πNN coupling constants from NN elastic data between 210 and 800 MeV

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High partial waves for pp and np elastic scattering are examined critically from 210 to 800 MeV. Non-one-pion-exchange contributions are compared with predictions from theory. There are some discrepancies, but sufficient agreement that values of the πNN coupling constants g_0^2 for π^0 exchange and g_c^2 for charged π exchange can be derived. Results are $g_0^2=13.94\pm0.17\pm0.07$ and $g_c^2=13.69\pm0.15\pm0.24$, where the first error is statistical and the second is an estimate of the systematic error arising from uncertainties in the normalization of total cross sections and $d\sigma/d\Omega$.

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I. INTRODUCTION

There has recently been controversy over the magnitude of the πNN coupling constant. Prior to this controversy, the accepted value for many years was that determined by Bugg, Carter, and Carter [1]: $f^2=0.0790(10)$, where the error is given in parentheses. Defining

$$g^2=f^2(2M/\mu_c)^2, \quad (1)$$

where M is the mass of the proton and μ_c the mass of the charged pion, this value of f^2 corresponds to g^2 = 14.28(18). This determination, based on the fixed t dispersion relations for the $B^{(+)}$ amplitude in πN elastic scattering, refers to exchange of charged pions at the nucleon pole: $\pi^- p\rightarrow n$. The result of Ref. [1] is known to be slightly high because it was influenced by some data at 310 MeV now believed to be wrong. A fresh analysis by Markopoulou-Kalamara and Bugg [2], using all new data from 90 to 310 MeV, and including Coulomb effects such as the mass and width differences of Δ^{++} and Δ^0, gives the updated result $f^2=0.0771(14), g^2=13.94(25)$.

Since 1987, the Nijmegen group has derived significantly lower values of g^2 from NN elastic data up to 350 MeV in a series of papers [3–6]. In their recent published work [6], they find equal coupling constants for exchange of neutral and charged pions within experimental error: f_0^2 =0.0750(5) or $g_0^2=13.56(9)$, $f_c^2=0.0748(3)$ or g_c^2 =13.52(5).

In 1990, Arndt *et al.* [7] obtained the value f^2 =0.0735(15), $g^2=13.28(27)$ from an analysis of new πN data. A new analysis by Arndt *et al.* [8], imposing dispersion relation constraints, now revises their determination upwards to $g^2=13.75(15)$.

Machleidt and Sammarruca [9] have compared results against precise information from the deuteron quadrupole moment and asymptotic D/S state ratio. They point out that the lower values of g^2 require lower values of f_ρ/g_ρ than are used by most theoretical groups.

In this mildly conflicting situation, we have examined NN data above the energy range considered by the Nijmegen group. From 210 to 800 MeV, measurements of Wolfenstein parameters have the virtue of being very precise and are a direct source of information on the spin dependence of high partial waves, hence g^2. Few of these data appeared in the Nijmegen analysis, so our results are to a considerable extent independent. A minor difficulty arises from inelasticity above 300 MeV, which introduces more free parameters into the phase shift analysis. However, the more serious issue is the separation of one-pion exchange (OPE) from exchange of 2π, 3π, ρ, ω, etc. These will be described here collectively as heavy boson exchange (HBE). In this paper, the conclusion is reached that NN data from 140 to 800 MeV are so precise that they do indeed provide useful determinations of g^2.

Pion exchange enters through the amplitude δ in the notation of Bystricky, Lehar, and Winternitz [10] having the form [11] $(\boldsymbol{\sigma}_1\cdot\mathbf{q})(\boldsymbol{\sigma}_2\cdot\mathbf{q})/(t-\mu^2)$, where $\mathbf{q}$ is transverse momentum and $t=-q^2$. It may be isolated using the spin dependence of NN elastic scattering. It turns out from phase shift analysis that this particular amplitude is well measured by Wolfenstein parameters D_{NN} for elastic scattering [12] and by K_{NN} and K_{LL} for np charge exchange. For example, in standard notation,

$$D_{NN}=\frac{|\alpha|^2+|\beta|^2-|\delta|^2-|\epsilon|^2+2|\gamma|^2}{|\alpha|^2+|\beta|^2+|\delta|^2+|\epsilon|^2+2|\gamma|^2}\,. \quad (2)$$

In this expression, the angular dependence of D_{NN} at small angles is dictated essentially by $|\beta|^2-|\delta|^2$. The idea is illustrated in Fig. 1. At 0°, amplitudes $\beta\propto(\boldsymbol{\sigma}_1\cdot\mathbf{n})(\boldsymbol{\sigma}_2\cdot\mathbf{n})$ and $\delta\propto(\boldsymbol{\sigma}_1.\mathbf{q})(\boldsymbol{\sigma}_2.\mathbf{q})$ are equal, since there is nothing to distinguish the normal $\mathbf{n}$ to the scattering plane and $\mathbf{q}$ in the plane of scattering. The amplitudes β, α, ϵ, and γ vary slowly with q^2 and can be extrapolated securely to $q=0$. The δ amplitude varies rapidly with q and crosses zero at about $t=-\mu^2$. Figures 1(c) and 1(d) show that the parameters D_{NN} and K_{NN} have striking dips and peaks, respectively, due to these zeros; these strong features establish the magnitude of the OPE contribution accurately. A detail is that the Cou-

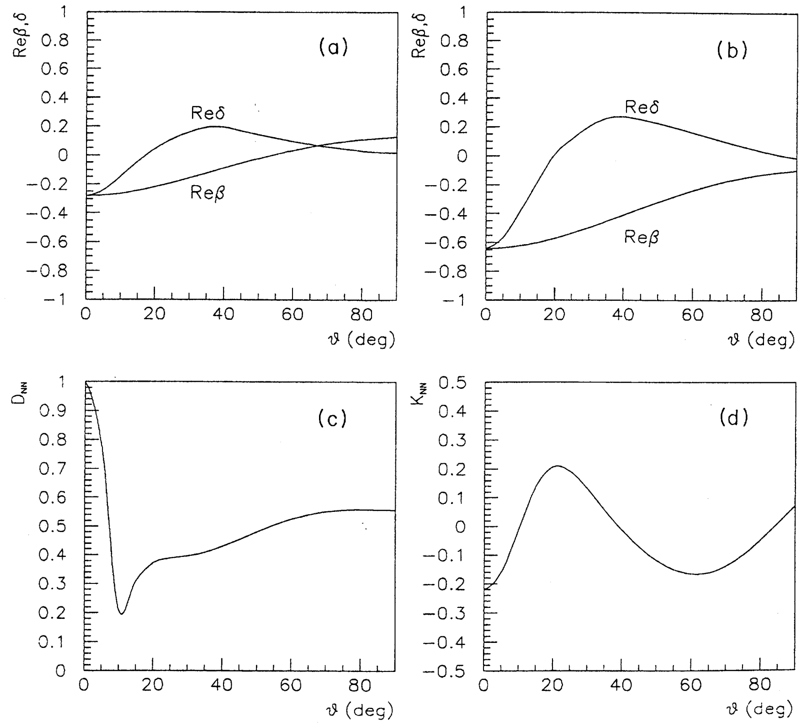

FIG. 1. Real parts of β and δ amplitudes for (a) pp elastic scattering, (b) np charge exchange at 515 MeV, (c) and (d) corresponding values of D_{NN} and K_{NN}. Amplitudes are in fm.

lomb amplitude dominates pp scattering below $q \simeq 40$ MeV/c, so for pp the vital q range is approximately 50–250 MeV/c.

The relevance of the D_{NN}, K_{NN}, and K_{LL} data to g^2 is readily checked by omitting them from the phase shift analysis; this produces a sharp increase in the statistical error on g^2 and a pronounced scatter in results at different energies. Other sources of information are less precise. It is well known that $d\sigma/d\Omega$ near $t=0$ or $u=0$ is sensitive to g^2. For pp scattering, however, the situation is obscured by the diffraction peak and by Coulomb interference. For np charge exchange, the absolute normalization of most data has errors of at least 5–10 %. If one steps g^2 through a series of values, the phase shift fits accommodate with little change in χ^2 by adjusting the normalization of $d\sigma/d\Omega$. For np charge exchange, there is a further difficulty. Interference of ${}^1S_0$, ${}^1D_2$ and ${}^1G_4$ with ${}^1P_1$ and ${}^1F_3$ generates a substantial non-OPE background, which is hard to pin down with the required accuracy. Precise determinaton of spin dependence plays a critical role in separating this background from OPE.

Unfortunately, direct analysis of data on Wolfenstein parameters turns out to be difficult, because of interferences and because it is necessary to describe the slowly varying components in a way consistent with the HBE contributions, which are partly determined by other input. We therefore use the full weight of partial wave analysis and the full database. The OPE contribution is then derived from the tensor components in high partial waves. These are of two sorts: (a) mixing parameters $\bar{\epsilon}$, which mix triplet states having $J=L\pm 1$, and (b) tensor combinations of H and I waves [13]:

$$H_T \propto -6\,{}^3H_4 + 11\,{}^3H_5 - 5\,{}^3H_6, \qquad (3)$$

$$I_T \propto -7\,{}^3I_5 + 13\,{}^3I_6 - 6\,{}^3I_7. \qquad (4)$$

In whatever way the analysis is done, the essential difficulty is to separate the exchange of heavier mesons from OPE. Actually σ and ω exchange contribute only to central and spin-orbit combinations, which are orthogonal to tensor components. This leaves ρ and 3π exchanges, which contribute a slowly varying component to tensor combinations. It is well known that ρ exchange cuts off the π tensor amplitude below $r \simeq 1$ fm. Machleidt [14] shows that, because of their large masses, ρ and 3π contribute mostly to low partial waves, notably ${}^3P_0$, ${}^3P_1$, and ${}^3P_2$, where they may be determined phenomenologically.

In this paper, we will use the HBE contributions as determined by the "Bonn peripheral model." This is an extension of the model for higher partial waves developed by Machleidt, Holinde, and Elster, in Sec. 5 of Ref. [15], above pion-production threshold. It uses the approach described in Appendix B (model II) of Ref. [14]. The ρ coupling is taken from the work of Höhler *et al.* [16] and the correlated 2π S-wave contribution from Durso *et al.* Only the ω coupling is adjusted so as to fit ${}^3F_4$ and ${}^1G_4$. The model also includes $\Delta(1232)$ isobars in intermediate states which are excited via π and ρ exchange and provide inelasticity as well as additional intermediate-range attraction. Phase shift predictions have been adjusted to the value $g^2=13.87$, which is the mean value in our conclusions.

The program of this work is to examine critically each of the high partial waves, so as to form an opinion of what is (or is not) understood, before passing judgment on which waves can be used to determine g^2. The predictions by the peripheral Bonn model turn out to be in generally good agreement with pp data for partial waves with $J \geq 4$, with the

TABLE I. Comparing predictions for $\bar{\epsilon}_4$ and $\bar{\epsilon}_6$ with experiment. Units are degrees.

T (MeV)	OPE	HBE	OPE+HBE	Experiment	Discrepancy	Parameter
210	−1.128	0.021	−1.107	−1.29 ± 0.09	−0.18±0.09	$\bar{\epsilon}_4$
325	−1.614	0.112	−1.502	−1.47 ± 0.07	0.03±0.07	
425	−1.939	0.262	−1.677	−1.66 ± 0.04	0.01±0.04	
515	−2.179	0.455	−1.724	−1.74 ± 0.07	−0.02±0.07	
580	−2.329	0.625	−1.704	−1.67 ± 0.06	0.03±0.06	
650	−2.473	0.827	−1.646	−1.44 ± 0.06	0.21±0.06	
720	−2.601	1.037	−1.564	−1.66 ± 0.04	−0.09±0.04	
800	−2.732	1.276	−1.456	−1.42 ± 0.04	0.04±0.04	
210	−0.332	0.000	−0.332			$\bar{\epsilon}_6$
325	−0.554	0.003	−0.551			
425	−0.717	0.010	−0.707			
515	−0.843	0.020	−0.823			
580	−0.918	0.031	−0.887	−0.99 ± 0.05	−0.10±0.05	
650	−1.003	0.046	−0.957			
720	−1.075	0.063	−1.022			
800	−1.149	0.087	−1.062	−0.98 ± 0.04	0.08±0.04	

exception of $^3H_4$. Corrections can be made for this small discrepancy, leaving what appears to be a reliable determination of neutral π exchange and the corresponding coupling constant g_0^2.

For np charge exchange, OPE is three times larger than for pp elastic scattering, so one might hope to derive precise values of the charged coupling constant g_c^2. The experimental data on Wolfenstein parameters are as good for K_{NN} in charge exchange as for D_{NN} in pp elastic scattering. Unfortunately, theory is not in such good shape, and there are clear discrepancies between HBE predictions and experiment for $I=0$ G and H waves. We find that it is still possible to make a reasonably accurate determination of g_c^2 from $\bar{\epsilon}_5$ and higher partial waves, but there is a significant systematic error arising from normalization of $d\sigma/d\Omega$ and $\sigma_{Tot}(np)$.

Section II provides analysis of pp scattering and arrives at a determination of g_0^2. Section III analyzes np data and g_c^2. Section IV comments on systematic errors. Section V presents conclusions.

II. HIGH PARTIAL WAVES FOR *pp* SCATTERING

There have been extensive and very accurate measurements of Wolfenstein parameters at TRIUMF [17–19] from 210 to 515 MeV, at PSI by the Geneva group up to 580 MeV [20] and at LAMPF from 485 to 800 MeV [21–24]. Where they overlap, these experiments agree well, so one can have confidence in the data. In fact, what is needed to determine g^2 is a product of Wolfenstein parameters and $d\sigma/d\Omega$, and the bigger problems reside in the latter, where absolute normalization presents experimental difficulties. Here the optical theorem is some help, by relating the imaginary part of the spin-averaged amplitude to the total cross section via the optical theorem. A precise determination of g^2 depends on high absolute accuracy in all these data. Results will be compared at eight energies from 210 to 800 MeV. Because data come from a variety of experimental groups using different techniques, one gets some idea of systematic errors. At the Gatchina energy of 970 MeV, there are no accurate measurements at the small angles required for present purposes. Likewise, around 142 MeV, the Harwell-Harvard-Orsay energy, there are no measurements of Wolfenstein parameters below 30°. So these two energies are omitted.

Table I compares predictions for mixing parameters $\bar{\epsilon}_4$ and $\bar{\epsilon}_6$ with the phase shift analysis of Bugg, using all currently available data. This analysis is a minor update of the analysis published by Bugg and Bryan [25], but adding a few new data points. It includes Coulomb barrier corrections and a Coulomb interaction allowing for the charge radius of the proton. For $\bar{\epsilon}_4$, agreement is remarkably good right up to 800 MeV. At the upper energies, HBE contributions are becoming uncomfortably large compared with OPE and it is obvious that errors in theoretical predictions for HBE could bias the determination of g_0^2. However, in view of the agreement for $\bar{\epsilon}_4$, one can have great confidence in $\bar{\epsilon}_6$, where HBE contributions are much smaller. Free fits to $\bar{\epsilon}_6$ at the best energies 580 and 800 MeV, where data are very extensive, give satisfactory agreement with predictions.

The agreement for $\bar{\epsilon}_4$ is so good that one might wonder whether this parameter has already been used in optimizing theoretical predictions. This is not so. We stress again that except for the ω (which does not create any tensor force and thus does not contribute directly to $\bar{\epsilon}_J$) all parameters in the theoretical model are taken from independent sources and have not been fitted to data. So the agreement of $\bar{\epsilon}_4$ with prediction up to 800 MeV is a real success of the HBE predictions and is not a circular argument.

Table II makes similar comparisons for $^1G_4$ and H waves. For $^1G_4$, agreement is excellent, but the HBE contribution is large compared with OPE, so one would not be justified in using this partial wave to determine g^2.

For $^3H_4$, there is a definite discrepancy beginning at 325 MeV. This raises the question of how far to trust HBE predictions for $^3K_6$. In a classical approximation, angular momentum L is related to impact parameter r and momentum k by $\sqrt{L(L+1)}=kr$. This suggests that agreement for $^3H_4$ up to 210 MeV implies agreement for $^3K_6$ up to a lab energy

TABLE II. Comparison of G, H, I, and K waves (degrees) with experiment.

T (MeV)	OPE	HBE	OPE+HBE	Experiment	Discrepancy	Parameter
210	0.700	0.280	0.980	1.07±0.07	0.09±0.07	$^1G_4$
325	0.905	0.761	1.666	1.68±0.05	0.01±0.05	
425	1.011	1.345	2.356	2.31±0.05	−0.04±0.05	
515	1.074	1.990	3.064	3.02±0.04	−0.04±0.04	
580	1.106	2.495	3.601	3.68±0.06	0.08±0.06	
650	1.132	2.964	4.098	4.26±0.06	0.16±0.06	
720	1.150	3.246	4.396	4.15±0.09	−0.25±0.09	
800	1.164	3.332	4.496	4.79±0.07	0.29±0.07	
580	0.542	0.292	0.834	0.87±0.04	0.03±0.04	$^1I_6$
800	0.620	0.531	1.151	1.24±0.05	0.09±0.05	
210	0.309	0.027	0.336	0.31±0.05	−0.03±0.05	$^3H_4$
325	0.539	0.061	0.600	0.49±0.05	−0.11±0.05	
425	0.723	0.073	0.796	0.50±0.05	−0.29±0.05	
515	0.875	0.052	0.927	0.37±0.05	−0.56±0.05	
580	0.977	0.006	0.983	0.51±0.04	−0.47±0.04	
650	1.079	−0.083	0.996	0.59±0.06	−0.41±0.06	
720	1.176	−0.231	0.945	0.48±0.06	−0.46±0.06	
800	1.279	−0.481	0.798	0.49±0.04	−0.31±0.04	
580	0.314	0.049	0.363	0.40±0.04	0.03±0.04	$^3K_6$
800	0.444	0.055	0.499	0.42±0.04	−0.08±0.04	
325	−1.286	0.183	−1.103	−1.19±0.06	−0.09±0.06	$^3H_5$
425	−1.645	0.333	−1.312	−1.36±0.05	−0.05±0.05	
515	−1.926	0.501	−1.425	−1.56±0.06	−0.13±0.06	
580	−2.109	0.635	−1.474	−1.58±0.07	−0.11±0.07	
650	−2.289	0.772	−1.517	−1.66±0.08	−0.15±0.08	
720	−2.454	0.870	−1.584	−1.28±0.07	0.30±0.07	
800	−2.626	0.917	−1.709	−1.48±0.07	0.22±0.07	
580	−0.847	0.103	−0.744	−0.75±0.07	−0.01±0.07	$^3K_7$
800	−1.124	0.194	−0.930	−0.96±0.07	−0.03±0.07	
210	0.121	0.063	0.184	0.20±0.04	0.02±0.04	$^3H_6$
325	0.230	0.185	0.415	0.42±0.04	0.00±0.04	
425	0.323	0.331	0.654	0.83±0.04	0.18±0.04	
515	0.402	0.475	0.877	0.81±0.02	−0.07±0.02	
580	0.456	0.587	1.043	1.05±0.03	0.01±0.03	
650	0.512	0.707	1.219	1.37±0.05	0.15±0.05	
720	0.564	0.823	1.387	1.64±0.05	0.25±0.05	
800	0.621	0.943	1.564	1.79±0.02	0.23±0.02	
580	0.167	0.105	0.272	0.40±0.06	0.13±0.06	$^3K_8$
800	0.246	0.207	0.453	0.50±0.02	0.04±0.02	

$T=210\times(6\times7)/(4\times5)\simeq450$ MeV. Calculations of HBE support this rough classical notion for high partial waves. Using this prescription, the discrepancies for $^3H_4$ have been used to estimate small corrections for $^3K_6$ from 515 to 800 MeV; see Table III. In practice, it turns out that this refinement has an effect on g^2 well below statistical errors.

For $^3H_5$ and $^3H_6$, agreement is satisfactory up to 580 MeV; above this energy, small systematic discrepancies begin to appear. It implies that $^3K_7$ and $^3K_8$ should be reliable up to 800 MeV and this appears to be so experimentally within two standard deviations.

Table IV summarizes the measure of agreement in the various partial waves. A check indicates agreement, a cross disagreement, and C indicates that a correction has been applied; L indicates that the HBE contribution is too large for comfort (a subjective judgement to be discussed shortly).

Table V gives values of g_0^2, depending on a variety of assumptions. In the first column, $\bar{\epsilon}_4$ and $^3H_5$ are used, together with all parameters from $\bar{\epsilon}_6$ upwards. (At 210 MeV, $^3H_4$ is also used.) In column 2, $\bar{\epsilon}_4$ is used but H waves are left free. In column 3, only $\bar{\epsilon}_6$ and higher waves are used; this is very conservative, almost certainly too conservative at 210 and 325 MeV.

Arithmetic for g_0^2. There is satisfactory consistency over

TABLE III. Corrected HBE values from an impact parameter prescription. Units are degrees.

T (MeV)	$^3K_6$	$^1H_5$	$^3I_5$	$^3I_6$
210	0.004	0.004	−0.030	0.032
325	0.013	−0.021	−0.083	0.088
425	0.026	−0.086	−0.130	0.149
515	0.018	−0.182	−0.130	0.150
580	0.007	−0.283	−0.105	0.121
650	−0.014	−0.421	−0.071	0.083
720	−0.026	−0.590	−0.036	0.041
800	−0.056	−0.821	0.000	0.000

TABLE V. Fitted values of g_0^2 from three assumptions for HBE.

T (MeV)	Using $\bar{\epsilon}_4$ and $^3H_5$	Using $\bar{\epsilon}_4$	$\bar{\epsilon}_6$ upwards
210	15.01 ± 0.46	15.79 ± 0.48	15.13 ± 0.75
325	14.23 ± 0.33	13.99 ± 0.35	14.70 ± 0.57
425	13.95 ± 0.26	13.41 ± 0.34	13.44 ± 0.38
515	14.02 ± 0.21	14.61 ± 0.25	14.58 ± 0.34
580	13.75 ± 0.35	13.74 ± 0.60	14.45 ± 0.68
650	12.86 ± 0.44	12.17 ± 0.48	12.88 ± 0.52
720	14.01 ± 0.45	14.10 ± 0.45	10.81 ± 0.83
800	13.69 ± 0.16	13.44 ± 0.16	13.61 ± 0.22

most of Table V. In trying to derive a mean value for g_0^2, it is necessary to steer a middle course between (a) using only the very high partial waves, hence incurring an increased error, or (b) risking that errors in HBE affect g_0^2. We shall illustrate the arithmetic with a variety of possible choices. First we give what we regard as the best choice.

We choose to use HBE contributions up to the energies where they become 20–25 % of OPE. The reasoning is as follows. We shall find errors on g^2 of about ±2%, including systematic errors. It seems reasonable to believe HBE contributions to 10% of their magnitudes in view of the excellent agreement for $\bar{\epsilon}_4$ (and later $\bar{\epsilon}_3$ and $\bar{\epsilon}_5$). This is the origin of our subjective judgment L in Table IV. It means that we use $\bar{\epsilon}_4$, $^3H_5$, and $^1I_6$ up to 515 MeV. We cut off $^3H_4$ above 210 MeV, because of the systematic discrepancies in Table II. The OPE contributions to $^3H_6$ are too small for this to be a reliable source of information, leaving only $^3H_5$ amongst the H waves. It implies taking g_0^2 from column 1 of Table V up to 515 MeV and column 3 thereafter. This choice results in a weighted mean $g_0^2=13.91\pm0.13$. However, results scatter about the mean with variance $\sigma^2=3.9$. This implies that the error should be scaled up by $1/\sigma$ to ±0.22.

The main culprit in this large value of σ is the value of g_0^2 in column 3 at 720 MeV, and this leads us to discuss the reliability of the database at different energies. The database is extensive at 210, 325, 425, 515, and 580 MeV and even more extensive at 800 MeV. There is a good case for selecting these energies and ignoring 650 and 720 MeV. At 650 MeV, the database can be described as adequate, but rather few cross-checks remain. At 720 MeV, it is decidedly sparse, and freeing $\bar{\epsilon}_4$ leads to considerable latitude in the phase shifts. The value $g_0^2=10.81\pm0.83$ at this energy in the third column of Table IV changes to 14.04 ±0.45 in column 2. Our conclusion is that the fit at 720 MeV is not sufficiently stable.

Omitting 720 MeV, the result is a weighted mean

$$g_0^2=13.94\pm0.17. \tag{5}$$

It is our judgement that this is the most reliable from Table V. The reader can easily make any other particular choice. However, let us just illustrate what happens if we are slightly less optimistic in trusting HBE contributions. If we drop the value of g_0^2 from column 1 at 515 MeV and substitute that from column 3, g_0^2 changes to 14.00±0.21. If we drop the value of g_0^2 from column 1 at 425 MeV as well and substitute that from column 3, g_0^2 changes to 13.94±0.21. Conversely, if we are more optimistic about HBE for $\bar{\epsilon}_4$ at 580 MeV, $g_0^2\rightarrow13.91\pm0.16$. If 650 MeV data were to be rejected, g_0^2 would increase to 13.99±0.15.

We give one final illustration of arithmetic. Up to the present point we have derived results from columns 1 and 3 of Table V. Suppose we replace column 1 by column 2. This could be justified as follows. Above 210 MeV, $^3H_4$ shows a significant disagreement with HBE. Hence so does the tensor combination of H waves. So it may make sense to drop all H waves from 325 to 515 MeV, and substitute column 2 for column 1 at these energies. With this choice, Eq. (5) changes to $g_0^2=13.97\pm0.24$. The mean value has barely changed from Eq. (5), but the error has increased by 50%, because of

TABLE IV. Summary of the measure of agreement between HBE and experiment for pp; √ indicates agreement, × disagreement, L indicates that HBE is uncomfortably large, and C indicates that an empirical correction has been applied using data on $^3H_4$.

T (MeV)	$\bar{\epsilon}_4$	$^3H_4$	$^1G_4$	$^3H_5$	$^3H_6$	$\bar{\epsilon}_6$	$^3K_6$	$^1I_6$	$^3K_7$	$^3K_8$
210	√	√	L	√	L	√	√	√	√	√
325	√	√	L	√	L	√	√	√	√	
425	√	×	L	√	L	√	√	√	√	√
515	√	×	L	√	L	√	C	√	√	√
580	L	×	L	L	L	√	C	L	√	√
650	L	×	L	L	L	√	C	L	√	√
720	L	×	L	L	L	√	C	L	√	√
800	L	×	L	L	L	√	C	L	√	√

TABLE VI. Comparison of fitted values of $\bar{\epsilon}_3$ with HBE. Units are degrees.

T (MeV)	OPE+HBE	Experiment	Discrepancy
142	4.829	4.63 ± 0.13	−0.20±0.13
210	6.211	6.00 ± 0.07	−0.21±0.07
325	7.445	7.36 ± 0.07	−0.08±0.07
425	7.882	7.98 ± 0.11	0.10±0.11
515	8.004	7.99 ± 0.13	−0.01±0.13
650	7.913	8.05 ± 0.13	0.14±0.13
800	7.635	8.17 ± 0.12	0.52±0.12

TABLE VII. Comparison of fitted values of $\bar{\epsilon}_5$ with HBE. Units are degrees.

T(MeV)	OPE	HBE	OPE+HBE	Experiment	Discrepancy
142	1.205	−0.011	1.194		
210	1.905	−0.026	1.879	1.83±0.07	−0.05±0.07
325	2.881	−0.067	2.814	2.71±0.05	−0.11±0.05
425	3.555	−0.119	3.436	3.34±0.08	−0.10±0.08
515	4.060	−0.183	3.877	3.79±0.09	−0.09±0.09
650	4.681	−0.313	4.368	4.37±0.10	0.00±0.10
800	5.234	−0.509	4.725	4.86±0.11	0.13±0.11

disregarding $^3H_5$. If the small discrepancy in $^3H_5$ above 515 MeV is remedied by a modification to HBE, and if the larger discrepancy in $^3H_4$ is likewise remedied, the value of g_0^2 goes back to the value 13.94 of Eq. (5), but with a smaller error of ±0.13.

We feel that Eq. (5) is a reasonable compromise amongst the possibilities. The majority of values in Table V lie above the Nijmegen result 13.56±0.09, with the exception of those at 650 and 720 MeV, where the database is least secure. We shall address the question of systematic errors arising from normalization uncertainties in Sec. IV.

As stated in the Introduction, the OPE amplitude is being determined essentially between q = 50 and 250 MeV/c, i.e., at a mean value of $t \simeq -\mu^2$. One has to worry about the effect of a form factor. We write the OPE amplitude proportional to

$$\frac{g^2}{t-\mu^2}\,\frac{\mu^2-\Lambda^2}{t-\Lambda^2}=g^2\left(\frac{1}{t-\mu^2}-\frac{1}{t-\Lambda^2}\right). \tag{6}$$

The Bonn fit to lower partial-wave phase parameters requires Λ = 1.3 to 1.7 GeV/c^2 [14]. It is then straightforward to make a partial wave decomposition of the term $g^2/(t-\Lambda^2)$. The result, using $\Lambda=1.4$ GeV/c^2, is a perturbation to $\bar{\epsilon}_4$ of +0.04° at 800 MeV and less for lower energies and higher partial waves. This is negligible. Physically it corresponds to the fact that the distant pole at $\Lambda=1.4$ GeV/c^2 affects only low partial waves. We remark, however, that the perturbation for $\bar{\epsilon}_3$ is −0.38° at 800 MeV with Λ = 1.4 GeV/c^2. In the next section, we shall find excellent agreement between $\bar{\epsilon}_3$ and experiment up to 650 MeV. This

TABLE VIII. Comparison of fitted values of $I=0$ G and H waves with HBE. Units are degrees.

T (MeV)	OPE	HBE	OPE+HBE	Experiment	Discrepancy	Parameter
142			−1.676			$^3G_3$
210			−2.860	−3.20±0.16	−0.34±0.16	
325			−4.647	−3.95±0.13	0.70±0.13	
450			−5.835	−5.26±0.19	0.58±0.19	
515			−6.593	−5.84±0.17	0.75±0.17	
650			−7.222	−6.96±0.20	0.26±0.20	
800			−7.346	−6.38±0.16	0.97±0.16	
800	−2.278	−0.091	−2.369	−2.20±0.10	0.17±0.10	$^3I_5$
142	3.263	0.209	3.472			$^3G_4$
210	4.931	0.427	5.358	5.84±0.14	0.48±0.14	
325	7.241	0.776	8.017	7.84±0.20	−0.18±0.20	
425	8.856	0.934	9.790	8.71±0.15	−1.08±0.15	
515	10.081	0.900	10.981	9.67±0.16	−1.31±0.16	
650	11.618	0.496	12.114	10.80±0.20	−1.31±0.20	
800	13.014	−0.433	12.581	10.20±0.18	−2.38±0.18	
800	5.199	0.300	5.499	5.25±0.16	−0.25±0.16	$^3I_6$
142	−0.467	0.195	−0.272			$^3G_5$
210	−0.825	0.454	−0.371			
325	−1.410	1.031	−0.379	−0.52±0.20	−0.14±0.20	
425	−1.872	1.609	−0.263	−0.70±0.14	−0.44±0.14	
515	−2.250	2.158	−0.092	−0.36±0.17	−0.27±0.17	
650	−2.756	2.999	0.243	0.11±0.12	−0.13±0.12	
800	−3.245	3.901	0.656	0.18±0.19	−0.48±0.19	
800	−1.219	0.676	−0.543	−0.69±0.07	−0.15±0.07	$^3I_7$
800	−2.495	−0.095	−2.590	−3.39±0.09	−0.80±0.09	$^1H_5$

is an independent check that Λ cannot be substantially less than 1.4 GeV/c^2, since the correction varies roughly as $1/\Lambda^2$.

III. np DATA AND g_c^2

At the outset, there is one important general comment. In the earlier phase shift analysis of Bugg and Bryan [25], there was rather little sensitivity to g^2. In that analysis, high partial waves were left as free as the data permitted, so as to obtain a solution as empirical as possible. At that time, the HBE predictions of Machleidt were not available. Here, our objective is different. We wish to use high partial waves to determine g^2 as well as possible. Therefore, we examine each of them critically in turn, and constrain them to OPE+HBE wherever this appears reasonable.

Table VI compares predictions for $\bar{\epsilon}_3$ with experiment. Agreement is satisfactory, except at 800 MeV, where prediction is dropping away from experiment. However, contributions from HBE are too large to allow direct use of $\bar{\epsilon}_3$ in determining g_0^2. Nonetheless, one can have great confidence in using $\bar{\epsilon}_5$, which is also well determined experimentally. Table VII shows values of $\bar{\epsilon}_5$ from the phase shift analysis of Bugg and Bryan.

For G and H waves, the story is not so nice; see Table VIII. Above 325 MeV, there are large discrepancies between experiment and Machleidt's predictions for $^3G_3$ and $^3G_4$ and a hint of disagreement for $^3G_5$. The former discrepancies are certainly real. They are visible in TRIUMF K_{SS}, K_{LS}, and A_{NN} data at 425 and 515 MeV and in independent LAMPF data for the same parameters at higher energies. An experimental cross-check is that there is no apparent problem with K_{NN} data, which were measured with the same technique and at the same time as K_{SS} and K_{LS}. Where they overlap near 500 MeV, TRIUMF and LAMPF data agree for A_{NN} and K_{NN}. We note that the fit to $^3G_4$ at higher energies could be improved by a drastic reduction of g_c^2 (to 11.41 $\pm$0.19 from 800 MeV data). However, this would result in terrible predictions for $\bar{\epsilon}_3$ and $\bar{\epsilon}_5$. Furthermore, $^3G_4$ at lower energies ($T\leqslant 325$ MeV) would deteriorate substantially. Thus there is no choice for g_c^2 that would consistently improve $^3G_4$ at all energies. So the conclusion is that something is wrong with HBE predictions for G waves and the disagreement must be taken seriously.

At 800 MeV, $^1H_5$ comes out significantly negative of OPE; $^1F_3$ shows the same feature from 325 to 800 MeV [25]. Until the problem with the G waves and $^1H_5$ is understood, one cannot have complete confidence in deriving g_c^2 from np data.

Table IX summarizes the measure of agreement between predictions and $I=0$ partial waves. From the impact parameter prescription [13], one can estimate perturbations to be applied to I and K waves for the discrepancies in Table VIII. The corrections, given in Table III, are small and have the effect of lowering g_c^2 slightly above 515 MeV. At 800 MeV, the experimental determination of $^3I_6$ is compatible with the estimated correction. At that energy, experiment is capable of determining partial waves up to $^3K_8$. The result of this analysis is a determination of g_c^2 shown in Table X using $\bar{\epsilon}_5$ and higher partial waves. The weighted mean value of g_c^2 is

$$g_c^2=13.67\pm 0.29, \qquad (7)$$

where the error has again been inflated from the statistical value ± 0.154 in order to cover fluctuations about the mean. Corrections for the pion form factor are again completely negligible.

In view of the excellent agreement between $\bar{\epsilon}_3$ and prediction, an alternative procedure is to use only $\bar{\epsilon}_5$, which is extremely well determined by K_{NN} data and also by K_{LL} from 485 to 800 MeV. We mention in particular the data of Abegg *et al.* [26] which lead to a very tight constraint on $\bar{\epsilon}_5$. Using this parameter alone leads to

$$g_c^2=13.69\pm 0.15. \qquad (8)$$

This value has the virtue of being independent of the small corrections applied to $^1H_5$, $^3I_5$, and $^3I_6$. It has a smaller error than Eq. (7) because the fluctuations in ϵ_5 are smaller than those in higher partial waves. It is our preferred value. However, the error in Eq. (8) is purely statistical and we shall show in the next section that systematic errors are likely to be larger.

TABLE IX. Summary of the measure of agreement between HBE and experiment for $I=0$ phase shifts; √ indicates agreement, × disagreement, and C indicates that a correction has been applied to HBE predictions using experimental data from G waves.

T (MeV)	$\bar{\epsilon}_3$	$^3G_3$	$^3G_4$	$^3G_5$	$\bar{\epsilon}_5$	$^3I_5$	$^1H_5$	$^3I_6$	$^3I_7$
210	√	√	?	√	√	√	√	√	√
325	√	×	√	√	√	√	C	√	√
425	√	×	×	?	√	√	C	√	√
515	√	×	×	√	√	C	C	C	√
650	√	?	×	√	√	C	C	C	√
800	×	×	×	×	√	C	C	C	√

IV. SYSTEMATIC ERRORS

The errors discussed so far are statistical, apart from a judgment about what HBE values to trust. We now estimate systematic errors arising from $d\sigma/d\Omega$ and Wolfenstein parameters, which we know to be the data decisive in determining g^2. For pp, it turns out that they are about ± 0.07 for g_0^2, i.e., rather less than statistical errors. For g_c^2 the situation is the reverse. Because of systematic uncertainties in normalization, there is a systematic uncertainty which we estimate as ± 0.24.

TABLE X. Fitted values of g_c^2.

T (MeV)	g_c^2
142	13.13 ± 1.70
210	11.84 ± 0.78
325	13.99 ± 0.45
425	12.68 ± 0.42
515	14.46 ± 0.43
650	14.23 ± 0.34
800	13.47 ± 0.31

For pp scattering, the OPE amplitude is determined mostly by $D_{NN} d\sigma/d\Omega$. The measurement of D_{NN} is made using a polarimeter which is calibrated directly using the polarized beam. At the very small momentum transfers relevant to OPE, any normalization errors cancel, so this source of systematic errors can probably be neglected. At the most pessimistic, it should be $\leq \pm 0.5\%$.

The larger problem lies in $d\sigma/d\Omega$. Here we estimate a systematic error of $\pm 1\%$. From 500 to 800 MeV, recent measurements of Simon *et al.* [27] have statistical errors of $\pm 0.5\%$ and an absolute accuracy of about $\pm 0.5\%$. From 500 to 580 MeV, there are similarly precise data of Chatelain *et al.* [28], and for 300–500 MeV precise 90° data of Ottewell *et al.* [29] with normalization errors of $\pm 1.8\%$. There are many measurements of the shape of $d\sigma/d\Omega$, but with larger normalization errors. These can all be tied together with the pp total cross section data of Schwaller *et al.* [30] from 179 to 555 MeV, having systematic errors of $\pm 0.8\%$ and statistics of $\pm 1\%$. One finds in phase shift analysis that there are no particular conflicts amongst all these data, so we conclude with an overall impression of a $\pm 1\%$ error in $d\sigma/d\Omega$. This estimate has been checked independently by dropping in turn various sets of data from the phase shift analysis and deliberately altering normalizations. This procedure leads to an identical estimate of the systematic error. As regards OPE, the formula for D_{NN}, Eq. (2), depends essentially on $|\beta|^2 - |\delta|^2$, so a 1% systematic error translates into a $\pm 0.5\%$ error in the scale of the OPE amplitude, i.e., an uncertainty in g_0^2 of $\simeq \pm 0.07$.

For np charge exchange, the question of absolute normalization has been reviewed recently by McNaughton *et al.* [31]. For Wolfenstein parameters, the normalization error is estimated as $\pm 1.8\%$. However, the situation for $d\sigma/d\Omega$ is less satisfactory. There are few measurements with good absolute normalization. The difficulty lies in knowing the absolute flux of the neutron beam. Keeler *et al.* [32] took great pains over this and claim an absolute accuracy of $\pm 1.6\%$. However, they admit to uncertainties of $\pm 3\%$ for the relative normalizations of forward scattering (where the neutron is detected) and charge exchange (where the proton is detected). Carlini *et al.* [33] present data at 800 MeV with good absolute normalization in the forward hemisphere.

The situation is confused by substantial discrepancies over np total cross sections. These disagree amongst themselves and also disagree with the differential cross sections we have just described. Lisowski *et al.* [34] present data from 40 to 770 MeV with very high statistics; they claim absolute normalization of $\leq \pm 1\%$. These data were obtained with $CH_2 - C$ difference and a continuous neutron spectrum. Similar measurements from 120 to 580 MeV have been presented by Grundies *et al.* [35]. These data all lie 6% lower than several measurements with monoenergetic neutron beams and liquid hydrogen targets, a more attractive technique as regards absolute normalization. Our phase shift solutions settle midway between the Lisowski *et al.* results and the rest, but can be driven to fit either without too great a penalty in χ^2. Keeler *et al.* $d\sigma/d\Omega$ data show a preference against the Lisowski *et al.* data. If the latter are dropped, g_c^2 rises systematically at all energies from 142 to 800 MeV and averages to 13.90. It therefore seems necessary to allow $\pm 3\%$ normalization uncertainty for $d\sigma/d\Omega$. Adding in quadrature the $\pm 1.8\%$ uncertainty in K_{NN}, we arrive at $\pm 3.5\%$ normalization error in $|\beta|^2 - |\delta|^2$, i.e., $\pm 1.75\%$ systematic uncertainty in the OPE amplitude. This translates to a systematic error in g_c^2 of ± 0.24.

In the lower energy range fitted by the Nijmegen group, the situation is similar. There are some very accurate total cross section measurements below 60 MeV. However, the classic problem has always been the determination of the ${}^1P_1$ phase shift (and $\bar{\epsilon}_1$ which is strongly correlated to it). Interference between ${}^1S_0$ and ${}^1P_1$ generates a forward-backward asymmetry in $d\sigma/d\Omega$ which is hard to measure experimentally because of the need to detect neutrons in the forward hemisphere and protons in the backward hemisphere. Furthermore, in the forward hemisphere, the neutron detection efficiency varies with angle and requires meticulous calibration. The error in ${}^1P_1$ leads to corresponding uncertainty in the normalization of the u-channel peak, which is relevant to the determination of g_c^2. It has been almost universal practice for experimental groups to normalize $d\sigma/d\Omega$ data to total cross section measurements; it has also been common practice to relate the forward and backward hemispheres through phase shift analysis. There are only two measurements, at 25.8 and 50 MeV [36], where forward and backward cross sections are joined experimentally. There is a danger that many data sets appear consistent because of a common normalization procedure, which may be subject to systematic error. Furthermore, there are few measurments of spin dependence below 200 MeV, except the polarization parameter. Therefore we see the possibility that determinations of g_c^2 from low enery np data may be subject to systematic error of at least the same magnitude as we find from 210 to 800 MeV. We regard it as important to identify exactly what data determine g^2, so that possible systematic errors can be assessed. From 210 MeV upwards, the situation is improved by the precise measurements by several groups of K_{SS} and other Wolfenstein parameters; ${}^1P_1$ and ${}^1F_3$ are very sensitive to K_{SS}.

V. CONCLUDING REMARKS

NN data give consistent determinations of both g_0^2 and g_c^2 using data from 140 to 800 MeV. The essential source of the information lies in precise measurements of Wolfenstein parameters D_{NN}, K_{NN}, and K_{LL}, together with $d\sigma/d\Omega$. We have given many illustrations of arithmetic. Mean values lie in the range g_0^2 = 13.91 to 14.00 and statistical errors from 0.17 to 0.24, depending on which partial waves are selected in calculating g_0^2. Our preferred values are

$$g_0^2 = 13.94 \pm 0.17 \pm 0.07,$$

$$g_c^2 = 13.69 \pm 0.15 \pm 0.24.$$

These are consistent with absence of charge independence within the errors. The choice of determinations in Tables V and X is somewhat subjective, but the interested reader can easily make his or her own choice.

The result for g_0^2 is somewhat larger than that of the Nijmegen group, $g_0^2 = 13.56 \pm 0.09$. It is marginally above Arndt's latest value $g_c^2 = 13.75 \pm 0.15$. Since this paper was submitted, Arndt *et al.* [37] have published an analysis of NN elastic data where they make a determination of g^2. They do not claim any great accuracy from this source. Their analysis does not contain the HBE predictions of Machleidt.

Further precise measurements of Wolfenstein parameters in the 140–300 MeV range, below the inelastic threshold, should allow even further improvement in accuracy for g_0^2, and such measurements are in progress at IUCF [38]. At these energies, HBE corrections to $\bar{\epsilon}_4$, G and H waves are surely small and accurately determined from a global fit to low partial waves and higher energy data. Errors of ± 0.005 on both Wolfenstein parameters and $d\sigma/d\Omega$ may be achievable and would determine g_0^2 with an accuracy of about ± 0.06. For np measurements, prospects of further improvements are not good, because of the great difficulty of measuring $d\sigma/d\Omega$ absolutely in charge exchange.

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[1] D.V. Bugg, A.A. Carter, and J.R. Carter, Phys. Lett. **44B**, 278 (1973).
[2] F.G. Markopoulou-Kalamara and D.V. Bugg, Phys. Lett. B **318**, 565 (1993).
[3] R.R. Bergervoet, P.C. van Campen, T.A. Rijken, and J.J. de Swart, Phys. Rev. Lett. **59**, 2255 (1987).
[4] J.R. Bergervoet *et al.*, Phys. Rev. C **41**, 1435 (1991).
[5] R.A.M. Klomp, V.G.J. Stoks, and J.J. de Swart, Phys. Rev. C **44**, 1258 (1991).
[6] V. Stoks, R. Timmermans, and J.J. de Swart, Phys. Rev. C **47**, 512 (1993).
[7] R.A. Arndt, Z. Li, L.D. Roper, and R.L. Workman, Phys. Rev. Lett. **65**, 157 (1990).
[8] R.A. Arndt, R.L. Workman, and M. Pavan, Phys. Rev. C **49**, 2729 (1994).
[9] R. Machleidt and F. Sammarruca, Phys. Rev. Lett. **66**, 564 (1991).
[10] J. Bystricky, F. Lehar, and P. Winternitz, J. Phys. (Paris) **39**, 1 (1978).
[11] M.H. MacGregor, M.J. Moravcsik, and H.P. Stapp, Annu. Rev. Nucl. Sci. **10**, 291 (1960).
[12] D.V. Bugg *et al.*, J. Phys. G **4**, 1025 (1978).
[13] R. Dubois *et al.*, Nucl. Phys. **A377**, 554 (1982).
[14] R. Machleidt, Adv. Nucl. Phys. **19**, 189 (1989).
[15] R. Machleidt, K. Holinde, and C. Elster, Phys. Rep. **149**, 1 (1987).
[16] G. Höhler, F. Kaiser, R. Koch, and E. Pietarinen, *Handbook of Pion-Nucleon Scattering* [Phys. Data 12-1, 1 (1979)]; J.W. Durso, A.D. Jackson, and B.J. VerWest, Nucl. Phys. **A345**, 471 (1980).
[17] D. Axen *et al.*, Lett. Nuovo Cimento **20**, 151 (1977).
[18] D. Axen *et al.*, Phys. Rev. C **21**, 988 (1980).
[19] A.S. Clough *et al.*, Phys. Rev. C **21**, 998 (1980).
[20] Y. Onel *et al.*, Phys. Rev. D **40**, 35 (1989), and references therein.
[21] C.L. Hollas *et al.*, Phys. Rev. C **30**, 1251 (1984).
[22] M.W. McNaughton *et al.*, Phys. Rev. C **25**, 1967 (1982).
[23] M.W. McNaughton *et al.*, Phys. Rev. C **26**, 249 (1982).
[24] M.W. McNaughton *et al.*, Phys. Rev. C **44**, 2267 (1991).
[25] D.V. Bugg and R.A. Bryan, Nucl. Phys. **A540**, 449 (1992).
[26] R. Abegg *et al.*, Phys. Rev. C **38**, 2173 (1988).
[27] A.J. Simon *et al.*, Phys. Rev. C **48**, 662 (1993).
[28] P. Chatelain *et al.*, J. Phys. G **8**, 643 (1982).
[29] D. Ottewell *et al.*, Nucl. Phys. **A412**, 189 (1984).
[30] P. Schwaller *et al.*, Phys. Lett. **35B**, 243 (1971).
[31] M.W. McNaughton *et al.*, Phys. Rev. C **45**, 2564 (1992); **48**, 256 (1993).
[32] R.K. Keeler *et al.*, Nucl. Phys. **A377**, 529 (1982).
[33] R. Carlini *et al.*, Phys. Rev. Lett. **41**, 1341 (1978).
[34] P.W. Lisowski *et al.*, Phys. Rev. Lett. **49**, 255 (1982).
[35] V. Grundies, J. Franz, E. Rössle, and H. Schmitt, Phys. Lett. **158B**, 15 (1985).
[36] T.C. Montgomery *et al.*, Phys. Rev. C **16**, 499 (1977).
[37] R.A. Arndt, I.I. Strakovsky, and R.L. Workman, Phys. Rev. C **50**, 2731 (1994).
[38] S. Bowyer and S. Wissink (private communication).