Boson-fermion mapping of collective fermion-pair algebras

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We construct finite Dyson boson-fermion mappings of general collective algebras extended by single-fermion operators. A key element in the construction is the implementation of a similarity transformation which transforms boson-fermion images obtained directly from the supercoherent state method. In addition to the general construction, we give detailed applications to SO(2N), SU(ℓ+1), SO(5), and SO(8) algebras.

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I. INTRODUCTION

The systematic construction of boson mappings for algebras defined by collective bifermion operators, or more generally the construction of boson realizations of Lie algebras, has by now been achieved in a rather complete fashion for which a comprehensive literature exists. (See Refs. [1–3] and references therein.) As shown by Dobaczewski [1], one of the most transparent ways to achieve this construction exploits generalized coherent states [4], a formalism which has also recently been shown [5,6] to be readily amenable to the construction of boson-fermion mappings from supercoherent states.

We refer to Ref. [5] for a background discussion and motivation concerning the construction of boson-fermion mappings of fermion systems in a many-body context. Here it suffices to recall that these methods aim at mapping (collective) fermion states with an even number of particles onto boson states, while those with an odd number of particles are mapped onto boson-fermion states. In this way odd boson-fermion states can be constructed in which collectivity and the Pauli exclusion principle are consistently included in the same theoretical framework.

In Ref. [5] it was possible to establish a general prescription for constructing such mappings and in particular it was shown that similarity transformations can be applied to modify properties of boson-fermion images. However, one important feature of these images was missing, namely, the odd states were represented by complicated many-fermion constructs instead of one-fermion states. Here we report on the construction of a mapping which regains this desired property and therefore corresponds to what has been hitherto proposed in most phenomenological models for odd-fermion states. Some aspects of our construction have already been implemented in Ref. [6] where we discuss dynamical supersymmetry in some specific fermion models.

It is thus the main objective of this paper to present the general construction mentioned above and to give some examples. The paper is organized as follows. In Sec. II we begin by recalling some basic definitions and notation and then we proceed by constructing the similarity transformation which is the key element of the present approach. We then discuss general properties of this similarity transformation when applied to the single-fermion images. Section III is devoted to examples concerning the SO(2N), SU(ℓ+1), SO(5), and SO(8) algebras, respectively, and conclusions are presented in Sec. IV.

II. DYSON BOSON-FERMION MAPPING OF THE COLLECTIVE ALGEBRA

We consider the boson-fermion mapping of a collective algebra defined by the collective fermion-pair creation operators

$$A^i=\tfrac{1}{2}\chi^i_{\mu\nu}a^\mu a^\nu, \tag{2.1}$$

labeled by the collective index $i=1,\dots,M$. Together with the corresponding collective fermion-pair annihilation operators $A_i=(A^i)^+$, all linearly independent commutators $[A_i,A^j]$, and the single-fermion operators a^ν and a_ν, they are assumed to form a closed collective superalgebra. [Here $\mu,\nu=1,\dots,N$ and $M\leqslant N(N-1)/2$.] These closure conditions read

$$\begin{aligned}
&[[A_i,A^j],A_k]=c^{jl}_{ik}A_l,\\
&[A^i,a_\nu]=\chi^i_{\mu\nu}a^\mu,\\
&[A^i,a^\nu]=0,\\
&\{a^\mu,a_\nu\}=\delta^\mu_\nu,\\
&\{a^\mu,a^\nu\}=0,
\end{aligned} \tag{2.2}$$

where c^{jl}_{ik} are structure constants and an implicit summation over repeated indices is assumed. With the single-fermion operators anticommuting to the identity, the odd and even parts of this superalgebra are composed of single-fermion and bifermion operators, respectively.

Following Ref. [7], we assume that the collective pairs are orthogonal and normalized to a common number g, i.e.,

$$\langle 0|A_iA^j|0\rangle\equiv\tfrac{1}{2}\chi^{\mu\nu}_i\chi^j_{\mu\nu}=g\,\delta^j_i \tag{2.3}$$

$[\chi^{\mu\nu}_i=(\chi^i_{\mu\nu})^*]$ which gives the symmetry properties of structure constants

$$c_{ik}^{jl}=c_{ik}^{lj}=c_{ki}^{jl}=(c_{jl}^{ik})^{*}. \qquad (2.4)$$

A Dyson type boson-fermion mapping of this algebra was derived in Ref. [5] using a collective Usui operator

$$U=\langle 0|\exp(B^{i}A_{i}+\alpha^{\mu}a_{\mu})|0) \qquad (2.5)$$

suggested by the supercoherent state method. This operator transforms collective even-fermion states, as well as collective states with additional individual fermions, into an ideal space composed of collective bosons, $B^{i}=B_{i}^{\dagger}$, $[B_{j},B^{i}]=\delta_{j}^{i}$, and of ideal fermions α^{μ}.

The mapping of operators $\mathscr{O}\leftrightarrow\hat{O}$ can be obtained from the equation $\mathscr{O}U=U\hat{O}$, which gives the following mapping of the collective superalgebra (2.2):

$$A^{j}\leftrightarrow gB^{j}-\tfrac{1}{2}c_{ik}^{jl}B^{i}B^{k}B_{l}-\chi_{\mu\rho}^{j}\chi_{i}^{\nu\rho}B^{i}\alpha^{\mu}\alpha_{\nu}+\mathscr{A}^{j}$$
$$=[\Lambda,B^{j}]+\mathscr{A}^{j}, \qquad (2.6a)$$

$$A_{j}\leftrightarrow B_{j}, \qquad (2.6b)$$

$$[A_{i},A^{j}]\leftrightarrow g\,\delta_{i}^{j}-c_{ik}^{jl}B^{k}B_{l}-\chi_{\mu\rho}^{j}\chi_{i}^{\nu\rho}\alpha^{\mu}\alpha_{\nu}, \qquad (2.6c)$$

$$a^{\nu}\leftrightarrow\alpha^{\nu}+\chi_{i}^{\nu\rho}B^{i}\alpha_{\rho}, \qquad (2.6d)$$

$$a_{\nu}\leftrightarrow\alpha_{\nu}, \qquad (2.6e)$$

where we introduced collective ideal fermion pairs,

$$\mathscr{A}^{j}=\tfrac{1}{2}\chi_{\mu\nu}^{j}\alpha^{\mu}\alpha^{\nu},\quad \mathscr{A}_{j}=(\mathscr{A}^{j})^{\dagger}, \qquad (2.7)$$

and the operator Λ,

$$\Lambda=gB^{l}B_{l}-\tfrac{1}{4}\,c_{mn}^{kl}B^{m}B^{n}B_{l}B_{k}-\chi_{\mu\rho}^{k}\chi_{l}^{\nu\rho}B^{l}B_{k}\alpha^{\mu}\alpha_{\nu}, \qquad (2.8)$$

invariant with respect to the core subalgebra.

At this point it is important to recall that a key element in the boson-fermion mapping formalism is the existence of a *physical subspace* of the ideal space. This subspace is the one whose states can be put into a one-to-one correspondence with the original fermion space. In Dyson type mappings (as opposed to, e.g., Schwinger type mappings [8]) this one-to-one correspondence is guaranteed by simply operating (repeatedly) with bifermion and single-fermion images onto the ideal space vacuum. At the same time the nature of the physical states obtained from this construction will clearly depend on the structure of the images and may not necessarily correspond to some preconceived physically desirable structure. This point was discussed in Ref. [5] and is illustrated by the images (2.6).

Consider, e.g., the image of the collective pair operator A^{i}, Eq. (2.6a), which contains the corresponding ideal collective pair operator $\mathscr{A}^{i}$. Operating with this image onto the ideal space vaccum $|0)$ gives $A^{i}|0\rangle\leftrightarrow(\mathscr{A}^{i}+gB^{i})|0)$, and the collective one-pair states are therefore not completely bosonized as one would desire from a physical point of view. It is necessary, therefore, to transform the mapping (2.6) into a form which will suitably address this problem, namely, result in a description where collective fermion pairs are represented by bosons only, while all other fermion degrees of freedom are simply accommodated as ideal fermions. Furthermore, the ideal states should accommodate an arbitrary number of ideal fermions, representing noncollective fermions, but of course still subject to reigning space limitations.

An attempt was made in the Ref. [5] to find a transformation leading to a mapping which reduces to the standard Dyson mapping (with bosons only) if the odd degrees of freedom are dropped. The drawback of the transformation suggested was that the mapping resulted in images of single-fermion operators given as infinite series. At the same time, the odd states were mapped onto rather complicated boson-fermion states.

In Ref. [6] we presented a similarity transformation which resolves this problem, i.e., gives even collective ideal states which are completely bosonized and at the same time yields the odd states described by single ideal fermions. In that paper we proved by induction that a specific similarity transformation meets these requirements. Here we proceed differently, by showing how the same transformation can be derived from properties of some Hamiltonian-like operators.

A. Construction of the transformation

We rewrite the right-hand sides of expressions (2.6a) and (2.6c) in a shorthand notation as

$$A^{j}\leftrightarrow R^{j}+\mathscr{A}^{j}, \qquad (2.9a)$$

$$A_{j}\leftrightarrow B_{j}, \qquad (2.9b)$$

$$[A_{i},A^{j}]\leftrightarrow[B_{i},R^{j}+\mathscr{A}^{j}]. \qquad (2.9c)$$

First, we observe that because $[B_{i},\mathscr{A}^{j}]=0$ we may simply drop the term $\mathscr{A}^{j}$ in Eqs. (2.9a) and (2.9c), and the commutation relations of the collective algebra will still be satisfied, i.e., the operator R^{j} alone when commuted with B_{j} gives the right-hand side of Eq. (2.6c). We may, therefore, envisage a similarity transformation (X, say) that will achieve just this modification, namely,

$$X^{-1}(R^{j}+\mathscr{A}^{j})X=R^{j}, \qquad (2.10a)$$

$$X^{-1}B_{j}X=B_{j}, \qquad (2.10b)$$

and which could then be applied to the right-hand sides of expressions (2.6d) and (2.6e) to find the corresponding single-fermion images.

To evaluate X we first multiply expression (2.9a) by B_{j} and sum over j. This gives the operator

$$H\equiv R^{j}B_{j}+\mathscr{A}^{j}B_{j}, \qquad (2.11)$$

where the first term on the right-hand side conserves the number of bosons and ideal fermions separately, while the second term decreases the number of bosons by 1 and increases the number of ideal fermions by 2. It is clear that this operator has an upper (or lower) triangular structure in the basis characterized by the numbers of ideal bosons (or fermions). Consequently, it has the same spectrum as the operator

$$H_{0}\equiv R^{j}B_{j} \qquad (2.12)$$

and, therefore, H and H_0 are related by a similarity transformation of the type introduced by Geyer [9]

$$X=\sum_{k=0}^{\infty} X_k=\sum_{k=0}^{\infty}\left(\frac{1}{\hat{H}_0-H_0}\mathscr{A}^j B_j\right)^k_{\wedge}, \qquad (2.13)$$

i.e.,

$$X^{-1}HX=H_0. \qquad (2.14)$$

Here the lone caret "$\wedge$" is read together with a positional operator [10] and determines at which position the careted operator, $\hat{H}_0$ in this case, is evaluated.

The denominator in expression (2.13) can be written in a convenient form much more directly linked to the structure of the ideal fermion (core) algebra. To show this we first identify

$$\begin{aligned} C_B&=gB^lB_l-\tfrac{1}{2}c^{kl}_{mn}B^mB^nB_kB_l\\ &=gB^lB_l-\tfrac{1}{2}c^{kl}_{mn}B^mB_kB^nB_l+\tfrac{1}{2}c^{kl}_{ml}B^mB_k \end{aligned} \qquad (2.15)$$

as the invariant operator of the boson core algebra, and

$$C_F=\mathscr{A}^l\mathscr{A}_l \qquad (2.16)$$

as the invariant operator of the ideal fermion core algebra. The invariant operator of the boson-fermion core algebra may be conveniently expressed if we perform an *auxiliary bosonization* of the ideal fermion-pair algebra by using auxiliary bosons b^i and b_j which commute with the other boson operators B,

$$\mathscr{A}^j\leftrightarrow gb^j-\tfrac{1}{2}c^{jl}_{ik}b^ib^kb_l, \qquad (2.17a)$$

$$\mathscr{A}_j\leftrightarrow b_j, \qquad (2.17b)$$

$$[\mathscr{A}_i,\mathscr{A}^j]\leftrightarrow g\,\delta^j_i-c^{jl}_{ik}b^kb_l. \qquad (2.17c)$$

This greatly facilitates the construction of the boson-fermion invariant operator, since the similar structures of C_B and C_F (when the latter is expressed in terms of the auxiliary bosons) allow one to write in analogy of expression (2.15)

$$\begin{aligned} C_{BF}=&\,g(B^lB_l+b^lb_l)-\tfrac{1}{2}c^{kl}_{mn}(B^mB_k+b^mb_k)(B^nB_l+b^nb_l)\\ &+\tfrac{1}{2}c^{kl}_{ml}(B^mB_k+b^mb_k). \end{aligned} \qquad (2.18)$$

This can be further rewritten as

$$\begin{aligned} C_{BF}&=C_F+B^l[\mathscr{A}_l,\mathscr{A}^k]B_k-\tfrac{1}{2}c^{kl}_{mn}B^mB^nB_kB_l\\ &=C_F+H_0, \end{aligned} \qquad (2.19)$$

with C_F now again given by expression (2.16). It follows that $H_0=C_BF-C_F$, and since

$$[C_{BF},\mathscr{A}^jB_j]=0 \qquad (2.20)$$

the desired result is

$$\hat{H}_0-H_0=C_F-\hat{C}_F, \qquad (2.21)$$

which explicitly demonstrates that the denominator in (2.13) depends only on the invariant operators of the ideal fermion core algebra, i.e.,

$$X=\sum_{k=0}^{\infty} X_k=\sum_{k=0}^{\infty}\left(\frac{1}{C_F-\hat{C}_F}\mathscr{A}^j B_j\right)^k_{\wedge}. \qquad (2.22)$$

We were not able to derive a similar closed form of the inverse transformation X^{-1}, but in fact this is not necessary provided we know how to commute boson-fermion operators with X. Indeed, $\mathscr{O}'$ is the similarity transform of the operator $\mathscr{O}$ if it satisfies the equation $\mathscr{O}X=X\mathscr{O}'$ where only the operator X appears. By inspection of the structure of X, however, it is possible to write down the lowest-order terms in an expansion for X^{-1},

$$\begin{aligned} X^{-1}=1&-\frac{1}{C_F-\hat{C}_F}\mathscr{A}^jB_{j\wedge}\\ &+\left(\frac{1}{C_F-\hat{C}_F}\mathscr{A}^jB_{j\wedge}\right)\left(\frac{1}{C_F-\hat{C}_F}\mathscr{A}^iB_{i\wedge}\right)\\ &-\frac{1}{C_F-\hat{C}_F}\mathscr{A}^jB_j\frac{1}{C_F-\hat{C}_F}\mathscr{A}^iB_{i\wedge}\ldots, \end{aligned} \qquad (2.23)$$

where brackets demarcate where the careted operators are to be evaluated in a term with more than one of these caret indicators.

It should be noted that X and X^{-1} have an identical triangular structure, namely, they contain terms which decrease (increase) boson (fermion) numbers by 0, 1, 2, etc. (0, 2, 4, etc.).

After deriving the expression for X we may now verify Eqs. (2.10). A proof of these identities by induction has been given in Ref. [6] and will not be repeated here. While it is clear that the transformation law (2.14) for H is a consequence of the transformation laws (2.10) for $R^j+\mathscr{A}^j$ and B_j, it was, however, only by considering the Hamiltonian H that we were actually able to derive the similarity transformation in the form of Eqs. (2.13) and (2.22).

B. Single-fermion images

To find the Dyson boson-fermion images of single-fermion operators we apply the transformation (2.22) to (2.6e) and (2.6f). From Eqs. (2.22) and (2.23) it follows that the transformed annihilation operator

$$X^{-1}\alpha_\nu X=\alpha_\nu-\frac{1}{C_F-\hat{C}_F}\mathscr{A}^l B_{l\wedge}\alpha_\nu+\alpha_\nu\frac{1}{C_F-\hat{C}_F}\mathscr{A}^l B_{l\wedge}-\frac{1}{C_F-\hat{C}_F}\mathscr{A}^l B_l\frac{1}{C_F-\hat{C}_F}\mathscr{A}^m B_{m\wedge}\alpha_\nu$$
$$+\alpha_\nu\frac{1}{C_F-\hat{C}_F}\mathscr{A}^l B_l\frac{1}{C_F-\hat{C}_F}\mathscr{A}^m B_{m\wedge}-\left(\frac{1}{C_F-\hat{C}_F}\mathscr{A}^l B_{l\wedge}\right)\alpha_\nu\left(\frac{1}{C_F-\hat{C}_F}\mathscr{A}^m B_{m\wedge}\right)$$
$$+\left(\frac{1}{C_F-\hat{C}_F}\mathscr{A}^l B_{l\wedge}\right)\left(\frac{1}{C_F-\hat{C}_F}\mathscr{A}^m B_{m\wedge}\right)\alpha_\nu+\cdots \tag{2.24}$$

decreases the number of ideal fermions by 1 and increases the number of ideal fermions by 1, 3, 5 etc. The same is true for the transformed creation operator

$$X^{-1}(\alpha^\nu+\chi_n^{\nu\rho}B^n\alpha_\rho)X=\alpha^\nu+\chi_n^{\nu\rho}B^n\alpha_\rho-\frac{1}{C_F-\hat{C}_F}\mathscr{A}^l B_{l\wedge}\chi_n^{\nu\rho}B^n\alpha_\rho+\chi_n^{\nu\rho}B^n\alpha_\rho\frac{1}{C_F-\hat{C}_F}\mathscr{A}^l B_{l\wedge}-\frac{1}{C_F-\hat{C}_F}\mathscr{A}^l B_{l\wedge}\alpha^\nu$$
$$+\alpha^\nu\frac{1}{C_F-\hat{C}_F}\mathscr{A}^l B_{l\wedge}-\frac{1}{C_F-\hat{C}_F}\mathscr{A}^l B_l\frac{1}{C_F-\hat{C}_F}\mathscr{A}^m B_{m\wedge}\chi_n^{\nu\rho}B^n\alpha_\rho$$
$$+\chi_n^{\nu\rho}B^n\alpha_\rho\frac{1}{C_F-\hat{C}_F}A^l B_l\frac{1}{C_F-\hat{C}_F}\mathscr{A}^m B_{m\wedge}-\left(\frac{1}{C_F-\hat{C}_F}\mathscr{A}^l B_{l\wedge}\right)\chi_n^{\nu\rho}B^n\alpha_\rho\left(\frac{1}{C_F-\hat{C}_F}\mathscr{A}^m B_{m\wedge}\right)$$
$$+\left(\frac{1}{C_F-\hat{C}_F}\mathscr{A}^l B_{l\wedge}\right)\left(\frac{1}{C_F-\hat{C}_F}\mathscr{A}^m B_{m\wedge}\right)\chi_n^{\nu\rho}B^n\alpha_\rho+\cdots\quad. \tag{2.25}$$

In Eqs. (2.24) and (2.25) we show all terms changing the number of ideal fermions by 1 and increasing this number by 3.

The question arises whether the terms in the single-fermion operator images which increase the ideal fermion number by more than 1 can contribute to matrix elements in the full ideal boson-fermion space. Although we were not able to settle this question on the operator level, we will demonstrate that the answer is negative for at least a wide class of states characterized by the condition

$$\mathscr{A}_l|\psi)=0 \tag{2.26}$$

for all those l which refer to the collective pairs singled out for mapping onto bosons as in (2.26a).

On the one hand this is a physically relevant condition, since the mapping should be designed to eliminate collective ideal fermion pairs in favor of bosons. At the same time condition (2.26) does not limit the considerations to the physical subspace of the ideal boson-fermion space, as is often the case with ideal space relations which are the counterparts of operator identities in the original space. (See also an explicit example in Sec. III C.) We note that the condition (2.26) is automatically fulfilled for the most practically interesting cases of $n_F=0$ and $n_F=1$. For ideal space states with two ideal fermions, the condition implies, as anticipated above, that the ideal fermions should form pairs orthogonal to all the collective pairs mapped onto bosons.

Condition (2.26) and its consequences for the single-fermion images are explicitly discussed in Appendix I. Here we only collect the final expressions which define the Dyson boson-fermion mapping of the general collective algebra characterized by (anti)commutation relations (2.2):

$$A^j\leftrightarrow gB^j-\tfrac{1}{2}c_{ik}^{jl}B^iB^kB_l-\chi_{\mu\rho}^j\chi_i^{\nu\rho}B^i\alpha^\mu\alpha_\nu, \tag{2.27a}$$

$$A_j\leftrightarrow B_j, \tag{2.27b}$$

$$[A_i,A^j]\leftrightarrow g\,\delta_i^j-c_{ik}^{jl}B^kB_l-\chi_{\mu\rho}^j\chi_i^{\nu\rho}\alpha^\mu\alpha_\nu, \tag{2.27c}$$

$$a^\nu\leftrightarrow\alpha^\nu+\chi_i^{\nu\rho}B^i\alpha_\rho-\frac{1}{C_F-\hat{C}_F}\mathscr{A}^l B_{l\wedge}\chi_n^{\nu\rho}B^n\alpha_\rho+\chi_n^{\nu\rho}B^n\alpha_\rho\frac{1}{C_F-\hat{C}_F}\mathscr{A}^l B_{l\wedge}\ldots, \tag{2.27d}$$

$$a_\nu\leftrightarrow\alpha_\nu-\frac{1}{C_F-\hat{C}_F}\mathscr{A}^l B_{l\wedge}\alpha_\nu+\alpha_\nu\frac{1}{C_F-\hat{C}_F}\mathscr{A}^l B_{l\wedge}\ldots, \tag{2.27e}$$

where the ellipses refer in general to higher-order terms increasing the number of ideal fermions by 3, 5, etc. These terms cancel when acting on a wide class of states characterized by condition (2.26).

C. Physical subspace

We now discuss the structure of the physical states and show how the condition (2.26) is satisfied for them. Simultaneously, it becomes clear that the higher-order terms in the single-fermion operator image cannot contribute to the physical states. First we note that the similarity transformation (2.22) does not change any state in which there are no bosons B^j, and therefore the single-fermion states are mapped onto single ideal fermion states,

$$a^\nu|0\rangle\leftrightarrow\alpha^\nu|0). \tag{2.28}$$

Since the images of pair creation operators (2.27a) do not change the ideal fermion number, collective odd states $|\Psi_{\rm odd}\rangle$ will be mapped onto ideal states with one ideal fermion only. Especially from a physical point of view, this result is a clear improvement over the solutions found in Ref. [5], where collective odd states were mapped onto ideal states with many-fermion components.

Repeated application of (2.27d) shows that the two-fermion states are mapped as

$$a^\mu a^\nu|0\rangle \leftrightarrow \left(\alpha^\mu\alpha^\nu + \chi_i^{\mu\nu}B^i - \frac{1}{C_F}\chi_i^{\mu\nu}\mathscr{A}^i\right)|0\rangle, \quad (2.29)$$

and in general contain the noncollective pair of ideal fermions $\alpha^\mu\alpha^\nu|0\rangle$. However, when the collective pair A^j is formed by summing the pairs $a^\mu a^\nu$ with collective amplitudes $\frac{1}{2}\chi^j_{\mu\nu}$, the ideal noncollective pairs above recombine and since [see Eq. (2.3)] $C_F\mathscr{A}^j|0\rangle = g\mathscr{A}^j|0\rangle$ the first and last terms above cancel. Only the boson state $gB^j|0\rangle$ therefore remains as the image of a collective fermion pair state. Again, since the images (2.27a) conserve the ideal fermion number, the same recombination mechanism is also valid for any even state. It is also clear that $\mathscr{A}_l$ acting on the right-hand side of (2.29) gives zero, in accordance with the condition (2.26).

Consider now the images of 3-fermion states. From (2.25) and (2.29) we observe that the terms increasing the ideal fermion number by 3 could also contribute to a three-fermion state constructed by an application of (2.25) on (2.29). These terms cancel as follows from the proof in the preceding section, because here these terms would act on a state with no ideal fermion. The three-fermion state therefore contains contributions of only those terms shown explicitly in (2.27d) and can be written as

$$a^\tau a^\mu a^\nu|0\rangle \leftrightarrow \Bigg(\alpha^\tau\alpha^\mu\alpha^\nu + \alpha^\tau\chi^l_{\rho\tau}B^i + \alpha^\nu\chi_i^{\tau\mu}B^i + \alpha^\mu\chi_i^{\nu\tau}B^i - \alpha^\tau\frac{1}{C_F}\chi_i^{\mu\nu}\mathscr{A}^i - \frac{1}{C_F}\chi_i^{\tau\mu}\mathscr{A}^i\alpha^\nu - \frac{1}{C_F}\chi_i^{\nu\tau}\mathscr{A}^i\alpha^\mu$$
$$+\frac{1}{g}\frac{1}{C_F}\chi^l_{\rho\tau}\chi_i^{\nu\mu}\chi^i_{\rho\sigma}\mathscr{A}^l\alpha^\sigma\Bigg)|0\rangle. \quad (2.30)$$

It is simple to verify that $\chi^i_{\mu\nu}a^\tau a^\mu a^\nu|0\rangle$ maps onto $R^i\alpha^\tau|0\rangle$, but it is more involved to show that the image of, say, $\chi^i_{\tau\mu}a^\tau a^\mu a^\nu|0\rangle$, obtained by explicitly combining the three indicated images, reduces to $R^i\alpha^\nu|0\rangle$, as it should if the mapping is consistent. This calculation can again be facilitated by performing an auxiliary bosonization of the ideal fermion operators using (A4). Once more the higher-order terms do not contribute when acting as in (2.30) as it fulfills the condition $\mathscr{A}_l|\psi\rangle = 0$, which can best be recognized in the auxiliary bosonized picture.

So far we have discussed states constructed from the single-fermion operator (2.27d). However, because the pair creation operator image (2.27a) does not change the ideal fermion number and commutes with (2.27d) (which can be checked explicitly) our previous analysis is valid for any physical state obtained by the application of (2.27a) on the states (2.28), (2.29), and (2.30). In particular, the terms increasing the ideal fermion number by 3, 5, etc. do not contribute to these physical states.

One property of the single-fermion images that has so far not been discussed concerns the question whether *anticommutators* among the original fermion operators a^μ and a_ν are preserved on the ideal space by their images. Common wisdom has so far held that it is only on the physical subspace that the images can in fact preserve these anticommutators [2,11–13]. This conclusion mostly follows from the particular construction of images and the subsequent method of verifying these relations in the ideal space, which typically leads to results of the type $\{(a_\nu)_I,(a^\mu)_I\}P = \delta^\mu_\nu P$, where the subscript I denotes a general image and P the projection operator to the physical subspace of the full ideal space.

One of the advantages of our construction through supercoherent states is that it is easily verified that the images (2.6d) and (2.6e) preserve anticommutation relations on the *full* ideal space. Subsequent images obtained from these original images through similarity transformation will naturally retain this property, as long as the transformation X, which defines the similarity transform $\mathscr{O}'$ of operator $\mathscr{O}$ through $\mathscr{O}X = X\mathscr{O}'$, is nonsingular, as is the case in our applications. We return to this point in the following examples.

III. APPLICATIONS AND EXAMPLES

The mapping derived in the previous section also covers the noncollective algebra SO(2N). As we discuss in Sec. III A, the similarity transformation may in this case be expressed in a more compact form. Other examples, pertaining to collective algebras, are presented in Secs. III B–III D.

A. SO(2N) mapping

The Dyson boson-fermion mapping obtained in Ref. [5] from the supercoherent state method for SO(2N) is

$$a^\mu a^\nu \leftrightarrow B^{\mu\nu} - B^{\mu\rho}B^{\nu\theta}B_{\rho\theta} - B^{\mu\rho}\alpha^\nu\alpha_\rho + B^{\nu\rho}\alpha^\mu\alpha_\rho + \alpha^\mu\alpha^\nu, \quad (3.1a)$$

$$a_\nu a_\mu \leftrightarrow B_{\mu\nu}, \quad (3.1b)$$

$$a^\mu a_\nu \leftrightarrow B^{\mu\theta}B_{\nu\theta} + \alpha^\mu\alpha_\nu, \quad (3.1c)$$

$$a^\nu \leftrightarrow \alpha^\nu + B^{\nu\rho}\alpha_\rho, \quad (3.1d)$$

$$a_\nu \leftrightarrow \alpha_\nu. \quad (3.1e)$$

The invariant operator of the ideal fermion core algebra depends in this case only on the number operator $n=\alpha^\rho\alpha_\rho$, namely,

$$C_F=\tfrac{1}{2}\alpha^\mu\alpha^\nu\alpha_\nu\alpha_\mu=\tfrac{1}{2}n(n-1). \tag{3.2}$$

This simplifies the similarity transformation significantly. In particular, we have

$$C_F-\hat{C}_F=\tfrac{1}{2}(n-\hat{n})(n+\hat{n}-1), \tag{3.3}$$

and the transformation (2.22) can be summed to the following closed form:

$$X=\frac{(2\hat{n}-1)!!}{(n+\hat{n}-1)!!}\exp[\tfrac{1}{2}\alpha^\mu\alpha^\nu B_{\mu\nu}]_\wedge. \tag{3.4}$$

Finally, the transformed Dyson boson-fermion mapping is

$$a^\mu a^\nu\leftrightarrow B^{\mu\nu}-B^{\mu\rho}B^{\nu\theta}B_{\rho\theta}-B^{\mu\rho}\alpha^\nu\alpha_\rho+B^{\nu\rho}\alpha^\mu\alpha_\rho, \tag{3.5a}$$

$$a_\nu a_\mu\leftrightarrow B_{\mu\nu}, \tag{3.5b}$$

$$a^\mu a_\nu\leftrightarrow B^{\mu\theta}B_{\nu\theta}+\alpha^\mu\alpha_\nu, \tag{3.5c}$$

$$a^\nu\leftrightarrow\alpha^\nu+B^{\nu\rho}\alpha_\rho-\frac{1}{2n-3}\alpha^\nu n+\frac{1}{2n-1}\alpha^\tau B^{\nu\rho}B_{\rho\tau} -\frac{1}{(2n-1)(2n-3)}\alpha^\sigma\alpha^\tau\alpha_\rho B^{\nu\rho}B_{\sigma\tau}, \tag{3.5d}$$

$$a_\nu\leftrightarrow\alpha_\nu-\frac{1}{(2n-1)(2n-3)}\alpha^\sigma\alpha^\tau\alpha_\nu B_{\sigma\tau}+\frac{1}{2n-1}\alpha^\tau B_{\nu\tau}. \tag{3.5e}$$

We observe that the single-fermion images are finite and comprised of terms changing the ideal fermion number by 1 only. Moreover, the mapping is such that an attempt to create a fermion pair using successive applications of (3.5d) is equivalent to the application of (3.5a).

It is not difficult to verify, as anticipated in the previous section, that the images (3.5d) and (3.5e) preserve anticommutators on the full ideal space, i.e., as *operator* identities. An efficient way to do this is first to form a product of two single-fermion images and then to *symmetrize* with respect to the indices. Upon symmetrization some terms in the product will immediately yield zero on their own, because of their antisymmetry, while others will conspire to yield either zero or unity, depending on the anticommutator being verified.

B. SU(ℓ+1) mapping

Let us suppose that the collective operators form an $(\Omega+1)$-dimensional symmetric representation of the unitary algebra SU(ℓ+1), i.e., one has ℓ collective pairs A_i. This model can be realized in an $(\ell+1)\times\Omega$ system of states, forming $\ell+1$ degenerate levels, by introducing the pairs

$$A^i=\sum_{m=1}^{\Omega}a^+_{im}a^+_{0m}, \tag{3.6}$$

where the index 0 refers to one of the levels, and $i=1,\ldots,\ell$. The simplest example is provided by the well-known quasispin SU(2) algebra obtained for $\ell=1$. By normalizing the collective pairs so that $g=\Omega$, one obtains Ω-independent structure constants:

$$c^{jl}_{ik}=\delta^j_i\delta^l_k+\delta^j_k\delta^l_i. \tag{3.7}$$

The mapping of this algebra derived in Ref. [5] from supercoherent states is

$$A^j\leftrightarrow -B^jN_B+B^i[\mathscr{A}_i,\mathscr{A}^j]+\mathscr{A}^j, \tag{3.8a}$$

$$A_j\leftrightarrow B_j, \tag{3.8b}$$

$$[A_i,A^j]\leftrightarrow\delta^j_i(\Omega-N_B)-B^jB_i-(\Omega\delta^j_i-[\mathscr{A}_i,\mathscr{A}^j]), \tag{3.8c}$$

$$a^\nu\leftrightarrow\alpha^\nu+B^i[\mathscr{A}_i,\alpha^\nu], \tag{3.8d}$$

$$a_\nu\leftrightarrow\alpha_\nu, \tag{3.8e}$$

where $N_B=B^kB_k$ is the boson number operator.

Again we may express the similarity transformation in a compact form as the invariant operator of the fermion core algebra depends only on the ideal fermion number operator $n=\alpha^\rho\alpha_\rho$, namely,

$$C_F=\tfrac{1}{2}n(\Omega+1-\tfrac{1}{2}n). \tag{3.9}$$

We then get

$$C_F-\hat{C}_F=\tfrac{1}{2}(n-\hat{n})[\Omega+1-\tfrac{1}{2}(n+\hat{n})], \tag{3.10}$$

and the transformation (2.13) is

$$X=\frac{[\Omega-\frac{1}{2}(n+\hat{n})]!}{(\Omega-\hat{n})!}\exp[\mathscr{A}^iB_i]_\wedge. \tag{3.11}$$

The transformed Dyson boson-fermion mapping is obtained in the form

$$A^j\leftrightarrow -B^jN_{\rm B}+B^i[\mathscr{A}_i,\mathscr{A}^j], \tag{3.12a}$$

$$A_j\leftrightarrow B_j, \tag{3.12b}$$

$$[A_i,A^j]\leftrightarrow\delta^j_i(\Omega-N_{\rm B})-B^jB_i-(\Omega\delta^j_i-[\mathscr{A}_i,\mathscr{A}^j]), \tag{3.12c}$$

$$a^\nu\leftrightarrow\alpha^\nu+B^i[\mathscr{A}_i,\alpha^\nu]-\frac{1}{\Omega-n+2}\mathscr{A}^i[\mathscr{A}_i,\alpha^\nu] +\frac{1}{\Omega-n+1}B^i[[\mathscr{A}_i,\alpha^\nu],\mathscr{A}^l]B_l +\frac{1}{(\Omega-n+1)(\Omega-n+2)}\mathscr{A}^lB^i[\mathscr{A}_i,\alpha^\nu]B_l, \tag{3.12d}$$

$$a_\nu \leftrightarrow \alpha_\nu + \frac{1}{(\Omega-n+1)(\Omega-n+2)} \mathscr{A}^l B_l \alpha_\nu + \frac{1}{\Omega-n+1}[\alpha_\nu, \mathscr{A}^l] B_l . \quad (3.12e)$$

As in the SO(2N) case we observe that the single-fermion images are finite and contain terms changing the ideal fermion number by 1 only. For the SU(2) algebra the mapping (3.12) reduces to the one derived previously in Ref. [12]. Furthermore, the images (3.12d) and (3.12e) preserve anticommutation relations on the full ideal space, as can be verified after some algebra. In Ref. [12] the construction of these same images allowed a simple argument to prove that these relations were preserved at least on the physical subspace. That we could anticipate and finally make a more general and complete statement about the anticommutation relations here once again illustrates the versatility of the (super)coherent state method.

C. SO(5) mapping

Probably the simplest model with an invariant operator C_F does not just trivially depend on the fermion number, as in SO(2N) and SU(ℓ+1), is the SO(5) model [2]. It assumes two single-j shells with the same degeneracy $\Omega=2j+1$, where three kinds of monopole pairs can be formed. Let us denote these shells by p and h and introduce the following pairs with the normalization g in (2.3) chosen to be equal to Ω:

$$S_+ = \sqrt{\Omega}(a^p a^h)^{(0)}, \quad S_- = (S_+)^+, \quad S_0 = \tfrac{1}{4}(n_p + n_h - 2\Omega), \quad (3.13a)$$

$$L_+ = \sqrt{\Omega/2}(a^p a^p)^{(0)}, \quad L_- = (L_+)^+, \quad L_0 = \tfrac{1}{2}(n_p - \Omega), \quad (3.13b)$$

$$K_+ = \sqrt{\Omega/2}(a^h a^h)^{(0)}, \quad K_- = (K_+)^+, \quad K_0 = \tfrac{1}{2}(n_h - \Omega), \quad (3.13c)$$

where the fermion creation operators are coupled to angular momentum zero. The algebra is closed by monopole single-particle operators:

$$T_+ = -\sqrt{\Omega}(a^p \tilde{a}_h)^{(0)}, \quad T_- = (T_+)^+, \quad T_0 = \tfrac{1}{4}(n_p - n_h), \quad (3.14)$$

where the tilde denotes the time-reversed operator.

The boson-fermion mapping derived from the supercoherent state method is (see also Ref. [14] where a different normalization is used)

$$S_+ \leftrightarrow B^f[\Omega - \tfrac{1}{2}(N_f + n_p + n_h) - N_p - N_h] - B^p B^h B_f - B^p \mathscr{T}_- - B^h \mathscr{T}_+ + \mathscr{S}_+ , \quad (3.15a)$$

$$S_- \leftrightarrow B_f , \quad (3.15b)$$

$$S_0 \leftrightarrow \tfrac{1}{2}(N_p + N_h + N_f + \tfrac{1}{2} n_p + \tfrac{1}{2} n_h - \Omega), \quad (3.15c)$$

$$L_+ \leftrightarrow B^p(\Omega - N_p - N_f - n_p) - \tfrac{1}{2} B^f B^f B_h - B^f \mathscr{T}_+ + \mathscr{L}_+ , \quad (3.15d)$$

$$L_- \leftrightarrow B_p , \quad (3.15e)$$

$$L_0 \leftrightarrow \tfrac{1}{2}(2N_p + N_f + n_p - \Omega), \quad (3.15f)$$

$$K_+ \leftrightarrow B^h(\Omega - N_h - N_f - n_h) - \tfrac{1}{2} B^f B^f B_p - B^f \mathscr{T}_- + \mathscr{K}_+ , \quad (3.15g)$$

$$K_- \leftrightarrow B_h , \quad (3.15h)$$

$$K_0 \leftrightarrow \tfrac{1}{2}(2N_h + N_f + n_h - \Omega), \quad (3.15i)$$

$$T_+ \leftrightarrow B^p B_f + B^f B_h + \mathscr{T}_+ , \quad (3.15j)$$

$$T_- \leftrightarrow B^f B_p + B^h B_f + \mathscr{T}_- , \quad (3.15k)$$

$$T_0 \leftrightarrow \tfrac{1}{2}(N_p + \tfrac{1}{2} n_p - N_h - \tfrac{1}{2} n_h), \quad (3.15l)$$

$$a^p \leftrightarrow \alpha^p + \frac{1}{\sqrt{2}} B^f \tilde{\alpha}_h + B^p \tilde{\alpha}_p , \quad (3.15m)$$

$$a_p \leftrightarrow \alpha_p , \quad (3.15n)$$

$$a^h \leftrightarrow \alpha^h + \frac{1}{\sqrt{2}} B^f \tilde{\alpha}_p + B^h \tilde{\alpha}_h , \quad (3.15o)$$

$$a_h \leftrightarrow \alpha_h . \quad (3.15p)$$

The ideal-fermion-pair and the single-particle operators are defined in the same way as in (3.13) and (3.14) with the a operators replaced by α.

The similarity transformation is given by

$$X = \sum_{k=0}^{\infty} \left(\frac{1}{C_F - \hat{C}_F} W \right)^k_\wedge , \quad (3.16)$$

with

$$W = \mathscr{S}_+ B_f + \mathscr{L}_+ B_p + \mathscr{K}_+ B_h \quad (3.17)$$

and

$$C_F = \mathscr{S}_+ \mathscr{S}_- + \mathscr{L}_+ \mathscr{L}_- + \mathscr{K}_+ \mathscr{K}_- . \quad (3.18)$$

Consequently, the transformed mapping of the bifermion operators is identical to that given in Eqs. (3.15a)–(3.15p), except for the fact that the ideal fermion pairs $\mathscr{S}_+$, $\mathscr{L}_+$, and $\mathscr{K}_+$ disappear from Eqs. (3.15a), (3.15d), and (3.15g), respectively. On the other hand, the mapping of single-fermion operators now reads

$$a^p \leftrightarrow \alpha^p + \frac{1}{\sqrt{2}} B^f \tilde{\alpha}_h + B^p \tilde{\alpha}_p - \frac{1}{C_F - \hat{C}_F} W_\wedge \left(\frac{1}{\sqrt{2}} B^f \tilde{\alpha}_h + B^p \tilde{\alpha}_p \right) + \left(\frac{1}{\sqrt{2}} B^f \tilde{\alpha}_h + B^p \tilde{\alpha}_p \right) \frac{1}{C_F - \hat{C}_F} W_\wedge + \cdots , \tag{3.19a}$$

$$a_p \leftrightarrow \alpha_p - \frac{1}{C_F - \hat{C}_F} W_\wedge \alpha_p + \alpha_p \frac{1}{C_F - \hat{C}_F} W_\wedge + \cdots , \tag{3.19b}$$

$$a^h \leftrightarrow \alpha^h + \frac{1}{\sqrt{2}} B^f \tilde{\alpha}_p + B^h \tilde{\alpha}_h - \frac{1}{C_F - \hat{C}_F} W_\wedge \left(\frac{1}{\sqrt{2}} B^f \tilde{\alpha}_p + B^h \tilde{\alpha}_h \right) + \left(\frac{1}{\sqrt{2}} B^f \tilde{\alpha}_p + B^h \tilde{\alpha}_h \right) \frac{1}{C_F - \hat{C}_F} W_\wedge + \cdots , \tag{3.19c}$$

$$a_h \leftrightarrow \alpha_h - \frac{1}{C_F - \hat{C}_F} W_\wedge \alpha_h + \alpha_h \frac{1}{C_F - \hat{C}_F} W_\wedge + \cdots \quad . \tag{3.19d}$$

It is not difficult to perform explicit calculations in this model and present explicit results for some particular configurations. For example, it is straightforward to show that the image of a_p (3.19b) acting on the $|n_F=0\rangle$ subspace has the form

$$a_p|n_F=0\rangle \leftrightarrow \frac{1}{\Omega}([\alpha_p, \mathscr{S}_+]B_f + [\alpha_p, \mathscr{L}_+]B_p)|n_F=0\rangle , \tag{3.20}$$

where $|n_F=0\rangle$ means that all the fermions form collective pairs only. The image of the same operator acting on the subspace $|n_F(p)=1\rangle$ is as follows:

$$a_p|n_F(p)=1\rangle \leftrightarrow \Bigg(\alpha_p - \frac{1}{\Omega}(\mathscr{S}_+ B_f + \mathscr{L}_+ B_p + \mathscr{K}_+ B_h)\alpha_p + \frac{1}{(\Omega-1)(\Omega+\frac{1}{2})}\{(\Omega\alpha_p \mathscr{S}_+ - \alpha_p \mathscr{L}_+ \mathscr{T}_-)B_f + [(\Omega - \tfrac{1}{2})\mathscr{K}_+ \alpha_p - \alpha_p \mathscr{S}_+ \mathscr{T}_-]B_h\} + \frac{1}{\Omega - 1} \alpha_p \mathscr{L}_+ B_p \Bigg) |n_F(p)=1\rangle , \tag{3.21}$$

where $|n_F(p)=1\rangle$ means that one fermion is unpaired and occupies the level p. In Appendix II we show how the general structure of the single-fermion images specifies to the case of the SO(5) model.

D. SO(8) mapping

The SO(8) model [15] is defined by collective pairs

$$F^+_{JM} = \sqrt{\tfrac{1}{2}} \sum_{j_1 j_2} (-1)^{J+i+k+j_1} \frac{\hat{j}_1 \hat{j}_2}{\hat{k}} \begin{Bmatrix} j_1 & j_2 & J \\ i & i & k \end{Bmatrix} (a^+_{j_1} a^+_{j_2})^{(J)}_M , \tag{3.22a}$$

$$P_{JM} = -\sqrt{2\Omega} \sum_{j_1 j_2} (-1)^{J+i+k+j_1} \frac{\hat{j}_1 \hat{j}_2}{\hat{k}} \begin{Bmatrix} j_1 & j_2 & J \\ i & i & k \end{Bmatrix} \times (a^+_{j_1} \tilde{a}_{j_2})^{(J)}_M , \tag{3.22b}$$

with $i=\frac{3}{2}$ and k integer. In (3.22a) only S^+ $(J=0)$ and D^+ $(J=2)$ pairs are allowed, while in (3.22b) J takes values 0, 1, 2, and 3.

In order to generalize the model to odd systems we may add creation and annihilation operators $a^\dagger_{jm}$ and a_{jm}. The boson-fermion mapping of this algebra derived from supercoherent states is

$$F^+_{JM} \leftrightarrow B^\dagger_{JM} - \frac{2}{\hat{k}^2} \sum_{J_1 J_2 J_3 J'} \hat{J}_1 \hat{J}_2 \hat{J}_3 \hat{J}' \begin{Bmatrix} i & i & J \\ i & i & J_3 \\ J_2 & J_1 & J' \end{Bmatrix} [(B^\dagger_{J_1} B^\dagger_{J_2})^{(J')} \tilde{B}_{J_3}]^{(J)}_M + \frac{2}{\hat{i}\hat{k}^2} (-1)^{J_2} \hat{J}_1 \hat{J}_2 \begin{Bmatrix} J_1 & J_2 & J \\ i & i & i \end{Bmatrix} (B^\dagger_{J_1} \mathscr{P}_{J_2})^{(J)}_M + \mathscr{F}^\dagger_{JM} , \tag{3.23a}$$

$$F_{JM} \leftrightarrow B_{JM} , \tag{3.23b}$$

$$P_{JM}\leftrightarrow \frac{2\sqrt{2\Omega}}{\hat{k}}(-1)^{J+2i}\sum_{J_1J_2}\hat{J}_1\hat{J}_2\begin{Bmatrix} J_1 & J_2 & J \\ i & i & i \end{Bmatrix}(B^\dagger_{J_1}\tilde{B}_{J_2})^{(J)}_M+\mathscr{P}_{JM}, \tag{3.23c}$$

$$a^+_{jm}\leftrightarrow \alpha^\dagger_{jm}+\frac{\sqrt{2}}{\hat{k}}\hat{J}_1\hat{j}_1(-1)^{j_1+i+k}\begin{Bmatrix} j_1 & j & J_1 \\ i & i & k \end{Bmatrix}(B^\dagger_{J_1}\tilde{\alpha}_{j_1})^{(j)}_m, \tag{3.23d}$$

$$a_{jm}\leftrightarrow \alpha_{jm}, \tag{3.23e}$$

where $\tilde{B}_{JM}=(-1)^{J-M}B_{J,-M}$ and $\tilde{a}_{jm}=(-1)^{j-m}a_{j,-m}$. The ideal fermion-pair operators $\mathscr{F}^\dagger_{JM}$ and $\mathscr{P}_{JM}$ are given by Eqs. (3.24a) and (3.24b), respectively, with the fermion operators a_{jm} replaced by ideal fermion operators α_{jm}.

In this case we cannot simplify the similarity transformation

$$X=\sum_{k=0}^{\infty}\left(\frac{1}{C_F-\hat{C}_F}\mathscr{F}^\dagger_J\cdot\tilde{B}_J\right)^k_\wedge \tag{3.24}$$

as

$$C_F-\hat{C}_F=\frac{1}{\Omega}\{\tfrac{1}{2}(n-\hat{n})[\Omega+6-\tfrac{1}{2}(n+\hat{n})]+\hat{C}_{2\mathrm{spin}_F(6)}-C_{2\mathrm{spin}_F(6)}\}, \tag{3.25}$$

with n the ideal fermion number operator and $C_{2\mathrm{spin}_F(6)}=\frac{1}{4}(P_1\cdot P_1+P_2\cdot P_2+P_3\cdot P_3)$. The transformed mapping then reads

$$F^+_{JM}\leftrightarrow B^\dagger_{JM}-\frac{2}{\hat{k}^2}\sum_{J_1J_2J_3J'}\hat{J}_1\hat{J}_2\hat{J}_3\hat{J}'\begin{Bmatrix} i & i & J \\ i & i & J_3 \\ J_2 & J_1 & J' \end{Bmatrix}[(B^\dagger_{J_1}B^\dagger_{J_2})^{(J')}\tilde{B}_{J_3}]^{(J)}_M+\frac{2}{\hat{i}\hat{k}^2}(-1)^{J_2}\hat{J}_1\hat{J}_2\begin{Bmatrix} J_1 & J_2 & J \\ i & i & i \end{Bmatrix}(B^\dagger_{J_1}\mathscr{P}_{J_2})^{(J)}_M, \tag{3.26a}$$

$$F_{JM}\leftrightarrow B_{JM}, \tag{3.26b}$$

$$P_{JM}\leftrightarrow \frac{2\sqrt{2\Omega}}{\hat{k}}(-1)^{J+2i}\sum_{J_1J_2}\hat{J}_1\hat{J}_2\begin{Bmatrix} J_1 & J_2 & J \\ i & i & i \end{Bmatrix}(B^\dagger_{J_1}\tilde{B}_{J_2})^{(J)}_M+\mathscr{P}_{JM}, \tag{3.26c}$$

for the collective pair operators while for the single-fermion operators we obtain

$$a^+_{jm}\leftrightarrow \alpha^\dagger_{jm}+B^\dagger_J\cdot[\tilde{\mathscr{F}}_J,\alpha^\dagger_{jm}]-\frac{1}{C_F-\hat{C}_F}\mathscr{F}^\dagger_{J_1}\cdot\tilde{B}_{J_1\wedge}B^\dagger_J\cdot[\tilde{\mathscr{F}}_J,\alpha^\dagger_{jm}]+B^\dagger_J\cdot[\tilde{\mathscr{F}}_J,\alpha^\dagger_{jm}]\frac{1}{C_F-\hat{C}_F}\mathscr{F}^\dagger_{J_1}\cdot\tilde{B}_{J_1\wedge}\cdots, \tag{3.27a}$$

$$a_{jm}\leftrightarrow \alpha_{jm}-\frac{1}{C_F-\hat{C}_F}\mathscr{F}^\dagger_{J_1}\cdot\tilde{B}_{J_1\wedge}\alpha_{jm}+\alpha_{jm}\frac{1}{C_F-\hat{C}_F}\mathscr{F}^\dagger_{J_1}\cdot\tilde{B}_{J_1\wedge}\cdots. \tag{3.27b}$$

The commutator $B^\dagger_{J_1}\cdot[\tilde{\mathscr{F}}_{J_1},\alpha^\dagger_{jm}]$ in (3.27a) just gives the second term in (3.23d) and the ellipses refer to the same class of terms as in Eqs. (2.27d) and (2.27e).

Let us now consider the matrix elements of the mapped single-fermion operators (3.27a) and (3.27b) between even and odd states $|\Psi\rangle$ and $|\Psi'\rangle$ corresponding to nuclei with particle numbers differing by 1. It is assumed that the even state contains collective pairs only. These matrix elements give then the spectroscopic factors and can be written in the following form:

$$\langle\Psi|a^+_{jm}|\Psi'\rangle=(\Psi|\left[\alpha^\dagger_{jm}+\frac{\sqrt{2}}{\hat{k}}\hat{J}_1\hat{j}_1(-1)^{j_1+i+k}\begin{Bmatrix} j_1 & j & J_1 \\ i & i & k \end{Bmatrix}(B^\dagger_{J_1}\tilde{\alpha}_{j_1})^{(j)}_m+\frac{2}{\hat{k}^2}\sum_{J_1j_2J_3j'}\hat{J}_1\hat{j}_2\hat{J}_3\hat{j}'\begin{Bmatrix} k & i & j_2 \\ i & i & J_1 \\ j & J_3 & j' \end{Bmatrix}\times[(B^\dagger_{J_1}\alpha^\dagger_{j_2})^{(j')}\tilde{B}_{J_3}]^{(j)}_M\right]|\Psi'), \tag{3.28a}$$

$$\langle\Psi|\tilde{a}_{jm}|\Psi'\rangle=(\Psi|\left[\tilde{\alpha}_{jm}-\frac{\sqrt{2}}{\hat{k}}\hat{J}_1\hat{j}_1(-1)^{j+i+k}\begin{Bmatrix} j_1 & j & J_1 \\ i & i & k \end{Bmatrix}(\alpha^\dagger_{j_1}\tilde{B}_{J_1})^{(j)}_m\right]|\Psi'). \tag{3.28b}$$

Note that unlike the phenomenological interacting boson-fermion model (IBFM) case [16] the tensorial form of the appropriate single-fermion transfer operators, as functions of supergenerators and ideal fermions [compare Eqs. (3.5) or their collective counterparts], is here *uniquely* fixed by the mapping. For example, in the U(6/4) case [$k=0$ in Eqs. (3.28)] the operators have the $\sigma_1=\frac{1}{2}$ spin(6) tensorial character.

It should be mentioned that a boson-fermion analysis of the SO(8) model had previously been presented by Frank *et al.* [17] from the point of view of group contractions. Although this led to the emergence of an IBFM type structure, only some truncated Holstein-Primakoff images of the SO(8) generators were presented, which makes direct comparison with our exact Dyson images (3.28) difficult. Furthermore, the construction of single-fermion images is not considered at all in Ref. [17], while the identification of possible supersymmetric structures is only speculated about. We refer to Ref. [6] for further discussion concerning supersymmetry in this context, as well as for a concrete example.

Finally, it is important to realize that the operator images of a^+_{jm} and $\tilde{a}_{jm}$ which are effective in Eqs. (3.28) cannot simply be compounded to obtain the image of an interaction term of the type a^+a^+aa, say, as these images are valid only for the subspace with zero and one ideal fermions only. This type of inconsistent application in some semimicroscopic applications is also analyzed and discussed in Ref. [18].

IV. CONCLUSIONS

In this paper we have derived a generalized Dyson boson-fermion mapping of the most general collective fermion-pair algebra extended by single-fermion operators. The mapping is given as finite non-Hermitian boson-fermion images of fermion pairs and single-fermion operators, both expressed in terms of ideal boson and ideal fermion annihilation and creation operators.

The constructed mapping exploits an important freedom available in the boson-fermion space, namely, that suitable similarity transformations can be devised to shift between components of the boson sector and the ideal fermion-pair sector of the ideal space. The principal achievement of the present paper lies in finding an explicit form of such a similarity transformation which leads to collective even states being mapped onto boson states only, while collective odd states are mapped onto boson-fermion states with one ideal fermion only. In algebraic models we are thus able to account fully for the effects of the Pauli correlations between the odd fermion and the collective fermion pairs.

Although our results are based on stringent conditions of algebra closure, they may serve as guidelines for realistic cases where the exact closure need not be fulfilled. In particular, by using our boson-fermion mapping we have been able to derive microscopically some supersymmetric structures [6] which previously have only been introduced in a phenomenological way. Similarly, we can explicitly obtain the tensorial structure of the single-fermion operators defining the spectroscopic factors in boson-fermion models.

We have derived a general formula for the required similarity transformation as a power series in terms of a particular operator invariant with respect to the ideal-fermion core subalgebra. In two cases, for the SO(2N) and SU($\ell+1$) models, we can sum the series and give finite expressions for the images of single-fermion operators. We also showed that in the general case simple physical requirements allow the series to be truncated to low-order terms when one is, e.g., interested in calculating spectroscopic factors.

Our two-stage construction of boson-fermion images, viz. a first image deduced from a supercoherent state, followed by a similarity transformation, allowed us to demonstrate that the single-fermion images constructed in this way generally preserve anticommutation relations on the full ideal space, unlike previous constructions of many workers in this field where these relations were valid on the physical subspace only. We anticipate that our more general result will become important especially when boson-fermion calculations are carried out in the full ideal space as part of the program discussed and advocated in Ref. [7].

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APPENDIX A: STRUCTURE OF SINGLE-FERMION IMAGES IN THE GENERAL CASE

We consider here the second-order terms in the annihilation operator (2.24) containing two boson operators B,

$$\Bigg[\alpha_\nu\frac{1}{C_F-\hat{C}_F}\mathscr{A}^l B_l\frac{1}{C_F-\hat{C}_F}\mathscr{A}^m B_{m\wedge}-\frac{1}{C_F-\hat{C}_F}\mathscr{A}^l B_l\frac{1}{C_F-\hat{C}_F}\mathscr{A}^m B_{m\wedge}\alpha_\nu$$
$$-\left(\frac{1}{C_F-\hat{C}_F}\mathscr{A}^l B_{l\wedge}\right)\alpha_\nu\left(\frac{1}{C_F-\hat{C}_F}\mathscr{A}^m B_{m\wedge}\right)+\left(\frac{1}{C_F-\hat{C}_F}\mathscr{A}^l B_{l\wedge}\right)\left(\frac{1}{C_F-\hat{C}_F}\mathscr{A}^m B_{m\wedge}\right)\alpha_\nu\Bigg]|\psi)$$
$$=\Bigg(\alpha_\nu\frac{1}{C_F}\mathscr{A}^l B_l\frac{1}{C_F}\mathscr{A}^m B_m-\frac{1}{C_F}\mathscr{A}^l B_l\frac{1}{C_F}\mathscr{A}^m B_m\alpha_\nu-\frac{1}{C_F-\hat{C}_F}\mathscr{A}^l B_{l\wedge}\alpha_\nu\frac{1}{C_F}\mathscr{A}^m B_m$$
$$+\frac{1}{C_F-\hat{C}_F}\mathscr{A}^l B_{l\wedge}\frac{1}{C_F}\mathscr{A}^m B_m\alpha_\nu\Bigg)|\psi), \tag{A1}$$

where (2.26) is assumed. We furthermore investigate

$$C_F\alpha_\nu \frac{1}{C_F}\mathscr{A}^l B_l \frac{1}{C_F}\mathscr{A}^m B_m\,|\psi\rangle$$

$$=\mathscr{A}^i\alpha_\nu\mathscr{A}_i\frac{1}{C_F}\mathscr{A}^l B_l\frac{1}{C_F}\mathscr{A}^m B_m\,|\psi\rangle. \tag{A2}$$

To evaluate

$$\mathscr{A}_i\frac{1}{C_F}\mathscr{A}^l B_l\frac{1}{C_F}\mathscr{A}^m B_m\,|\psi\rangle \tag{A3}$$

we perform an auxiliary bosonization of the fermion operators $\mathscr{A}$ in the spirit of mapping (2.17). This time, however, we introduce, in addition to the auxiliary bosons b, also auxiliary fermions $\bar{\alpha}$ commuting with the bosons B and b and consider the mapping (2.17) in the alternative form

$$\mathscr{A}^j\leftrightarrow[\bar{\Lambda},b^j], \tag{A4a}$$

$$\mathscr{A}_j\leftrightarrow b_j, \tag{A4b}$$

$$[\mathscr{A}_i,\mathscr{A}^j]\leftrightarrow g\,\delta_i^j - c_{ik}^{jl}b^k b_l - \chi^j_{\mu\rho}\chi_i^{\nu\rho}\bar{\alpha}^\mu\bar{\alpha}_\nu, \tag{A4c}$$

$$\alpha^\nu\leftrightarrow\bar{\alpha}^\nu+\chi_i^{\nu\rho}b^i\bar{\alpha}_\rho-\frac{1}{C_f-\hat{C}_f}\bar{\mathscr{A}}^l b_l\wedge\chi_n^{\nu\rho}b^n\bar{\alpha}_\rho$$
$$+\chi_n^{\nu\rho}b^n\bar{\alpha}_\rho\frac{1}{C_f-\hat{C}_f}\bar{\mathscr{A}}^l b_l\wedge\ldots, \tag{A4d}$$

$$\alpha_\nu\leftrightarrow\bar{\alpha}_\nu-\frac{1}{C_f-\hat{C}_f}\bar{\mathscr{A}}^l b_l\wedge\bar{\alpha}_\nu+\bar{\alpha}_\nu\frac{1}{C_f-\hat{C}_f}\bar{\mathscr{A}}^l b_l\wedge\ldots, \tag{A4e}$$

with $\bar{\mathscr{A}}^l=\frac{1}{2}\chi^l_{\mu\nu}\bar{\alpha}^\mu\bar{\alpha}^\nu$, $C_f=\bar{\mathscr{A}}^l\bar{\mathscr{A}}_l$, and

$$\bar{\Lambda}=g b^l b_l-\tfrac{1}{4}c_{mn}^{kl}b^m b^n b_l b_k-\chi^k_{\mu\rho}\chi_l^{\nu\rho}b^l b_k\bar{\alpha}^\mu\bar{\alpha}_\nu$$
$$=(C_F)_{bf}+\tfrac{1}{4}c_{mn}^{kl}b^m b^n b_l b_k$$
$$=\tfrac{1}{2}(C_F)_{bf}+\tfrac{1}{2}b^l[\bar{\mathscr{A}}_l,\bar{\mathscr{A}}^k]b_k, \tag{A5}$$

where $(C_F)_{bf}$ is expressed using the operators b and $\bar{\alpha}$. The condition (2.26) in the b, $\bar{\alpha}$ space takes the form $b_l|\psi\rangle_{bf}=0$, that is, there are no bosons b contained in $|\psi\rangle_{bf}$.

It follows that (1.7) is mapped onto

$$b_i\frac{1}{(C_F)_{bf}-(\hat{C}_F)_{bf}}[\bar{\Lambda},b^l]B_l\frac{1}{(C_F)_{bf}-(\hat{C}_F)_{bf}}$$
$$\times[\bar{\Lambda},b^k]B_k\hat{}\,|n_b=0)$$
$$=b_i\tfrac{1}{2}\,b^l B_l b^k B_k|n_b=0)=B_i b^k B_k|n_b=0). \tag{A6}$$

This is the image of the original ideal space state $B_i(1C_F)\mathscr{A}^k B_k|\psi\rangle$, which follows after operating with $\bar{\Lambda}\bar{\Lambda}^{-1}$ on the final state above and completing the commutator $[\bar{\Lambda},b^k]$ to identify the image of $\mathscr{A}^k$. Consequently, we find from (A2) the relation

$$\alpha_\nu\frac{1}{C_F}\mathscr{A}^l B_l\frac{1}{C_F}\mathscr{A}^m B_m\,|\psi\rangle=\frac{1}{C_F}\mathscr{A}^i B_i\alpha_\nu\frac{1}{C_F}\mathscr{A}^m B_m\,|\psi\rangle. \tag{A7}$$

We note that it is not possible to repeat a similar derivation for the first-order terms as C_F acting on $\alpha_\nu(1/C_F)\mathscr{A}^l B_l|\psi\rangle$ has in general zero eigenvalues—this is evident for $|\psi\rangle\equiv|n_F=0)$, for example.

Returning to Eq. (A1) and using (A7), we have after some manipulation

$$\left(\frac{C_F-\hat{C}_F}{C_F-\hat{C}_F}\frac{1}{C_F}\mathscr{A}^i B_i\wedge\alpha_\nu\frac{1}{C_F}\mathscr{A}^m B_m-\frac{1}{C_F-\hat{C}_F}\mathscr{A}^l B_l\wedge\alpha_\nu\frac{1}{C_F}\mathscr{A}^m B_m+\frac{1}{C_F(C_F-\hat{C}_F)}\mathscr{A}^i B_i\wedge\mathscr{A}^m B_m\,\alpha_\nu\right)|\psi\rangle$$
$$=\left(-\frac{1}{C_F(C_F-\hat{C}_F)}\mathscr{A}^i B_i\wedge\mathscr{A}^k\,\alpha_\nu\,\mathscr{A}_k\frac{1}{C_F}\mathscr{A}^m B_m+\frac{1}{C_F(C_F-\hat{C}_F)}\mathscr{A}^i B_i\wedge\mathscr{A}^m B_m\,\alpha_\nu\right)|\psi\rangle$$
$$=0, \tag{A8}$$

because, using (A4) again, we have $\mathscr{A}_k(1/C_F)\mathscr{A}^m B_m|\psi\rangle=B_k|\psi\rangle$.

It is now straightforward to prove the cancellation for all higher-order terms by induction using

$$b_l\frac{1}{(C_F)_{bf}-(\hat{C}_F)_{bf}}[\bar{\Lambda},b^{i_1}]B_{i_1}\cdots\frac{1}{(C_F)_{bf}-(\hat{C}_F)_{bf}}[\bar{\Lambda},b^{i_n}]B_{i_n}\wedge|n_b=0)=b_l\frac{1}{n!}b^{i_1}B_{i_1}\cdots b^{i_n}B_{i_n}|n_b=0)$$
$$=B_l\frac{1}{(n-1)!}b^{i_1}B_{i_1}\cdots b^{i_{n-1}}B_{i_{n-1}}|n_b=0). \tag{A9}$$

Since annihilation and creation operators are linked by the commutation relations (2.2) it follows that the terms increasing the number of ideal fermions α by 3, 5, etc. also cancel in the creation operator (2.25) when acting on the class of states characterized by (2.26).

Nevertheless, it is instructive to see explicitly how the terms increasing the ideal fermion number by 3 in the creation operator image actually cancel. These terms appear in expression (2.25) which shows that the relevant boson operator parts have, respectively, the structure $B^k B_l B_i$ and that of a single-boson annihilation operator. The terms of the former type cancel as the structure is of exactly the same form as the second-order part of the annihilation operator image for which we have demonstrated the cancellation above. The latter part can be written as

$$\left(\alpha^\nu \frac{1}{C_F}\mathscr{A}^l B_l - \frac{1}{C_F-\hat{C}_F}\mathscr{A}^l B_{l\wedge}\alpha^\nu - \frac{1}{C_F-\hat{C}_F}\mathscr{A}^l{}_\wedge[\mathscr{A}_l,\alpha^\nu]\frac{1}{C_F}\mathscr{A}^m B_m + \frac{1}{C_F(C_F-\hat{C}_F)}\mathscr{A}^m B_{m\wedge}\mathscr{A}^l[\mathscr{A}_l,\alpha^\nu]\right.$$
$$\left. + \frac{1}{C_F(C_F-\hat{C}_F)}\mathscr{A}^l{}_\wedge\mathscr{A}^m B_m[\mathscr{A}_l,\alpha^\nu]\right)|\psi)$$
$$=\left(\frac{1}{C_F}\mathscr{A}^l[\mathscr{A}_l,\alpha^\nu]\frac{1}{C_F}\mathscr{A}^m B_m - \frac{1}{C_F(C_F-\hat{C}_F)}\mathscr{A}^m B_{m\wedge}\mathscr{A}^k\mathscr{A}_k\alpha^\nu - \frac{1}{C_rmF-\hat{C}_F}\mathscr{A}^l{}_\wedge[\mathscr{A}_l,\alpha^\nu]\frac{1}{C_F}\mathscr{A}^m B_m\right.$$
$$\left. + \frac{1}{C_F(C_F-\hat{C}_F)}\mathscr{A}^m B_{m\wedge}\mathscr{A}^l[\mathscr{A}_l,\alpha^\nu] + \frac{1}{C_F(C_F-\hat{C}_F)}\mathscr{A}^l{}_\wedge\mathscr{A}^m B_m[\mathscr{A}_l,\alpha^\nu]\right)|\psi)$$
$$=\left(\frac{C_F-\hat{C}_F}{C_F-\hat{C}_F}\frac{1}{C_F}\mathscr{A}^l{}_\wedge[\mathscr{A}_l,\alpha^\nu]\frac{1}{C_F}\mathscr{A}^m B_m - \frac{1}{C_F-\hat{C}_F}\mathscr{A}^l{}_\wedge[\mathscr{A}_l,\alpha^\nu]\frac{1}{C_F}\mathscr{A}^m B_m + \frac{1}{C_F(C_F-\hat{C}_F)}\mathscr{A}^l{}_\wedge\mathscr{A}^m B_m[\mathscr{A}_l,\alpha^\nu]\right)|\psi)$$
$$=\left(-\frac{1}{C_F(C_F-\hat{C}_F)}\mathscr{A}^l{}_\wedge\mathscr{A}^k\mathscr{A}_k[\mathscr{A}_l,\alpha^\nu]\frac{1}{C_F}\mathscr{A}^m B_m + \frac{1}{C_F(C_F-\hat{C}_F)}\mathscr{A}^l{}_\wedge\mathscr{A}^m B_m[\mathscr{A}_l,\alpha^\nu]\right)|\psi)=0\,, \qquad \text{(A10)}$$

using

$$\left[\alpha^\nu,\frac{1}{C_F}\right]=\frac{1}{C_F}[C_F,\alpha^\nu]\frac{1}{C_F} \qquad \text{(A11)}$$

and

$$C_F[\mathscr{A}_l,\alpha^\nu]\frac{1}{C_F}\mathscr{A}^m B_m|\psi)=\mathscr{A}^m B_m[\mathscr{A}_l,\alpha^\nu]|\psi) \qquad \text{(A12)}$$

derived from expressions (A4) and the definition (2.16).

APPENDIX B: STRUCTURE OF SINGLE-FERMION IMAGES IN THE SO(5) CASE

Here we exemplify the result of Appendix I in the case of the SO(5) model discussed in Sec. III C. We show that higher-order terms in the single-fermion images cancel for the second-order part of the annihilation operator acting in the space where no ideal fermions are present. To be explicit, we show that

$$\left[\alpha_p\frac{1}{C_F-\hat{C}_F}W\frac{1}{C_F-\hat{C}_F}W_\wedge - \left(\frac{1}{C_F-\hat{C}_F}W_\wedge\right)\alpha_p\left(\frac{1}{C_F-\hat{C}_F}W_\wedge\right)\right]|n_F=0)=0\,. \qquad \text{(B1)}$$

The left-hand side follows from (A1), W is given by (3.17), and C_F by (3.18). It can be derived that

$$\frac{1}{C_F-\hat{C}_F}W\frac{1}{C_F-\hat{C}_F}W_\wedge|n_F=0)=\left(\frac{1}{\Omega(\Omega-1)}(\mathscr{L}_+\mathscr{S}_+B_fB_p+\mathscr{K}_+\mathscr{S}_+B_fB_h+\tfrac{1}{2}\mathscr{L}_+\mathscr{L}_+B_pB_p+\tfrac{1}{2}\mathscr{K}_+\mathscr{K}_+B_hB_h)\right.$$
$$+\frac{1}{\Omega(\Omega-1)(2\Omega+1)}(\Omega\mathscr{S}_+\mathscr{S}_++\mathscr{K}_+\mathscr{L}_+)B_fB_f$$
$$\left.+\frac{1}{\Omega(\Omega-1)(2\Omega+1)}(\mathscr{S}_+\mathscr{S}_++(2\Omega-1)\mathscr{K}_+\mathscr{L}_+)B_pB_h\right)|n_F=0) \qquad \text{(B2)}$$

and

$$\left(\frac{1}{C_F-\hat{C}_F}W_\wedge\right)\alpha_p\left(\frac{1}{C_F-\hat{C}_F}W_\wedge\right)|n_F=0)=\Bigg(\frac{1}{\Omega(\Omega-1)}[(\mathscr{L}_+[\alpha_p,\mathscr{S}_+]+\mathscr{S}_+[\alpha_p,\mathscr{L}_+])B_fB_p+\mathscr{K}_+[\alpha_p,\mathscr{S}_+]B_fB_h$$
$$+\mathscr{L}_+[\alpha_p,\mathscr{L}_+]B_pB_p]+\frac{1}{\Omega(\Omega-1)(\Omega+\frac{1}{2})}$$
$$\times(\Omega\mathscr{S}_+[\alpha_p,\mathscr{S}_+]+\tfrac{1}{2}\mathscr{K}_+[\alpha_p,\mathscr{L}_+])B_fB_f+\frac{1}{\Omega(\Omega-1)(\Omega+\frac{1}{2})}$$
$$\times\{\mathscr{S}_+[\alpha_p,\mathscr{S}_+]+(\Omega-\tfrac{1}{2})\mathscr{K}_+[\alpha_p,\mathscr{L}_+]\}B_pB_h\Bigg)|n_F=0)\quad. \tag{B3}$$

From (B2) and (B3) it is apparent that (B1) is fulfilled.

It is interesting to note that the above derivation is valid even for spurious states in the boson-fermion space. Consider, e.g., $\Omega=1$. The two-boson configuration

$$(B^fB^f+B^pB^h)|0) \tag{B4}$$

is then a spurious state [7]. We observe that expressions (3.21), (2.2), and (2.6) are singular for $\Omega=1$. However, for $\Omega=1$ these equations are simply not applicable as they stand, because

$$(\mathscr{S}_+\mathscr{S}_++\mathscr{L}_+\mathscr{K}_+)|0)=0 \tag{B5}$$

and

$$(\mathscr{S}_+\alpha^h+\mathscr{K}_+\alpha^p)|0)=0. \tag{B6}$$

Consequently, instead of (B2) we have here

$$\frac{1}{C_F-\hat{C}_F}W\frac{1}{C_F-\hat{C}_F}W_\wedge|n_F=0)$$
$$=\frac{4}{3\Omega(2\Omega+1)}(\tfrac{1}{2}\mathscr{S}_+\mathscr{S}_++\mathscr{K}_+\mathscr{L}_+)$$
$$\times(-\tfrac{1}{2}B_fB_f+B_pB_h)|n_F=0), \tag{B7}$$

and (2.6) should be replaced by

$$\frac{1}{C_F-\hat{C}_F}W_\wedge\alpha_p\frac{1}{C_F-\hat{C}_F}W_\wedge|n_F=0)$$
$$=\frac{4}{3\Omega(2\Omega+1)}(\mathscr{S}_+[\alpha_p,\mathscr{S}_+]+\mathscr{K}_+[\alpha_p,\mathscr{L}_+])$$
$$\times(-\tfrac{1}{2}B_fB_f+B_pB_h)|n_F=0), \tag{B8}$$

from which we can see that the second-order contributions to the single-fermion annihilation operator image cancel even in this case. Moreover, it is apparent that the operators that act on $|n_F=0)$ in both (B7) and (B8) give a zero state when they act on the spurious state (B4).

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