag{10a}$$

and

$$G'_q=\sum_{n=q}^{N} Q_n(M,N)(n/N)^q \quad . \tag{10b}$$

Putting $n=q+k$, $k=0,1\ldots$, we have,

$$F'_q=M^{q-1}\sum_{k=0}^{N-q} N^{-q}Q_{q+k}(M,N)\frac{(q+k)^q}{k!} \tag{11a}$$

and

$$G'_q=\sum_{k=0}^{N-q} N^{-q}Q_{q+k}(M,N)(q+k)^q \quad . \tag{11b}$$

Averaging over the resulting expansion of modified SFM's gives

$$\langle F'_q\rangle=M^{q-1}\sum_{k=0}^{\infty}\frac{B_{q,k}(M)(q+k)!}{k!} \quad , \tag{12a}$$

$$\langle G'_q\rangle=\sum_{k=0}^{\infty} B_{q,k}(M)(q+k)^q \quad . \tag{12b}$$

Here we have expanded moments in terms of the basic functions

$$B_{q,k}(M)=\langle N^{-q}Q_{q+k}(M,N)\rangle \quad . \tag{13}$$

The quantities $B_{q,k}(M)$ have been introduced by Hwa and Pan [13] to investigate their connection with the multifractal moments.

Figures 4(a)–4(c) show the behavior of $\bar{Q}_n(M,N)$ as a function of M for various values of n and for $N=4$, 5, and 6, respectively. As the resolution is increased indefinitely, it is expected that $\bar{Q}_0(M,N)\to M-N$, $\bar{Q}_1(M,N)\to N$, $\bar{Q}_n(M,N)\to 0$ for $n\geq 2$. These features are evident in Figs. 4(a)–4(c), which show that, for $M\gg N$, $\bar{Q}_1$ becomes independent of M and $\bar{Q}_2$, $\bar{Q}_3$, etc., drop to zero. For large M, $\bar{Q}_n(M,N)$ shows a power-law dependence on M:

$$\bar{Q}_n(M,N)\propto M^{\omega(n,N)} \quad . \tag{14}$$

The slope values are obtained by least-squares fits to the linear regimes of the double-logarithm plots in Figs. 4(a)–4(c). As expected, we have found $\omega(0,N)\simeq 1$, $\omega(1,N)\simeq 0$, and $\omega(n,N)<0$ for $n\geq 2$. The observed small deviation of $\omega(0,N)$ from 1 and $\omega(1,N)$ from 0 is due to the limited value of M (≤ 40) considered by us. Figure 5 shows the

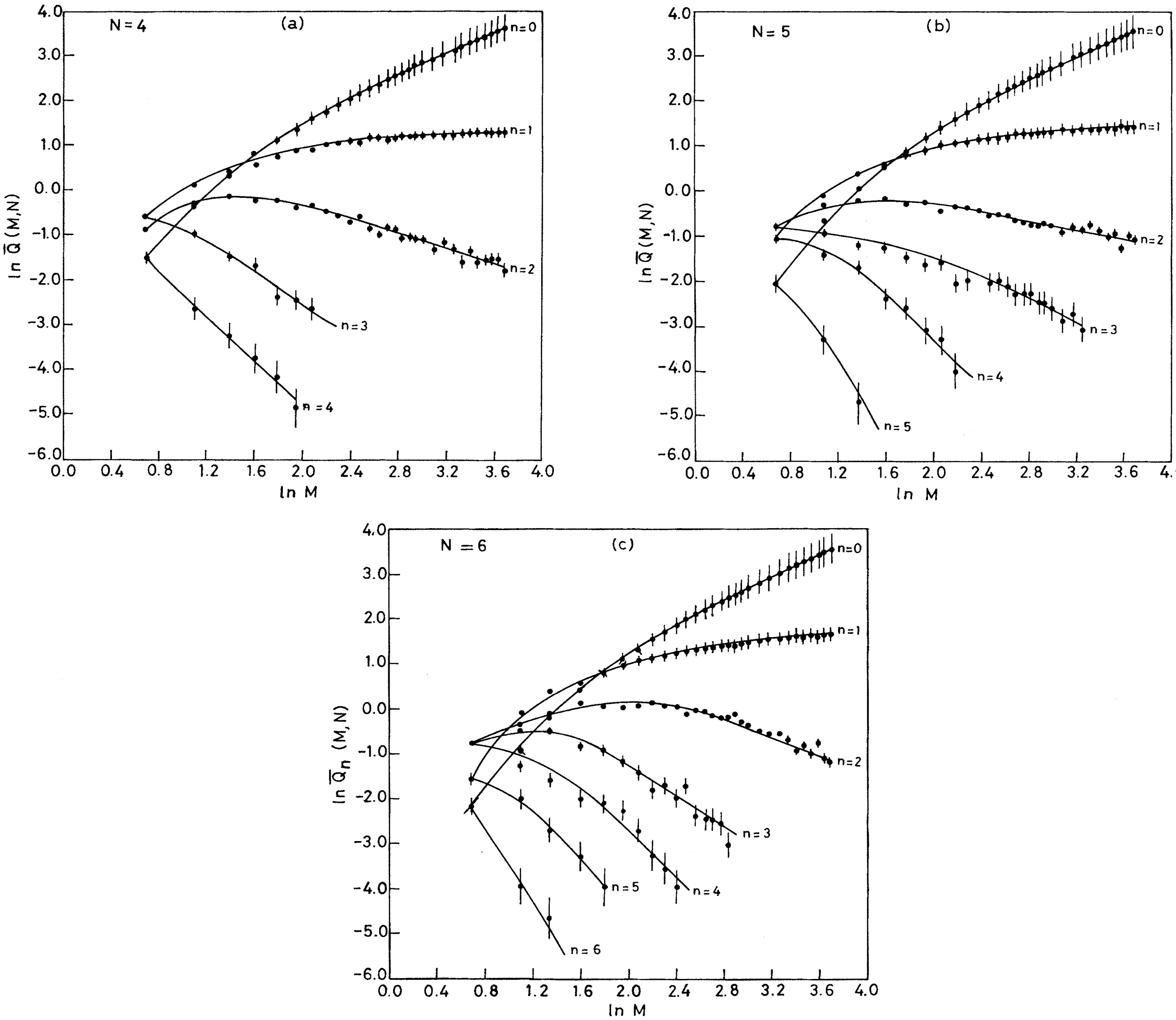

FIG. 4. (a)–(c) $\ln\bar{Q}(M,N)$ as a function of $\ln M$ for $N=4-6$, respectively. Curves are drawn to guide the eye, and least-squares fittings are for large M.

functional dependence of experimental values of $\omega(n,N)$ as a function of n for $N=4$, 5, and 6.

The fractal interpretation of $B_{q,k}(M)$ is evident from Figs. 6(a)–6(d), which illustrate the M dependences of $B_{q,k}(M)$ for $q=0$, 1, 2, and 3 and $k=0$, 1, 2, 3, and 4. Least-squares fitting of points in the linear region of the log-log plot yields

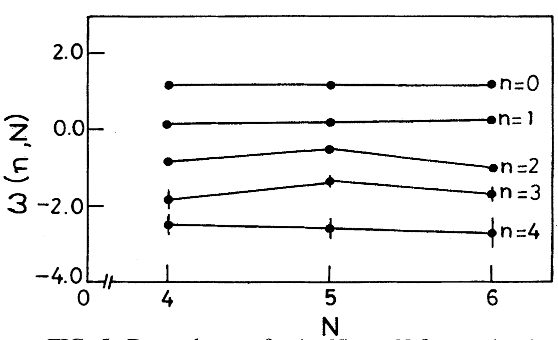

FIG. 5. Dependence of $\omega(n,N)$ on N for $n=1-4$.

the trajectory function $\lambda(q,k)$, which is a fractal dimension of the set

$$B_{q,k}(M) \propto M^{\lambda(q,k)} \quad . \tag{15}$$

The behavior of $B_{q,k}(M)$ for low values of q and k can be predicted as $\omega(0,N)=1$, which implies $\lambda(0,0)=1$, $\omega(1,N)=0$ implies $\lambda(0,1)=\lambda(1,0)=0$, and $\omega(n,N)<0$ for $n>1$ implies $\lambda(q,k)<0$ for $q+k>1$. The variation of $\lambda(q,k)$ with q for $k=0,1,\ldots,4$ is shown in Fig. 7(a). It is seen that $\lambda(q,k)$ is nearly constant for fixed $q+k\leq 3$. This can be seen more explicitly in Fig. 7(b), which illustrates the variation of $\lambda(q,k)$ with $q+k$. As is evident from Fig. 7(b), $\lambda(q,k)$ decreases with $q+k$, making in turn only the first few terms contribute significantly in the expansion of $\langle F_q' \rangle$. The first few terms of this expansion are

$$\langle F_q' \rangle = M^{q-1} B_{q,0}(M) q! \left(\frac{1+B_{q,1}(M)(q+1)+\cdots}{B_{q,0}(M)} \right) \quad . \tag{16}$$

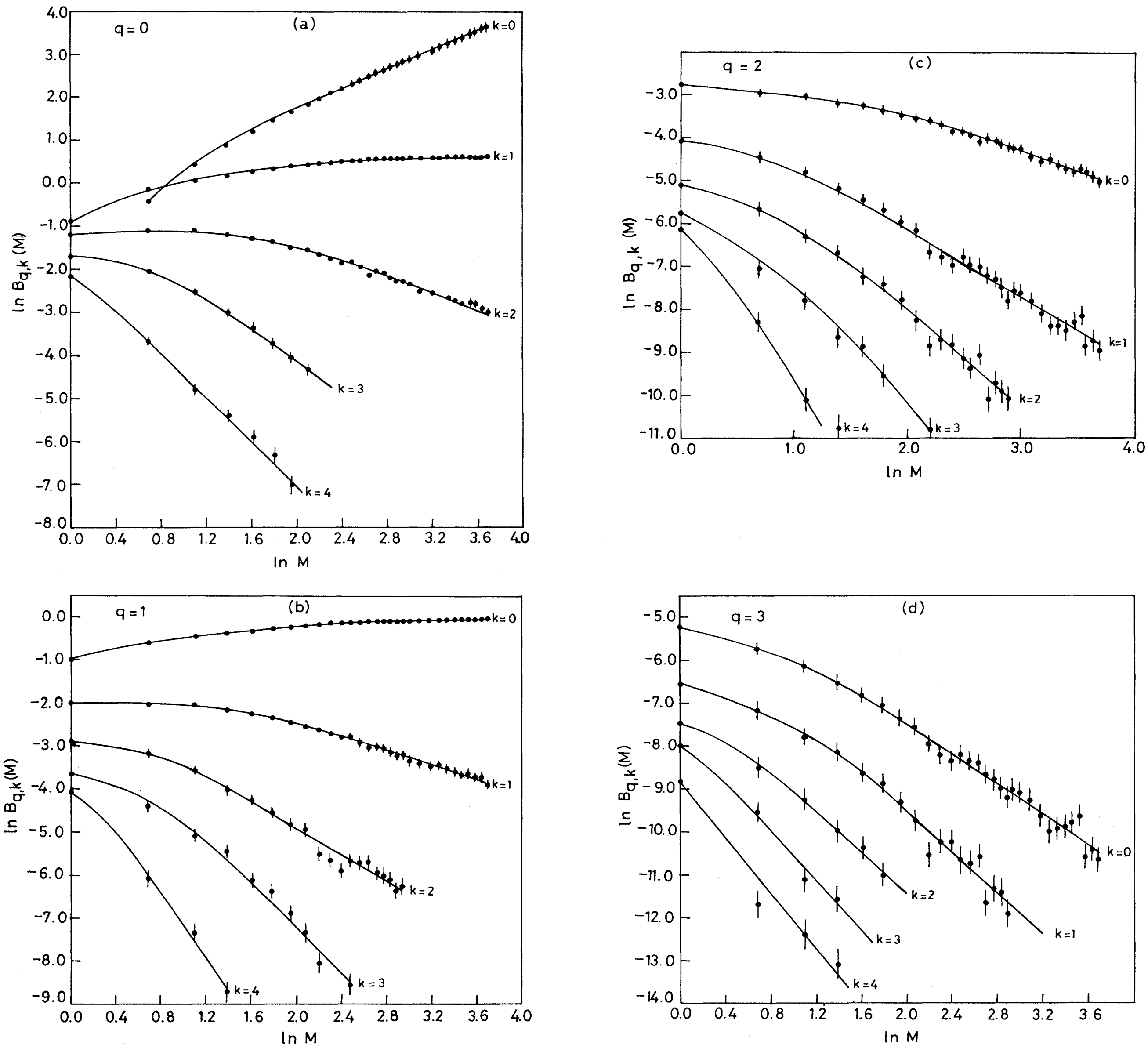

FIG. 6. (a)–(d) M dependence of $B_{q,k}(M)$ for $q=0-3$ and $k=0-4$. Best fits have been done for the linear regions, and the curves are drawn to guide the eye.

To a first approximation, we get

$$a_q \simeq q-1+\lambda(q,0) \ . \tag{17}$$

The experimental values of $\lambda(q,0)$ are given in Table I, and the above relation is well reproduced. Thus expansion in terms of the functions $B_{q,k}(M)$ provides a straightforward fractal interpretation of intermittency.

Expansion of the $\langle G_q' \rangle$ moments in terms of the basic functions $B_{q,k}(M)$ provides a fractal interpretation for multifractality. Table I lists the generalized dynamical dimensions calculated from $\lambda(q,0)$, and these are found to be in good agreement with the dimensions (also listed in Table I) obtained directly from the slopes of Fig. 3. Comparing the various values of different parameters listed in Table I, the dynamical mass exponents τ_q^{dyn} and intermittency indices a_q are seen to satisfy approximately the relation

$$a_q \simeq q-1-\tau_q^{\text{dyn}} \ . \tag{18}$$

Figure 8 illustrates the q dependences of a_q and $q-1-\tau_q^{\text{dyn}}$. The approximate equivalence of this relation arises because N is finite here. Thus intermittency and multifractal analyses are interconnected and complementary in character.

IV. CONCLUSIONS

Our analysis of intermittency and multifractality in the present interactions has led to the following conclusions.

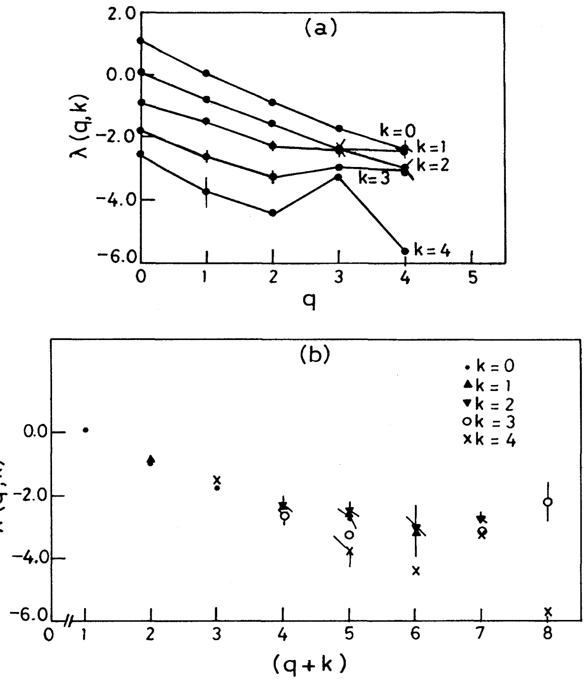

FIG. 7. (a) $\lambda(q,k)$ as a function of q. (b) $\lambda(q,k)$ as a function of $q+k$ for $q=0-4$ and $k=0-4$.

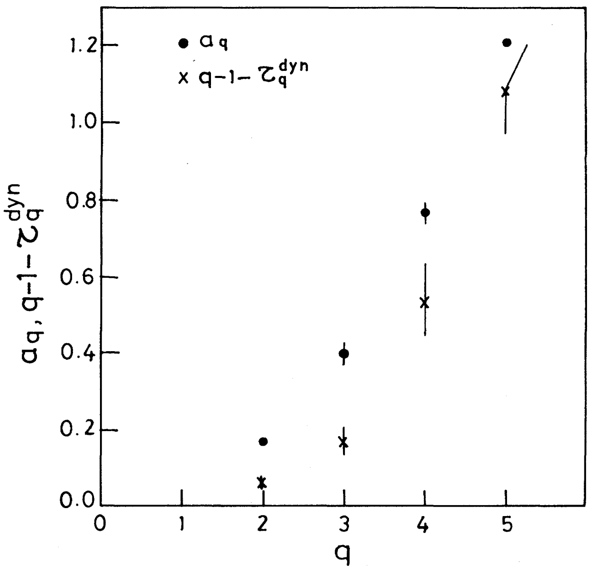

FIG. 8. Comparison of a_q and $q-1-\tau_q^{\rm dyn}$ as functions of order q.

The one-dimensional F_q and G_q' moments are found to exhibit power-law dependence on the bin size. The mass exponents τ_q characterizing the power-law behavior of G_q' moments have further been corrected for statistical contamination to yield $\tau_q^{\rm dyn}$. The observed q dependence of intermittency exponents Φ_q and the dynamical mass exponents $\tau_q^{\rm dyn}$ shows that our intermittency and multifractal analyses both support a self-similar cascading mechanism for particle production process.

The modified SFM F_q' and G_q' moments have been expanded in terms of a set of basic functions $B_{q,k}(M)$, each of which exhibits a power-law behavior at large M. The trajectory functions $\lambda(q,k)$ are found to behave linearly for small values of $q+k$. The first term in the expansion contributes to the fractal dimension D_q, which when corrected for statistical contamination is found to be in excellent agreement with Renyi dimensions obtained directly from the slopes of a $\ln\langle G_q'\rangle$ vs $\ln M$ plot.

Last, the relation between the fractal and intermittency analyses has been explored. The dynamical mass exponent $\tau_q^{\rm dyn}$ and the intermittency index a_q (which appears in scaling relation of F_q' moments) are found to be approximately related as $a_q=q-1-\tau_q^{\rm dyn}$. Consequently, for each q value, the dynamical Renyi dimensions $D_q^{\rm dyn}$ are complementary to the anomalous fractal dimensions d_q. In other words $d_q+D_q^{\rm dyn}=1$.

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