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# Stability of bound states of negative pions and neutrons

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The problem of stable bound states of a system composed of $N$ neutrons and $Z$ negative pions is discussed. Calculations performed within a theoretical model indicate that one needs at least 5 pions and around 8 neutrons in order to get binding. The stability of the system against either pion or neutron emission is investigated, as well as the stability against its possible weak decay modes. Results for several values of $N$ and $Z$ are presented.

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The possibility that a system consisting of negative pions and neutrons may under certain conditions form a stable bound state was first discussed in Refs. [1,2]. The word "pineuts" to name these objects was invented by Van Dantzig and de Boer [3–5].

Since negative pions cannot be absorbed by neutrons, that implies that pineuts will decay only through weak interactions. The dominant decay mode is the one that proceeds through the process

$$\pi^- \to \mu^- + \bar{\nu}\,, \tag{1}$$

which has a lifetime of $\tau \sim 10^{-8}$ s. If this decay mode were closed, the next process would be

$$\pi^- \to e^- + \bar{\nu}\,, \tag{2}$$

which has a lifetime $\tau \sim 10^{-4}$ s. Thus, if we assume that the $Z$ pions bound in a pineut have all the free pion lifetime, then the lifetime of pineuts will be of the order of $10^{-8}/Z$ s if the decay mode (1) is open. If the decay mode (1) is closed then the decay goes through the process of Eq. (2) and the lifetime of pineuts will be of the order of $10^{-4}/Z$ s. It has been argued recently [6,7] that the $\pi^- \to e^- + \bar{\nu}$ decay branch might be considerably enhanced in a dense medium if angular momentum is transfered to the surrounding nucleons in the decay process, since for the free pion this decay branch is helicity suppressed. We are presently investigating this point [8].

In an earlier experiment, de Boer *et al.* [9] used a 600 MeV proton beam scattered on $^9$Be to search for the pineuts $(\pi^-)^Z n^N$ with $Z=1$, $N$=2–6 and $Z=2$, $N=3$ with negative results. The simplest possible pineut, i.e., $(\pi^-)^1 n^2$ has been searched extensively [10–14] using the reactions $\pi^- d \to \pi^+ X$ and $\pi^- d \to \gamma X$ also with negative results. Theoretically, it is also now accepted that this pineut does not exist [15,16]. More recent experiments [17,6] used a heavy ion beam with energies of several GeV/nucleon that was scattered on a heavy ion target. For example, Hemmick *et al.* [17] used a $^{28}$Si beam of 14.6 GeV/nucleon scattered on Al, Sn, Cu, and Pb, while de Boer *et al.* [6] used a $^{238}$U target on which they scattered $^{40}$Ar at 1.8 GeV/nucleon and $^{139}$La at 1.3 GeV/nucleon. From these searches it has now been established that the pineuts $(\pi^-)^Z n^N$ with $Z$=1,2 and $N$=2–4 do not exist.

The original model and the basic idea of pineuts with an arbitrary number of neutrons and pions was proposed in Ref. [2]. We will use here that model to calculate the binding energies of pineuts as a function of $N$ (the number of neutrons) and $Z$ (the number of pions). The theoretical model of Ref. [2] provides a prescription to calculate (a) the interaction of the pion with the neutrons, (b) the interaction between the neutrons themeselves, and (c) the Coulomb interaction between the pions. The strong interaction between the pions is neglected since two negative pions must necessarily be in an isospin 2 state. Therefore, since the $\pi\pi$ phase shifts for isospin 2 are very small [18] one expects this interaction to be negligible. Thus, the energy of a pineut consisting of $N$ neutrons and $Z$ pions is

$$E = E_N - ZB + E_{\rm Coul}\,, \tag{3}$$

where $E_N$ is the self-energy of the $N$ neutrons, $B$ is the binding energy of each pion, and $E_{\rm Coul}$ is the repulsive Coulomb energy of the $Z$ pions.

The shifts and widths of the energy levels of pionic atoms due to the pion-nucleus strong interaction have traditionally been fitted using the truncated Klein-Gordon equation [19,20]

$$[\nabla^2 + k_0^2 - 2\omega V(\vec{r})]\Psi(\vec{r}) = 0\,, \tag{4}$$

where $V(\vec{r})$ is the pion-nucleus optical potential [19,20]. The optical potential of a pion with a piece of neutron matter is obtained by taking the limit of the optical potential of pionic atoms [19–22] when the proton density is equal to zero. This gives [2]

$$V(\vec{r}) = \frac{4\pi}{2\mu}[\beta_0\rho_n(r) - \beta_1\vec{\nabla}\cdot\rho_n(r)\vec{\nabla}] \tag{5}$$

with

$$\beta_0 = 0.147(1+\mu/M)\mu^{-1}\,, \tag{6}$$

$$\beta_1 = -0.38(1+\mu/M)^{-1}\mu^{-3}\,, \tag{7}$$

and $\mu$ is the mass of the pion, $M$ is the mass of the nucleon, while $\rho_n(r)$ is the neutron density. The optical potential (5) has a pathological behavior so that it has to be regularized. Thus, we use instead of Eq. (5) the

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regularized form in momentum space

$$V(\vec{k},\vec{k}') = \frac{4\pi}{2\mu}(\beta_0 + \beta_1 \vec{k}\cdot\vec{k}')\rho_n(\vec{k}-\vec{k}') \times \frac{M^2}{M^2+k^2}\frac{M^2}{M^2+k'^2}\,, \tag{8}$$

where we have introduced a cutoff with a cutoff parameter $M$ equal to the mass of the nucleon as it is appropriate for the $\pi N$ amplitude with isospin 3/2 (see, for example, Refs. [23,24]).

For the density of the neutrons we assume a Fermi distribution

$$\rho_n(r) = \frac{\rho_0}{1+e^{(r-R)/d}} \tag{9}$$

with

$$R = r_0 N^{1/3}\,. \tag{10}$$

The parameter $\rho_0$ in Eq. (9) is determined by the condition

$$\int d\vec{r}\rho_n(r) = N\,, \tag{11}$$

while the other two parameters $r_0$ and $d$ will be determined by a variational condition as will be discussed later.

The self-energy of a piece of neutron matter is calculated from the results of infinite neutron matter [25] by applying the local density approximation [2]. Jackson *et al.* [25] calculated the self-energy of infinite neutron matter for the cases of the Reid soft-core potential [26] and the Bethe-Johnson potential [27]. Since the Reid soft-core potential is less repulsive at short distances, one can get more easily pineut solutions with it. Therefore, in all the pineut calculations of this paper (unless otherwise explicitly stated), we have used the more repulsive Bethe-Johnson potential.

Our calculation of the Coulomb energy differs from Ref. [2] (there we assumed a uniform charge density) and therefore we will describe it next. Since the $Z$ pions are all in the state of lowest energy which is an $S$ state, this gives rise to a spherically symmetric charge density given by

$$\rho(r) = -eZ|\psi(\vec{r})|^2\,, \tag{12}$$

where $\psi(\vec{r})$ is the wave function of the pion obtained from the solution of the Klein-Gordon equation (4). The Coulomb energy is calculated in the standard way by considering a spherical shell of radius $r$ with a differential of charge $dq = \rho(r)4\pi r^2 dr$. If we call $Q(r)$ to the charge contained inside a sphere of radius $r$,

$$Q(r) = \int_0^r \rho(r')4\pi r'^2 dr'\,, \tag{13}$$

then the Coulomb energy is

$$E_{\rm Coul} = \int_0^\infty \frac{\rho(r)4\pi r^2 dr Q(r)}{r}\,. \tag{14}$$

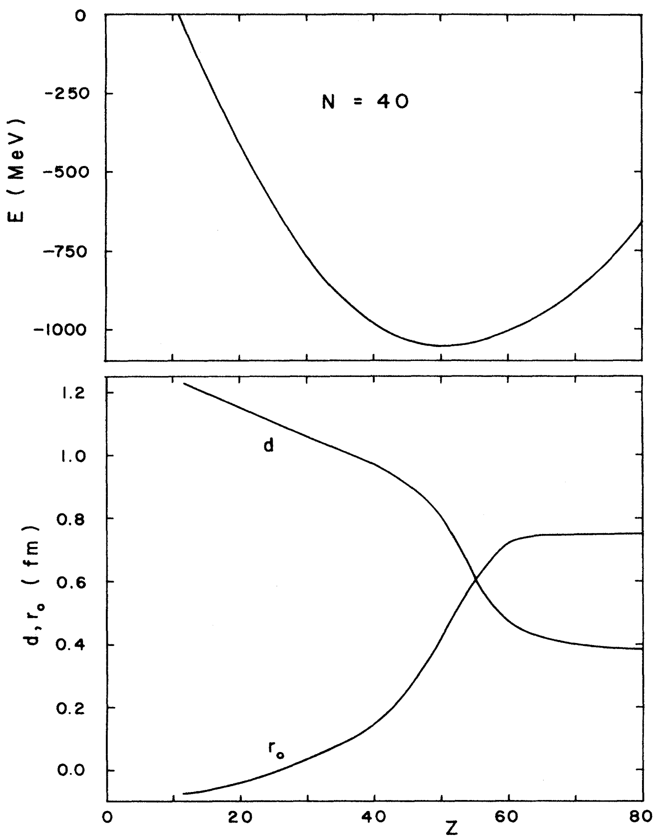


FIG. 1. Energy of the pineut and the parameters $r_0$ and $d$ of the density of the neutrons, as functions of $Z$ for $N = 40$.

The energy of the pineut is given by Eq. (3) and is a function of the two parameters $r_0$ and $d$ of the density of the piece of neutron matter, i.e.,

$$E(r_0,d) = E_N(r_0,d) - ZB(r_0,d) + E_{\rm Coul}(r_0,d), \tag{15}$$

therefore, one has to find the values of the parameters $r_0$ and $d$ that minimize the energy. Thus, the energy of the physical pineut is obtained by applying the variational conditions

$$\frac{\partial E(r_0,d)}{\partial r_0} = 0\,, \tag{16}$$

$$\frac{\partial E(r_0,d)}{\partial d} = 0\,, \tag{17}$$

which determine simultaneously the parameters $r_0$ and $d$ as well as the energy of the pineut. Figure 1 shows as an example the results of the case $N = 40$. There it is shown, as a function of $Z$, the energy of the pineut and the two parameters of the density $r_0$ and $d$.

Before showing our final results, we like to discuss three important points of our model.

The first point refers to the parameters $\beta_0$ and $\beta_1$ that were derived in Eqs. (5)–(7) using the results of pionic atoms. The parameters $\beta_0$ and $\beta_1$ used in the regularized form of the optical potential (8) do not have to be changed in order to reproduce the pionic atom data. This

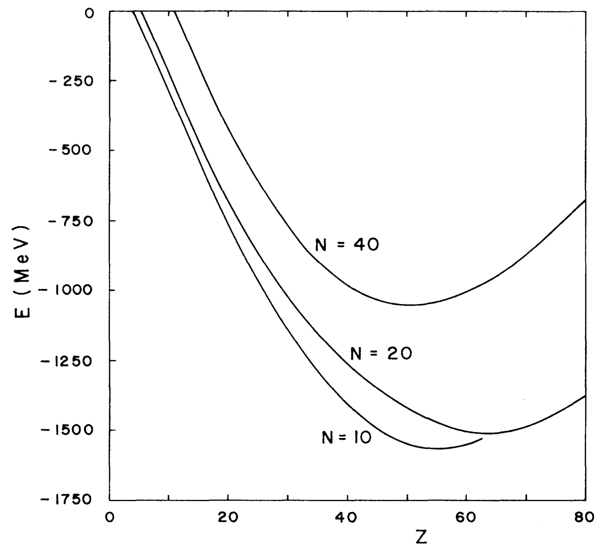


FIG. 2. Energies of pineuts as functions of $Z$ for $N = 10$, $N = 20$, and $N = 40$.

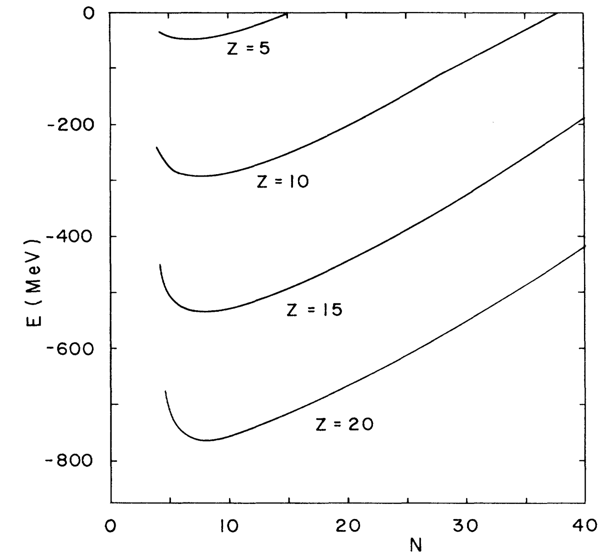


FIG. 3. Energies of pineuts as functions of $N$ for $Z = 5$, $Z = 10$, $Z = 15$, and $Z = 20$.

is due to the fact that the hydrogenic wave functions of the pion in momentum space are confined to the region of very low momenta (about 0.1 $\text{fm}^{-1}$) and therefore in the calculation of the shifts and widths there is very little effect from the cutoff. We have calculated the shift and width of the $1s$ level of the pionic atom with 20 protons and 20 neutrons using the parameters of Ref. [19], and found that the results with and without cutoff differ among themeselves by less than 1%.

The second point concerns the sensitivity of the binding energy of a pineut with respect to the cutoff parameter of the optical potential (8). We found that larger cutoffs give larger binding energies for the pineut and vice versa. In particular, if we lower the cutoff parameter to one-half of the nucleon mass, then our standard model with the neutron-neutron interaction of Bethe-Johnson predicts no pineuts. However, if we use this lower cutoff together with the neutron-neutron interaction of the Reid soft-core potential, pineuts will still be predicted. For example, for $N = 10, 20, 30, 40$, they will appear if $Z \geq 18, 16, 19, 22$, respectively.

The third point has to do with the Coulomb interaction. The Klein-Gordon equation solved to find the pion wave function does not include the Coulomb interaction with the $Z - 1$ other pions. This will tend to push the pions apart and weaken the strong binding of the pions to the neutrons; as a tradeoff, the larger pion wave function could lead to a decrease in $E_{\text{Coul}}$, the Coulomb energy. As an estimate of this effect, we considered the pineut with $N = Z = 20$ and added to the optical potential the screened Coulomb potential $V(r) = (Z - 1)e^2 \exp(-r/\rho)/r$ with a screening radius $\rho = 5$ fm (since we solve the Klein-Gordon equation in momentum space we cannot use the unscreened Coulomb potential). The addition of this term changed the binding energy of the pineut from 672 MeV to 648 MeV, i.e., it lowered it by 24 MeV. The contribution from pion binding was lowered by 26 MeV while the Coulomb energy was lowered by only 2 MeV. Thus, there is not much of a tradeoff. Notice, however, that the total energy changed by less than 5%.

We show in Fig. 2 the binding energies of pineuts as a function of $Z$ (number of pions) for a constant value of $N$ (number of neutrons) where we have considered the three cases $N = 10$, $N = 20$, and $N = 40$. In all three cases as $Z$ is increased, the binding energy increases until about 50 or 60 pions and afterwards it decreases with increasing $Z$. This behavior is originated by the repulsive Coulomb interaction between the pions. As one can see from Eqs. (12)–(14), the Coulomb energy is proportional to $Z^2$ and therefore for large $Z$ the Coulomb repulsion dominates over the attractive interaction of the pions with the piece of neutron matter. Therefore, in the region of large $Z$ the pineuts are unstable against pion emission since by emitting one or more pions they can move into a state with lower energy. Thus, only pineuts with $Z$ less than about 50 are stable against pion emission. Similarly, as one can see in Fig. 2, for a constant value of $Z$ $E(N = 40) > E(N = 20) > E(N = 10)$ so that the pineuts with large $N$ are unstable against neutron emission, since by emitting one or more neutrons they can move into a state with lower energy. In order to find the pineuts that are stable against both pion and neutron emission, one has to consider a constant $Z$ smaller than about 50 and keep lowering $N$ until a minimum in the energy is found.

We show in Fig. 3 the energies as functions of $N$ for the pineuts with $Z = 5$, $Z = 10$, $Z = 15$, and $Z = 20$. As one can see, the minimum of the energy lies at about 8 neutrons so that below this value of $N$ the pineuts are stable against both pion and neutron emission. Thus, they will decay only through the weak processes indicated by Eqs. (1) or (2).

The muonic decay mode of Eq. (1) will be open if the energy spacing between the pineuts $NZ$ and $N(Z - 1)$

is less than 34 MeV. As one can get from Fig. 3, the energy spacing is about 45 MeV so that the decay mode (1) is closed and the decay mode (2) will be the one that proceeds.

Another result that is obtained from Fig. 3 is that one needs at least 5 pions in order to produce a pineut. This explains the negative results of the experimental searches [6,9–14,17], which have been restricted to $Z \leq 2$.

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