F-spin mixing and $M1$ properties of the low-lying states in the neutron-proton interacting boson model

J. Dobeš
Institute of Nuclear Physics, Czech Academy of Sciences, CS 250 68 Řež, Czech Republic
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The F-spin mixing and $M1$ properties of the low-lying bands in the neutron-proton interacting boson model (IBM-2) are studied. Application of the intrinsic state formalism in the leading order in $1/N$ and of the perturbative approach makes it possible to obtain analytical expressions in which a dependence on F-spin breaking terms is clearly exhibited. A simple explanation of many features observed in numerical studies is given. Comparison between the IBM-2 results and the treatment of $M1$ properties with the extended $M1$ operator in the IBM-1 approach is discussed.

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I. INTRODUCTION

An explicit recognition of neutron and proton degrees of freedom is deemed to be influential in an explanation of magnetic properties of even-even nuclei within the context of collective nuclear models [1]. Pertaining to the interacting boson model (IBM) [2], the neutron-proton IBM-2 version is thus employed in describing $M1$ data. In the IBM-2, the concept of F spin indicates a degree of neutron-proton symmetry. The low-lying states have a predominant component with the maximal value of F spin, $F_{\max}=\frac{1}{2}(N_\nu+N_\pi)$, where N_ν and N_π are the neutron and proton boson numbers, respectively. For obtaining nonzero $M1$ transitions rates between these states, F-spin breaking should be invoked with the $M1$ operator connecting components differing by the F-spin label. Therefore, the $M1$ transition rates represent important quantities to examine the F-spin mixing [3–6]. Recently, it has been argued that information on F-spin admixtures can also be extracted from analysis of magnetic moments [7–9].

In most investigations, the F-spin mixing is studied numerically by diagonalizing the IBM-2 Hamiltonian [4–6,8,10]. Such studies, being exact, might not always be quite transparent and systematic. Analytical expressions can be obtained utilizing the mean field (intrinsic state) formalism [11]. This has been applied to explore particular features of the F-spin mixing and $M1$ matrix elements in several studies [3,7,9,12]. In some of them, the F-spin mixing is treated nonperturbatively [7,12]. Because of the smallness of F-spin admixtures in the low-lying states, one can use the perturbative approach as well [3,9]. None of the mentioned analytical studies can, however, be viewed to be complete as concerns the F-spin mixing and $M1$ properties within and between the lowest-lying ground-state (g.s.), β, and γ bands.

In this paper, we shall also use the mean field formalism. The leading order terms in the $1/N$ expansion ($N=N_\nu+N_\pi$ is the total boson number) are only considered and the F-spin mixing is treated perturbatively. These simplifications enable us to obtain expressions in which an explicit dependence of the F-spin admixtures and $M1$ matrix elements on model parameters may easily be inspected. All the $M1$ matrix elements concerning the set of the g.s., β, and γ bands are discussed.

II. INTRINSIC STATES

We start from the F-spin symmetric ($F=F_{\max}$), axially symmetric ground-state band that is written in the intrinsic frame as

$$|\text{g.s.},F_{\max},K=0^+\rangle=(N_\pi!N_\nu!)^{-1/2}(\Gamma_\pi^\dagger)^{N_\pi}(\Gamma_\nu^\dagger)^{N_\nu}|0\rangle \quad , \tag{1}$$

where

$$\Gamma_\rho^\dagger=\frac{1}{\sqrt{1+\beta^2}}(s_\rho^\dagger+\beta d_{\rho 0}^\dagger),$$

with $\rho=\pi$ and ν for protons and neutrons, respectively, and β denoting the deformation parameter. On this ground state, one builds two Tamm-Dancoff excitations with $K=0^+$. These are combined to get the F-spin symmetric β band and F-spin mixed-symmetry ($F=F_{\max}-1$) $K=0^+$ band

$$|\beta,F_{\max},K=0^+\rangle=[N(1+\beta^2)]^{-1/2}[(\beta s_\pi^\dagger-d_{\pi 0}^\dagger)\Gamma_\pi+(\beta s_\nu^\dagger-d_{\nu 0}^\dagger)\Gamma_\nu]|\text{g.s.},K=0^+\rangle \quad , \tag{2}$$

$$|F_{\max}-1,K=0^+\rangle=[N(1+\beta^2)]^{-1/2}\left[\sqrt{\frac{N_\nu}{N_\pi}}(\beta s_\pi^\dagger-d_{\pi 0}^\dagger)\Gamma_\pi-\sqrt{\frac{N_\pi}{N_\nu}}(\beta s_\nu^\dagger-d_{\nu 0}^\dagger)\Gamma_\nu\right]|\text{g.s.},K=0^+\rangle \quad . \tag{3}$$

Similarly, the F-spin symmetric γ band and mixed-symmetry $K=2^+$ band are given by

$$|\gamma,F_{\max},K=2^+\rangle=(N)^{-1/2}[d^\dagger_{\pi 2}\Gamma_\pi+d^\dagger_{\nu 2}\Gamma_\nu]|\text{g.s.},K=0^+\rangle\ ,\tag{4}$$

$$|F_{\max}-1,K=2^+\rangle=(N)^{-1/2}\left[\sqrt{\frac{N_\nu}{N_\pi}}d^\dagger_{\pi 2}\Gamma_\pi-\sqrt{\frac{N_\pi}{N_\nu}}d^\dagger_{\nu 2}\Gamma_\nu\right]|\text{g.s.},K=0^+\rangle\ .\tag{5}$$

From the $K=1^+$ Tamm-Dancoff excitations, the mixed-symmetry one only is physically relevant:

$$|F_{\max}-1,K=1^+\rangle=(N)^{-1/2}\left[\sqrt{\frac{N_\nu}{N_\pi}}d^\dagger_{\pi 1}\Gamma_\pi-\sqrt{\frac{N_\pi}{N_\nu}}d^\dagger_{\nu 1}\Gamma_\nu\right]|\text{g.s.},K=0^+\rangle\ .\tag{6}$$

The symmetric $K=1^+$ combination is spurious, corresponding to the rotation of the ground state. Note that the set of intrinsic states (1)–(6) is orthonormal.

Intrinsic matrix elements are obtained by calculating the operator value between the angular momentum unprojected intrinsic states (1)–(6). From the intrinsic elements, one gets the matrix elements in the laboratory system after the angular momentum projection. In the lowest order, the Alaga rule then results [13]. That rule, however, might not be applicable for the $M1$ operator as it gives a zero result for many matrix elements (for example, between states of two $K=0^+$ bands or between states of $K=0^+$ and $K=2^+$ bands). We shall also need Hamiltonian elements between states of bands with different K. One should therefore consider higher order terms coming from the angular momentum projection.

Generally, the reduced matrix element of a tensor operator of rank λ in the laboratory frame is related to the intrinsic matrix elements by the standard textbook expression [13]

$$\langle J_1K_1\|\hat{O}_\lambda\|J_2K_2\rangle=[n(J_1,K_1)n(J_2,K_2)]^{-1/2}\frac{1}{2}\hat{J}_1\hat{J}_2^2\sum_\mu(J_2K_1-\mu\lambda\mu|J_1K_1)\int d\cos\vartheta\, d^{J_2}_{K_1-\mu K_2}(\vartheta)\times\langle K_1|\hat{O}_{\lambda\mu}\exp(-i\vartheta\hat{J}_y)|K_2\rangle,\tag{7}$$

with the normalization factor

$$n(J,K)=\frac{1}{2}\hat{J}^2\int d\cos\vartheta\, d^J_{KK}(\vartheta)\langle K|\exp(-i\vartheta\hat{J}_y)|K\rangle\ .\tag{8}$$

For the IBM intrinsic states, we shall evaluate these elements using techniques discussed in Refs. [14,15]. Namely, the fact that the integrals in Eqs. (7) and (8) are of the type

$$\int_0^\pi d\vartheta g(\vartheta)\left(\frac{1+\beta^2d_{00}^2(\vartheta)}{1+\beta^2}\right)^{N'}=\int_0^{\pi/2}d\vartheta(g(\vartheta)+g(\pi-\vartheta))\left(\frac{1+\beta^2d_{00}^2(\vartheta)}{1+\beta^2}\right)^{N'}$$

is utilized. Here, N' is close to the total boson number N assumed to be sufficiently large. The expression $[1+\beta^2d_{00}^2(\vartheta)]^{N'}$ is peaking at $\vartheta=0$ and one approximates

$$\left(\frac{1+\beta^2d_{00}^2(\vartheta)}{1+\beta^2}\right)^{N'}\approx\exp\left(-\frac{3}{2}\frac{\beta^2}{1+\beta^2}N'\vartheta^2\right)\ .$$

After expanding $g(\vartheta)+g(\pi-\vartheta)$ in powers of θ, the calculated matrix element is obtained as a series in the expansion parameter $1/N$ [or more strictly $(1+\beta^2)/\beta^2N$]. We retain only the first nonzero term from this expansion in the following calculations. Note that such an approach may not be correct for $\beta\to 0$.

III. F-SPIN MIXING MATRIX ELEMENTS

The general IBM-2 Hamiltonian does not conserve F spin. In the low-lying states, however, the symmteric $F=F_{\max}$ component prevails. The admixtures of the $F=F_{\max}-1$ states into symmetric states can be estimated perturbatively. We consider the IBM-2 Hamiltonian in the frequently used form

$$H=\epsilon_\pi d^\dagger_\pi\cdot\tilde{d}_\pi+\epsilon_\nu d^\dagger_\nu\cdot\tilde{d}_\nu+\kappa Q_\pi\cdot Q_\nu+M,\tag{9}$$

with the quadrupole operator

$$Q_\rho=(s^\dagger_\rho\tilde{d}_\rho+d^\dagger_\rho s_\rho)^{(2)}+\chi_\rho(d^\dagger_\rho\tilde{d}_\rho)^{(2)}\tag{10}$$

and with the Majorana force

$$M=\xi_2(d^\dagger_\nu s^\dagger_\pi-d^\dagger_\pi s^\dagger_\nu)^{(2)}\cdot(s_\pi\tilde{d}_\nu-s_\nu\tilde{d}_\pi)^{(2)}-2\sum_{k=1,3}\xi_k(d^\dagger_\nu d^\dagger_\pi)^{(k)}\cdot(\tilde{d}_\nu\tilde{d}_\pi)^{(k)}\ .$$

After slightly laborious but straightforward calculations in which rotations and angular momentum algebra in the F-spin space can be of help, one gets the following matrix elements of H between the $F=F_{\max}$ states (1), (2), and (4) and $F=F_{\max}-1$ states (3), (5), and (6):

$$\langle\text{g.s.},F_{\max},JK=0^+|H|F_{\max}-1,JK=0^+\rangle=\sqrt{\frac{N_\pi N_\nu}{N}}H_{g0}\ ,\tag{11a}$$

$$\langle \beta, F_{\max}, JK=0^+|H|F_{\max}-1, JK=0^+\rangle = \frac{\sqrt{N_\pi N_\nu}}{N} H_{\beta 0} \ , \tag{11b}$$

$$\langle \gamma, F_{\max}, JK=2^+|H|F_{\max}-1, JK=2^+\rangle = \frac{\sqrt{N_\pi N_\nu}}{N} H_{\gamma 2} \ , \tag{11c}$$

$$\langle \text{g.s.}, F_{\max}, JK=0^+|H|F_{\max}-1, JK=1^+\rangle = -\frac{\sqrt{N_\pi N_\nu}}{N^{3/2}} \sqrt{\frac{J(J+1)}{3}} \frac{\sqrt{1+\beta^2}}{\beta^2} H_{g1} \ , \tag{11d}$$

$$\langle \beta, F_{\max}, JK=0^+|H|F_{\max}-1, JK=1^+\rangle = -\frac{\sqrt{N_\pi N_\nu}}{N^2} \sqrt{\frac{J(J+1)}{3}} \frac{\sqrt{1+\beta^2}}{\beta^2} H_{\beta 1} \ , \tag{11e}$$

$$\langle \gamma, F_{\max}, JK=2^+|H|F_{\max}-1, JK=1^+\rangle = \frac{\sqrt{N_\pi N_\nu}}{N^2} \frac{\sqrt{(J-1)(J+2)}}{3} \frac{1+\beta^2}{\beta^2} H_{\gamma 1} \ . \tag{11f}$$

Here,

$$H_{g0} = \frac{\beta}{1+\beta^2}\left\{\Delta\epsilon + \beta N \Delta\chi + \frac{\kappa \Delta N}{1+\beta^2}\left[2-2\beta^2 + \sqrt{\frac{1}{14}}(\chi_\pi+\chi_\nu)\beta(\beta^2-3) + \tfrac{2}{7}\chi_\pi\chi_\nu\beta^2\right]\right\} \ , \tag{12a}$$

$$H_{\beta 0} = \frac{1}{1+\beta^2}\left\{\beta^2\Delta\epsilon - \beta N \Delta\chi + \frac{\kappa \Delta N}{1+\beta^2}\left[1+8\beta^2-\beta^4 + \sqrt{\frac{1}{14}}(\chi_\pi+\chi_\nu)\beta(1-7\beta^2) + \tfrac{2}{7}\chi_\pi\chi_\nu\beta^2(\beta^2-1)\right]\right\} \ , \tag{12b}$$

$$H_{\gamma 2} = \frac{1}{1+\beta^2}\left\{-\Delta\epsilon + 2\beta N \Delta\chi + \frac{\kappa \Delta N}{1+\beta^2}\left[3\beta^2-1 - \sqrt{\frac{8}{7}}(\chi_\pi+\chi_\nu)\beta(1+2\beta^2) + \tfrac{2}{7}\chi_\pi\chi_\nu\beta^4\right]\right\} \ , \tag{12c}$$

$$H_{g1} = \frac{\beta}{1+\beta^2}\left\{\Delta\epsilon + \beta N \Delta\chi + \frac{\kappa \Delta N}{1+\beta^2}\left[1-3\beta^2 + \sqrt{\frac{2}{7}}(\chi_\pi+\chi_\nu)\beta(\beta^2-1) + \tfrac{1}{14}\chi_\pi\chi_\nu\beta^2(3-\beta^2)\right]\right\} \ , \tag{12d}$$

$$H_{\beta 1} = \Delta\epsilon + \frac{\kappa \Delta N}{(1+\beta^2)^2}\left[1+2\beta^2-3\beta^4 + \sqrt{\frac{1}{14}}(\chi_\pi+\chi_\nu)\beta(\beta^4-4\beta^2-1) + \tfrac{2}{7}\chi_\pi\chi_\nu\beta^4\right] \ , \tag{12e}$$

$$H_{\gamma 1} = \frac{\beta}{1+\beta^2}\left\{3N\Delta\chi + \kappa\Delta N\left[\frac{3}{\sqrt{14}}(\chi_\pi+\chi_\nu) - \tfrac{3}{14}\chi_\pi\chi_\nu\beta\right]\right\} \ . \tag{12f}$$

We have introduced the notation in Eqs. (12)

$$\Delta\epsilon = \epsilon_\nu - \epsilon_\pi \ ,$$

$$\Delta\chi = \sqrt{\tfrac{1}{14}}\kappa(\chi_\pi - \chi_\nu) \ ,$$

$$\Delta N = N_\pi - N_\nu \ .$$

We will refer to the F-spin mixing induced by the terms proportional to $\Delta\epsilon$ and $\Delta\chi$ as $\Delta\epsilon$ and $\Delta\chi$ breakings, respectively. The terms proportional to ΔN are usually attributed as due to $\Delta\kappa$ breaking because these reflect breaking of the F-spin symmetry by departures from the F-spin conserving interaction $\kappa(Q_\pi+Q_\nu)\cdot(Q_\pi+Q_\nu)$ (for $\chi_\pi=\chi_\nu$).

IV. $M1$ MATRIX ELEMENTS

The $M1$ operator in the IBM-2 form is written as

$$T(M1) = \sqrt{\frac{3}{4\pi}}(g_\pi L_\pi + g_\nu L_\nu) \ , \tag{13}$$

where g_π and g_ν are the proton and neutron boson gyromagnetic factors, respectively, and L_π and L_ν are the angular momentum operators:

$$L_\rho = \sqrt{10}(d_\rho^\dagger \tilde{d}_\rho)^{(1)}, \quad \rho = \pi, \nu.$$

One transcribes the $T(M1)$ operator as a sum of the F-spin scalar and vector operators

$$T(M1) = T_s(M1) + T_v(M1) \ , \tag{14}$$

$$T_s(M1) = \sqrt{\frac{3}{4\pi}}\frac{1}{2}(g_\pi+g_\nu)(L_\pi+L_\nu) \ , \tag{14a}$$

$$T_v(M1) = \sqrt{\frac{3}{4\pi}}\frac{1}{22}(g_\pi-g_\nu)(L_\pi-L_\nu) \ . \tag{14b}$$

Using this decomposition and the F-spin SU(2) Wigner-Eckart theorem, one easily deduces that the reduced $M1$ matrix element between two F-spin symmetric states is related to the reduced matrix element of the total angular momentum operator $L=L_\pi+L_\nu$ by

$$\langle aF_{\max}\|T(M1)\|bF_{\max}\rangle=\sqrt{\frac{3}{4\pi}}\left(\frac{N_\pi}{N}g_\pi+\frac{N_\nu}{N}g_\nu\right)\times\langle aF_{\max}\|L\|bF_{\max}\rangle \ . \quad (15)$$

For the F-spin symmetric states in the angular momentum diagonal representation, there are therefore no $M1$ transitions and the gyromagnetic factors are constant:

$$g_{F_{\max}}=\frac{N_\pi}{N}g_\pi+\frac{N_\nu}{N}g_\nu \ . \quad (16)$$

Any departure of the gyromagnetic factor from the $g_{F_{\max}}$ value and/or the nonzero $M1$ transition develops within the IBM-2 context by components with $F<F_{\max}$. Of course, in the matrix element connecting the $F=F_{\max}$ and $F=F_{\max}-1$ states, the F-spin vector part of the $T(M1)$ operator only is effective.

Straightforward calculations give the matrix elements of the $T(M1)$ operator between the $F=F_{\max}$ states (1), (2), and (4) and $F=F_{\max}-1$ states (3), (5), and (6). One gets in the leading order

$$\langle \text{g.s.},F_{\max},JK=0^+\|T(M1)\|F_{\max}-1,JK=0^+\rangle=G\,\frac{1}{\beta}\,\hat{J}\sqrt{J(J+1)}\frac{1}{N} \ , \quad (17a)$$

$$\langle \beta,F_{\max},JK=0^+\|T(M1)\|F_{\max}-1,JK=0^+\rangle=G\,\frac{1+\beta^2}{\beta^2}\,\hat{J}\sqrt{J(J+1)}\frac{1}{N^{3/2}} \ , \quad (17b)$$

$$\langle \gamma,F_{\max},J_1K=2^+\|T(M1)\|F_{\max}-1,J_2K=2^+\rangle=-G\cdot 2\hat{J}_2(J_2 2 1 0|J_1 2)\frac{1}{\sqrt{N}} \ , \quad (17c)$$

$$\langle \text{g.s.},F_{\max},J_1K=0^+\|T(M1)\|F_{\max}-1,J_2K=1^+\rangle=-G\sqrt{6}\frac{\beta}{\sqrt{1+\beta^2}}\,\hat{J}_2(J_2 1 1 -1|J_1 0) \ , \quad (17d)$$

$$\langle \beta,F_{\max},J_1K=0^+\|T(M1)\|F_{\max}-1,J_2K=1^+\rangle=0+G\cdot O\left(\frac{1}{N^{3/2}}\right) \ , \quad (17e)$$

$$\langle \gamma,F_{\max},J_1K=2^+\|T(M1)\|F_{\max}-1,J_2K=1^+\rangle=G\sqrt{2}\hat{J}_2(J_2 1 1 1|J_1 2)\frac{1}{\sqrt{N}} \ , \quad (17f)$$

$$\langle \text{g.s.},F_{\max},J_1K=0^+\|T(M1)\|F_{\max}-1,J_2K=2^+\rangle=-G\sqrt{\frac{2}{3}}\frac{\sqrt{1+\beta^2}}{\beta}\,\hat{J}_2(J_2 1 1 -1|J_1 0)\sqrt{(J_2+2)(J_2-1)}\,\frac{1}{N} \ , \quad (17g)$$

$$\langle \beta,F_{\max},J_1K=0^+\|T(M1)\|F_{\max}-1,J_2K=2^+\rangle=-G\sqrt{\frac{2}{3}}\frac{\sqrt{1+\beta^2}}{\beta^2}\,\hat{J}_2(J_2 1 1 -1|J_1 0)\sqrt{(J_2+2)(J_2-1)}\,\frac{1}{N^{3/2}} \ , \quad (17h)$$

$$\langle \gamma,F_{\max},J_1K=2^+\|T(M1)\|F_{\max}-1,J_2K=0^+\rangle=0+G\cdot O\left(\frac{1}{N^{5/2}}\right) \ , \quad (17i)$$

where we have introduced

$$G=(g_\nu-g_\pi)\sqrt{\frac{3}{4\pi}}\sqrt{\frac{N_\pi N_\nu}{N}} \ .$$

V. ORTHOGONALIZATION OF THE ANGULAR MOMENTUM PROJECTED INTRINSIC STATES

The intrinsic states (1)–(6) form a set of orthonormalized states. This may not, however, be true when the angular momentum projected states are considered. Of course, in the leading order the overlap of different angular momentum projected states is zero, but the higher order terms can give a contribution that must be accounted for in the present study. Particularly, the following overlaps are relevant:

$$\langle \text{g.s.},F_{\max},JK=0^+|\beta,F_{\max},JK=0^+\rangle=\frac{1}{\beta}\frac{1}{\sqrt{N}} \ , \quad (18a)$$

$$\langle F_{\max}-1,JK=0^+|F_{\max}-1,JK=1^+\rangle=-\frac{\sqrt{1+\beta^2}}{\beta^2}\sqrt{\frac{J(J+1)}{3}}\frac{1}{N} \ , \quad (18b)$$

$$\langle F_{\max}-1,JK=2^{+}|F_{\max}-1,JK=1^{+}\rangle=\frac{1+\beta^2}{3\beta^2}\sqrt{(J+2)(J-1)}\frac{1}{N} \quad . \tag{18c}$$

The matrix elements of H and $T(M1)$ operators displayed in Secs. III and IV are not elements between an orthogonal set of states. To keep a relation to the true physical states, one has to orthogonalize the angular momentum projected state. For example, for the symmetric ground state band and β band states, we consider (again in the leading relevant order)

$$|\text{g.s.}_{\text{orth}},F_{\max},JK=0^{+}\rangle=|\text{g.s.},F_{\max},JK=0^{+}\rangle-\xi_{g\beta}\frac{1}{\beta}\frac{1}{\sqrt{N}}|\beta,F_{\max},JK=0^{+}\rangle \quad , \tag{19a}$$

$$|\beta_{\text{orth}},F_{\max},JK=0^{+}\rangle=|\beta,F_{\max},JK=0^{+}\rangle-\eta_{g\beta}\frac{1}{\beta}\frac{1}{\sqrt{N}}|\text{g.s.},F_{\max},JK=0^{+}\rangle \quad . \tag{19b}$$

The condition of orthogonality gives

$$\xi_{g\beta}+\eta_{g\beta}=1$$

and the actual values of the mixing parameters $\xi_{g\beta}$ and $\eta_{g\beta}$ are determined by the Hamiltonian H. Here, one can, however, argue that the deformation parameter β is chosen so that the projected ground-state wave functions do not contain any mixed components and we have $\xi_{g\beta}=0$ and $\eta_{g\beta}=1$.

Similarly, we write the orthogonalized mixed-symmetry states as

$$|\text{orth},F_{\max}-1,JK=0^{+}\rangle=|F_{\max}-1,JK=0^{+}\rangle+\xi_{01}\frac{\sqrt{1+\beta^2}}{\beta^2}\sqrt{\frac{J(J+1)}{3}}\frac{1}{N}|F_{\max}-1,JK=1^{+}\rangle \quad , \tag{20a}$$

$$|\text{orth},F_{\max}-1,JK=2^{+}\rangle=|F_{\max}-1,JK=2^{+}\rangle-\xi_{21}\frac{1+\beta^2}{3\beta^2}\sqrt{(J+2)(J-1)}\frac{1}{N}|F_{\max}-1,JK=1^{+}\rangle \quad , \tag{20b}$$

$$|\text{orth},F_{\max}-1,JK=1^{+}\rangle=|F_{\max}-1,JK=1^{+}\rangle+\eta_{01}\frac{\sqrt{1+\beta^2}}{\beta^2}\sqrt{\frac{J(J+1)}{3}}\frac{1}{N}|F_{\max}-1,JK=0^{+}\rangle$$
$$-\eta_{21}\frac{1+\beta^2}{3\beta^2}\sqrt{(J+2)(J-1)}\frac{1}{N}|F_{\max}-1,JK=2^{+}\rangle, \tag{20c}$$

with

$$\xi_{01}+\eta_{01}=1 \quad ,$$
$$\xi_{21}+\eta_{21}=1 \quad .$$

Again, the values of mixing parameters ξ and η follow from the actual Hamiltonian.

VI. $M1$ MATRIX ELEMENTS BETWEEN LOW-LYING BANDS

As has already been discussed above, the $M1$ transitions between the predominantly $F_{\max}$ symmetric states and the departure

$$\Delta g=g-g_{F_{\max}} \tag{21}$$

of g factors in these states from the $F_{\max}$ value should be accounted for by the presence of the mixed-symmetry $F_{\max}-1$ components in the wave functions. We may formally introduce the operator

$$T_p(M1)=T_v(M1)-P_{F_{\max}}T_v(M1)P_{F_{\max}}-P_{F_{\max}-1}T_v(M1)P_{F_{\max}-1} \quad , \tag{22}$$

where P denotes the projection operator on the respective space. The matrix elements of the $T_p(M1)$ operator are then directly related to the strengths of $M1$ transitions and to the Δg factors.

We estimate admixtures perturbatively. The amplitudes α_i in the wave function

$$|F_{\max}\rangle+\sum_i \alpha_i|(F_{\max}-1)_i\rangle$$

are given by

$$\alpha_i=\frac{1}{E_{F_{\max}}-E_{(F_{\max}-1)_i}}\langle F_{\max}|H|(F_{\max}-1)_i\rangle \quad ,$$

where E denotes energies of the respective states. Then the matrix element of the $T_p(M1)$ operator between the states labeled by a and b is

$$\langle a\|T_p(M1)\|b\rangle=\sum_i \alpha_i^b\langle aF_{\max}\|T(M1)\|(F_{\max}-1)_i\rangle+\sum_i \alpha_i^a\langle(F_{\max}-1)_i\|T(M1)\|bF_{\max}\rangle \quad . \tag{23}$$

In calculation of amplitudes of the $F_{\max}-1$ components in the predominantly $F_{\max}$ state and the subsequent estimate of the $M1$ matrix elements, the orthogonalized states (19) and (20) have to be, of course, used. By combining the expressions (11), (12), (17), (19), and (20), the results are easily obtained:

$$\langle \text{g.s.},JK=0^+\|T_p(M1)\|\text{g.s.},JK=0^+\rangle=G_1\frac{1}{\beta}\hat{J}\sqrt{J(J+1)}\frac{2}{N}\left[\frac{1}{E_{\text{g.s.}-E_{K=0^+}}}\eta_{01}H_{g0}+\frac{1}{E_{\text{g.s.}-E_{K=1^+}}}(H_{g1}-\eta_{01}H_{g0})\right] \quad , \tag{24a}$$

$$\begin{aligned}\langle \beta,JK=0^+\|T_p(M1)\|\beta,JK=0^+\rangle=G_1\frac{2}{\beta^2}\hat{J}\sqrt{J(J+1)}\frac{1}{N^2}&\left\{\frac{1}{E_\beta-E_{K=0^+}}(1+\beta^2-\eta_{01})\left(H_{\beta0}-\frac{1}{\beta}H_{g0}\right)+\frac{1}{E_\beta-E_{K=1^+}}\right.\\&\left.\times\left[-H_{\beta1}+\frac{1}{\beta}H_{g1}+\eta_{01}\left(H_{\beta0}-\frac{1}{\beta}H_{g0}\right)\right]\right\} \quad ,\end{aligned} \tag{24b}$$

$$\langle \gamma,J_1K=2^+\|T_p(M1)\|\gamma,J_2K=2^+\rangle=-4G_1\hat{J}_2(J_2210|J_12)\frac{1}{N}\frac{1}{E_\gamma-E_{K=2^+}}H_{\gamma2} \quad , \tag{24c}$$

$$\begin{aligned}\langle \text{g.s.},JK=0^+\|T_p(M1)\|\beta,JK=0^+\rangle=G_1\frac{1}{\beta^2}\hat{J}\sqrt{J(J+1)}\frac{1}{N^{3/2}}&\\ \times\left\{\frac{1}{E_{\text{g.s.}}-E_{K=0^+}}(1+\beta^2-\eta_{01})H_{g0}+\frac{1}{E_{\text{g.s.}}-E_{K=1^+}}(-H_{g1}+\eta_{01}H_{g0})\right.&\\ +\frac{1}{E_\beta-E_{K=0^+}}\beta\eta_{01}\left(H_{\beta0}-\frac{1}{\beta}H_{g0}\right)&\\ \left.+\frac{1}{E_\beta-E_{K=1^+}}\beta\left[H_{\beta1}-\frac{1}{\beta}H_{g1}-\eta_{01}\left(H_{\beta0}-\frac{1}{\beta}H_{g0}\right)\right]\right\} \quad ,&\end{aligned} \tag{24d}$$

$$\begin{aligned}\langle \text{g.s.},J_1K=0^+\|T_p(M1)\|\gamma,J_2K=2^+\rangle=G_1\frac{\sqrt{1+\beta^2}}{\beta^2}\hat{J}_1\sqrt{J_1(J_1+1)}(-)^{J_1-J_2}(J_1111|J_22)\sqrt{\frac{2}{3}}\frac{1}{N^{3/2}}&\\ \times\left[\frac{1}{E_{\text{g.s.}}-E_{K=0^+}}(1-\eta_{01})H_{g0}+\frac{1}{E_{\text{g.s.}}-E_{K=1^+}}(-H_{g1}+\eta_{01}H_{g0})\right.&\\ \left.+\frac{1}{E_\gamma-E_{K=2^+}}\beta\eta_{21}H_{\gamma2}+\frac{1}{E_\gamma-E_{K=1^+}}\beta(H_{\gamma1}-\eta_{21}H_{\gamma2})\right] \quad ,&\end{aligned} \tag{24e}$$

$$\begin{aligned}\langle \beta,J_1K=0^+\|T_p(M1)\|\gamma,J_2K=2^+\rangle=G_1\frac{\sqrt{1+\beta^2}}{\beta^2}\hat{J}_1\sqrt{J_1(J_1+1)}(-)^{J_1-J_2}(J_1111|J_22)\sqrt{\frac{2}{3}}\frac{1}{N^2}&\\ \times\left[\frac{1}{E_\beta-E_{K=0^+}}(1-\eta_{01})\left(H_{\beta0}-\frac{1}{\beta}H_{g0}\right)\right.&\\ +\frac{1}{E_\beta-E_{K=1^+}}\left(\eta_{01}H_{\beta0}-H_{\beta1}+\frac{1}{\beta}H_{g1}-\eta_{01}\frac{1}{\beta}H_{g0}\right)&\\ \left.+\frac{1}{E_\gamma-E_{K=2^+}}(1-\eta_{21})H_{\gamma2}+\frac{1}{E_\gamma-E_{K=1^+}}(-H_{\gamma1}+\eta_{21}H_{\gamma2})\right] \quad ,&\end{aligned} \tag{24f}$$

where we have defined

$$G_1=G\sqrt{\frac{N_\pi N_\nu}{N}} \ .$$

VII. DISCUSSION

Both the F-spin conserving and F-spin mixing parts of the IBM-2 Hamiltonian control the magnitude of the $T_p(M1)$ matrix elements. The F-spin mixing part renders matrix elements connecting the $F_{\rm max}$ and $F_{\rm max}-1$ spaces. On the other hand, the F-spin conserving part manages energy differences of the unperturbed states appearing in the mixing amplitudes. The F-spin conserving part determines also the mixing amplitudes η_{01} and η_{21} arising from the nonorthogonality of the angular momentum projected $F_{\rm max}-1$ states.

Particularly the dependence on the amplitudes η_{01} and η_{21} makes Eqs. (24) slightly complicated and not too transparent. The expressions are simplified when the equality of the energies of the $F_{\rm max}-1$ states is assumed, $E_{K=0^+}=E_{K=1^+}=E_{K=2^+}=E_{F_{\rm max}-1}$. Then the dependence on η_{01} and η_{21} disappears. In practice, this approximation means that the energy differences between the $K=0^+$, 1^+, and 2^+ mixed-symmetry states are smaller as compared to their difference from the g.s., β, and/or γ bands. Such a situation occurs quite frequently. We will call this simplification an approximation of equal $F_{\rm max}-1$ energies and will use it in the expressions below.

Another simplification employed in the following consists in neglecting the $\Delta\kappa$-breaking terms. These terms are proportional to the strength κ of the quadrupole-quadrupole interaction, that is, usually up to a few tens keV, and to the ΔN. On the other hand, microscopic estimates provide a value of $\Delta\epsilon$ of about hundreds keV and the value of $\Delta\chi$ is generally in the range of tens keV. The $\Delta\chi$ term, however, appears in combination with the total boson number N in the mixing matrix elements. As a result, $\Delta\kappa$ breaking is usually less important than $\Delta\epsilon$ and $\Delta\chi$ breakings. Actual calculations indeed confirm this conjecture [5,6].

A. g.s. band *g* factors

The g.s.$\rightarrow$g.s. $T_p(M1)$ matrix element is proportional to the element of the angular momentum operator. The Δg factor is therefore independent of the spin J in the ground-state band. Neglecting $\Delta\kappa$ breaking, one gets

$$\Delta g(\text{g.s.})=(g_\nu-g_\pi)\frac{2N_\nu N_\pi}{N^2}\left[\frac{1}{E_{\text{g.s.}}-E_{K=0^+}}\eta_{01}+\frac{1}{E_{\text{g.s.}}-E_{K=1^+}}(1-\eta_{01})\right]\frac{1}{1+\beta^2}(\Delta\epsilon+\beta N\Delta\chi) \ . \qquad (25)$$

Two mechanisms contribute to Δg(g.s.), namely, $\Delta K=0$ and $\Delta K=1$ mixings of the respective $K=0^+$ and $K=1^+$ mixed-symmetry states into the g.s. band. The mixing amplitude for the $\Delta K=1$ mixing is by factor of the order $1/N$ smaller than that for the $\Delta K=0$ mixing. On the other hand, the $T_p(M1)$ matrix element connecting the $F=F_{\rm max}-1$ and $F=F_{\rm max}$ states is by factor of the order N larger for the $K=1^+\rightarrow$g.s. transition than that for the the $K=0^+\rightarrow$g.s. transition.

As a result, the actual interplay of $\Delta K=0$ and $\Delta K=1$ mixings in Δg(g.s.) is controlled by the energy differences in denominators and by the amplitude η_{01}. With $\eta_{01}=1$ and $\Delta\chi=0$, the above expression agrees with that of Ref. [9].

In Refs. [8,9], the influence of $\Delta\epsilon$ breaking on the g factor $g(2_1^+)$ has been discussed in detail. Equation (25), however, suggests that also $\Delta\chi$ breaking may have an effect especially in cases of deformed nuclei with $\beta>1$ and with large boson numbers N.

B. β band *g* factors

The $\beta\rightarrow\beta$ $T_p(M1)$ matrix element is proportional to the element of the angular momentum operator. The Δg factor is again independent of the spin J in the β band and both the $\Delta K=0$ and 1 mixings contribute to Δg. In the approximation of equal $F_{\rm max}-1$ energies and with $\Delta\kappa$ breaking neglected, one gets

$$\Delta g(\beta)=(g_\nu-g_\pi)\frac{N_\nu N_\pi}{N^3}\frac{1}{E_\beta-E_{F_{\rm max}-1}}\frac{(-2)}{(1+\beta^2)\beta^2}\times[(1+\beta^2-\beta^4)\Delta\epsilon+\beta(1+2\beta^2)N\Delta\chi] \ . \qquad (26)$$

Here, for $\beta\approx1$, $\Delta\chi$ breaking is more important than $\Delta\epsilon$ breaking. Generally, however, $\Delta g(\beta)$ is suppressed by factor of the order $1/N$ as compared to Δg(g.s.). We do not expect it to be very pronounced except perhaps for small deformations β. The g factors of the β band should thus be close to the $g_{F_{\rm max}}$ value. Accurate knowledge of these factors would be useful in determining $g_{F_{\rm max}}$. Unfortunately, these data are experimentally very difficult to obtain.

C. γ band *g* factors and $\gamma\rightarrow\gamma$ transitions

The $\gamma\rightarrow\gamma$ $T_p(M1)$ matrix elements are the only ones among these discussed in the present section that obey the Alaga rule [$\mu=K_2-K_1$ in Eq. (7)]. Only the $\Delta K=0$ mixing of the $K=2^+$, $F=F_{\rm max}-1$ state into the γ band contributes in the leading order. From the Alaga rule, one gets the J dependence in the Δg factor. With $\Delta\kappa$ breaking neglected, we have

$$\Delta g(\gamma,J)=(g_\nu-g_\pi)\frac{N_\nu N_\pi}{N^2}\frac{1}{E_\gamma-E_{K=2^+}}\frac{8}{1+\beta^2}\times\frac{1}{J(J+1)}(\Delta\epsilon-2\beta N\Delta\chi). \tag{27}$$

The $\Delta g(\gamma)$ is of the same order in N as Δg(g.s.). Note that the sign by $\Delta\chi$ is reversed in $\Delta g(\gamma)$ in comparison with Δg(g.s.). This explains the differences in behavior of the g.s. band and γ band magnetic moments with the F-spin admixtures given by $\Delta\chi$ breaking as discussed in Ref. [7]. As concerns, however, the F-spin admixtures given by $\Delta\epsilon$ breaking, $\Delta g(\gamma)$ and Δg(g.s.) should have a similar dependence on $\Delta\epsilon$. Again, this feature is observed in numerical analysis [10].

The $\Delta g(\gamma)$ factor decreases sharply with increasing J. If accurate data on γ band magnetic moments would be available, the dependence on J could be helpful in separating $g_{F_{\max}}$ and Δg contributions to the total gyromagnetic ratio g. In this task also the intraband $\gamma\to\gamma M1$ matrix elements might be useful as

$$\langle\gamma,JK=2^+\|T(M1)\|\gamma,J+1K=2^+\rangle=-\sqrt{\frac{3}{16\pi}}J\sqrt{(J-1)(J+1)(J+3)}\Delta g(\gamma,J)\ . \tag{28}$$

D. $\beta\to$g.s. transitions

The interband $\beta\to$g.s. $M1$ matrix elements are by the order of $1/\sqrt{N}$ weaker than the intraband $\gamma\to\gamma$ ones. Their dependence on J is identical to the angular momentum operator matrix element in the state with spin J. F-spin mixing with $\Delta K=0$ and 1 both into the g.s. and β bands contributes to the $M1$ matrix element. In the approximation of equal $F_{\max}-1$ energies and with $\Delta\kappa$ breaking neglected, we have

$$\langle\text{g.s.},JK=0^+\|T(M1)\|\beta,JK=0^+\rangle=\sqrt{\frac{3}{4\pi}}(g_\nu-g_\pi)\frac{N_\nu N_\pi}{N^{5/2}}\hat{J}\sqrt{J(J+1)}\frac{1}{1+\beta^2}\times\left[\frac{1}{E_{\text{g.s.}}-E_{F_{\max}-1}}(\beta\Delta\epsilon+\beta^2N\Delta\chi)+\frac{1}{E_\beta-E_{F_{\max}-1}}(\beta\Delta\epsilon-N\Delta\chi)\right]\ . \tag{29}$$

Here, for $\beta\approx1$, we expect the contribution from $\Delta\epsilon$ breaking to be more important than that from $\Delta\chi$ breaking.

E. $\gamma\to$g.s. transitions

The interband $\gamma\to$g.s. $M1$ matrix elements are by the order of $1/\sqrt{N}$ weaker than the intraband $\gamma\to\gamma$ ones. This finding agrees of course with the discussion of Ref. [3] and qualitatively is confirmed by calculations [5]. Also the dependence on the initial and final spins J_2 and J_1 is identical to that of Ref. [3] (except the phase factor that is erroneously missing in Ref. [3]). There is, however, one noticeable difference in the present discussion. In Ref. [3], the spin dependence follows from consideration of the $\Delta K=1$ mixing of the $K=1^+$ band both into the g.s. and γ bands. Not surprisingly such a spin dependence is also provided by the $\Delta K=1$ mixing considered in the general rotational model frame [16]. In the present treatment, we include also the $\Delta K=0$ mixing of the $K=0^+$, $F=F_{\max}-1$ state into the g.s. band and of the $K=2^+$, $F=F_{\max}-1$ state into the γ band. The $\Delta K=0$ mixing gives a contribution of the same order as the $\Delta K=1$ one. The spin dependence is the same both in the $\Delta K=0$ and 1 cases. It is thus difficult to disentangle the two mixing mechanisms experimentally.

The different spin dependence is obtained with the $\Delta K=2$ mixing of the $K=2^+$, $F=F_{\max}-1$ state into the g.s. band and of the $K=0^+$, $F=F_{\max}-1$ state into the γ band. Again, an agreement with the general rotational formula of Ref. [16] is obtained. The contribution from the $\Delta K=2$ mixing to the $\gamma\to$g.s. $M1$ matrix elements is, however, by order of $1/N$ smaller than contributions from the $\Delta K=0$ and 1 mixings and we generally do not expect it to be very important. Experimental data indeed disfavor the $\Delta K=2$ mixing [17,18].

There might be, however, cases in which the influence of the $\Delta K=2$ mixing is magnified. This occurs when the respective mixed bands lie close to each other and the small energy difference gives preference to the mixing mechanism that is otherwise suppressed in the $1/N$ expansion. Such a situation is present in calculations of Ref. [9] for ^{154}Sm in which the mixed-symmetry $K=0^+$ band very close to the γ band is advocated. Then the $2^+_\gamma\to2^+_{\text{g.s.}}M1$ transition is influenced mainly by the $\Delta K=2$ mixing whereas the $3^+_\gamma\to2^+_{\text{g.s.}}M1$ transition goes through the $\Delta K=0$ and 1 mixings (there is no $3^+,K=0^+$ state to mix into the γ band). The $\Delta K=0$ and 1 mixing formula (24e) gives the ratio of the reduced $M1$ matrix element of the $2^+_\gamma\to2^+_{\text{g.s.}}$ transition to that of the $3^+_\gamma\to2^+_{\text{g.s.}}$ transition equal to $1/\sqrt{2}$ (phases here and in the following discussion are fixed by positive signs of the respective $E2$ matrix elements). On the other hand, calculations for ^{154}Sm with three sets of parameters denoted as a, b, and c in Ref. [9] give the respective reduced $M1$ matrix elements (in μ_N) 0.0575, 0.0272, and 0.0252 for the $2^+_\gamma\to2^+_{\text{g.s.}}$ transition and -0.0004, 0.0012, and 0.0019 for the $3^+_\gamma\to2^+_{\text{g.s.}}$ transition. The $\gamma\to$g.s. matrix elements thus represent sensitive quantities to decide whether the conjecture of Davis and Navrátil about the low-lying mixed-symmetry $K=0^+$ band is appropriate. There are no respective data available for ^{154}Sm.

In the approximation of equal $F_{\max}-1$ energies and with $\Delta\kappa$ breaking neglected, contributions due to the mixing of

$F=F_{\max}-1$ states into the g.s. band subtract in Eq. (24e) and only mixing into the γ band appears in the simplified expression

$$\langle \text{g.s.},J_1K=0^+\|T(M1)\|\gamma,J_2K=2^+\rangle = \sqrt{\frac{3}{4\pi}}(g_\nu-g_\pi)\frac{N_\nu N_\pi}{N^{5/2}}\hat{J}_1\sqrt{J_1(J_1+1)}(-)^{J_1-J_2} \times (J_1 1 1 1|J_2 2)\sqrt{\frac{6}{1+\beta^2}}\frac{1}{E_\gamma-E_{F_{\max}-1 g}}N\Delta\chi \ . \quad (30)$$

Here, the $\gamma\to$g.s. $M1$ matrix elements are not influenced by $\Delta\epsilon$ breaking as really confirmed in actual calculations [10].

F. $\gamma\to\beta$ transitions

For completeness, we present also the simplified expression for the $\gamma\to\beta$ $M1$ matrix elements. Such quantities are experimentally extremely difficult to obtain. In the approximation of equal $F_{\max}-1$ energies and with $\Delta\kappa$ breaking neglected, we have

$$\langle \beta,J_1K=0^+\|T(M1)\|\gamma,J_2K=2^+\rangle = \sqrt{\frac{3}{4\pi}}(g_\nu-g_\pi)\frac{N_\nu N_\pi}{N^3}\hat{J}_1\sqrt{J_1(J_1+1)}(-)^{J_1-J_2}(J_1 1 1 1|J_2 2) \times\sqrt{\frac{2}{3}}\frac{(-1)}{\beta^2\sqrt{1+\beta^2}}\left(\frac{1}{E_\beta-E_{F_{\max}-1}}+\frac{1}{E_\gamma-E_{F_{\max}-1}}\right)(\Delta\epsilon+\beta N\Delta\chi). \quad (31)$$

G. $E2/M1$ mixing ratios

Directly experimentally measurable quantities involving the $M1$ matrix elements are the $\delta(E2/M1)$ mixing ratios

$$\delta(E2/M1)=0.00835E_{\text{trans}}\Delta(E2/M1) \ ,$$

where the transition energy E_{trans} is given in units of MeV and the reduced mixing ratio Δ,

$$\Delta(E2/M1)=\frac{\langle J_1\|T(E2)\|J_2\rangle}{\langle J_1\|T(M1)\|J_2\rangle} \ ,$$

is calculated with $E2$ and $M1$ matrix elements in units of e fm^2 and μ_N, respectively. The $T(E2)$ operator in the IBA-2 is

$$T(E2)=e_\pi Q_\pi+e_\nu Q_\nu \ ,$$

where the quadrupole operators Q are given by Eq. (10). Here, the χ_ν and χ_ν values could in principle differ from values used in the Hamiltonian (9). Introducing the $F=F_{\max}$ projected values

$$e_{F_{\max}}=\frac{N_\pi}{N}e_\pi+\frac{N_\nu}{N}e_\nu \ ,$$

$$\chi_{F_{\max}}=\left(\frac{N_\pi}{N}e_\pi\chi_\pi+\frac{N_\nu}{N}e_\nu\chi_\nu\right)\Big/ e_{F_{\max}} \ ,$$

one gets the reduced $E2$ matrix elements in the leading order in $1/N$ as [19]

$$\langle \gamma,J_1K=2^+\|T(E2)\|\gamma,J_2K=2^+\rangle=\hat{J}_2(J_2 2 2 0|J_1 2)\frac{2\beta N}{1+\beta^2}e_{F_{\max}}\left(1-\beta\sqrt{\frac{1}{14}}\chi_{F_{\max}}\right) \ , \quad (32a)$$

$$\langle \text{g.s},J_1K=0^+\|T(E2)\|\beta,J_2K=0^+\rangle=\hat{J}_2(J_2 0 2 0|J_1 0)\frac{\sqrt{N}}{1+\beta^2}e_{F_{\max}}\left(\beta^2-1+\beta\sqrt{\frac{2}{7}}\chi_{F_{\max}}\right) \ , \quad (32b)$$

$$\langle \text{g.s},J_1K=0^+\|T(E2)\|\gamma,J_2K=2^+\rangle=\hat{J}_2(J_2 2 2-2|J_1 0)\sqrt{\frac{2N}{1+\beta^2}}e_{F_{\max}}\left(1+\beta\sqrt{\frac{2}{7}}\chi_{F_{\max}}\right) \ , \quad (32c)$$

$$\langle \beta,J_1K=0^+\|T(E2)\|\gamma,J_2K=2^+\rangle=\hat{J}_2(J_2 2 2-2|J_1 0)\sqrt{\frac{2}{1+\beta^2}}e_{F_{\max}}\left(\beta-\sqrt{\frac{2}{7}}\chi_{F_{\max}}\right) \ . \quad (32d)$$

With explicit formulas for the Clebsch-Gordan coefficients, the spin dependence of the reduced mixing ratios Δ for transitions within the set of the g.s, β, and γ bands is simply factorized as

$$\Delta(E2/M1,J_2\rightarrow J_1)=-\frac{1}{B\ f(J_2,J_1)}\ , \tag{33}$$

with

$$f(J_2,J_1)=\sqrt{\tfrac{1}{40}}\sqrt{(J_2+J_1+3)(J_1-J_2+2)(J_2-J_1+2)(J_2+J_1-1)}. \tag{33a}$$

The expressions for quantities B are easily obtained by combination of the previous results. We give them in the simplified case of the approximation of equal $F_{\max}-1$ energies and with $\Delta\kappa$ breaking neglected:

$$B(\gamma\rightarrow\gamma)=-G_2\frac{1}{E_\gamma-E_{K=2^+}}\,\frac{(\Delta\epsilon-2\beta N\Delta\chi)}{3\beta(1-\beta\sqrt{\tfrac{1}{14}}\chi_{F_{\max}})}\ , \tag{34a}$$

$$B(\beta\rightarrow\text{g.s.})=G_2\frac{1}{\beta^2-1+\beta\sqrt{\tfrac{2}{7}}\chi_{F_{\max}}}\left[\frac{1}{E_{\text{g.s.}}-E_{F_{\max}-1}}(\beta\Delta\epsilon+\beta^2N\Delta\chi)+\frac{1}{E_\beta-E_{F_{\max}-1}}(\beta\Delta\epsilon-N\Delta\chi)\right], \tag{34b}$$

$$B(\gamma\rightarrow\text{g.s.})=G_2\frac{1}{E_\gamma-E_{F_{\max}-1}}\,\frac{N\Delta\chi}{1+\beta\sqrt{\tfrac{2}{7}}\chi_{F_{\max}}}\ , \tag{34c}$$

$$B(\gamma\rightarrow\beta)=-G_2\left(\frac{1}{E_\beta-E_{F_{\max}-1}}+\frac{1}{E_\gamma-E_{F_{\max}-1}}\right)\frac{\Delta\epsilon+\beta N\Delta\chi}{3\beta^2(\beta-\sqrt{\tfrac{2}{7}}\chi_{F_{\max}})}\ . \tag{34d}$$

with the notation

$$G_2=\sqrt{\frac{30}{4\pi}}\,\frac{(g_\nu-g_\pi)}{e_{F_{\max}}}\,\frac{N_\nu N_\pi}{N^3}\ .$$

The reduced mixing ratios Δ for the above discussed transitions are all of the same order in the parameter N. The spin dependence of Δ is determined essentially by the geometry of the rotational collectivity in the IBM and it is the same as in other model approaches based on the rotational picture [20]. Of course, predictions for the values of B may be specific for the particular model.

VIII. $M1$ PROPERTIES IN THE IBM-1

The inclusion of the neutron and proton degrees of freedom is essential for explanation of $M1$ properties in the interacting boson model. Still, there is a potentiality to describe the $M1$ properties of the low-lying states with the predominant $F=F_{\max}$ component by considering only the $F_{\max}$ (IBM-1) space with an effective $M1$ operator [20,21]. Such an operator up to two-body terms is written most generally as [22]

$$T^{\text{IBM}-1}(M1)=\sqrt{\frac{3}{4\pi}}\,g_{F_{\max}}L+T_p^{\text{IBM-1}}(M1)\ , \tag{35}$$

$$T_p^{\text{IBM-1}}(M1)=\sqrt{\frac{3}{4\pi}}\,\{AL+[(B_1(s^\dagger\tilde{d}+d^\dagger s)^{(2)}+B_2(d^\dagger\tilde{d})^{(2)})L]^{(1)}+Cd^\dagger\tilde{d}L\}\ . \tag{35a}$$

One now attempts at specifying the parameters A, B_1, B_2, and C so that the $T_p^{\text{IBM-1}}$ operator simulates as much as possible the action of the T_p operator (23) in the proton-neutron version of the IBM.

For the transitions within the set of g.s., β, and γ bands, the $T_p^{\text{IBM-1}}$ operator provides the geometrical dependence on the initial and final spins J_1 and J_2 identical to that obtained above for the IBM-2 T_p operator [20]. One again obtains the reduced mixing ratios as Eq. (33) with the quantities B equal in the $1/N$ leading order to

$$B^{\text{IBM-1}}(\gamma\to\gamma)=\sqrt{\frac{3}{4\pi}}\frac{1}{e_{F_{\max}}}\frac{B_1-\beta\sqrt{\frac{1}{14}}B_2}{1-\beta\sqrt{\frac{1}{14}}\chi_{F_{\max}}}\,, \quad (36a)$$

$$B^{\text{IBM-1}}(\beta\to\text{g.s.})=\sqrt{\frac{3}{4\pi}}\frac{1}{e_{F_{\max}}}\frac{(\beta^2-1)B_1+\beta\sqrt{\frac{2}{7}}B_2-\beta\sqrt{10}C}{\beta^2-1+\beta\sqrt{\frac{2}{7}}\chi_{F_{\max}}}\,, \quad (36b)$$

$$B^{\text{IBM-1}}(\gamma\to\text{g.s.})=\sqrt{\frac{3}{4\pi}}\frac{1}{e_{F_{\max}}}\frac{B_1+\beta\sqrt{\frac{2}{7}}B_2}{1+\beta\sqrt{\frac{2}{7}}\chi_{F_{\max}}}\,, \quad (36c)$$

$$B^{\text{IBM-1}}(\gamma\to\beta)=\sqrt{\frac{3}{4\pi}}\frac{1}{e_{F_{\max}}}\frac{\beta B_1-\sqrt{\frac{2}{7}}B_2}{\beta-\sqrt{\frac{2}{7}}\chi_{F_{\max}}}\,. \quad (36d)$$

When we do not consider $\gamma\to\beta$ transitions, the parameters B_1, B_2, and C can be found so that the effective IBM-1 B quantities agree with the IBM-2 ones. On the level of the leading order mean field approximation, one can thus construct an effective $M1$ IBM-1 operator that gives the $\gamma\to\gamma$, $\beta\to$g.s., and $\gamma\to$g.s. transitions identical to the proton-neutron IBM-2 results.

For the factors Δg in the gyromagnetic ratios, the operator $T_p^{\text{IBM-1}}$ gives in the leading order

$$\Delta g(\text{g.s.})=A+\sqrt{\frac{2}{5}}\frac{\beta N}{1+\beta^2}\left(B_1-\beta\sqrt{\frac{1}{14}}B_2+\beta\sqrt{\frac{5}{2}}C\right)\,, \quad (37a)$$

$$\Delta g(\beta)=\Delta g(\text{g.s.})\,, \quad (37b)$$

$$\Delta g(\gamma,J)=\Delta g(\text{g.s.})-\frac{12}{J(J+1)}\sqrt{\frac{2}{5}}\frac{\beta N}{1+\beta^2}\left(B_1-\beta\sqrt{\frac{1}{14}}B_2\right)\,. \quad (37c)$$

Even if we take parameters of the operator $T_p^{\text{IBM-1}}$ as free ones and do not fix them by transition rates, the expressions for the Δg factors in the IBM-1 do not match generally those given in Eqs. (25), (26), and (27) for the IBM-2 formalism. In the g.s. and β bands, the IBM-1 gives equal and spin independent g factors. On the other hand, it was argued above that the IBM-2 Δg factor should be smaller in the absolute value in the β band than in the g.s. band. Also the spin dependence given in the IBM-2 for the Δg in the γ band is not obtained within the IBM-1 context unless Δg(g.s.) is zero. The IBM-1 formalism with an effective two-body $M1$ operator of Eq. (35) is not thus able to simulate diversity in the g factors that might be present in the neutron-proton version of the model. To achieve this, one should then include three-body terms into the effective $M1$ operator. However, we should perhaps note that accurate experimental data on magnetic moments of excited band states are rather sparse.

IX. CONCLUSIONS

We have studied the F-spin mixing and $M1$ properties of the low-lying states in the IBM-2. Application of the intrinsic state formalism in the leading order in $1/N$ and of the perturbative approach makes it possible to obtain analytical expressions in which a dependence on the various F-spin breaking terms is clearly exhibited. We can thus give a simple explanation of many features observed in numerical studies.

Of course, the present formulas are only approximate ones and might not always perfectly agree with exact numerical solutions. An example has been discussed in which numerical results are driven by the higher order terms in the $1/N$ expansion because of proximity of the particular symmetric and mixed-symmetry states. The intrinsic state formalism also loses its applicability in the vibrational regime. Nevertheless, the analytical expressions can be useful to give a first guess for a detailed numerical analysis.

The intrinsic state formalism also enables a simple comparison between the original IBM-2 results and the treatment of $M1$ properties with the extended $M1$ operator in the IBM-1 approach. We have found that the IBM-1 procedure could explain the $M1$ transition rates but, on the other hand, it might get into trouble as concerns magnetic moments.

The F-spin conserving part of the IBM-2 Hamiltonian (9) can be fixed by properties as the energy levels (including energies of mixed-symmetry states) and $E2$ values that are not influenced much by the F-spin mixing. We are then left with four parameters to characterize $M1$ matrix elements, namely, the mixing parameters $\Delta\epsilon$ and $\Delta\chi$ and boson gyromagnetic factors g_ν and g_π. On the other hand, the above discussion shows that the $M1$ elements between the low-lying predominantly symmetric states depend essentially on three combinations of these parameters: $g_{F_{\max}}$, $(g_\nu-g_\pi)\Delta\epsilon$, and $(g_\nu-g_\pi)\Delta\chi$. We cannot thus find the four parameters unambiguously when inspecting the $M1$ properties of the low-lying states only. Even an analysis of a chain of neighboring nuclei under assumption that the boson gyromagnetic factor g_ρ does not vary in nuclei with the same boson number N_ρ should be done carefully. To make a full specification, one has to use additional information such as the total $M1$ strength.

Finally, we should perhaps note that degrees of freedom outside the IBM-2 space may influence $M1$ properties in collective even-even nuclei and thus complicate practical analyses of F-spin mixing.

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