Structural properties of hot deformed ^{122}Xe at high spins

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The ^{122}Xe nucleus is investigated using statistical theory for studying intrinsic properties as a function of temperature and spin assuming the nucleus to move in a triaxially deformed Nilsson potential. The single particle level density parameter is extracted as a function of temperature for various spins and is found to reach a constant value at higher temperature. The excitation energy is calculated as a function of angular momentum for different entropies and temperatures. The spin cutoff parameter as a function of angular momentum is determined for various temperatures from the rotational energy. The shell correction as a function of angular momentum is determined. The variation of proton and neutron Fermi energies with temperature and angular momentum is studied. The effect of the variation of nuclear level density with excitation energy on nuclear temperature is verified for different spins.

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The influence of rotation on nuclear structure [1–3] and shape has opened up a large field for studying intrinsic properties of nuclei. The production of hot nuclei has caused considerable interest in the study of nuclear shapes and recently a study of such properties in the $A = 120$ region was done by Ragnarsson *et al.* [4]. The interplay of angular momentum generated by the alignment of single particle spins or by the increase of collective rotational angular momentum due to deformation with temperature is very important in understanding the nuclear structure. The experimental searches [5–7] for a high spin shape transition in the barium nuclei are inconclusive. In ^{121}I nuclei, oblate to prolate shape hindrance has been studied and for $Z = 53$ iodine nuclei shape coexistence has been reported [8,9]. A dynamical triaxial deformation in the mass region $A = 120$–140, in particular, Xe and Ba nuclei, is observed [10]. In this work, an attempt has been made to study the intrinsic properties of the ^{122}Xe nucleus as functions of temperature, spin, and deformation.

The statistical theory given in Ref. [11] is used for this purpose and the applicability of thermodynamical concepts at very high temperature is known from Ref. [12]. The main assumption for statistical analysis is microscopic equilibrium which means that all the states with the same excitation energy E^* are equally populated. The single particle level density parameter is studied as a function of temperature and spin [13] and is found to reach a constant value at higher temperatures for all spins.

The constant entropy lines are drawn in the excitation energy E^* versus the angular momentum plane. From these lines it is evident that backbending and yrast traps are absent in ^{122}Xe.

By calculating the rotational energy as a function of angular momentum, the spin cutoff parameter is determined for various spins at different temperatures. The variation of proton and neutron Fermi energies with temperature and angular momentum is studied. The shell correction [14] as a function of angular momentum is calculated for this nucleus by extending the work of Ramamurthy, Kapoor, and Kataria [15] from nonrotating to rotating nuclei.

Since the inputs for the statistical theory are the microscopic single particle levels [16] corresponding to the triaxially deformed Nilsson harmonic oscillator potential [17], the results exhibit the effect of shell structure of the system at different deformations. The deformed potential [18,19] used here is given by

$$V = -\kappa\hbar\omega_0[2\mathbf{l}_t \cdot \mathbf{S} + \mu(l_t^2 - \langle l_t^2\rangle_N)], \tag{1}$$

where ω_0 is the harmonic oscillator parameter which incorporates the principle of volume conservation for nuclei deformed from spherical shapes. The intrinsic nucleon spin is represented by $\mathbf{S}$ while $\mathbf{l}_t$ represents the orbital angular momentum in the stretched coordinate basis.

The κ, μ pair used for generating the single particle levels are as given in Ref. [18] and differ for different oscillator shells corresponding to $N = 4$, 5, and 6 for protons. The parameters are appropriate since they reproduce the experimental bandhead energies more accurately than the standard parameter set given in Ref. [20]. However, for neutrons, the standard parameter set given in Ref. [20] is used.

The deformation parameters δ and θ are varied [21] in the range $\delta = 0.0$ to 0.6 with $\Delta\delta = 0.1$ and $\theta = -180°$ to $-120°$ with $\Delta\theta = 20°$. The levels generated are up to $N = 8$ which is found to be sufficient for the range of temperatures used in this calculation. The necessity of renormalizing the total energy is avoided here since only the energy differences are involved and not the actual magnitude of energies.

We start with the grand canonical partition function for a system of N neutrons and Z protons in a state of total angular momentum M along the direction of the

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rotation axis. The statistical properties of the system are contained in the grand partition function

$$Q(\alpha_Z, \alpha_n, \beta, \gamma) = \sum \exp(-\beta E_i + \alpha_Z Z_i + \alpha_n N_i + \gamma M_i). \tag{2}$$

The Lagrangian multipliers α_Z, α_n, and γ conserve the proton number, neutron number, and total angular momentum M along the Z axis for a given temperature $T = 1/\beta$.

In terms of single particle energies ε_i and the spin projections m_i the conservation equations are

$$\begin{aligned} \langle Z \rangle &= \sum n_i^Z = \sum [1 + \exp(-\alpha_Z + \beta\varepsilon_i^Z + \gamma m_i^Z)]^{-1}, \\ \langle N \rangle &= \sum n_i^n = \sum [1 + \exp(-\alpha_n + \beta\varepsilon_i^n + \gamma m_i^n)]^{-1}, \\ \langle E \rangle &= \sum n_i^n \varepsilon_i^n + n_i^Z \varepsilon_i^Z, \\ \langle M \rangle &= \sum n_i^n m_i^n + n_i^Z m_i^Z, \end{aligned} \tag{3}$$

where n_i is the occupation probability of the ith shell. These equations fix the Lagrangian multipliers α_Z, α_n, and γ. The Fermi energies for protons and neutrons are given by the Lagrangian multipliers α_Z and α_n, respectively.

The entropy of the system is obtained as

$$S = S_Z + S_n,$$

where

$$S_Z = -\sum [n_i^Z \ln n_i^Z + (1 - n_i^Z) \ln(1 - n_i^Z)] \tag{4}$$

with a similar equation for neutrons.

The excitation energy $E^*(M, T, \delta, \theta)$ is obtained using the equation

$$E^*(M, T, \delta, \theta) = E(M, T, \delta, \theta) - E_0, \tag{5}$$

where $E_0 = E(0, 0, \delta, \theta)$ is the ground state energy of the nucleus. The level density [22] of the system $\rho(M, E^*)$ as obtained in Ref. [23] is

$$\rho(M, E^*, \delta, \theta) = \exp S(M, E^*, \delta, \theta)/S_{\max}. \tag{6}$$

The normalization factor $S_{\max}$ depends on the dimensionality of phase space which is the number of eigenstates used, and the single particle level density parameter $a(M, T, \delta, \theta)$ as a function of angular momentum M and temperature T is extracted using the equation

$$a(M, T, \delta, \theta) = S^2(M, T, \delta, \theta)/4E^*(M, T, \delta, \theta). \tag{7}$$

In our calculations the temperature is varied from 0.2 to 1 MeV and for each temperature the excitation energy and the entropy are computed as a function of the deformation parameters δ and θ for various spins. The free energy $F = E - TS$ is then minimized with respect to δ and θ. For a given spin, the entropy is different for different temperatures. Constant entropy lines which are equal to constant level density lines are drawn in the plane of E^* and M.

The rotational energies are calculated using the relation

$$E_{\rm rot} = E(M, T) - E(0, T) \tag{8}$$

for minimized deformation parameters δ and θ for different angular momenta at a particular temperature T. The rotational energy is found to increase with angular momentum whereas its dependence on temperature is small.

The spin cutoff parameter is estimated from the rotational energy for various temperatures from the expression

$$\sigma^2 = T(I/\hbar^2), \tag{9}$$

where I is the moment of inertia and is given by the equation

$$(I/\hbar^2) = M/(dE_{\rm rot}/dM). \tag{10}$$

The effect of the variation of nuclear level density [24] with excitation energy on nuclear temperature is found to be in accordance with the relation

$$1/T = d/dE^*[\ln \rho(E^*)] \tag{11}$$

for different spins.

The occupation probabilities n_i for protons and neutrons are displayed in Figs. 1 and 2 as function of the single particle energies for various temperatures and spins. In Fig. 1 the occupation probability of protons and neutrons is plotted for the spin $M = 0$ at different temperatures. It is evident that at low temperatures the occupation probability is unity up to the Fermi energy and thereafter it becomes zero. As the temperature increases, the occupation probabilities for the levels below the Fermi energy decrease from unity and for the levels above the Fermi energy increase from zero. However, this

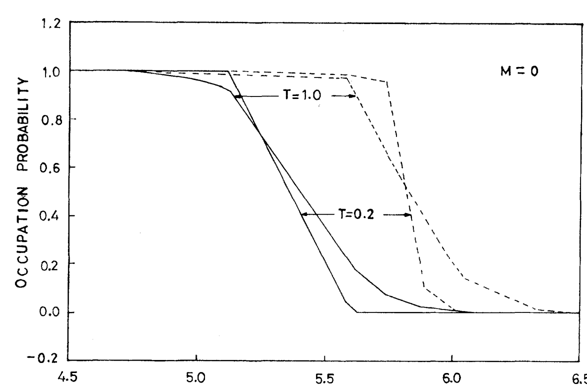

FIG. 1. Occupation probabilities for protons and neutrons at two different temperatures as a function of single particle energies corresponding to the spin $M = 0$ for ^{122}Xe. The continuous curves are for protons and the dashed curves are for neutrons.

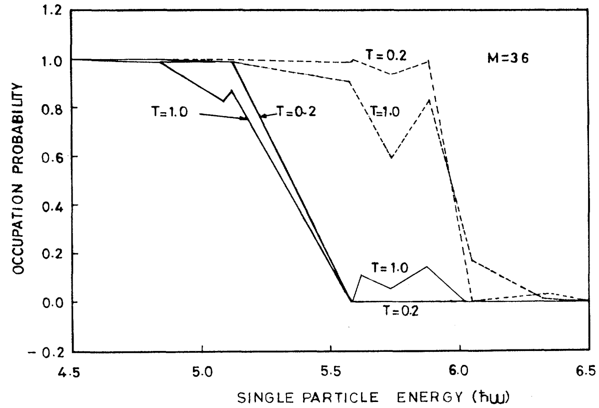

FIG. 2. As in Fig. 1 corresponding to the spin $M = 36$ for ^{122}Xe.

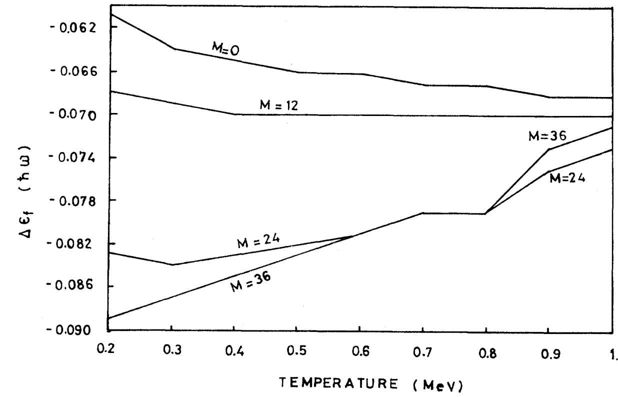

FIG. 4. As in Fig. 3 for neutrons in ^{122}Xe.

happens only for a few levels around the Fermi energy. The case for neutrons is similar. In Fig. 2 the same is depicted for spin $M = 36\hbar$. It is evident that for higher spins at higher temperatures more fluctuations are seen for both protons and neutrons.

These curves are helpful in comprehending the way of generating the net spin of the system which can be obtained from the graph as

$$M = \int dn_i^Z m_i^Z + \int dn_i^n m_i^n . \tag{12}$$

In Figs. 3 and 4, the change in Fermi energy $\Delta\varepsilon_f$ is plotted against temperature for protons and neutrons, respectively, for various spins. It is found from Fig. 3 that the proton Fermi energy decreases very much for low temperatures and at higher temperatures it almost reaches the original value. However, for very high spins $M = 36\hbar$ it increases with temperature.

The decrease in neutron Fermi energy with temperature for various spins is displayed in Fig. 4. At higher spins this decrease is more and at higher temperatures the Fermi energy reaches a constant value.

Figure 5 shows the variation of the Lagrangian multiplier γ as a function of angular momentum for various temperatures. At low temperatures the Lagrangian multiplier falls very rapidly with angular momentum whereas at higher temperatures it remains almost constant. The sensitiveness of the Lagrangian multiplier at low temperatures may be due to the pairing effects which we have not considered.

The single particle level density parameter as a function of temperature and angular momentum is presented in Figs. 6 and 7 for the nucleus ^{122}Xe. These curves are drawn by minimizing the free energy for triaxial deformation. From Fig. 6 it is evident that at low temperatures the single particle level density parameter increases steeply and at higher temperature of $T = 3$ MeV it reaches almost the constant value $a \simeq A/10$ predicted experimentally [25,26]. The effect of rotation on the single particle level density parameter is very pronounced at low temperatures. The single particle level density fluctuations are different for different angular momentum states at low temperatures because the shell structure plays a major role at these temperatures. Figure 7 shows that for a given temperature the single particle level density parameter decreases with increase in angu-

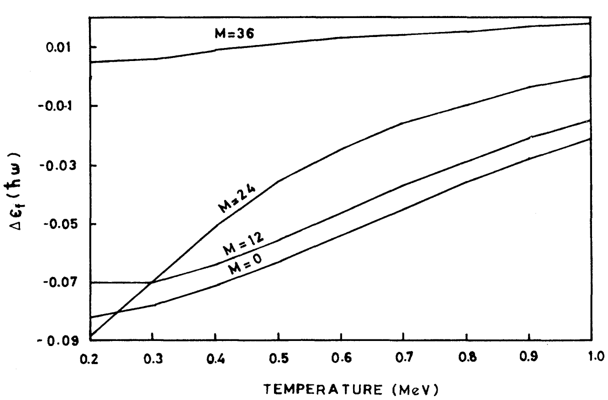

FIG. 3. The variation of Fermi energy of protons in ^{122}Xe as a function of temperature and spin.

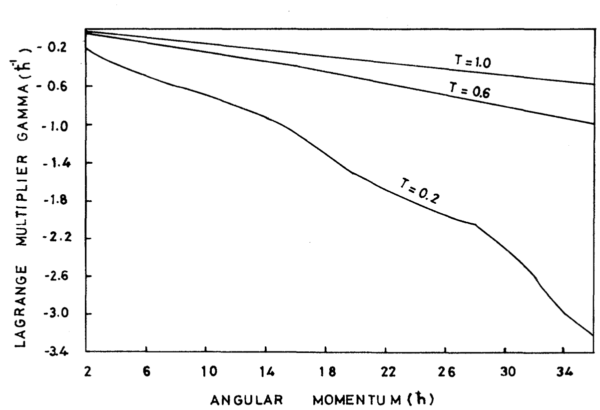

FIG. 5. Spin conserving Lagrangian multiplier γ as a function of angular momentum for various temperatures in ^{122}Xe.

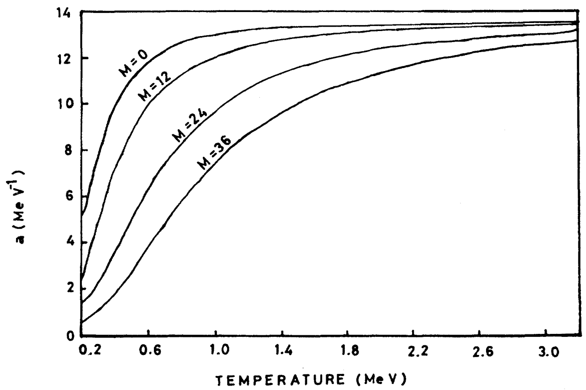

FIG. 6. Single particle level density parameter as a function of temperature for various spins of the nucleus ^{122}Xe. The numbers on the curves refer to the angular momentum of the system.

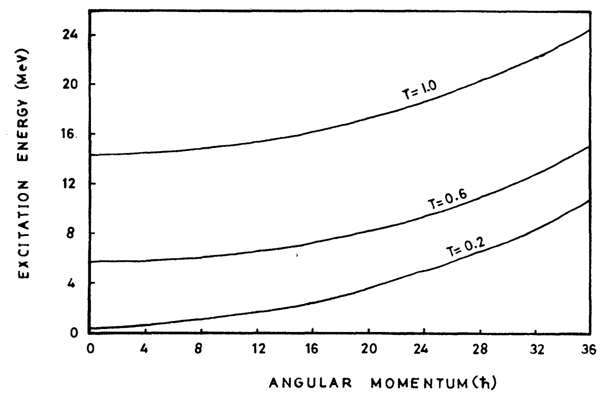

FIG. 8. Excitation energy as a function of angular momentum for different temperatures in ^{122}Xe.

lar momentum. However, at a very high temperature of $T = 3$ MeV it is almost constant for all spins.

Figures 8 and 9 show the constant temperature and constant entropy lines, respectively, in the excitation energy versus angular momentum plane drawn by minimizing the free energy for various deformations. It is to be noted that the excitation energy calculated includes the rotational energy. No yrast trap is seen in these and there is no possibility of the system being trapped in the pocket of a yrast trap in the process of deexcitation along the constant entropy lines. Both the constant temperature and constant entropy lines exhibit similar behavior. As the temperature increases the entropy as well as the excitation energy increases and these constant entropy lines are useful in determining the phase space available for the system. These lines are found to be roughly at constant energy above the yrast line as stated in Refs. [27,28].

The spin cutoff parameter determined from the rotational energy is plotted in Fig. 10 as a function of an-

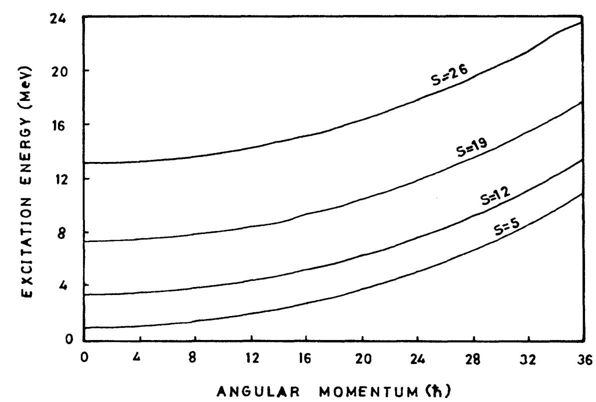

FIG. 9. Constant entropy lines for ^{122}Xe.

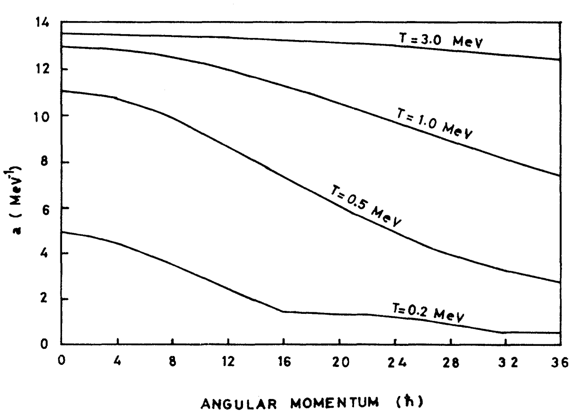

FIG. 7. Single particle level density parameter as a function of angular momentum for various temperatures in ^{122}Xe.

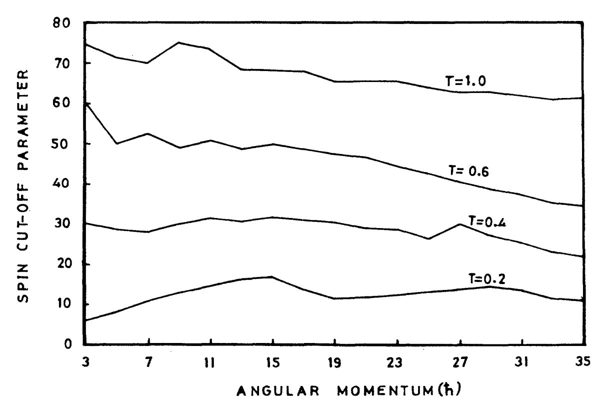

FIG. 10. Spin cutoff parameter as a function of angular momentum for different temperatures in ^{122}Xe.

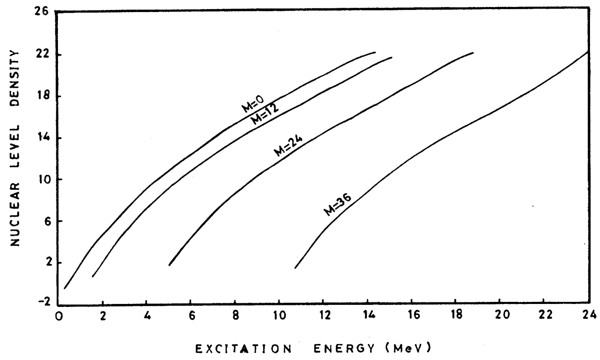

FIG. 11. The dependence of nuclear level density on excitation energy for various angular momenta of the nucleus ^{122}Xe.

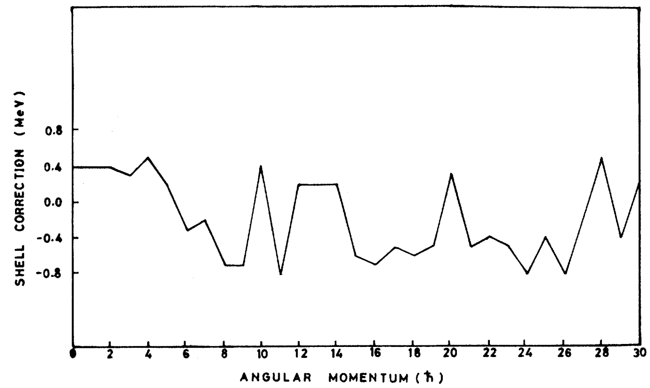

FIG. 12. Shell correction as a function of angular momentum for equilibrium deformation in ^{122}Xe.

gular momentum for different temperatures. The spin cutoff parameter at $T = 0.4$ MeV shows a minimum at $M = 25\hbar$ and this may be suggestive of the state at $M = 25\hbar$ being distorted by the noncollective states [29].

Assuming that all states with the excitation energy E^* are equally populated, the nuclear level density can be expressed as a function of excitation energy. Figure 11 shows the nuclear level density as a function of E^* for various spins. It is found that the nuclear level density increases with excitation energy for all spins. For building up higher spins at a given nuclear level density a higher excitation energy is needed. The nuclear temperature estimated using Eq. (11) is found to be in accordance with the temperature used in this calculation.

The shell correction is calculated as a function of angular momentum for minimized free energy values. For this purpose the thermodynamical method for nonrotating nuclei suggested by Ramamurthy, Kapoor, and Kataria [15] is extended to rotating nuclei. While calculating the shell correction for rotating nuclei the rotational energy has to be subtracted from the excitation energy given by Eq. (5). The excitation energy without the rotational energy is

$$E_{\text{exc}}(M,T) = E(M,T) - E(M,0). \qquad (13)$$

By plotting S^2 versus E_{exc} for a particular angular momentum M and using the relation

$$S^2 = 4a[E_{\text{exc}}(M,T) + \Delta E_{\text{shell}}], \qquad (14)$$

the shell correction ΔE_{shell} for a particular M is calculated. The shell correction ΔE_{shell} is the intercept on the E_{exc} axis when large temperature values of S^2 are extrapolated towards low temperatures. This extrapolation is valid because at large temperatures the shell correction vanishes. By repeating the procedure for different angular momenta the shell correction is calculated as a function of angular momentum. The variations in shell correction energy are shown in Fig. 12. It is found that the shell correction shows fluctuations with angular momentum.

Since there is a conceptual difficulty in comprehending the nuclear level density of a deformed excited nucleus because of the possibility of the excited nucleus to have different deformations [30], the calculations are performed by minimizing the free energy for various deformations δ and θ. It is observed that the free energy minimum for the xenon nucleus occurs for the oblate deformed shape corresponding to $\delta = 0.2$ and $\theta = 180°$ at all temperatures and spins. This means that the shape of ^{122}Xe is oblate and remains the same even at high spins.

These oblate states built from the spin alignment of specific single particle orbitals are observed to be specially favored states. However, a prolate collective to oblate noncollective shape change around $M = 20\hbar$ has been reported [29]. The value of the shell correction at $M = 20\hbar$ may be supportive of this shape change due to dynamical triaxial deformation. Similar shape changes may be expected around $M = 10\hbar$ and $M = 28\hbar$. Clearly more experimental and theoretical investigations are needed to study the competition and interaction between these two different shapes.

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