Charge symmetry breaking in 500 MeV nucleon-trinucleon scattering

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Elastic nucleon scattering from the ^{3}He and ^{3}H mirror nuclei is considered as a possible test of charge symmetry violation. The differential cross sections at 500 MeV are calculated using a microscopic, momentum-space optical potential including the full coupling of two spin-1/2 particles and an exact treatment of the Coulomb force. The charge-symmetry-breaking effects investigated arise from a violation within the nuclear structure, from the p-nucleus Coulomb force, and from the mass differences of the charge symmetric states. Measurements likely to reveal reliable information are noted.

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I. INTRODUCTION

Charge symmetry (CS) is an approximate symmetry of the strong force arising from the nearly equal masses of the up and down current quarks. At the nuclear level this means that, apart from electromagnetic and weak effects, the interactions of nuclear systems must be the same as the interactions of their charge-symmetric counterparts as long as the spin-space states involved remain identical. Yet even if the nucleon-nucleon strong forces respected this symmetry, in an experimental measurement we would expect some CS violation to arise from the proton-nucleus Coulomb force, from the different masses for the neutron and proton, and from the different masses of charge-symmetric (mirror) nuclei. If the nucleon-nucleon strong force does not obey CS [1], we would expect still further violation within the nuclear structure and within the nucleon–nucleus interaction.

A useful approach to testing CS in scattering from the trinucleon system is to remove some experimental uncertainties by forming the ratio of cross sections which should be identically equal if CS were obeyed. For the pion-trinucleon system, charge symmetry predicts [2]

$$r1=\frac{d\sigma/d\Omega(\pi^+-{}^3\mathrm{H})}{d\sigma/d\Omega(\pi^--{}^3\mathrm{He})}\equiv 1, \qquad (1)$$

$$r2=\frac{d\sigma/d\Omega(\pi^--{}^3\mathrm{H})}{d\sigma/d\Omega(\pi^+-{}^3\mathrm{He})}\equiv 1. \qquad (2)$$

When Pillai *et al.* [3,4] measured these ratios they found large deviations from 1 and concluded that substantial CS violation must be occurring. The analysis by Kim *et al.* [5] showed that the π-trinucleon Coulomb force is largely responsible for these ratios differing from 1, specifically, the ratios get relatively large when the denominators, $d\sigma/d\Omega(\pi^{\mp}-{}^3\mathrm{He})$, have minima, and these minima are sensitive to Coulomb-nuclear interference. In addition, Kim *et al.* concluded that meaningful predictions of these ratios required a theory which could reproduce accurately the individual cross sections, and not just the ratios.

In a further study of the pion system, Gibbs and Gibson [6] concluded that a statistically good least-squares fit to the ratios $r1$ and $r2$ requires CS violation within the nuclear structure. They fit the pion scattering data by introducing differences in the root-mean-square nuclear radii,

$$R_n({}^3\mathrm{H})-R_p({}^3\mathrm{He})=-0.030\pm0.008\ \mathrm{fm}, \qquad (3)$$

$$R_n({}^3\mathrm{He})-R_p({}^3\mathrm{H})=0.035\pm0.007\ \mathrm{fm}, \qquad (4)$$

differences which would vanish if CS were good within the nuclear structure [7]. While these differences are relatively small compared to the nuclear radius of ~ 2 fm, and probably close to the level of uncertainty in strong-interaction calculations, they are large compared to the neutron-proton relative mass difference, and are approximately the same size as the CS violation found by including the Coulomb force in Faddeev calculations of nuclear structure. It is, accordingly, interesting to see if other hadron probes can confirm this degree of CS violation.

For the nucleon-trinucleon system, charge symmetry demands equal cross sections for p-^{3}H and n-^{3}He reactions, and, independently, for the p-^{3}He and n-^{3}H reactions:

$$r1\overset{\mathrm{def}}{=}\frac{d\sigma/d\Omega(p-{}^3\mathrm{H})}{d\sigma/d\Omega(n-{}^3\mathrm{He})}\equiv 1, \qquad (5)$$

$$r2\overset{\mathrm{def}}{=}\frac{d\sigma/d\Omega(n-{}^3\mathrm{H})}{d\sigma/d\Omega(p-{}^3\mathrm{He})}\equiv 1, \qquad (6)$$

$$\mathscr{R}\overset{\mathrm{def}}{=}r1\times r2\equiv 1, \qquad (7)$$

where $\mathscr{R}$ is called a "super-ratio." In this paper we are concerned with the calculation of these ratios and their sensitivities to CS violation. A direct CS violation within the nucleon-nucleon interaction [1] is not considered here.

II. THE CALCULATION

Our calculation is based on a solution of the Lippmann-Schwinger equation with a microscopic, nonlocal, momentum-space optical potential including all spin $\frac{1}{2}\times\frac{1}{2}$ couplings and an exact inclusion of the Coulomb potential [8–10]. The potential is the sum of nuclear (nuc) and Coulomb (coul) parts:

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$$V(\mathbf{k}',\mathbf{k},E) = V^{\mathrm{nuc}}(\mathbf{k}',\mathbf{k},E) + V^{\mathrm{coul}}(\mathbf{k}',\mathbf{k}), \tag{8}$$

$$\begin{aligned} V^{\mathrm{nuc}}(\mathbf{k}',\mathbf{k},E) \simeq N\{ & (t^{Nn}_{a+b} + t^{Nn}_{e}\vec{\sigma}^{N}_{n})\rho^{n}_{mt}(q=|\mathbf{k}'-\mathbf{k}|) + [t^{Nn}_{a-b}\vec{\sigma}^{N}_{n}\vec{\sigma}^{3}_{n} + t^{Nn}_{e}\vec{\sigma}^{3}_{n} + t^{Nn}_{c+d}\vec{\sigma}^{N}_{m}\vec{\sigma}^{3}_{m} + t^{Nn}_{c-d}\vec{\sigma}^{N}_{l}\vec{\sigma}^{3}_{l} + t^{Nn}_{c+d}(\vec{\sigma}^{N}_{m}\vec{\sigma}^{3}_{l} \\ & + \vec{\sigma}^{N}_{l}\vec{\sigma}^{3}_{m})]\rho^{n}_{sp}(q)\} + Z\{(t^{Np}_{a+b} + t^{Np}_{e}\vec{\sigma}^{N}_{n})\rho^{n}_{mt}(q) + [t^{Np}_{a-b}\vec{\sigma}^{N}_{n}\vec{\sigma}^{3}_{n} + t^{Np}_{e}\vec{\sigma}^{3}_{n} + t^{Np}_{c+d}\vec{\sigma}^{N}_{m}\vec{\sigma}^{3}_{m} + t^{Np}_{c-d}\vec{\sigma}^{N}_{l}\vec{\sigma}^{3}_{l} + t^{Np}_{c+d}(\vec{\sigma}^{N}_{m}\vec{\sigma}^{3}_{l} \\ & + \vec{\sigma}^{N}_{l}\vec{\sigma}^{3}_{m})]\rho^{n}_{sp}(q)\}. \end{aligned} \tag{9}$$

Here the t's are elementary, two-nucleon T matrices with their superscripts indicating the nucleons involved and with their subscripts indicating the spin dependences [11]. The σ's are Pauli spinors with their superscripts indicating the beam nucleon and trinucleon target involved, and with their subscripts indicating σ's projections onto the three independent scattering vectors, $\hat{\mathbf{n}} \propto \mathbf{k}\times\mathbf{k}'$, $\hat{\mathbf{m}} \propto \mathbf{k}-\mathbf{k}'$, and $\hat{\mathbf{l}} \propto \mathbf{k}+\mathbf{k}'$. The ρ's are four independent form factors describing the distribution of matter (mt) and spin (sp) within the nucleus.

The nuclear form factors are a key ingredient of the optical potential and possibly the most interesting path through which CS violation enters our calculation. If CS were good for the trinucleon structure, the matter and spin (α) form factors would obey the relation

$$\rho^{p}_{\alpha}(^{3}\mathrm{He}) = \rho^{n}_{\alpha}(^{3}\mathrm{H}), \rho^{p}_{\alpha}(^{3}\mathrm{H}) = \rho^{n}_{\alpha}(^{3}\mathrm{He}). \tag{10}$$

The relations (9) reflect the CS of mirror nuclei: the distribution of the two "like" nucleons is the same in both nuclei, as is the separate distributions of the "unlike" nucleons, and this is independent of whether the nucleons are neutrons or protons.

If we ignore meson-exchange currents, the matter and spin form factors are related to the charge (ch) and magnetic (mg) form factors of the trinucleon system with the finite proton size removed [8],

$$\rho^{p}_{\mathrm{mt}}(\mathrm{He}) = F_{\mathrm{ch}}(\mathrm{He})/f, \tag{11}$$

$$\rho^{n}_{\mathrm{mt}}(\mathrm{He}) = F_{\mathrm{ch}}(\mathrm{H})/f, \tag{12}$$

$$\rho^{n}_{\mathrm{sp}}(\mathrm{He}) = [\mu_{p}^{2}F_{\mathrm{mg}}(\mathrm{H}) - \mu_{n}^{2}F_{\mathrm{mg}}(\mathrm{He})]/[f(\mu_{p}^{2}-\mu_{n}^{2})], \tag{13}$$

$$\rho^{p}_{\mathrm{sp}}(\mathrm{He}) = \mu_{p}\mu_{n}[F_{\mathrm{mg}}(\mathrm{H}) - F_{\mathrm{mg}}(\mathrm{He})]/[2f(\mu_{p}^{2}-\mu_{n}^{2})]. \tag{14}$$

Here f is the charge form factor of an elementary proton and $\mu_{p,n}$ the nucleon magnetic moments (to reduce noise we assume the elementary neutron charge form factor vanishes). In previous work we have applied (12)–(14) with realistic charge and magnetic form factors. For the present calculation, however, we make some simplifying assumptions which permit a more convenient variation of nuclear radii and which help eliminate noise from what is already a numerically challenging calculation. Specifically, we assume that the distribution of spin for the nonpaired nucleon is the same as its distribution of matter,

$$\rho^{n}_{\mathrm{sp}}(\mathrm{He}) = \rho^{n}_{\mathrm{mt}}(\mathrm{He}), \tag{15}$$

while the spin distribution for the paired nucleons vanish,

$$\rho^{p}_{\mathrm{sp}}(\mathrm{He}) = 0. \tag{16}$$

To enable convenient variations of the nuclear radii, we use analytic expressions [12] for the form factors. (Performing the calculations with numerical, Faddeev form factors [13], changes the predictions somewhat, but not the conclusions.)

The nuclear RMS radii used in our calculation are given in Table I. The values in row one obey charge symmetry, while the values in rows two and three gives break CS. The 1.88 fm value derives from the ^{3}He charge form factor [12] and the 1.70 fm value from the ^{3}H charge form factor [14,15]. The value of 1.76 fm for $R_{p}(^{3}\mathrm{H})$ in row three arises from a recent measurement [16], and is significantly larger than the values of previous measurements. It would be valuable to have it confirmed with a nucleon probe.

To obtain sufficient numerical accuracy for the large momentum transfers which occur with protons, we discretized the coupled-channels Lippmann-Schwinger equation over as many as 64 grid points, decomposed the potentials and T matrices into 64 partial waves, and included NN phase shifts up to $l=4$ (higher NN partial waves tend to introduce numerical noise). To include the singular, momentum-space Coulomb potential, we used a cutoff radius of 7 fm, and verified that our results are stable for small variations about this radius [10]. With all these effects included at the requisite high precision, the calculation is numerically intensive, and so we modified the computer code LPOTp to run on a parallel computer [17].

III. RESULTS

The calculations we report here are for 500 MeV nucleon scattering. Nearby energies yield similar predictions. In the top part of Fig. 1 we show the predicted differential cross sections for proton and neutron elastic scattering from ^{3}He and ^{3}H with all CS-breaking effects included. The neutron scattering cross sections (dotted and long-dashed curves) do not contain a forward Coulomb peak, but aside from that, the p ^{3}He and n ^{3}H cross sections, and the p ^{3}H and n ^{3}He cross sections, respectively, are nearly equal (as expected from approximate CS). We also note that the proton scatter-

TABLE I. The rms radii of the matter distributions for the trinucleon system. The first row are the charge symmetric values; the others include CS breaking.

	$R_{p}(^{3}\mathrm{H})$	$R_{n}(^{3}\mathrm{H})$	$R_{p}(^{3}\mathrm{He})$	$R_{n}(^{3}\mathrm{He})$
CS	1.700	1.880	1.880	1.700
CSB1	1.700	1.850	1.880	1.735
CSB2	1.760	1.850	1.880	1.795

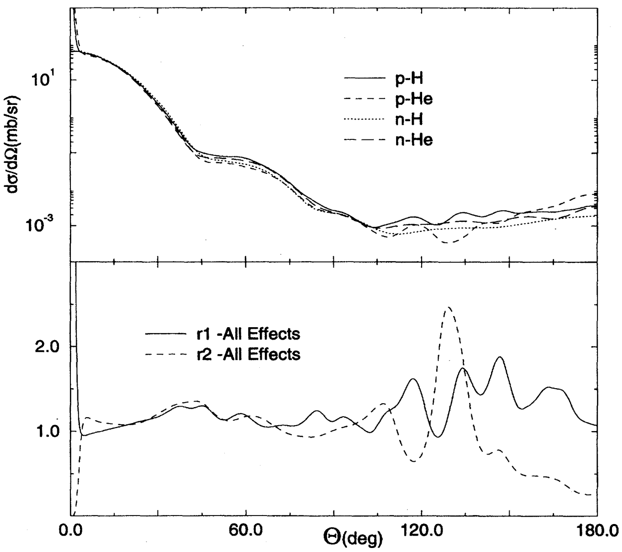

FIG. 1. Top: Differential cross sections for p and n scattering from ^3He and ^3H at 500 MeV as a function of CM scattering angle. Bottom: The ratios $r1$ and $r2$ given by Eqs. (5) and (6). All CS-violating effects are included in this figure.

ing cross sections (solid and short-dashed curves) develop distinct oscillations for angles larger than $\sim 100°$. These oscillations are a consequence of Coulomb-nuclear interference, and in contrast to pion scattering where the large-angle ratios all equal 1, nucleon oscillations occur because the nuclear cross section have fallen off to a level comparable to the background Coulomb cross section.

An important consequence of the backward-hemisphere oscillations in nucleon-trinucleon scattering is that the ratios (5),(6) deviate considerably from 1, as we show in the bottom of Fig. 1. Unfortunately, while we expect this large of an experimental CS-violation signal to occur, we suspect that the back-angle part is too sensitive to details of Coulomb-nuclear interference and to uncertainties in the numerical Coulomb procedure [10] to produce reliable information. Consequently, we consider only forward-hemisphere measurement of the ratios as reliable, and henceforth show only them.

We notice in the bottom of Fig. 1 that the forward-hemisphere ratios $r1$ and $r2$ differ smoothly and significantly from 1. Furthermore, as shown in Figs. 2 and 3, when the superratio $\mathscr{R}$ (7) is formed, we obtain a large signal in the forward hemisphere, quite a bit larger in fact than the one for pion scattering.

In the top of Fig. 2 we show the predicted superratio when all CS violation effects are included. The solid and dashed curves correspond to the smaller and larger nuclear radii given in rows 2 and 3 of Table I. We see that the super-ratio is sensitive, but not overly sensitive, to the uncertainty in nuclear radii. If an experiment could measure the ratio to this level of precision, an independent measurement of nuclear size should be possible.

In the bottom of Fig. 2 we show the predicted super-ratio including the CS violation arising from only the (n,p) and $(^3He,^3H)$ mass differences (solid curve), and from only the nuclear structure (dashed curve). The mass difference effect

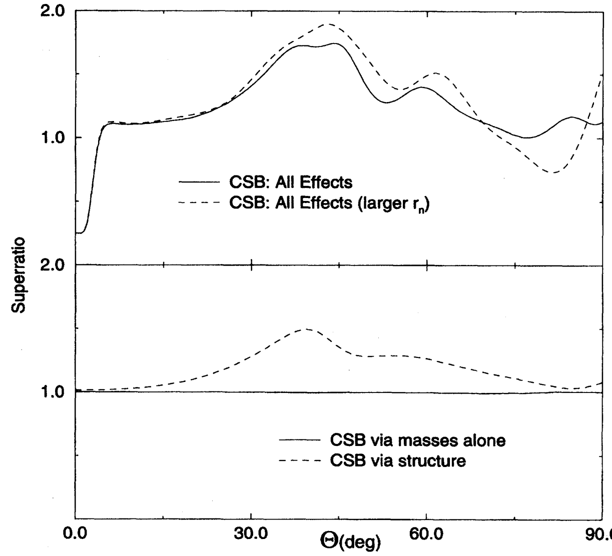

FIG. 2. Top: The super-ratio R, (6), with all CS-violating effects included. The solid and dashed curves correspond to smaller and large nuclear radii given in lines 2 and 3, respectively, of Table I. Bottom: The super-ratio R including the CS violation arising from only the (n,p) and $(^3He,^3H)$ mass differences (solid curve), and from CS violation arising from only the nuclear structure (dashed curve).

essentially disappears in comparison to the nuclear structure one. Since a violation in the nuclear structure is of more interest than the mass difference one, this is an encouraging finding.

In the top of Fig. 3 we see the sensitivity of the super-ratio

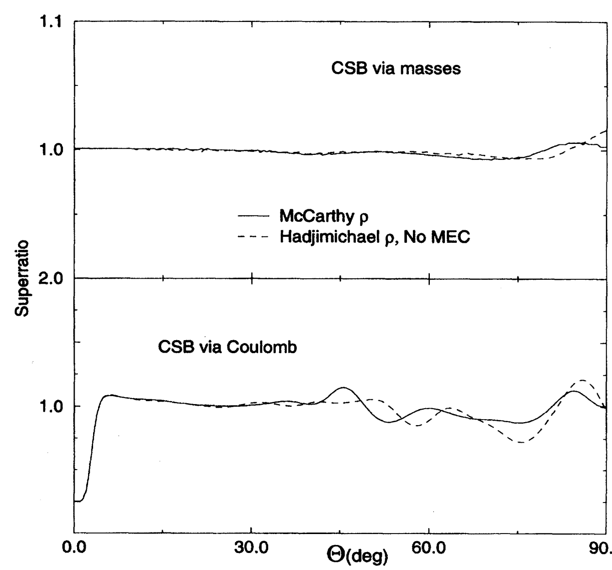

FIG. 3. Top: The sensitivity of the super-ratio R to use of nuclear form factors given by analytic fits to electron scattering data (solid curve) and given by numerical solutions to Faddeev equations (dashed curve). In this case, CS violation from only the mass differences are included. Bottom: Same as on top, only now CS violation arises from only the p-nucleus Coulomb force.

$\mathscr{R}$ to the use of nuclear form factors given by analytic fits to electron scattering data (solid curve) and given by numerical solutions to Faddeev equations (dashed curve). This figure shows that the super-ratio in this angular region is not sensitive to details of the nuclear form factors but is sensitive to the nuclear size. In addition, by comparing Figs. 2 and 3 we see the main CS violation arises from the nuclear structure, with a somewhat smaller violation arising from the p-nucleus Coulomb force. The CS violation from Coulomb and structure effects are seen to combine in the 40° region in Fig. 2 to produce a large effect.

IV. CONCLUSION

We have calculated the ratios of differential cross sections for 500 MeV proton and neutron elastic scattering from ^{3}He and ^{3}H. We used a microscopic, momentum-space optical potential and included the Coulomb force and all spin couplings exactly. We found that at large angles the utility of these ratios as a measure of charge symmetry breaking is low due to highly sensitive Coulomb-nuclear interference. However, the forward-hemisphere ratio appears to be reliable yet still sensitive to the important CS violation mechanisms.

We predict that most of the CS violation in nucleon-trinucleon scattering should arise from CS violation at the nuclear structure level, with about 1/3 of the effect arising from the proton-nucleus Coulomb interaction. A measurement of the ratios of cross sections for nucleon-trinucleon scattering at the 10% level in the forward hemisphere would be a valuable adjoint to the analogous pion measurements.

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