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Integral characteristic parameters of the giant $M1$ resonance

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The dipole magnetization of a heavy spherical nucleus is studied with macroscopic standpoint. The semiclassical model under consideration focuses on the giant $M1$ resonance as a result of long wavelength oscillations of the collective magnetization current induced in the surface massive layer of finite depth. The macroscopic picture of the excited collective flow is found to be like that for the torsional elastic vibrations of the peripheral layer against the central spherical region inert with respect to external perturbation. The emphasis is placed on calculation of scaling behavior of integral characteristic parameters of magnetic dipole resonance.

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Studying the giant $M1$ resonance in medium and heavy nuclei has been and still is the subject of current investigations [1]. First experiments carried out in the past decade by means of (e,e') and (p,p') scattering [2–4] have allowed establishing that the energy of magnetic dipole resonance is centered at $E(M1) \approx 41\,A^{-1/3}$ MeV. An important finding of the nuclear resonance fluorescence measurements is the regularity of the $B(M1)$ factors: the total excitation probability of the $M1$ resonance is smoothly enhanced throughout the periodic table [5–7]. In this Brief Report an attempt is made to discover the scaling behavior of total excitation strength and other integral characteristic parameters of the magnetic dipole resonance, that is, their dependence upon atomic Z and mass A numbers.

In what follows we study the dynamics of dipole nuclear magnetization, based on the continuum method which has been used with success for systematizing the data on both the electric [8,9] and magnetic [10–12] resonances of the multipole order $\lambda \geq 2$ (see also Sec. 2 of Ref. [13]). It is noteworthy that the continuum approach in question presumes that spherical nucleus reacts as an elastic solid globe rather than as a liquid drop. The elastic behavior of a heavy nucleus in the energy range of giant resonances, as was first pointed out in Ref. [8], has a quantum nature in the sense that restoring elastic force originates from anisotropic distortions of the nuclear Fermi sphere [8,9,13]. Specifically, in the semiclassical model under consideration a heavy nucleus of radius $R = r_0 A^{1/3}$ with spin zero in the ground state is viewed as a spherical piece of an elastic Fermi continuum with the uniform densities of charge $n_e = e(Z/A)n_0$ and mass $\rho_0 = mn_0$ (m is the nucleon mass). In this Brief Report we use the simplest parametrization for the particle density $n_0 = (2/3\pi^2)k_F^3 = 3/(4\pi r_0^3)$, where $k_F = mp_F/\hbar$ and p_F is the Fermi momentum. Dynamics of nuclear excitations is described in terms of harmonic fluctuations of the velocity field $\delta\mathbf{V}(\mathbf{r},t) = \mathbf{a}(\mathbf{r})\,\dot\phi(t)$, where $\mathbf{a}(\mathbf{r})$ is the field of instantaneous displacements. The collective flow accompanying the giant magnetic response is described by the toroidal solution to the vector Laplace equation:

$$\Delta\delta\mathbf{V} = 0, \qquad \mathrm{div}\,\delta\mathbf{V} = 0. \tag{1}$$

A magnetic resonance is considered as an eigenmode of the oscillator Hamilton function:

$$H = \frac{J\dot\phi^2}{2} + \frac{C\phi^2}{2}, \tag{2}$$

where the inertia J and stiffness C are given by

$$J = \int \rho_0\, a_i\, a_i\, d\tau, \qquad C = \frac{1}{2}\int \mu \left(\frac{\partial a_i}{\partial x_j} + \frac{\partial a_j}{\partial x_i}\right)^2 d\tau. \tag{3}$$

Here $\mu = \rho_0(\hbar^2\, k_F^2/5m^2)$ is interpreted to be the nuclear shear modulus. The details of calculation of J and C can be found in [12]. The position of energy peak and the total excitation probability are calculated as follows

$$E(M1) = \hbar\sqrt{\frac{C}{J}}, \qquad B(M1) = 3\langle|\mathcal{M}(M1)|^2\rangle_t, \tag{4}$$

where

$$\mathcal{M}(M1) = \sqrt{\frac{3}{16\pi c^2}}\int \mathbf{j}\cdot[\mathbf{r}\times\nabla]\, r\,\sin\theta\, d\tau, \tag{5}$$

is the collective dipole moment[1] induced by oscillations of the electric current density $\mathbf{j} = n_e\delta\mathbf{V} = n_e\,\mathbf{a}\dot\phi$; hereafter c stands for the light velocity. In Eq. (4) symbol $\langle\cdots\rangle_t$ means the time averaging. In particular, $\langle\dot\phi\rangle_t = \frac{1}{2}\phi_0^2\omega^2$ and $\phi_0 = (\hbar\omega/2C)^{1/2}$ [14]. The method allows one to evaluate another integral characteristic such as the magnetic oscillator strength $S(M1)$ and the magnetic polarizability α_M related to the energy and the total excitation probability as follows:

$$S(M1) = E(M1)B(M1), \qquad \alpha_M = -\frac{9\pi}{8}\frac{B(M1)}{E(M1)}. \tag{6}$$

These quantities are often regarded as a macroscopic analog of sum rules [15,16].

[1] In this Brief Report all analytic calculations are carried out in the frame with fixed polar axis.

The phenomenological method developed in [9,11] allows one to evaluate the spread width of a giant resonance caused by the two-body collisional damping of collective motions. Making use of this method the following simple relation

$$\Gamma(M1) = g\,[E(M1)]^2, \qquad g = \frac{5\eta}{\rho_0 \hbar v_F^2}, \tag{7}$$

can be inferred (see for details [11]). The parameter $\eta \approx 0.03 \pm 0.01 \times 0.948\,\hbar\,\mathrm{fm}^{-3}$ stands for the viscosity coefficient.

In the above calculation scheme the main ingredient is the velocity field (more precisely, the field of instantaneous displacements) subjected to Eq. (1). Having found the solution of this latter equation the method permits one to estimate, in fact, a complete set of the measurable integral characteristics of $M1$ resonance.

Our consideration is essentially relayed on conclusions stemming from the single-particle shell model. As is well known in this model the magnetic dipole resonance is interpreted to be caused by transitions between spin-orbital partners ($j_1 = l \pm s \to j_2 = l \mp s$) with high angular momenta l. From this model it follows that the probability of the nucleon localization is shifted from center to surface with increasing orbital moment: the higher the l, the closer the nucleon happens to be localized to the nucleus surface. This means that $M1$ resonance is dominated by coherent motion of nucleons localized mostly into the surface layer of the nucleus. Based on this observation we explore the following collective mechanism of dipole magnetization. It is assumed that a heavy nucleus being irradiated by the electromagnetic field turns out to be decomposed into two subsystems so that only the peripheral layer of the nucleus is involved in the motion, whereas a central spherical region of the radius R_c stays unperturbed. This perturbation gives rise to the long wavelength torsional oscillations of charged flow taking place only in the peripheral layer of finite depth. It is presumed that such a picture of nuclear decomposition appears and exists only in the course of excitation and should be considered as an essentially *dynamical* macroscopic model of the process of dipole magnetization.

Following the above prescription we look for the solution to Eq. (1) in the form of toroidal vector field

$$\delta\mathbf{V}(\mathbf{r},t) = \mathbf{rot}\,\mathbf{r}\chi(r)\,\cos\theta\,\dot\phi(t). \tag{8}$$

The explicit view of the velocity field [in fact, the radial function $\chi(r)$] can be uniquely established based on the following boundary conditions. The coordinate dependence of velocity on the nucleus edge is assumed to be the same as in the case of rigid rotational oscillations

$$\delta\mathbf{V}(\mathbf{r},t) = [\mathbf{r} \times \mathbf{\Omega}_0(t)]_{r=R}, \tag{9}$$

where $\mathbf{\Omega}_0(t) = \mathbf{e}_z\,\dot\phi(t)$ with $\phi(t) = \phi_0 \sin\omega t$. Considering that external perturbation produces motions in the surface spherical layer of finite depth $\Delta R = R - R_c$, whereas the internal spherical region of radius R_c unaffected by the perturbation stays at rest, we put

$$\delta\mathbf{V} = 0\big|_{r \le R_c}. \tag{10}$$

As a result one obtains

$$\chi(r) = K\left(\frac{r}{R_c^3} - \frac{1}{r^2}\right), \qquad K = \frac{R^3 R_c^3}{R^3 - R_c^3}. \tag{11}$$

Spherical components of the field of instantaneous displacements are written as

$$a_r^1 = a_\theta^1 = 0, \qquad a_\phi^1 = K\left(\frac{r}{R_c^3} - \frac{1}{r^2}\right)\sin\theta. \tag{12}$$

This field bears a strong resemblance to that for torsional elastic oscillations of the surface layer against central spherical region which stays at rest.

It is worthwhile to emphasize that the density of perturbed electric current may be represented in the well-known form of the magnetization current

$$\mathbf{j} = n_e \delta\mathbf{V} = c\,\mathbf{rot}\,\mathbf{M},$$

where

$$\mathbf{M}(\mathbf{r},t) = -\mathbf{r}\left[\frac{n_e}{c} K\left(\frac{r}{R_c^3} - \frac{1}{r^2}\right)\cos\theta\right]\dot\phi(t). \tag{13}$$

From the macroscopic electrodynamics it is known that the magnetization current (in contrast with the current of conductivity) is not accompanied by transportation of charge in the course of excitation. Notice that similar property is attributed to the quantum-mechanical current of spin magnetization. However, it is not our purpose here to speculate about the microscopic origin of the giant $M1$ resonance. The considered mechanism of dipole magnetization carries an essentially collective content, since the model in question is formulated in terms of macroscopic electrodynamics of continuum medium.

Analytic calculations are easily performed with use of, instead of R_c, the parameter $x = R_c/R$ measuring the part of nuclear mass

$$\Delta M = M - M_c = M(1 - x^3),$$
$$M = \frac{4\pi}{3}\rho_0 R^3, \quad M_c = \frac{4\pi}{3}\rho_0 R_c^3 \tag{14}$$

involved in the collective dynamics. In terms of x the torsional stiffness is given by

$$C = \frac{8\pi}{5}\rho_0\, v_F^2\, R^3 \frac{x^3}{1 - x^3}. \tag{15}$$

From this latter equation it follows that the nonzero value of C is achieved only due to effective splitting of nucleus into two spherical subsystems. Indeed, the parameter C is canceled if $R_c = 0$ (in this case $x = 0$). Inserting (12) into (3) the mass parameter takes the view

$$J = \frac{8\pi}{15}\rho_0\, R^5\left[\frac{1 - 5x^3 + 9x^5 - 5x^6}{(1 - x^3)^2}\right]. \tag{16}$$

From (16) it follows that when x=0 the torsional inertia is reduced to the moment of inertia of a rigid globe $J(x = 0) = (2/5)MR^2$. Having calculated the parameters C

and J we arrive at the following expression for energy:

$$E(M1) = \hbar \left(\frac{C_1}{J_1}\right)^{1/2} = \kappa A^{-1/3}\,\text{MeV}, \quad (17)$$

$$\kappa = \frac{\hbar^2 (9\pi)^{1/3}}{2mr_0^2}\left[\frac{3x^3(1-x^3)}{1-5x^3+9x^5-5x^6}\right]^{1/2}.$$

For the total excitation probability we obtain

$$B(M1) = \gamma Z^2 A^{-2/3} \mu_N^2,$$

$$\gamma = \frac{9\,(9\pi)^{1/3}}{80\pi} \times \sqrt{\frac{3x^3(1-x^3)}{(1-5x^3+9x^5-5x^6)^3}} \times \left[1 - \frac{5}{2}x^3 + \frac{3}{2}x^5\right]^2, \quad (18)$$

where μ_N stands for the nuclear magneton. As was mentioned above the semiclassical model in question, as any collective model, is aimed at revealing the scaling dependence of integral characteristic parameters. This dependence may be figured out if the parameter x is taken to be constant. This latter attitude reflects the fact that the process of the dipole magnetization has one feature in common for medium and heavy spherical nuclei. The continuum picture is unwarranted for light nuclei with a relatively small number of particles. As it follows from (18) when x is constant, the latter formula provides the observable $A^{-1/3}$ dependence of energy. Taking $x = 0.53$ and $r_0 = 1.15$ we exactly reproduce the energy of the $M1$ resonance in ^{90}Zr. With the above fixed parameters we arrive at the following scaling estimate for the energy centroid and the total excitation strength

$$E(M1) \approx 41\,A^{-1/3}\,\text{MeV},$$
$$B(M1) \approx 8.5 \times 10^{-2}\,Z^2 A^{-2/3} \mu_N^2.$$

It is worthwhile to stress that having fixed the constant x, the $B(M1)$ factor is calculated without recourse to any adjustable constants. In Table I a systematic comparison of the model predictions with the NRF data available is presented. With above given parameters the part of mass participating in the collective motion is evaluated to be $\Delta M = 0.85\,M$, that is, the giant $M1$ resonance is an essentially volume nuclear response. The model predicts the following scaling behavior of the magnetic oscillator strength and the magnetic polarizability [Eq. (6)]

$$S(M1) \approx 3.5\,Z^2 A^{-1} \mu_N^2\,\text{MeV},$$
$$\alpha_M \approx -7.5 \times 10^{-3}\,Z^2 A^{-1/3} \mu_N^2\,\text{MeV}^{-1}.$$

The spread width of the giant $M1$ resonance throughout the periodic table is expected to be

$$\Gamma(M1) \approx 20\,A^{-2/3}\,\text{MeV}.$$

Let us shortly discuss the collective characteristics of inelastic electron scattering. In the semiclassical approximation the transition current density may be defined as follows:

$$\rho_{1,1}(r) = \left\langle \left| \frac{1}{ec}\int \mathbf{j}(\mathbf{r},t)\cdot \mathbf{Y}_{11;1}\,d\Omega \right|^2 \right\rangle_t^{1/2}. \quad (19)$$

To calculate this quantity it is convenient to rewrite the above calculated magnetization current density, Eq. (13), as follows:

$$\delta\mathbf{j} = n_e\,\delta\mathbf{V} = n_e\,f(r)\,\mathbf{Y}_{11;1}\,\dot{\phi}_1(t),$$
$$f(r) = \frac{8\pi}{3}\frac{1}{1-x^3}\left(r - \frac{x^3R^3}{r^2}\right). \quad (20)$$

Substitution of (20) into (19) leads to

$$\rho_{1,1}(r) = G\,\frac{(r^3 - x^3R^3)}{r^2(1-x^3)}, \qquad G = \sqrt{\frac{2\pi\hbar\omega_1}{3J_1}}\,\frac{Zn_0}{cA}. \quad (21)$$

The magnetic form factor calculated in the plane wave Born approximation [17] is given by

$$|F^{M1}(q)|^2 = \left|\frac{\sqrt{12\pi}}{Z}\int \rho_{1,1}(r)\,j_1(qr)r^2\,dr\right|^2$$
$$= N\left[\frac{j_2(qR) + x^3(j_0(qR) - j_0(qR_c) - j_2(qR_c))}{qR}\right]^2,$$
$$N = \frac{8\pi^2}{A^2}\frac{E(M1)}{c^2J_1}\frac{n_0^2R^8}{(1-x^3)^2}. \quad (22)$$

This collective form factor can be compared with the integral experimental form factor of all the 1^+ states con-

TABLE I. Comparison of the model predictions for energy $E(M1)$ and total excitation probability $B(M1)$ of the giant $M1$ resonance with the Illinois data obtained from nuclear resonance fluorescence measurements [4–7]. The experimental data for magnetic polarizability α_M are taken from [14,15].

Nucleus	Experiment $E(M1)$ MeV	Experiment $B(M1)$ μ_N^2	Experiment $\lvert\alpha_M\rvert$ μ_N^2/MeV	Model $E(M1)$ MeV	Model $B(M1)$ μ_N^2	Model $\lvert\alpha_M\rvert$ μ_N^2/MeV	Model $\Gamma(M1)$ MeV
^{90}Zr	9.1	6.7	2.1	9.1	6.8	2.6	0.9
^{120}Sn	8.3	8.8	–	8.3	8.8	3.7	0.8
^{140}Ce	7.9	7.5	–	7.9	10.6	4.8	0.7
^{206}Pb	7.5	19.0	7.0	6.9	16.4	8.4	0.6
^{208}Pb	7.3	17.5	7.0	6.9	16.3	8.4	0.6

tributing to the giant $M1$ resonance in the inelastic electron scattering.

As a conclusion it is noteworthy that the presented scheme of calculation can be thought of as providing global phenomenological estimates for the integral characteristic parameters of the giant $M1$ resonance. The considered picture gives an intuitive feeling of the motions accompanying the giant magnetic dipole response, and discloses the collective mechanism by means of which a heavy nucleus can accommodate magnetic dipole moment. It is argued that the giant magnetic dipole resonance is a manifestation of the collective nonrigid rotations of a spherical nucleus. As can be seen from Table I the expected behavior of integral parameters throughout the periodic table reasonably agrees with overall trends in the data available. One may hope, therefore, that the analysis presented will be useful in further study of magnetic dipole response of heavy nuclei.

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