Regional regularities for the even-even nuclei: Medium to heavy systems

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By the concept of effective boson numbers, systematics of the $E_{2_1^+}$, $E_{4_1^+}$, $E_{6_1^+}$, $R=E_{4_1^+}/E_{2_1^+}$, $B(E2,0_1^+\rightarrow 2_1^+)$, and Q_0 for the even-even nuclei with $A\geqslant 70$ are presented. Some features for the excitation energies in the ground state band are observed and an intuitive explanation is given. Assuming correlation between the $E2$ and $M1$ transition strengths, the general trend of $B(M1)$ for the even-even actinide nuclei is predicted. The results in this paper are also compared with the previous results in the $N_\pi N_\nu$ scheme and other parametrizations.

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I. INTRODUCTION

With the development of experimental techniques, more and more nuclear data are accumulated and compiled [1,2]. Based on these data, systematics of various properties for even-even nuclei, e.g., the energy spectra, electromagnetic transition strengths, deformation, etc., appeared in the past few years [3–9]. Casten *et al.* showed [3] that the $N_\pi N_\nu$ [product of boson number in the interacting boson model (IBM-2)] scheme provides a very useful aid in understanding the nuclear structure over extended regions by providing a more economical classification, since a reasonable estimate of the n-p interaction is directly related to this value. However, the counting scheme of valence nucleons for the even-even nuclei near the subshells is oversimplified in previous studies [3–13]. Therefore, it is of interest to study the systematics by the application of effective boson numbers with which one can simulate the effects of the partial shell closures. This paper tackles the systematics of the $A\geqslant 70$ even-even nuclei, and the results are analyzed by the concept of effective boson numbers.

The assessment of a systematic is mainly based on whether the chosen parameters (variables) are reasonable and whether any new regularities are found. So we explain how we obtain the values of effective boson numbers and argue its validity first. The next two sections present a series of figures which plot the systematics of the $E_{2_1^+}$, $E_{4_1^+}$, $E_{6_1^+}$, $R=E_{4_1^+}/E_{2_1^+}$, $B(E2)$, and Q_0. A simple physical mechanism for the general properties of low-lying states is given, predictions on the systematics of $B(M1)$ for the actinide nuclei are also included, and we summarize and discuss the results in Sec. V.

II. EFFECTIVE BOSON NUMBERS IN IBM-2

Casten and Frank's work [3,4] showed that although the simple $N_\pi N_\nu$ scheme gives simplification to some extent if the normal major shells are considered, a substantial improvement can be achieved after the subshell closures are taken into account. Because of the existence of the subshell closures [7–14] and the Pauli effect, which we will come to later, it is not appropriate to take N_π and N_ν simply as the numbers of valence proton (neutron) pairs which are counted as hole pairs beyond the middle of a major shell. So one usually resorts to the concept of effective boson numbers.

The ambiguities of the counting scheme for valence nucleons near the subshells attracted much attention in the past few years. For example, it was shown that the effects of $Z=64$ subshell may be important for nuclei with $N\leqslant 88$ but disappear as $N\geqslant 90$ [3]. Microscopic calculations of N_π for the 50–82 major shell by Scholten [14] did yield $(N_\pi)_{\rm min}$ at $Z=64$ but $(N_\pi)_{\rm min}\approx 2.4$ instead of 0. Therefore, $Z=64$ should be treated as a subshell closure instead of a full shell closure. It was also suggested that the eradication of the subshell gap resulted from the strong n-p interaction between particles in spin-orbit partner orbits [10]. So many authors took an abrupt change in N_π at $N=88-90$. However, the shell-model single-particle level spacings are model space dependent. As a consequence, the disappearance of the $Z=64$ subshell could depend on N_ν in a smooth manner. In order to simulate the partially closure effects and the smooth neutron number dependence, Chuu, Han, and Hsieh performed the IBM-1 calculations for the Sm, Gd, Er, and Dy isotopes as testing examples [13]. In their calculations N_ν was counted as usual and N_π was regarded as an adjustable parameter. They found that the agreement between the theoretical results and the experimental data is satisfactory when partial closure effect is taken into account by a smooth variation of effective proton boson number. However, the $N=84$ isotones and the Nd isotopes where the $Z=64$ subshell closure play an important role were not calculated therein (because there exists low-lying mixed-symmetry states which the IBM-1 model cannot deal with, e.g., the 2_3^+ state), and an extensive calculation to simulate other subshells is not available.

By these discussions, we follow the idea of Ref. [10] and run the NPBOS code for all the nuclei with $A\geqslant 70$. The Hamiltonian is taken as

$$H=\sum_\rho \epsilon_\rho n_\rho+\kappa Q_\pi Q_\nu+M_{\pi\nu}, \tag{1}$$

where $n_\rho=d_\rho^\dagger d_\rho$, $\rho=\pi,\nu$,

$$Q_\rho=d_\rho^\dagger s_\rho+s_\rho^\dagger d_\rho+\chi(d_\rho^\dagger\tilde{d}_\rho)^{(2)},$$

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$$M_{\pi\nu}=\xi\left[(d_\nu^\dagger s_\pi^\dagger-s_\nu^\dagger d_\pi^\dagger)^{(2)}(s_\nu\tilde{d}_\pi-\tilde{d}_\nu s_\pi)^{(2)}-2\sum_{k=1,3}(d_\nu^\dagger d_\pi^\dagger)^{(k)}(\tilde{d}_\nu\tilde{d}_\pi)^{(k)}\right].$$

N_π for the nuclei near the $Z=40$ and 64 subshells (or N_ν for nuclei near $N=38$, 56, 96, 114 neutron subshells) is regarded as an adjustable integer parameter and N_ν (or N_π) is fixed as usual, and a unified electromagnetic transition operator is used for each isotope series. These effective boson numbers N_ν (or N_π) are determined by a careful calculation with the Hamiltonian equation (1) to get a best fit with the experimental data of energy spectra (and electromagnetic transition probabilities if the data are available). In other words, we chose the N_π (or N_ν) as the effective boson number when the N_π (or N_ν) yields the least overall root-mean-square deviation. The effective boson number N_π (N_ν) is obtained by these procedures.

We have also checked the sensitivity of the model Hamiltonian dependence of these effective boson numbers. Taking the $Z=64$ subshell as an example, in our calculation both the Hamiltonian equation (1) and the following Hamiltonian with more terms

$$H=\sum_{\rho=\pi,\nu}\epsilon_\rho n_\rho+\kappa Q_\pi Q_\nu+M_{\pi\nu}+\sum_{\rho=\pi,\nu}\sum_{l=0,2,4}C_\rho^{(l)}(d_\rho^\dagger d_\rho^\dagger)^{(l)}(\tilde{d}_\rho\tilde{d}_\rho)^{(l)} \tag{2}$$

are used to check whether the N_π depends on the particular form of Hamiltonian for Nd and Sm isotopes, and it is found that the effective boson number N_π obtained from the best fit procedures is the same. Table I presents the overall root-mean-square deviations (RMSD) of the energy spectra for some of the Nd isotopes. The RMSD-I and RMSD-II are the root-mean-square deviations by the Hamiltonian of Eqs. (1) and (2), respectively. By the RMSD in Table I, one can see that the fit is dependent on the boson number if the deformation of the nucleus is not large.

The saturation phenomenon of effective boson numbers N_π (N_ν), which was stated in our previous studies [6], is introduced phenomenologically in this paper which means the effective boson number dependence turns out to be weaker after a large critical value, i.e., the further increase of the N_π would not improve the fit between the experimental data and the calculated results significantly. As an example,

TABLE I. The RMSD of the Nd isotopes by the different Hamiltonian and effective boson numbers.

Nuclei				^{144}Nd	
N_π	1	2	3	4	5
RMSD−I	0.1025	0.0937	0.1039	0.1116	0.1335
RMSD−II	0.0906	0.0782	0.0891	0.0989	0.1083
Nuclei				^{146}Nd	
N_π	1	2	3	4	5
RMSD−I	0.0941	0.0849	0.0963	0.1111	0.1272
RMSD−II	0.0895	0.0721	0.0876	0.0957	0.1208
Nuclei				^{148}Nd	
N_π	1	2	3	4	5
RMSD−I	0.1305	0.1069	0.0971	0.1088	0.1307
RMSD−II	0.1104	0.0930	0.0749	0.0828	0.1033
Nuclei				^{150}Nd	
N_π	1	2	3	4	5
RMSD−I	0.1747	0.1358	0.1095	0.0848	0.0981
RMSD−II	0.1132	0.1045	0.0711	0.0520	0.0656
Nuclei				^{152}Nd	
N_π	1	2	3	4	5
RMSD−I	0.1508	0.1372	0.1086	0.0947	0.0820
RMSD−II	0.1219	0.1160	0.0834	0.0751	0.0678

TABLE II. The RMSD of the ^{158}Dy by the Hamiltonian Eq. (1). When the $N_\pi>6$, the fit depends on the effective boson numbers weakly.

N_π	3	4	5	6	7	8
RMSD−I	0.1242	0.1050	0.0942	0.0724	0.0747	0.0756

TABLE III. The effective boson numbers of nuclei near the $Z=64$ subshell.

Nuclei	^{142}Ce	^{144}Ce	^{146}Ce	^{148}Ce	^{150}Ce
N_π (this work)	3	3	3	4	4
N_π (Ref. [4])	3	3	3	?	4
Nuclei	^{144}Nd	^{146}Nd	^{148}Nd	^{150}Nd	^{152}Nd
N_π (this work)	2	2	3	4	5
N_π (Ref. [4])	2	2	2	?	5
Nuclei	^{146}Sm	^{148}Sm	^{150}Sm	^{152}Sm	^{154}Sm
N_π (this work)	2	2	3	5	6
N_π (Ref. [4])	1	1	1	?	6
N_π (Ref. [13])		4	5	6	6
Nuclei	^{148}Gd	^{150}Gd	^{152}Gd	^{154}Gd	^{156}Ge
N_π (this work)	2	3	4	5	6
N_π (Ref. [13])		2	5	7	7
Nuclei	^{150}Dy	^{152}Dy	^{154}Dy	^{156}Dy	^{158}Dy
N_π (this work)	2	3	5	6	6
N_π (Ref. [4])	8	8	8	8	8
N_π (Ref. [13])	4	5	7	8	8
Nuclei	^{152}Er	^{154}Er	^{156}Er	^{158}Er	^{160}Er
N_π (this work)	3	4	5	5	6
N_π (Ref. [4])	7	7	7	7	7
N_π (Ref. [13])		4	5	7	7
Nuclei	^{154}Yb	^{156}Yb	^{158}Yb	^{160}Yb	^{162}Yb
N_π (this work)	5	5	5	6	6
N_π (Ref. [4])	6	6	6	6	6

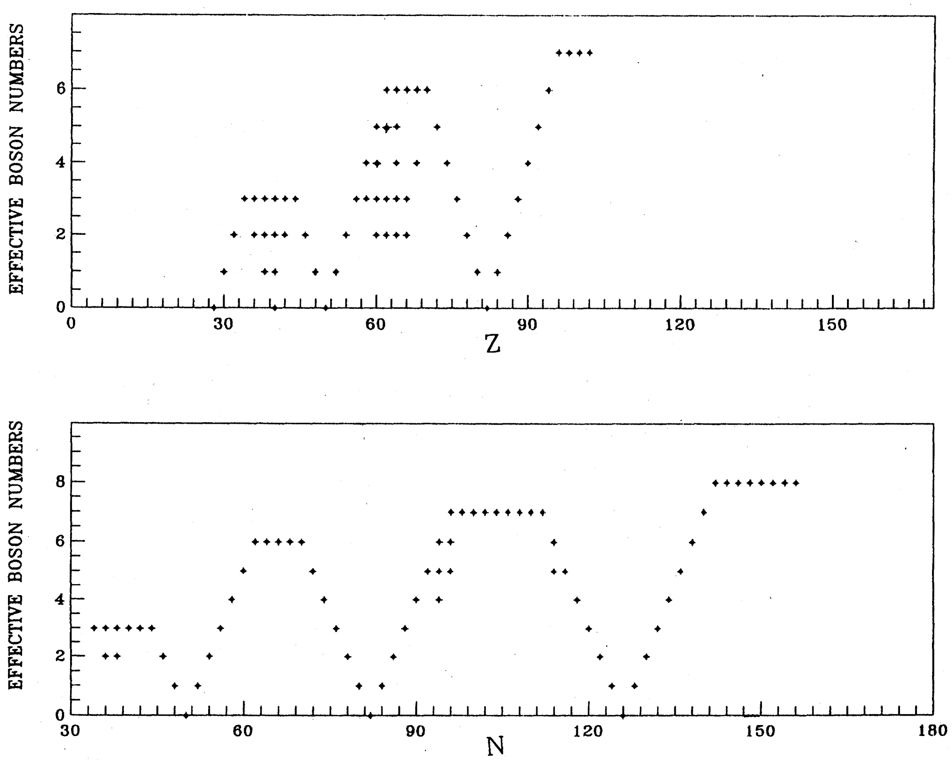

FIG. 1. Effective boson numbers for even-even nuclei of different proton and neutron numbers. N represents neutron number of nuclei, and Z the proton number of nuclei.

some of the RMSD-I of the ^{158}Dy by the different N_π are presented in Table II.

By Fig. 1, it is easy to see that when the number of valence nucleons outside a core (major shell closure) is small, N_π and N_ν are exactly the same as the valence nucleon pair numbers. The values of N_π are quite ambiguous near the so-called subshell $Z=40$ and 64 region, which is the very reason that the concept of effective boson numbers originates from the study of anomaly of properties for nuclei in these two regions. Besides these two regions, the small subshells of neutron [15], $N=38$, 56, 96, and 114, are also reproduced (the effect of the neutron subshell closure is quite small compared with $Z=40$ and 64 subshell).

As an example, effective proton boson numbers for nuclei near the $Z=64$ subshell are listed and compared with the previous results in Table III. Table IV presents the effective boson numbers for the $Z=40$ subshell and the small neutron subshells. The saturation values of the different major shells are presented in Fig. 1. For the nucleus not included in Tables III and IV, the effective boson number is taken as half of the valence "particle" ("hole") number before it reaches the saturation (maxima) value of the corresponding major shell, and the "saturation" value if the valence nucleon number is more than two times of the corresponding saturation boson number in Fig. 1. Figures 2–6 in this paper are plotted against the $N_\pi N_\nu$ obtained this way.

Now let us come to the reason for the saturation of effective boson number. What causes the saturation? We suggest that it is due to the Pauli effect, which is left out in the IBM. According to the fermion dynamical symmetry model [16] (referred to as the FDSM from now on), when the Pauli effect is ignored, the FDSM is reduced to the IBM. Because well-deformed nuclei have many valence nucleons outside the inert core, it is unreasonable to ignore the Pauli effect. According to the FDSM, the boson number in the IBM

TABLE IV. The effective boson numbers for nuclei near the $Z=40$ and $N=38$, 56, 96, and 114 subshells.

Nuclei	^{74}Kr	^{76}Kr	^{78}Kr	^{80}Kr	^{82}Kr	^{84}Kr	^{88}Kr
N_π	2	2	2	1	1	2	2
N_ν	2	2	2	3	2	1	1
Nuclei	^{90}Kr	^{92}Kr	^{80}Sr	^{82}Sr	^{84}Sr	^{86}Sr	^{90}Sr
N_π	2	1	2	1	1	1	1
N_ν	2	2	3	3	2	1	1
Nuclei	^{72}Se	^{92}Sr	^{94}Sr	^{96}Sr	^{98}Sr	^{100}Sr	^{82}Zr
N_π	1	1	1	1	3	3	2
N_ν	2	2	1	2	5	6	4
Nuclei	^{84}Zr	^{86}Zr	^{88}Zr	^{92}Zr	^{94}Zr	^{96}Zr	^{98}Zr
N_π	2	2	1	1	1	0	1
N_ν	3	1	1	1	2	3	4
Nuclei	^{100}Zr	^{102}Zr	^{88}Mo	^{90}Mo	^{94}Mo	^{96}Mo	^{98}Mo
N_π	3	3	1	2	2	2	2
N_ν	5	6	2	2	2	2	2
Nuclei	^{100}Mo	^{164}Yb	^{166}Yb	^{166}Hf	^{168}Hf	^{168}W	^{170}W
N_π	2	6	6	5	5	4	4
N_ν	2	5	6	1	3	1	3

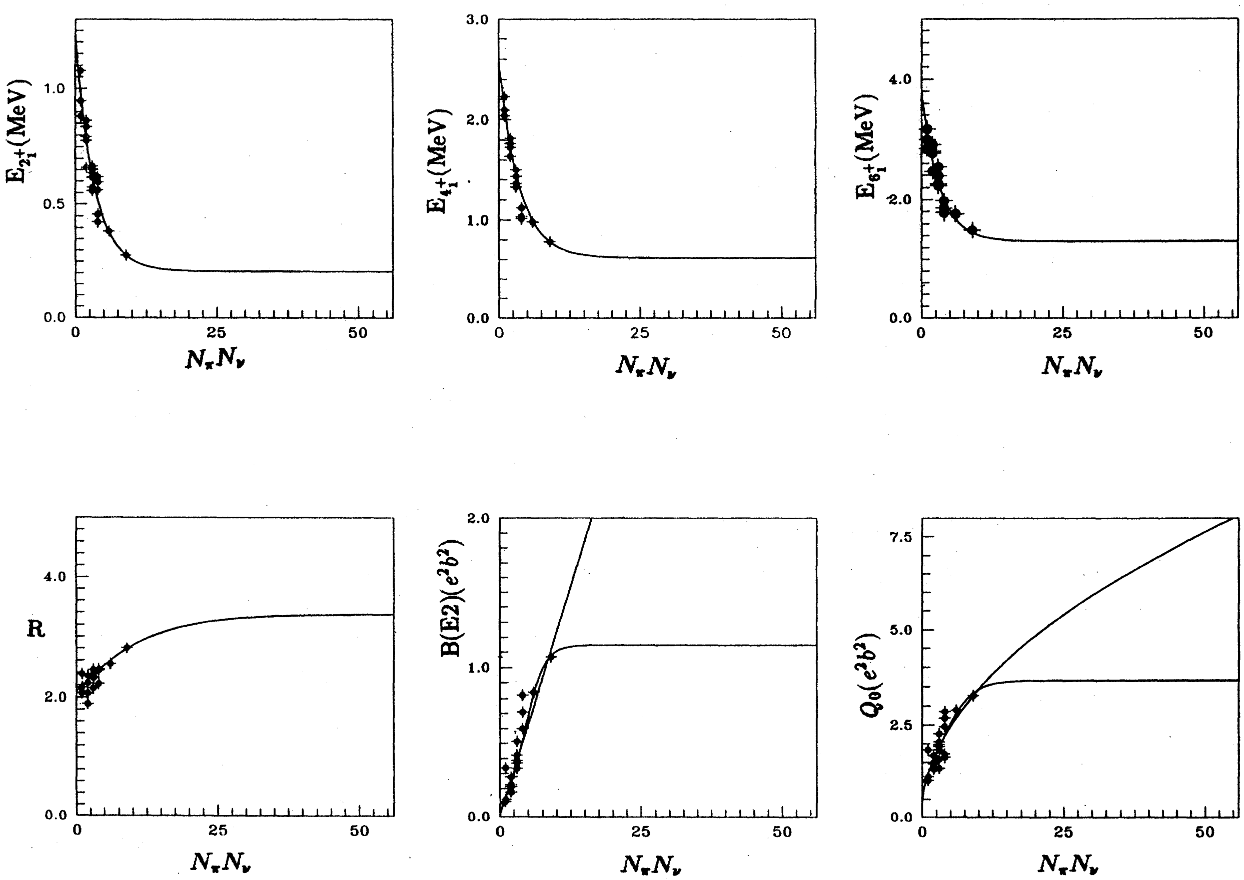

FIG. 2. $E_{2_1^+}$, $E_{4_1^+}$, and $E_{6_1^+}$ are fitted by $A[1+Be^{-N_\pi N_\nu/C}]$ (in MeV). $R=E_{4_1^+}/E_{2_1^+}$ is fitted by $A/(1+Be^{-CN_\pi N_\nu})$. $B(E2)$ is fitted with $a+bN_\pi N_\nu$ (in e^2 b^2, curve I) and $A/(1+Be^{-CN_\pi N_\nu})$ (in e^2 b^2, curve II). Q_0 is fitted by $[a+bN_\pi N_\nu]^{1/2}$ (in b, curve I) and $A/(1+Be^{-CN_\pi N_\nu})$ (in b, curve II). In this figure, $28<N$, $Z<50$, $A=0.21$, $B=4.98$, $C=3.34$, for $E_{2_1^+}$; $A=0.62$, $B=3.19$, $C=3.60$ for $E_{4_1^+}$; $A=1.30$, $B=1.94$, $C=3.15$ for $E_{6_1^+}$; $A=3.37$, $B=0.61$, $C=0.12$ for R; $a=0.02$, $b=1.22$; $A=1.15$, $B=12.2$, $C=0.58$ for $B(E2)$; $a=0.27$, $b=1.16$; $A=3.66$, $B=2.87$, $C=0.38$ for Q_0.

should be taken as the nucleon-pair number contributed to collectivity, and therefore it should be taken as the valence nucleon-pair number occupying the normal parity states instead of pair number of all valence nucleons [16], while the abnormal parity levels for both proton and neutron play the role of "siphoning" off the valence nucleons. So we get a saturation value for the effective boson numbers. According to the FDSM, there exists a maximum of fermion-pair numbers in a major shell, which can be easily determined as (we made an assumption that the abnormal parity level of well deformed even-even nuclei lies in the middle of the major shell for sake of simplicity, and there is a little difference of the maximum if we consider it more carefully) 5 (6) in the 50–82 major shell, 7–8 (7) in the 82–126 major shell, and 10–11 (8) in the 126–184 major shell (the value in the parenthesis is what we can easily see from Fig. 1). Apparently, these two values are consistent. We also noticed the work by Nakata and Arima on microscopic foundation of IBM [17], in which the effective boson numbers (for neutron the 50–82 shell only) are quite similar to ours, and the results also show a saturation due to the Pauli blocking effect, with saturation value 5 for "hole" boson number and 7 for "particle" boson number, consistent with ours (6).

III. SYSTEMATICS OF LOW-LYING ENERGY SPECTRA

It is well known that although simple systematics have been obtained by a simple $N_\pi N_\nu$ scheme [3], improvements have been achieved by introducing effective boson numbers [4]. However, the counting scheme for N_π and N_ν was oversimplified in the previous systematics, so it is highly desirable to have a unified treatment of systematics for the $A\geqslant 70$ even-even nuclei if it is possible.

Figures 2–6 present the $E_{2_1^+}$, $E_{4_1^+}$, $E_{6_1^+}$, and $R=(E_{4_1^+}/E_{2_1^+})$ systematics in the new $N_\pi N_\nu$ scheme. They are plotted against the $N_\pi N_\nu$ obtained above. Note that all the $A\geqslant 70$ even-even nuclei, except those with proton (neutron, or both) full shell closures, are included in Figs. 2–6. Compared with the work of Casten and Frank, Figs. 2–6 are much simpler. Two properties are noticed here.

(a) The $E_{2_1^+}$ etc. saturate before $N_\pi N_\nu$ saturates in the rare-earth and actinide region, i.e., they saturate much faster than $B(E2)$ and $B(M1)$. [$B(E2)$ or $B(M1)$ saturation is

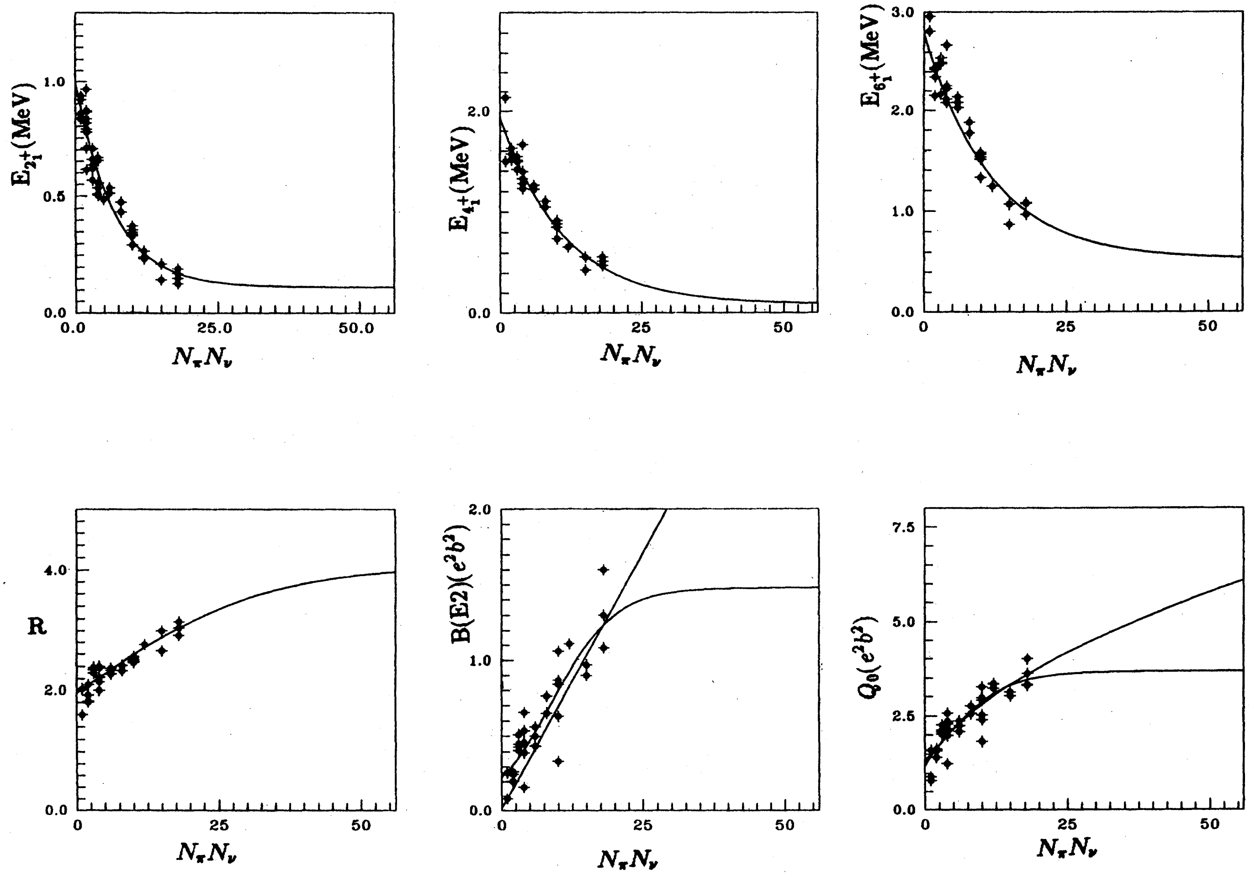

FIG. 3. The formulas are the same as Fig. 2. In this figure, $28<Z<50$, $50<N<82$, $A=0.11$, $B=7.83$, $C=6.68$ for $E_{2_1^+}$; $A=0.08$, $B=22.0$, $C=11.3$ for $E_{4_1^+}$; $A=0.53$, $B=4.31$, $C=11.5$ for $E_{6_1^+}$; $A=4.09$, $B=1.09$, $C=0.06$ for R; $a=0.02$, $b=0.07$; $A=1.48$, $B=5.37$, $C=0.18$ for $B(E2)$; $a=1.46$, $b=0.64$; $A=3.68$, $B=1.74$, $C=0.18$ for Q_0.

due to the N_π and N_ν saturation or the Pauli effect stated in the FDSM. Apparently, the physics of $E_{2_1^+}$ saturation is different from $B(E2)$ saturation.]

(b) The $E_{2_1^+}$, $E_{4_1^+}$, $E_{6_1^+}$, and R in Figs. 2–6 begin to saturate at almost the same $N_\pi N_\nu$ value (about 10–20).

Before we go on, one may ask why these two properties are not noticed before.

Let us first look at the $E_{2_1^+}$ systematics by Casten and Frank [3,4] for the even-even rare-earth nuclei. The systematics in their papers was separated into two cases: the ($N\leqslant 88$, $Z\leqslant 64$) region and the other region. The $E_{2_1^+}$ etc. decrease quite fast with the $N_\pi N_\nu$ and then saturate a function of $N_\pi N_\nu$ in the first case; and they saturate very "slowly" in the second case, almost as slow as the $B(E2)$ does. The existence of these two curves with different saturation "rates" of the rare-earth nuclei makes it impossible to compare with systematics in the actinide region. The difference of the systematics in the rare-earth region between ours and Casten and Frank's papers [3,4] *comes mainly from the* N_π *counting scheme for* $Z\geqslant 66$ *nuclei*. In this paper the effects of the $Z=64$ subshell are *also important for the* $Z\geqslant 66$ *nuclei* while Casten and Frank *neglected* it (Ref. [13] also suggested that the nuclei of $Z\geqslant 66$ are *affected by the* $Z=64$ *subshell substantially*; see Table III). So the $Z\geqslant 66$ line [3,4] of $E_{2_1^+}$, $E_{4_1^+}$, and $E_{6_1^+}$ in Casten and Frank's paper would be shifted to the left, and one would get the results similar to Figs. 2–6 in this paper. Besides, the systematics of Frank's paper does not include the $N=90$ isotones. We also notice misprint of data for the Nd isotopes in their figures.

Let J be the rotational inertia, β be the deformation parameter, and B be the inertia parameter. For well-deformed nuclei, we have $J=\alpha B\beta^2$ and $E_{2_1^+}=6h^2/J=\alpha'/(B\beta^2)$, where α and α' are constants. We know $B(E2)\propto\beta^2$ while $E_{2_1^+}$ depends on both β^2 and B, and B is not constant [21], so it is not surprising that $E_{2_1^+}$ does not saturate at the same $N_\pi N_\nu$ value as $B(E2)$ does. The same argument holds for $E_{2_1^+}$, $E_{4_1^+}$, $E_{6_1^+}$, and $R=(E_{4_1^+}/E_{2_1^+})$.

The $E_{2_1^+}$ etc. for the even-even rare-earth and actinide nuclei saturate at almost the same value of $(N_\pi N_\nu)$. This phenomenon can be understood intuitively if we assume that the $E_{2_1^+}$ etc. saturate when the n-p quadrupole force gradually dominates in the residual interaction.

Let V_{N-P} and V_{NN} be the total strength of the n-p interaction and identical nucleon pairing, respectively, and V_{np} and V_{nn} be the average strength of the n-p and identical nucleon pairing, respectively. Using the concept of effective boson number, we get

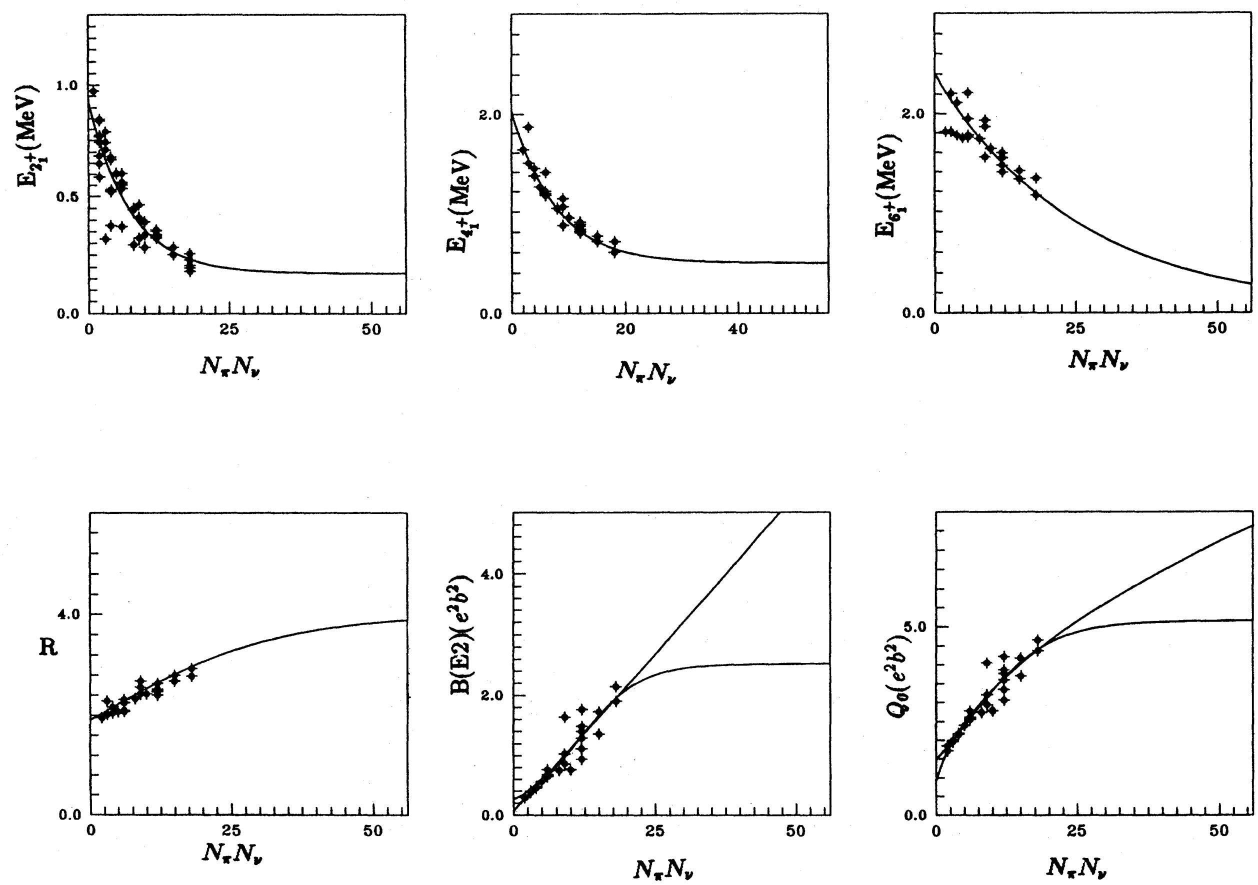

FIG. 4. The formulas are the same as Fig. 2. In this figure, $50<Z$, $N<82$, $A=0.17$, $B=4.36$, $C=7.14$ for $E_{2_1^+}$; $A=0.50$, $B=3.08$, $C=7.53$ for $E_{4_1^+}$; $A=0.05$, $B=52.1$, $C=24.4$ for $E_{6_1^+}$; $A=4.05$, $B=1.10$, $C=0.06$ for R; $a=0.07$, $b=0.10$, $A=2.52$, $B=8.64$, $C=0.19$ for $B(E2)$; $a=0.86$, $b=1.02$; $A=5.15$, $B=2.43$, $C=0.15$ for Q_0.

$$\frac{V_{N-P}}{V_{NN}}=\frac{2N_\pi N_\nu}{(N_\pi+N_\nu)}\left(\frac{V_{np}}{V_{nn}}\right).$$

By work of Jensen *et al.*, Wapstra *et al.*, and Brenner *et al.* [18], we can conclude that the V_{np}/V_{nn} value is almost the same constant for the above even-even nuclei [19]. As an example, one can take $V_{nn}\approx 1$ MeV, $V_{np}\approx 0.25$ MeV which are typical values in the rare-earth region. It is easy to get that the V_{N-P} begins to dominate in the residual interaction when the $(N_\pi N_\nu)\approx 16$ (taking $N_\pi=N_\nu$ for simplicity), and then stable deformation sets on, the lowest excitation states are rotational spectra. According to the simple assumption above, the $E_{2_1^+}$, $E_{4_1^+}$, $E_{6_1^+}$, and R begin to saturate consequently.

IV. SYSTEMATICS OF ELECTROMAGNETIC TRANSITION

Raman *et al.* reviewed and compiled the $B(E2)$ data for the even-even nuclei and predicted many $B(E2)$ values by various systematics. By the techniques of NRF, (p,p'), and (e,e') techniques, more and more $B(M1)$ data are also available now. Whether these accumulated data can be classified by simple empirical formulas is currently an interesting topic in the systematic studies [5,6,8,9,20]. It is of interest to present the systematics of $B(E2)$ and $B(M1)$ vs the product of the effective boson numbers $(N_\pi N_\nu)$.

Figures labeled with $B(E2)$ and Q_0 in Figs. 2–6 show that the $B(E2)$ and Q_0 (intrinsic electroquadrupole moment) change smoothly with the $(N_\pi N_\nu)$. We see that $B(E2)$ is nearly proportional to the $(N_\pi N_\nu)$ for the five regions, and Q_0 is approximately proportional to $(N_\pi N_\nu)^{1/2}$, which is consistent with the recent systematics [3] [Casten *et al.* (1993)]. It seems that there does not exist the saturation phenomenon for $B(E2)$ or Q_0 in the figures. In fact, the saturation data in Raman's paper are converged together at the $(N_\pi N_\nu)_{\max}$ in our Figs. 2–6. With the concept of boson numbers above, the strong saturation and correlation of $B(E2)$ and $B(M1)$ strength [8] for the even-even rare-earth and actinide nuclei is quite simple, because both the $B(E2)$ and $B(M1)$ increase with the $(N_\pi N_\nu)$, and they saturate when the $(N_\pi N_\nu)$ saturates.

The concept of effective boson numbers cannot only improve the systematics but also is useful to predict the unknown data. For example, the experimental data of $B(M1)$ for the even-even actinide nuclei are still scarce. We can use our systematics to estimate the saturation value and the saturation critical point of $B(M1)$. Under the SU(3) limit of IBM,

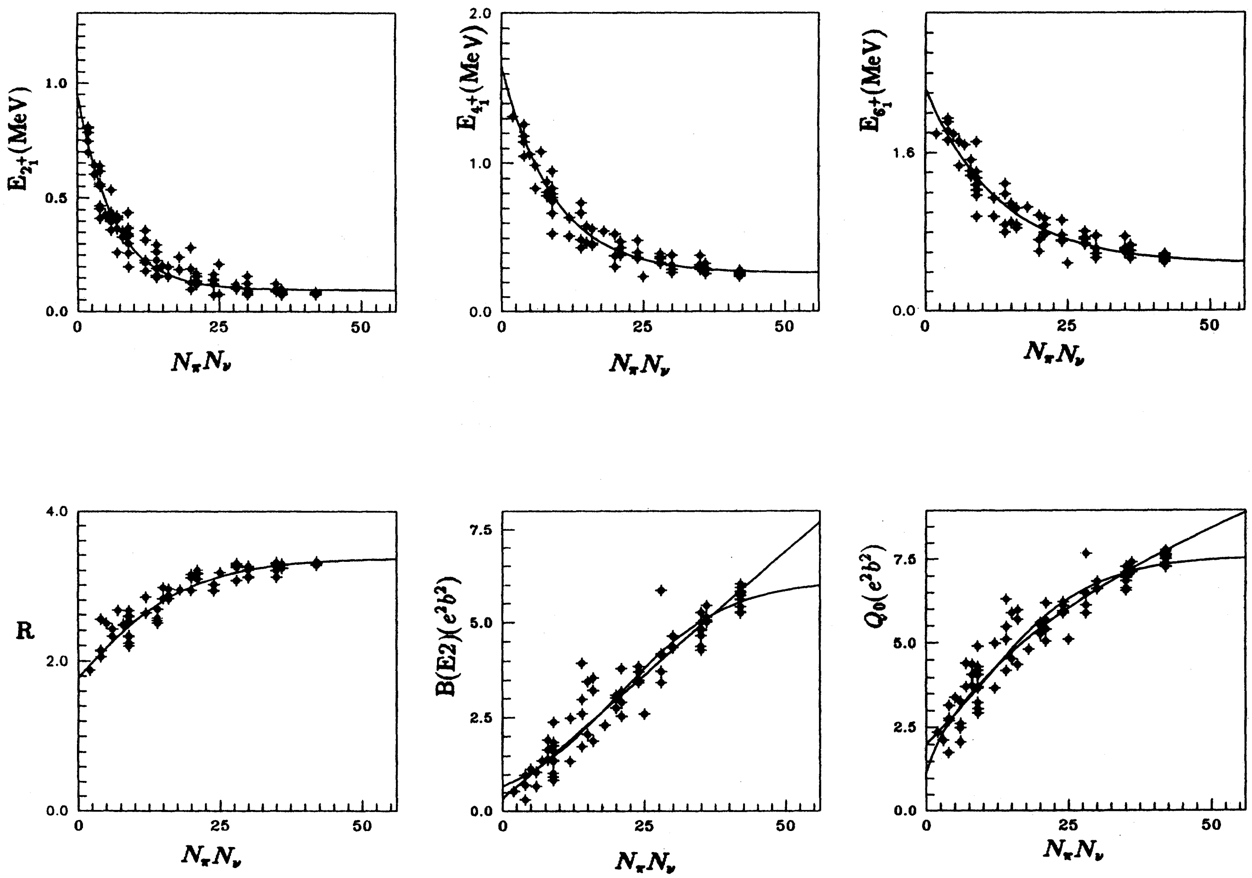

FIG. 5. The formulas are the same as Fig. 2. In this figure, $50<Z<82$, $82<N<126$, $A=0.11$, $B=7.83$, $C=6.68$ for $E_{2_1^+}$; $A=0.26$, $B=5.26$, $C=9.24$ for $E_{4_1^+}$; $A=0.48$, $B=3.72$, $C=12.8$ for $E_{6_1^+}$; $A=3.36$, $B=0.88$, $C=0.10$, for R; $a=0.35$, $b=0.13$; $A=6.14$, $B=7.93$, $C=0.10$ for $B(E2)$; $a=1.32$, $b=1.40$; $A=7.58$; $B=2.75$, $C=0.11$ for Q_0.

$$B(M1)=\frac{3g^2N_\pi N_\nu}{\pi(N_\pi+N_\nu)} ,$$

where $g=g_\pi-g_\nu$ is g factor, and $g^2\approx 0.85$ and 0.95 in the IBM-2 SU(3) limit for the rare-earth and actinide nuclei, respectively [13].[1] Substituting the saturation values in Fig. 1 for N_π and N_ν in Eq. (1), we get

$$\frac{B^{\rm sat}(M1,\text{actinide})}{B^{\rm sat}(M1,\text{rare-earth})}=\frac{g^2(\text{actinide})}{g^2(\text{rare-earth})}\,\frac{7\times 8}{7+8}\,\frac{6\times 7}{6+7} ,$$

taking $B^{\rm sat}(M1,\text{rare-earth})\approx 2.66\mu_N^2$, we get that $B^{\rm sat}(M1,\text{actinide})\approx 3.44\mu_N^2$, the saturation transitional point is

$$P=\frac{N_pN_n}{N_p+N_n}=\frac{2N_\pi^{\rm sat}N_\nu^{\rm sat}}{N_\pi^{\rm sat}N_\nu^{\rm sat}}=2\times 56/15=7.47,$$

[1]Some authors, e.g., Ref. [12], use $g^2(\text{actinide})/g^2(\text{rare-earth})$ a little larger than the value here; in Ref. [4], this ratio is even larger. So the predicted value of $B(M1)$ would be a little larger. On the other hand, there are also references where g is simply taken as 1, then the predicted $B(M1)$ would be a little smaller.

all consistent with the recent prediction [9].

V. DISCUSSION AND CONCLUSION

Systematics are obtained by introducing effective boson numbers; some difference of the general behavior with the previous systematics is noticed. A test of these results can be performed by other parametrizations. Recently, the parametrization V_{NP}, which is defined as $V_{NP}(N,Z)=\delta V_{np}(N,Z)N_nN_p$ and $\delta V_{np}(Z+1,N+1)=\frac{1}{4}\{[B(Z+2,N+2)-B(Z+2,N)]-[B(Z,N+2)-B(Z,N)]\}$, was used in the systematic study of energy spectra and $B(E2)$ for the even-even medium to heavy nuclei. Note that V_{NP} is regarded as an approximation of the total np interaction for the nucleus, and was extracted empirically without any ambiguities, and the subshell closures are unnecessary to be in particular consideration because the subshell effect is already included in the δV_{np} fluctuation [22]. As we know, the effective boson numbers are introduced to improve the proportionality of $(N_\pi N_\nu)$ with the general behavior of the np interaction, so the systematics of nuclear properties vs V_{NP} can be a qualitative test of the effective $(N_\pi N_\nu)$ scheme, and the results are quite consistent with the systematics in this paper. In the V_{NP} scheme, the energy spectra can be well described by one curve, and the correlation phenomenon of the excitation en-

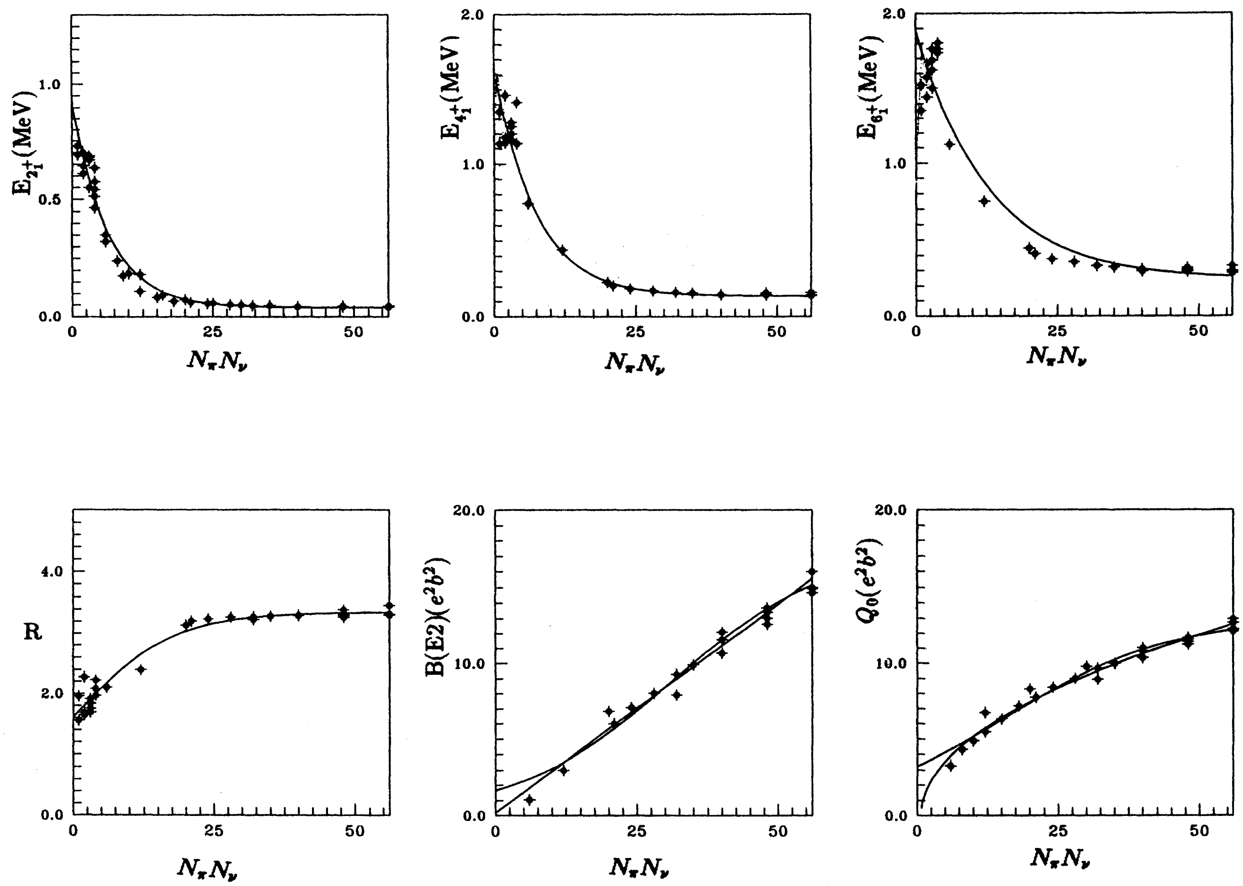

FIG. 6. The formulas are the same as Fig. 2. In this figure, $Z>82$, $N>126$, $A=0.04$, $B=22.7$, $C=6.30$ for $E_{2_1^+}$; $A=0.13$, $B=11.2$, $C=7.30$ for $E_{4_1^+}$; $A=0.24$, $B=6.76$, $C=12.6$ for $E_{6_1^+}$; $A=3.35$, $B=1.08$, $C=0.12$ for R; $a=0.18$, $b=0.27$; $A=17.6$, $B=9.68$, $C=0.07$ for $B(E2)$; $a=-2.02$, $b=2.88$, $A=13.0$, $B=3.03$, $C=0.07$ for Q_0.

ergy decreasing with the $(N_\pi N_\nu)$ in different major shells is verified therein [22], so it is consistent with the regularities found in the above $(N_\pi N_\nu)$ scheme.

To summarize, in this paper we present effective boson numbers of even-even nuclei near the subshells and midshells for the $A\geqslant 70$ even-even nuclei. By these effective boson numbers we tackle the systematics of $E_{2_1^+}$, $E_{4_1^+}$, $E_{6_1^+}$, R, $B(E2)$, and Q_0 in the $(N_\pi N_\nu)$ scheme. It is found that $E_{2_1^+}$, $E_{4_1^+}$, $E_{6_1^+}$ saturate almost at the same point of $(N_\pi N_\nu)$ in these regions, and we provide a plausible explanation by the competition between the pairing and the n-p quadrupole interaction. A simple systematics of $B(E2)$ is suggested in this scheme, and we predict the behavior of $B(M1)$ systematics for the even-even actinide nuclei. The systematics in this paper is also supported by the results in the V_{NP} scheme.

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