Instant form dynamics of one particle exchange models

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A general procedure for constructing instant form, one particle exchange models for two particle systems is developed. The procedure entails the construction of a mass operator which when used in conjunction with a free spin operator and a free Newton-Wigner position operator leads to an exactly Poincaré invariant model. The method is applied to a simple model for s-wave pion-nucleon scattering. This model is derived from a quantum field theory which describes the interaction between pions, nucleons, and sigma mesons through the virtual processes $N \Leftrightarrow N+\pi$, $\pi \Leftrightarrow \pi+\sigma$, and $N \Leftrightarrow N+\sigma$. The instant form version of this exchange model is compared with a front form version that was constructed previously. With the procedures used to ensure Poincaré invariance, the instant form two-particle potentials are of the same form as the front form potentials; however, the pion-nucleon propagators that appear in the two-particle Lippmann-Schwinger equations are not the same. The instant form and front form models are fit to the same s-wave pion-nucleon phase shifts, and the resulting parameters are compared.

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I. INTRODUCTION

Particle exchange models have played an important role in strong interaction physics since Yukawa proposed that the nucleon-nucleon interaction is due to the exchange of a massive particle [1]. Almost immediately, Yukawa's scalar meson field theory was extended to vector fields [2]; to pseudoscalar, pseudovector, and tensor fields [3], and to combinations of vector and pseudoscalar fields [4]. In recent years sophisticated exchange models have been constructed for the nucleon-nucleon system and the pion-nucleon system.

One boson exchange models of the nucleon-nucleon interaction usually include the exchange of π, η, ρ, ω, δ, and σ mesons [5]. In such models the use of the σ meson is often viewed as a phenomenological way of accounting for multimeson exchanges that are not included explicitly. More sophisticated models take into account processes such as 2π exchange and $\pi\rho$ exchange [5].

Particle exchange models for the pion-nucleon system often include contributions from direct and crossed nucleon (N), delta (Δ), Roper (N^*), etc., diagrams, as well as σ and ρ exchange [6–9]. The s-wave model considered here only includes the direct and crossed nucleon diagrams and σ exchange. Our purpose here is to develop a general procedure for constructing *instant form* models based on particle exchange mechanisms and to compare the results with those obtained previously [10] in a *front form* model based on the exchange mechanisms considered here.

The basic ingredients of a particle exchange model are its vertices. The nature of the coupling at a vertex is specified by a Lorentz invariant, Lagrangian density, as well as a form factor or vertex function. The purpose of the vertex function is to take into account the extension of a strong interaction vertex, which in general involves composite particles. In most cases these vertex functions are phenomenological.

These vertices can be related to the observables of a hadronic system in a manifestly covariant way by using the Bethe-Salpeter equation [11] or one of its three-dimensional reductions. The three-dimensional reductions that are most widely used are due to Blankenbecler and Sugar [12], and to Gross [13]. Tjon and co-workers [14] have employed both the Bethe-Salpeter equation and the Blankenbecler-Sugar equation. The most recent application of the Gross equation to the two-nucleon system is given in Ref. [15] and to the pion-nucleon system in Ref. [8].

Holinde and co-workers [5] have made extensive use of time-ordered perturbation theory in developing the Bonn meson exchange model for the nucleon-nucleon interaction, starting from a set of meson-nucleon vertices. Johnson's method of folded diagrams [16] has been used to eliminate the energy dependence of the amplitudes obtained from time-ordered perturbation theory. This leads to instantaneous interactions which can be conveniently used in calculating the properties of system with more than two nucleons. This approach is not manifestly covariant; nor does it fall within the framework of one of Dirac's *forms* of relativistic quantum mechanics [17]. It should be noted, however, that the potentials and Lippmann-Schwinger equation obtained in a recent *front form*, one boson exchange model of the two-nucleon system [18] turn out to be almost identical to those employed in the Bonn one boson exchange models [5].

The purpose of the present work is to introduce a framework for developing particle exchange models within the context of the *instant form* of relativistic quantum mechanics [17,19]. Relativistic quantum mechanics arises when it is required that the state vectors of a quantum mechanical system transform according to a unitary representation of the Poincaré group. The subgroup of continuous transformations, the so-called *proper* subgroup, involves ten generators, the four components P_μ ($\mu=0,1,2,3$) of the four-momentum operator and the six independent components $J_{\mu\nu}=-J_{\nu\mu}$ of the angular momentum tensor. These ten operators must satisfy a set of communication relations, which is usually referred to as the *Poincaré algebra*. Several subsets of these generators have the property that they satisfy a *closed* subset of these commutation relations and are therefore associated with a subgroup of the proper Poincaré transformations. Some of these subgroups are associated with three-

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dimensional hypersurfaces in Minkowski space that do not contain timelike directions. Each *form* of relativistic quantum mechanics is associated with such a hypersurface and its corresponding subgroup [17,19]. These subgroups are called kinematic subgroups [20] or stability groups [21]. The most obvious form, i.e., the *instant form,* is based on the hypersurface $t=\text{const}$, while the light front form is based on the *null plane* $ct+z=0$. In each form of relativistic quantum mechanics, the subset of generators associated with the form's hypersurface is chosen to be noninteracting, while the remaining generators contain interactions.

In the instant form the three-momentum **P** and the angular momentum **J** are noninteracting, while the Hamiltonian H and **K**, the generator of rotationless boosts, contain interactions. A rotationless boost is a Lorentz transformation which relates two inertial frames moving relative to each other with the corresponding spatial axes parallel. The instant form is like nonrelativistic quantum mechanics in that the state vectors are specified on the $t=\text{const}$ hypersurfaces; however, the nonrelativistic boost operators, which generate Galilean transformations, do not contain interactions.

In constructing instant form models, it is convenient to work with the set of operators $\{M,\mathbf{P},\mathcal{J},\mathbf{X}\}$; where M, $\mathcal{J}$, and $\mathbf{X}$ are the mass operator, the spin operator, and the Newton-Wigner position operator [19], respectively. These operators satisfy simpler commutation rules than the generators. In fact, the only nonzero commutators are given by (2.10) and (2.11). The ten generators of the Poincaré group can be expressed in terms of the set $\{M,\mathbf{P},\mathcal{J},\mathbf{X}\}$, and it can be shown that if the members of the set $\{M,\mathbf{P},\mathcal{J},\mathbf{X}\}$ satisfy the correct commutation relations, then the generators expressed in terms of them satisfy the Poincaré algebra.

Here we will obtain an instant form mass operator for the pion-nucleon system based on the virtual processes $N\Leftrightarrow N+\pi$, $\pi\Leftrightarrow\pi+\sigma$, and $N\Leftrightarrow N+\sigma$. When this mass operator M is combined with the **P**, $\mathcal{J}$, and **X** of the noninteracting pion-nucleon system, it leads to a set $\{M,\mathbf{P},\mathcal{J},\mathbf{X}\}$ which satisfies the correct commutation relations. This in turn guarantees that the model is exactly Poincaré invariant. The procedures used to develop this model are of a general nature and can be used to derive instant form models for the pion-nucleon system, which include other exchange processes, and to derive models for other systems such as the nucleon-nucleon system.

The outline of the paper is as follows. Those aspects of relativistic quantum mechanics that are necessary for an understanding of the instant form are reviewed and summarized in Sec. II. In Sec. III a two-particle basis is constructed in which the **P**, $\mathcal{J}$, and **X** of a noninteracting system have a particularly simple representation. The method for obtaining an instant form, one particle exchange model for the pion-nucleon system is presented in Sec. IV. Relative three-momentum variables are introduced, and a method for ensuring the Poincaré invariance of such a model is given. Phenomenological form factors for the πNN, $\sigma\pi\pi$, and σNN vertices are constructed in Sec. V. The numerical calculations are presented in Sec. VI, and a comparison of the instant form model and the corresponding front form model is made. A discussion of the results and suggestions for future work are given in Sec. VII.

Throughout units in which $\hbar=c=1$ are used.

II. GENERAL BACKGROUND

The elements (a,b) of the Poincaré a group consist of the Lorentz transformations a and spacetime translations b that appear in the inhomogeneous Lorentz transformations $x'=ax+b$. In the passive interpretation, x and x' refer to the spacetime coordinates of the same event in two different inertial frames, i.e., the x and x' frames, respectively.

The proper subgroup of the Poincaré group involves only continuous transformations. This subgroup is a ten-parameter group; four parameters are associated with translations in four-dimensional spacetime, while the other six are associated with "rotations" in spacetime. Three of these six can be associated with true rotations in three-dimensional space, while the other three can be associated with rotationless boosts. In what follows the expression *Poincaré group* refers only to the subgroup of continuous transformations.

In a relativistic quantum-mechanical model formulated on a Hilbert space, there exists a set of operators $U(a,b)$ which form a unitary representation of the Poincaré group. If $|\Psi\rangle$ and $|\Psi'\rangle$ are states associated with the x and x' frames, respectively, they are related by $|\Psi\rangle=U(a,b)|\Psi\rangle$. If $|\Phi\rangle$ and $|\Psi\rangle$ are two different states, clearly $|\langle\Phi'|\Psi'\rangle|^2=|\langle\Phi|\Psi\rangle|^2$; i.e., the probabilities are invariant.

For the infinitesimal transformation

$$a_{\mu\nu}=g_{\mu\nu}+\varepsilon_{\mu\nu} \quad (\varepsilon_{\mu\nu}=-\varepsilon_{\nu\mu}), \quad b_\mu=\varepsilon_\mu , \tag{2.1}$$

where $g_{\mu\nu}$ is the metric tensor and the epsilons are the infinitesimals, the corresponding unitary operator can be written in the form

$$U(a,b)=1+i\varepsilon_\mu P^\mu-\frac{i}{2}\varepsilon_{\mu\nu}J^{\mu\nu} \quad (J^{\mu\nu}=-J^{\nu\mu}) . \tag{2.2}$$

In order for the operators $U(a,b)$ to form a unitary representation of the Poincaré group, the ten Hermitian generators $\{P_\mu,J_{\mu\nu}\}$ must satisfy the commutation rules

$$[P_\mu,P_\nu]=0 , \tag{2.3a}$$

$$[J_{\mu\nu},P_\rho]=i(g_{\nu\rho}P_\mu-g_{\mu\rho}P_\nu) , \tag{2.3b}$$

$$[J_{\mu\nu},J_{\rho\lambda}]=i(g_{\mu\lambda}J_{\nu\rho}+g_{\nu\rho}J_{\mu\lambda}-g_{\mu\rho}J_{\nu\lambda}-g_{\nu\lambda}J_{\mu\rho}) . \tag{2.3c}$$

These commutation rules, which define the so-called *Poincaré algebra*, are valid for any choice of the components $\{P_\mu,J_{\mu\nu}\}$ and the corresponding metric $(g_{\mu\nu})$. In instant form dynamics, it is convenient to work with the metric

$$(g_{\mu\nu})=\begin{bmatrix}1&0&0&0\\0&-1&0&0\\0&0&-1&0\\0&0&0&-1\end{bmatrix} . \tag{2.4}$$

A convenient notation for the components of the four-momentum and the angular momentum tensor is given by

$$P=(P^\mu)=(H,\mathbf{P}), \quad \mathbf{J}=(J^{23},J^{31},J^{12}), \quad \mathbf{K}=(J^{01},J^{02},J^{03}) . \tag{2.5}$$

The Hermitian operators H, $\mathbf{P}$, $\mathbf{J}$, and $\mathbf{K}$ are the Hamiltonian, the three-momentum, the angular momentum, and the generator of rotationless boosts, respectively.

It follows from (2.1) and (2.2) that the generators $\mathbf{P}$ and $\mathbf{J}$ induce transformations on the hypersurface $t=\text{const}$. Accordingly, these generators are taken to be noninteracting in instant form dynamics. As H is the Hamiltonian, it contains an interaction. It follows from (2.5), (2.3b), and (2.4) that

$$[K_j,P^k]=-i\delta_{jk}H\ ; \tag{2.6}$$

therefore, the components of $\mathbf{K}$ must be interacting.

The most important operators associated with the internal structure of a system are the mass operator M and spin operator $\mathcal{J}$. The mass operator is defined by

$$M=(P\cdot P)^{1/2}=(H^2-\mathbf{P}^2)^{1/2}\ , \tag{2.7}$$

while the instant form spin operator is defined by [19]

$$\mathcal{J}=\frac{1}{M}(H\mathbf{J}-\mathbf{P}\times\mathbf{K})-\frac{\mathbf{P}(\mathbf{P}\cdot\mathbf{J})}{M(M+H)}\ . \tag{2.8}$$

The operators M and $\mathcal{J}\cdot\mathcal{J}$ are Casimir operators and, as such, commute with all of the generators of the group. Their eigenvalues are invariants and are used to label the irreducible representations of the Poincaré group. In instant form dynamics, it also convenient to introduce the Newton-Wigner position operator defined by [19]

$$\mathbf{X}=-\frac{1}{2}\left(\frac{1}{H}\mathbf{K}+\mathbf{K}\frac{1}{H}\right)-\frac{\mathbf{P}\times\mathcal{J}}{H(M+H)}\ . \tag{2.9}$$

Rather than work with the generators, it is often simpler to work with the set $\{M,\mathbf{P},\mathcal{J},\mathbf{X}\}$, since the only nonzero commutators of the members of this set are

$$[X^j,P^k]=i\delta_{jk}\ , \tag{2.10}$$

$$[\mathcal{J}^j,\mathcal{J}^k]=i\varepsilon_{jkl}\mathcal{J}^l\ . \tag{2.11}$$

Relations (2.7)–(2.9) can be inverted to give

$$H=(\mathbf{P}^2+M^2)^{1/2}\ , \tag{2.12}$$

$$\mathbf{J}=\mathbf{X}\times\mathbf{P}+\mathcal{J}\ , \tag{2.13}$$

$$\mathbf{K}=-\frac{1}{2}(H\mathbf{X}+\mathbf{X}H)-\frac{\mathbf{P}\times\mathcal{J}}{M+H}\ . \tag{2.14}$$

If the members of the set $\{M,\mathbf{P},\mathcal{J},\mathbf{X}\}$ satisfy the correct commutation relations, then the members of the set $\{H,\mathbf{P},\mathbf{J},\mathbf{K}\}$ defined by (2.12)–(2.14) satisfy the fundamental commutation relations (2.3) and thereby qualify as Poincaré generators.

III. BASIS STATES

In constructing Poincaré invariant models, it is desirable to work with a basis in which $\mathbf{P}$, $\mathcal{J}$, and $\mathbf{X}$ have simple representations. The model we are going to construct in Sec. IV assumes that $\mathcal{J}$ and $\mathbf{X}$ are the same as the operators for a noninteracting two-particle system. This suggests that our basis states should be chosen to be direct product states of single-particle basis states; however, it turns out to not be quite that simple. In order to get simple representations for $\mathcal{J}$ and $\mathbf{X}$, it is necessary to take for the basis states special linear combinations of the direct product states. The procedure for constructing the appropriate basis states is given in the review article of Keister and Polyzou [19]. Here we will simply quote the relevant results.

Our two-particle space is spanned by the direct product states

$$|p_1,h_1;p_2,h_2\rangle=|p_1,h_1\rangle\otimes|p_2,h_2\rangle\ , \tag{3.1}$$

where p_1 and p_2 are the on-mass-shell momenta of the particles and h_1 and h_2 are the three-components of their spins. We define a two-particle rest frame, the x_Λ frame, by the relations

$$x=l_c(\Lambda)x_\Lambda\ , \quad \Lambda=p/W\ , \tag{3.2}$$

$$p=p_1+p_2\ , \quad W=(p\cdot p)^{1/2}\ . \tag{3.3}$$

Here $l_c(\Lambda)$ is a so-called canonical or rotationless boost and is generated by $\mathbf{K}$ [19]. It maps the unit four-vector $(1,\mathbf{0})$ to Λ. Using this rest frame, we introduce a relative three-momentum variable $\mathbf{q}$ through the relations

$$\begin{aligned}(\omega_1(\mathbf{q}),\mathbf{q})&=(p_{1\Lambda}^{\mu})=p_{1\Lambda}=l_c^{-1}(\Lambda)p_1\ ,\\ (\omega_2(\mathbf{q}),-\mathbf{q})&=(p_{2\Lambda}^{\mu})=p_{2\Lambda}=l_c^{-1}(\Lambda)p_2\ .\end{aligned} \tag{3.4}$$

Clearly, $\mathbf{q}$ is the three-momentum of particle 1 in the rest frame defined by (3.2) and (3.3), and moreover the invariant mass of the two-particle state is given by

$$W=W(\mathbf{q})=\omega_1(\mathbf{q})+\omega_2(\mathbf{q})\ . \tag{3.5}$$

Unfortunately, the spin operator $\mathcal{J}$ and Newton-Wigner position $\mathbf{X}$ do not have a simple representation in the basis provided by the states (3.1). In order to obtain a simple representation for these operators, we take as our basis states the following linear combination of the direct product states [19]:

$$\begin{aligned}|\mathbf{q},h_1,h_2;\mathbf{p}\rangle=\sum_{h_1'h_2'}&|p_1,h_1';p_2,h_2'\rangle\\ &\times D^{(s_1)}_{h_1'h_1}\{r_c[l_c(\Lambda),p_{1\Lambda}/m_1]\}\\ &\times D^{(s_2)}_{h_2'h_2}\{r_c[l_c(\Lambda),p_{2\Lambda}/m_2]\}\ .\end{aligned} \tag{3.6}$$

Here $D^{(s)}(r)$ is the standard SU(2) matrix representation of a three-rotation r and r_c is a so-called Wigner rotation defined by

$$r_c(a,\lambda)\equiv l_c^{-1}(a\lambda)al_c(\lambda)\ . \tag{3.7}$$

So as to be consistent with Ref. [10], we choose for our states the covariant normalization

$$\langle \mathbf{q},h_1,h_2;\mathbf{p}|\mathbf{q}',h_1',h_2';\mathbf{p}'\rangle=\langle p_1,h_1|p_1',h_1'\rangle\langle p_2,h_2|p_2',h_2'\rangle=\delta_{h_1h_1'}\delta_{h_2h_2'}(2\pi)^3 2p^0(\mathbf{p},\mathbf{q})\delta^3(\mathbf{p}-\mathbf{p}')\Delta(\mathbf{q})\delta^3(\mathbf{q}-\mathbf{q}') \; , \tag{3.8}$$

where

$$p^0(\mathbf{p},\mathbf{q})=[\mathbf{p}^2+W^2(\mathbf{q})]^{1/2}, \quad \Delta(\mathbf{q})=(2\pi)^3 2\omega_1(\mathbf{q})\omega_2(\mathbf{q})/W(\mathbf{q}) \; . \tag{3.9}$$

In the basis provided by the states (3.6), the Newton-Wigner position operator and the spin operator have the representations [19]

$$\langle \mathbf{q},h_1,h_2;\mathbf{p}|\mathbf{X}=\left\{ i\nabla_p - \frac{i\mathbf{p}}{2[p^0(\mathbf{q},\mathbf{p})]^2}\right\}\langle \mathbf{q},h_1,h_2;\mathbf{p}| \; , \tag{3.10}$$

$$\langle \mathbf{q},h_1,h_2;\mathbf{p}|\mathscr{J}=\sum_{h_1'h_2'}\mathscr{J}_{h_1h_2,h_1'h_2'}(\mathbf{q})\langle \mathbf{q},h_1',h_2';\mathbf{p}| \; , \tag{3.11}$$

where

$$\mathscr{J}(\mathbf{q})\equiv I_1\otimes I_2(i\nabla_{\mathbf{q}}\times\mathbf{q})+\mathbf{S}_1\otimes I_2+I_1\otimes\mathbf{S}_2, \tag{3.12}$$

with I_i and $\mathbf{S}_i$ the unit matrix and spin matrix vector for particle i, respectively. The spin operators $\mathbf{S}_1$ and $\mathbf{S}_2$ are related to the individual spins by the Wigner rotations specified in (3.6) [19].

In Sec. IV we will construct an effective pion-nucleon interaction in the basis $\{|\mathbf{q},h_1,h_2;\mathbf{p}\rangle\}$, and verify that it commutes with the spin operator defined by (3.11) and (3.12), as well as with the Newton-Wigner position operator defined by (3.10). This will guarantee that we have a Poincaré invariant model.

IV. MODEL OF THE PION-NUCLEON SYSTEM

The model of the pion-nucleon system we will construct here is derived from the same quantum field theory as the one assumed in Ref. [10]. This quantum field theory describes the interaction between pions, nucleons, and sigma mesons through the virtual processes $N\Leftrightarrow N+\pi$, $\pi\Leftrightarrow\pi+\sigma$, and $N\Leftrightarrow N+\sigma$. In Ref. [10] we constructed an effective pion-nucleon interaction for use in the front form of relativistic quantum mechanics by extending to light front dynamics procedures developed by Okubo [22] and Glöckle and Müller [23]. Here we can revert to Glöckle and Müller's instant form version of the Okubo method. It turns out that just as in the front form, to second order in the coupling constants the pion-nucleon instant form potentials can be determined by using a slight variation of the standard Feynman diagram rules. The potentials can be obtained by first drawing the relevant second-order Feynman diagrams and then determining the four-momentum of the virtual particle in each diagram by assuming that the total four-momentum is conserved *either* at the vertex on the "right" *or* at the vertex on the "left," but not necessarily at both vertices. The potentials are obtained by adding together the two resulting Feynman amplitudes and dividing by 2. The detailed justification for this rule is given in Sec. V of Ref. [10]. The result for the pion-nucleon potential is

$$\tilde{V}_{tih,t'i'h'}(p_\pi,p_N;p_\pi',p_N') = \sum_{x=D,N,\sigma}\tilde{V}^x_{tih,t'i'h'}(p_\pi,p_N;p_\pi',p_N') \; , \tag{4.1}$$

where D, N, and σ indicate the direct nucleon contribution, the crossed nucleon contribution, and the σ exchange contribution, respectively, given by

$$\tilde{V}^D_{tih,t'i'h'}(p_\pi,p_N;p_\pi',p_N')=g^2_{\pi NN}(\chi_i^\dagger\tau_t^\dagger\tau_{t'}\chi_{i'})\bar{u}(p_N,h)i\gamma_5\frac{1}{2}\left[\frac{\not{p}+m_N}{p^2-m_N^2}+\frac{\not{p}'+m_N}{p'^2-m_N^2}\right]i\gamma_5 u(p_N',h') \; , \tag{4.2a}$$

$$\tilde{V}^N_{tih,t'i'h'}(p_\pi,p_N;p_\pi',p_N')=g^2_{\pi NN}(\chi_i^\dagger\tau_{t'}\tau_t^\dagger\chi_{i'})\bar{u}(p_N,h)i\gamma_5\frac{1}{2}\left[\frac{\not{p}-\not{p}_\pi-\not{p}_\pi'+m_N}{(p-p_\pi-p_\pi')^2-m_N^2}+\frac{\not{p}'-\not{p}_\pi-\not{p}_\pi'+m_N}{(p'-p_\pi-p_\pi')^2-m_N^2}\right]i\gamma_5 u(p_N',h') \; , \tag{4.2b}$$

$$\tilde{V}^\sigma_{tih,t'i'h'}(p_\pi,p_N;p_\pi',p_N')=(g_{\sigma\pi\pi}m_\sigma g_{\sigma NN})(\chi_i^\dagger\delta_{tt'}\chi_{i'})\bar{u}(p_N,h)\frac{1}{2}\left[\frac{1}{(p_N-p_N')^2-m_\sigma^2}+\frac{1}{(p_\pi-p_\pi')^2-m_\sigma^2}\right]u(p_N',h') \; . \tag{4.2c}$$

Here the g's are the coupling constants, the τ_t are the spherical components [24] of the nucleon's isospin vector $\boldsymbol{\tau}$, and the χ_i are the isospinors

$$\chi_{1/2}=\begin{bmatrix}1\\0\end{bmatrix}, \quad \chi_{-1/2}=\begin{bmatrix}0\\1\end{bmatrix} . \tag{4.3}$$

The Dirac spinors are normalized according to

$$\bar{u}(p_N,h)u(p_N,h')=2m_N\delta_{hh'} . \tag{4.4}$$

We now express these potentials in terms of the relative three-momentum variable $\mathbf{q}$ defined as in (3.4), where we identify particles 1 and 2 with the pion and nucleon, respectively, and to simplify the notation we write

$$\omega_1(\mathbf{q})=\omega(\mathbf{q})=(\mathbf{q}^2+m_\pi^2)^{1/2}, \quad \omega_2(\mathbf{q})=\varepsilon(\mathbf{q})=(\mathbf{q}^2+m_N^2)^{1/2} . \tag{4.5}$$

The potentials (4.2) are given in a basis like (3.1). In order to work with the simple representations of the spin and Newton-Wigner position operator, given by (3.10)–(3.12), we must transform to the basis defined by (3.6). The potentials in this basis are given by

$$\begin{aligned}V^x_{tih,t'i'h'}(\mathbf{q},\mathbf{q}';\mathbf{p}) &= \sum_{h''h'''} D^{(1/2)}_{hh''}\{r_c^{-1}[l_c(\Lambda),p_{N\Lambda}/m_N]\} \\ &\times \tilde{V}^x_{tih'',t'i'h'''}(p_\pi,p_N;p'_\pi,p'_N) \\ &\times D^{(1/2)}_{h'''h'}\{r_c[l_c(\Lambda'),p'_{N\Lambda'}/m_N]\} .\end{aligned} \tag{4.6}$$

We see from here and (4.2) that we are led to consider a linear combination of Dirac spinors. It can be shown that [10]

$$\sum_{h'} u(p_N,h')D^{(1/2)}_{h'h}\{r_c[l_c(\Lambda),p_{N\Lambda}/m_N]\} = S[l_c(\Lambda)]u(p_{N\Lambda},h) , \tag{4.7}$$

where for an arbitrary Lorentz transformation a the 4×4 matrix $S(a)$ satisfies

$$S^{-1}(a)\gamma^\mu S(a)=a^\mu{}_\nu\gamma^\nu . \tag{4.8}$$

We see that carrying out the transformation (4.6) is equivalent to making the replacements

$$u(p_N,h)\rightarrow S[l_c(\Lambda)]u(p_{N\Lambda},h), \quad u(p'_N,h')\rightarrow S[l_c(\Lambda')]u(p'_{N\Lambda'},h') , \tag{4.9}$$

in (4.2a)–(4.2c). The spinors associated with the two-particle rest frame are given by Eq. (3.7) of Bjorken and Drell [25], multiplied by $\sqrt{2m_N}$, so as to give the normalization (4.4). With the help of (3.4) and (4.5), we have, e.g.,

$$u(p_{N\Lambda},h)=[\varepsilon(\mathbf{q})+m_N]^{1/2}\begin{bmatrix}\chi_h\\ -\boldsymbol{\sigma}\cdot\mathbf{x}\chi_h\end{bmatrix} , \tag{4.10}$$

with

$$\mathbf{x}=\frac{\mathbf{q}}{\varepsilon(\mathbf{q})+m_N} \tag{4.11}$$

and where the two-component spinors are the same as the isospinors (4.3).

In expressing (4.2a)–(4.2c) in terms of the relative momentum variables, it is necessary to know the relation between the final and initial two-particle rest frame states. According to (3.2), they are related by

$$x_\Lambda=l_c^{-1}(\Lambda)l_c(\Lambda')x_{\Lambda'} . \tag{4.12}$$

Keeping in mind that the three-momentum is conserved, i.e., $\mathbf{p}=\mathbf{p}'$, we can show that

$$x_\Lambda=l_c(\Omega)x_{\Lambda'} , \tag{4.13}$$

where

$$\Omega=(p^0p'^0-\mathbf{p}^2,(p^0-p'^0)\mathbf{p})/[W(\mathbf{q})W(\mathbf{q}')] , \tag{4.14}$$

with

$$p^0=[\mathbf{p}^2+W^2(\mathbf{q})]^{1/2}, \quad p'^0=[\mathbf{p}^2+W^2(\mathbf{q}')]^{1/2} . \tag{4.15}$$

It is important to note that, on shell or when $\mathbf{p}=\mathbf{0}$,

$$\Omega=(1,\mathbf{0}) \quad (q=q' \text{ or } \mathbf{p}=\mathbf{0}) . \tag{4.16}$$

When the replacements (4.9) are made in (4.2a)–(4.2c), we encounter various matrices such as the unit matrix or the Feynman slash quantity $\not{p}_\pi$ sandwiched between $S^{-1}[l_c(\Lambda)]$ and $S[l_c(\Lambda')]$. These products can be worked out by using [25]

$$S^{-1}[l_c(\Lambda)]S[l_c(\Lambda')]=S[l_c(\Omega)]=\exp(\tfrac{1}{2}\mathbf{u}\alpha\zeta) , \quad \mathbf{u}=\boldsymbol{\Omega}/|\boldsymbol{\Omega}|, \quad \tanh(\zeta)=|\boldsymbol{\Omega}|/\Omega^0 , \tag{4.17}$$

as well as, for example,

$$S^{-1}[l_c(\Lambda)]\not{p}_\pi S[l_c(\Lambda)]=\not{p}_{\pi\Lambda}=\omega(\mathbf{q})\gamma^0-\mathbf{q}\cdot\boldsymbol{\gamma} . \tag{4.18}$$

The invariant denominators that appear in the potentials can be worked out, for example, in the x_Λ frame by using (3.4) and (4.12).

If we let $|t,i,h\rangle$ denote a basis state in the isospin-spin space of the πN system, then using the above results we find that the potentials defined by (4.6) and (4.2) can be written in the form

$$V^x_{tih,t'i'h'}(\mathbf{q},\mathbf{q}';\mathbf{p})=\langle t,i,h|V^x(\mathbf{q},\mathbf{q}';\mathbf{p})|t',i',h'\rangle , \tag{4.19}$$

where

$$V^D(\mathbf{q},\mathbf{q}';\mathbf{p})=g^2_{\pi NN}(3P_{1/2})\frac{1}{2}\left[\frac{N(\mathbf{q},\mathbf{q}';\mathbf{p})}{W'^2-m_N^2}+\frac{N^\dagger(\mathbf{q}',\mathbf{q};\mathbf{p})}{W^2-m_N^2}\right], \tag{4.20}$$

$$N(\mathbf{q},\mathbf{q}';\mathbf{p})=(\varepsilon+m_N)^{1/2}(\varepsilon'+m_N)^{1/2}\{\cosh(\zeta/2)[(W'-m_N)+(\boldsymbol{\sigma}\cdot\mathbf{x})(W'+m_N)(\boldsymbol{\sigma}\cdot\mathbf{x}')]+\sinh(\zeta/2)[(\boldsymbol{\sigma}\cdot\mathbf{x})(\boldsymbol{\sigma}\cdot\mathbf{u})(W'-m_N) +(W'+m_N)(\boldsymbol{\sigma}\cdot\mathbf{u})(\sigma\cdot\mathbf{x}')]\}, \tag{4.21}$$

$$V^N(\mathbf{q},\mathbf{q}';\mathbf{p})=g^2_{\pi NN}(-P_{1/2}+2P_{3/2})\frac{1}{2}\left[\frac{N(\mathbf{q},\mathbf{q}';\mathbf{p})}{D(\mathbf{q},\mathbf{q}';\mathbf{p})}+\frac{N^\dagger(\mathbf{q}',\mathbf{q};\mathbf{p})}{D(\mathbf{q}',\mathbf{q};\mathbf{p})}\right], \tag{4.22}$$

$$\begin{aligned}D(\mathbf{q},\mathbf{q}';\mathbf{p})&=m_N^2-(p_N-p'_\pi)^2\\ &=m_N^2-\varepsilon^2-\omega'^2+2\varepsilon\omega'\cosh(\zeta)+(\mathbf{q}+\mathbf{q}')^2+2\sinh(\zeta)(\omega'\mathbf{q}\cdot\mathbf{u}+\varepsilon\mathbf{q}'\cdot\mathbf{u})+2[\cosh(\zeta)-1]\mathbf{q}\cdot\mathbf{u}\mathbf{q}'\cdot\mathbf{u}\,,\end{aligned} \tag{4.23}$$

$$V^\sigma(\mathbf{q},\mathbf{q}';\mathbf{p})=-g_{\sigma\pi\pi}m_\sigma g_{\sigma NN}(\varepsilon+m_N)^{1/2}(\varepsilon'+m_N)^{1/2}\{\cosh(\zeta/2)[1-(\boldsymbol{\sigma}\cdot\mathbf{x})(\boldsymbol{\sigma}\cdot\mathbf{x}')]+\sinh(\zeta/2)[(\boldsymbol{\sigma}\cdot\mathbf{x})(\boldsymbol{\sigma}\cdot\mathbf{u})-(\boldsymbol{\sigma}\cdot\mathbf{u}) \times(\boldsymbol{\sigma}\cdot\mathbf{x}')]\}\frac{1}{2}\left[\frac{1}{D_1(\mathbf{q},\mathbf{q}';\mathbf{p})}+\frac{1}{D_2(\mathbf{q},\mathbf{q}';\mathbf{p})}\right], \tag{4.24}$$

$$\begin{aligned}D_1(\mathbf{q},\mathbf{u},\mathbf{q}')&=m_\sigma^2-(p_\pi-p'_\pi)^2\\ &=m_\sigma^2-\omega^2-\omega'^2+2\omega\omega'\cosh(\zeta)+(\mathbf{q}-\mathbf{q}')^2-2\sinh(\zeta)(\omega'\mathbf{q}\cdot\mathbf{u}-\omega\mathbf{q}'\cdot\mathbf{u})-2[\cosh(\zeta)-1]\mathbf{q}\cdot\mathbf{u}\mathbf{q}'\cdot\mathbf{u}\,,\end{aligned} \tag{4.25}$$

$$\begin{aligned}D_2(\mathbf{q},\mathbf{u},\mathbf{q}')&=m_\sigma^2-(p_N-p'_N)^2\\ &=m_\sigma^2-\varepsilon^2-\varepsilon'^2+2\varepsilon\varepsilon'\cosh(\zeta)+(\mathbf{q}-\mathbf{q}')^2-2\sinh(\zeta)(\varepsilon\mathbf{q}'\cdot\mathbf{u}-\varepsilon'\mathbf{q}\cdot\mathbf{u})-2[\cosh(\zeta)-1]\mathbf{q}\cdot\mathbf{u}\mathbf{q}'\cdot\mathbf{u}\,.\end{aligned} \tag{4.26}$$

Here $P_{1/2}$ and $P_{3/2}$ are projection operators onto the subspaces with total isospins of 1/2 and 3/2, respectively. In the above equations, we have set $\varepsilon=\varepsilon(\mathbf{q})$, $\varepsilon'=\varepsilon(\mathbf{q}')$, $\omega=\omega(\mathbf{q})$, $\omega'=\omega(\mathbf{q}')$, $W=\omega+\varepsilon$, and $W'=\omega'+\varepsilon'$. The complete effective pion-nucleon interaction is given in the basis $\{|\mathbf{q},t,i,h;\mathbf{p}\rangle\}$ by

$$\langle\mathbf{q},t,i,h;\mathbf{p}|V_{\pi N}|\mathbf{q}',t',i',h';\mathbf{p}'\rangle=(2\pi)^3\delta^3(\mathbf{p}-\mathbf{p}')\sum_{x=D,N,\sigma}\langle t,i,h|V^x(\mathbf{q},\mathbf{q}';\mathbf{p})|t',i',h'\rangle\,. \tag{4.27}$$

In constructing a *Poincaré invariant* model for the *interacting* pion-nucleon system, we will assume the noninteracting spin and Newton-Wigner position operator defined by (3.10)–(3.12). According to the results outlined in Sec. II, a necessary and sufficient condition for our model to be Poincaré invariant is that **P**, $\mathcal{J}$, and **X** commute with the mass operator M. The other commutation relations of the set $\{M,\mathbf{P},\mathcal{J},\mathbf{X}\}$ are automatically satisfied by our choice of $\mathcal{J}$ and **X**. We can now easily show that our model will be Poincaré invariant if and only if the matrix elements of the mass operator are of the form

$$\langle\mathbf{q},t,i,h;\mathbf{p}|M|\mathbf{q}',t',i',h';\mathbf{p}'\rangle=(2\pi)^3 2[p^0(\mathbf{p},\mathbf{q})p^0(\mathbf{p},\mathbf{q}')]^{1/2}\delta^3(\mathbf{p}-\mathbf{p}')\langle t,i,h|M(\mathbf{q},\mathbf{q}')|t',i',h'\rangle\,, \tag{4.28}$$

where $M(\mathbf{q},\mathbf{q}')$ is a rotationally invariant function of **q**, **q**′, and $\boldsymbol{\sigma}$; and is independent of **p**. An $M(\mathbf{q},\mathbf{q}')$ with these properties can be defined by

$$M(\mathbf{q},\mathbf{q}')=W(\mathbf{q})\Delta(\mathbf{q})\delta^3(\mathbf{q}-\mathbf{q}')+\frac{V_{\pi N}(\mathbf{q},\mathbf{q}')}{2[W(\mathbf{q})W(\mathbf{q}')]^{1/2}}, \tag{4.29}$$

with

$$V_{\pi N}(\mathbf{q},\mathbf{q}')=\sum_{x=D,N,\sigma}V^x(\mathbf{q},\mathbf{q}';\mathbf{0})\,. \tag{4.30}$$

According to (4.14) and (4.17), $\zeta=0$ when $\mathbf{p}=\mathbf{0}$. Looking at (4.20)–(4.26), we see that this greatly simplifies the expressions for the potentials.

V. FORM FACTORS

In order to take account of the extension of the πNN, σNN, and $\sigma\pi\pi$ vertices and to improve the behavior of the potentials at large momenta, it is necessary to introduce form factors or vertex functions. We will adopt a phenomenological form introduced by Gross, van Orden, and Holinde [15] in the context of a one boson exchange model of the two-nucleon system, i.e.,

$$f(t^2;m,\Lambda)=\frac{(\Lambda^2-m^2)^2+\Lambda^4}{(\Lambda^2-t^2)^2+\Lambda^4}, \tag{5.1}$$

where here t is a four-momentum, m is the mass of a π or σ meson, and Λ is a cutoff mass. This form is normalized according to

$$f(m^2;m,\Lambda)=1 \ . \tag{5.2}$$

In the direct nucleon contribution to the potential [Eq. (4.2a)], we make the replacements

$$\frac{1}{x^2-m_N^2}\to f\left[\frac{(p_\pi-p_N)^2-m_N^2}{2};m_\pi,\Lambda_{\pi NN}\right]\frac{1}{x^2-m_N^2}\times f\left[\frac{(p'_\pi-p'_N)^2-m_N^2}{2};m_\pi,\Lambda_{\pi NN}\right],\quad x=p,p' \ . \tag{5.3}$$

Here, as is common in models of the pion-nucleon system [26], we are assuming that the form factors depend on the square of the relative pion-nucleon four-momentum. In the crossed nucleon contribution [Eq. (4.2b)], we assume that the form factors are functions of $(p_\pi-p''_N)^2$ and $(p'_\pi-p''_N)^2$, where p''_N is the momentum of the virtual nucleon in the intermediate state. In the first and second terms on the right-hand side of (4.2b), we have $p''_N=p-p_\pi-p'_\pi$ and $p''_N=p'-p_\pi-p'_\pi$, respectively. Thus the replacements in (4.2b) are given by

$$\frac{1}{(x-p_\pi-p'_\pi)^2-m_N^2}\to f\left[\frac{(x-p_\pi-2p'_\pi)^2-m_N^2}{2};m_\pi,\Lambda_{\pi NN}\right]\times\frac{1}{(x-p_\pi-p'_\pi)^2-m_N^2}\times f\left[\frac{(x-p'_\pi-2p_\pi)^2-m_N^2}{2};\ m_\pi,\Lambda_{\pi NN}\right],\quad x=p,p' \ . \tag{5.4}$$

For the σ exchange contribution, we make the replacements

$$\frac{1}{(p_i-p'_i)^2-m_\sigma^2}\to f[(p_\pi-p'_\pi)^2;m_\sigma,\Lambda_{\sigma\pi\pi}]\frac{1}{(p_i-p'_i)^2-m_\sigma^2}\times f[(p_N-p'_N)^2;m_\sigma,\Lambda_{\sigma NN}],\quad i=\pi,N \ . \tag{5.5}$$

It is straightforward to check that on shell, i.e., when $p_\pi+p_N=p'_\pi+p'_N$, the form factors in (5.3)–(5.5) all become 1 when the denominators vanish. Thus the modified propagators have the correct residues.

We must now express the arguments of the form factors in terms of the relative three-momentum variables $\mathbf{q}$ and $\mathbf{q}'$. We can easily show that

$$[(p_\pi-p_N)^2-m_N^2]/2=m_\pi^2+[m_N^2-W^2(\mathbf{q})]/2 \ . \tag{5.6}$$

In our prescription for the mass operator, i.e., (4.29) and (4.30), we set $\mathbf{p}=\mathbf{0}$. We can easily show that, when $\mathbf{p}=\mathbf{0}$,

$$p'_{\pi\Lambda}=(\omega(\mathbf{q}'),\mathbf{q}'),\ \ p'_{N\Lambda}=(\varepsilon(\mathbf{q}'),-\mathbf{q}')\quad(\text{assumes }\mathbf{p}=\mathbf{0}) \ , \tag{5.7}$$

which when combined with (3.4) and (4.5) makes it trivial to express the arguments of the form factors in (5.4) and (5.5) in terms of $\mathbf{q}$ and $\mathbf{q}'$. It is important to note that the prescription (5.7) preserves the correct residues of the modified propagators. It should be noted that the treatment of the form factors here is not the same as in Ref. [10], as there separable functions of $\mathbf{q}$ and $\mathbf{q}'$ were simply introduced into each of the three potentials in (4.30).

VI. NUMERICAL RESULTS

Our model of the pion-nucleon system is formulated in terms of a mass operator defined by (4.28)–(4.30). Scattering theory is usually formulated in terms of a Hamiltonian operator; however, it can be formulated in terms of the mass operator, as shown for example in Appendix A of Ref. [19]. According to (4.28) and (4.29), we can separate our mass operator into a noninteracting mass operator and an interaction, i.e.,

$$M=M_0+U \ , \tag{6.1}$$

where

$$\langle\mathbf{q},t,i,h;\mathbf{p}|U|\mathbf{q}',t',i',h';\mathbf{p}'\rangle=(2\pi)^3 2[p^0(\mathbf{p},\mathbf{q})p^0(\mathbf{p},\mathbf{q}')]^{1/2}\delta^3(\mathbf{p}-\mathbf{p}')\frac{\langle t,i,h|V_{\pi N}(\mathbf{q},\mathbf{q}')|t',i',h'\rangle}{2[W(\mathbf{q})W(\mathbf{q}')]^{1/2}} \ . \tag{6.2}$$

The transition operator for our model is obtained by solving the Lippmann-Schwinger equation

$$T(z)=U+U\,\frac{1}{z-M_0}\,T(z) \ , \tag{6.3}$$

where z is a parameter with the dimension of mass. If we write

$$\langle\mathbf{q},t,i,h;\mathbf{p}|T(z)|\mathbf{q}',t',i',h';\mathbf{p}'\rangle=(2\pi)^3 2[p^0(\mathbf{p},\mathbf{q})p^0(\mathbf{p},\mathbf{q}')]^{1/2}\delta^3(\mathbf{p}-\mathbf{p}')\frac{\langle t,i,h|T_{\pi N}(\mathbf{q},\mathbf{q}';z)|t',i',h'\rangle}{2[W(\mathbf{q})W(\mathbf{q}')]^{1/2}} \tag{6.4}$$

and use the completeness relation implied by (3.8), we can show that the operator equation (6.3) gives rise to the integral equation

$$T_{\pi N}(\mathbf{q},\mathbf{q}';z)=V_{\pi N}(\mathbf{q},\mathbf{q}')+\int \frac{d^3q''}{\Delta(\mathbf{q}'')}\frac{V_{\pi N}(\mathbf{q},\mathbf{q}'')}{2W(\mathbf{q}'')[z-W(\mathbf{q}'')]}T_{\pi N}(\mathbf{q}'',\mathbf{q}';z) . \tag{6.5}$$

We can readily solve this singular integral equations numerically by using Kowalski's method [27].

As pointed out above, the exchange model we are using for the pion-nucleon system was used previously [10] as the basis for a front form model of this system. It is an interesting fact that the potentials obtained in the front form formulation are given by equations that are identical in appearance to (4.19)–(4.26); however, ζ and $\mathbf{u}$ are not given by (4.14) and (4.17), but rather by

$$\zeta\to\beta=\ln[W(\mathbf{q})/W(\mathbf{q}')], \quad \mathbf{u}\to\mathbf{e}_3=(0,0,1) . \tag{6.6}$$

In Ref. [10] we argued that in the front form formulation it is a reasonable approximation to let $\beta\to 0$, which has the effect of eliminating all of the $\mathbf{e}_3$-dependent terms. This leads to exactly the same pion-nucleon potential as (4.30), where we set $\mathbf{p}=\mathbf{0}$ in (4.19)–(4.26). Thus, with the approximations that have been made, the front form and instant form pion-nucleon potentials are identical. This does not imply, however, that the T matrices are identical, since the *front form* T-matrix equation is given by (6.5) with the replacement

$$\frac{1}{2W(\mathbf{q}'')[z-W(\mathbf{q}'')]}\to\frac{1}{z^2-W^2(\mathbf{q}'')} . \tag{6.7}$$

We note that both forms have the same residue at the singular point $z=W(\mathbf{q}'')$.

The difference between the two forms is due to the difference in the way interactions are added into the noninteracting operators. In the instant form, the three-momentum $\mathbf{P}$ is noninteracting and the interaction appears in $P^0=H=H_0+H_1$. In the front form, we work with the components

$$\begin{aligned}P&=(P^+,\mathbf{P}_\perp,P^-)\\&=((P^0+\mathbf{e}_3\cdot\mathbf{P})/\sqrt{2},\mathbf{P}-\mathbf{e}_3\cdot\mathbf{P},(P^0-\mathbf{e}_3\cdot\mathbf{P})/\sqrt{2}) ,\end{aligned} \tag{6.8}$$

where $\mathbf{e}_3$ is given by (6.6) and P^+ and $\mathbf{P}_\perp$ are taken to be noninteracting, while P^- contains an interaction, i.e., $P^-=P_0^-+P_1^-$. In the instant form, the perturbation series for the effective two-particle interaction is developed in powers of H_1, while in the front form it is developed in powers of P_1^- [10,28]. Since our two-particle, instant form mass operator is essentially the Hamiltonian in the c.m. frame, it naturally decomposes into noninteracting and interacting parts according to (6.1). In the front form, the mass-square operator is given by

$$\begin{aligned}M^2&=2P^+P^--\mathbf{P}_\perp^2=2P^+P_0^--\mathbf{P}_\perp^2+2P^+P_1^-\\&=M_0^2+2P^+P_1^- ,\end{aligned} \tag{6.9}$$

and so the two-particle, front form mass-square operator naturally decomposes into noninteracting and interacting parts according to

$$M^2=M_0^2+V_f . \tag{6.10}$$

It should be noted that as far as Poincaré invariance is concerned, there is nothing wrong with reinterpreting the instant form mass operator (4.29) as a front form mass operator [19]. The only technical difference is that in the front form the relative three-momentum variable $\mathbf{q}$ is defined by (3.4), but with the canonical boost replaced with a so-called *front form boost*. If this is done, the instant form and front form lead to identical S matrices. We have not adopted this procedure since we feel that it weakens the connection between the effective two-particle models and the underlying quantum field theory.

In order to assess the difference between the front form model developed in Ref. [10] and the instant form model developed here, we have fit the s-wave pion-nucleon scattering amplitudes using both (6.5) and (6.5) with the replacement (6.7). The experimental phase shifts were taken from the SAID WI94 analysis [29]. The pion-nucleon coupling constant, pion mass, and nucleon mass were taken to be

$$g_{\pi NN}^2/4\pi=13.50, \quad m_\pi=139.57 \quad \text{MeV},$$

$$m_N=938.92 \quad \text{MeV} . \tag{6.11}$$

The parameters that were varied are the sigma exchange potential strength $g_{\sigma\pi\pi}m_\sigma g_{\sigma NN}$, the sigma mass m_σ, and the cutoff masses $\Lambda_{\pi NN}$ and $\Lambda_{\sigma\pi\pi}=\Lambda_{\sigma NN}$. As indicated, we

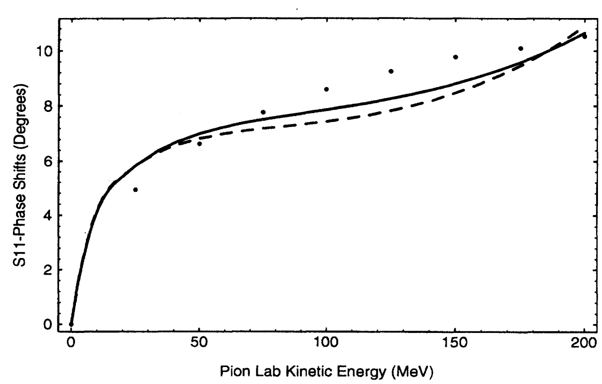

FIG. 1. $S11$ pion-nucleon phase shifts as a function of the pion laboratory kinetic energy. The solid and dashed lines are the instant and front form results, respectively. The dots are from the SAID WI 94 analysis.

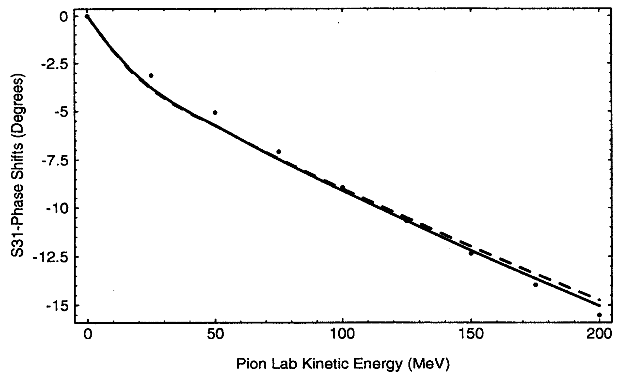

FIG. 2. $S31$ pion-nucleon phase shifts as a function of the pion laboratory kinetic energy. The solid and dashed lines are the instant and front form results, respectively. The dots are from the SAID WI94 analysis.

have chosen the two sigma exchange cutoff masses to be the same. The fits are shown in Figs. 1 and 2, and the resulting parameters are given in Table I. The coupling constant combination $g_{\sigma\pi\pi}g_{\sigma NN}/4\pi$ differs by only 0.06% between the two forms, the m_σ's differ by 4.6%, the $\Lambda_{\pi NN}$'s differ by 0.64%, while the cutoff masses for the σ vertices differ dramatically. It is encouraging that $g_{\sigma\pi\pi}g_{\sigma NN}/4\pi$ and m_σ are not sensitive to the form of relativistic quantum mechanics assumed; after all, these are supposed to be fundamental parameters of the underlying field theory. At low energies the difference (6.7) between the instant form and front form pion-nucleon propagators is negligible, but at high energies we expect the difference to be of some significance. This difference is clearly reflected in the σ exchange cutoff mass.

VII. DISCUSSION

It is clear that the method developed here can be applied to a model of the pion-nucleon system which, besides nucleon and σ exchange, also includes the contributions of the Δ and N^* direct and crossed diagrams, as well as ρ exchange. Such a model is presently being constructed in both the instant form and front form so as to test the sensitivity of the model parameters to the form assumed. It will be interesting to see if the differences between the two forms are reflected only in the cutoff masses, as was found here, or if the masses of the exchanged particles and the coupling constants are also affected.

TABLE I. Comparison of instant form and front form parameters.

Parameter	Instant form	Front form
$g_{\sigma\pi\pi}g_{\sigma NN}/4\pi$	4.81326	4.81033
m_σ (MeV)	505.865	482.753
$\Lambda_{\pi NN}$ (MeV)	1431.07	1421.94
$\Lambda_{\sigma\pi\pi}=\Lambda_{\sigma NN}$ (MeV)	6725.64	3577.70

The model considered here does not take into account inelasticity. An attempt is under way to take this into account by including coupling to $\pi\Delta$ and ηN channels, as has been done in the past by other authors [8,30]. A more ambitious treatment of inelasticity would also include the $\pi\pi N$ channel. The development of an exactly Poincaré invariant instant form or front form three-particle model of pion-nucleon scattering is nontrivial, but is certainly worth pursuing.

An exactly Poincaré invariant *front form*, one boson exchange model of nucleon-nucleon scattering has already been constructed and fit to the experimental amplitudes [18]. At present the corresponding instant form model is under construction, and here also it will be interesting to see how the differences in the forms are reflected in the model parameters.

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