Tests of time-reversal invariance in nuclear and particle physics

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(Received 12 September 1994)

There is a theorem that states that there can be no null test of time-reversal invariance (T symmetry) in exclusive reactions; that is, T symmetry does not require any *single* observable to vanish. This theorem is extended to inclusive reactions. Also, a general argument, that an experimental observable vanishes from T symmetry if a corresponding operator changes sign under time reversal, is shown to be invalid.

PACS number(s): 24.80.Dc, 11.30.Er, 13.10.+q, 13.60.Hb

I. INTRODUCTION

There have been statements throughout the literature, direct or implied, arguing that an experimental observable, the expectation value of an operator, must vanish from time-reversal invariance (T symmetry) if the corresponding operator changes sign under time reversal [1]. Such an operator and/or observable is then identified as being T noninvariant or T odd. If this were a valid general statement, i.e., independent of the interaction dynamics, it would follow that such a vanishing observable would provide a null test of T symmetry. The very existence of a null-test observable would be very important with respect to the level of experimental precision attainable in such a test of T symmetry. Since null tests of parity conservation exist, it has been possible to reach the remarkable precision of 2×10^{-8} in such tests [2,3], and so a comparable null test of T would permit an improvement in experimental precision of *several orders of magnitude* over that achieved in past tests.

However, ten years ago, a proof of the nonexistence of a null test of T in a two-particle in and two-particle out reaction was established. It states that in such a reaction there can be no *single* experimental observable that is required to vanish by T symmetry [4]. Thus the above often expressed view, that T symmetry requires a so-called T-odd observable to vanish, is in direct conflict with the theorem. It follows, then, that the finding of a nonzero value of *any* observable cannot be taken as a proof of T-symmetry violation without some additional condition(s) that must be provided by the interaction dynamics.

The theorem, however, does not encompass total cross-section observables, and it has been shown that, there, T-odd spin-correlation observables provide null tests of T symmetry [5].

Since the importance of the "no-null-test" (of T) theorem has not been widely appreciated [6], it is developed in Sec. II from a more experimentally oriented perspective than that of the formal theoretical approach of Ref. [4], and the characterization of T-odd amplitudes and observables is discussed. The theorem is also extended to include the accessible observables of the inclusive reactions $a + b \to c + X$. A parallel discussion of parity-nonconserving (P-odd) amplitudes and observables is presented, mainly for purposes of comparison and contrast, but the principal focus is on tests of T symmetry. In Sec. III the argument that the expectation value of a T-odd operator is required to vanish by T symmetry is discussed and shown to be faulty. Then, however, the dynamical conditions imposed by the electromagnetic and/or weak interactions are seen to provide limited null-test observables, because the level attainable in such a test of T symmetry is limited by nonvanishing second-order contributions and/or final-state interactions and not by experimental precision alone. Section IV provides a summary of the discussion and conclusions.

II. NONEXISTENCE OF A NULL TEST OF TIME-REVERSAL INVARIANCE

For a reaction or scattering with two particles in and two out, the underlying reason for the lack of a null test of T symmetry can be clearly stated by comparison, for example, with a null test of parity conservation (PC). In the latter case, one compares a transition amplitude or an experimental observable with the corresponding amplitude or observable for the *same*, but parity-transformed, process. Then, since PC requires that the corresponding amplitudes and observables be the same, any P-odd amplitude or observable vanishes [7]. The fundamental difference in a test of T is that one compares an observable in a reaction with a *different* observable in the inverse, i.e., time-reversed, reaction, so that the difference (or sum) of the two observables is required by T symmetry to be zero. Thus there is no single-observable null test of T in these reactions.

A. Reaction $a + b \to c + d$

For a more formal illustration of these remarks, consider a reaction with the simple spin structure $\frac{1}{2} + 0 \to \frac{1}{2} + 0$. The matrix of amplitudes in the 2×2 spin space can be expanded in terms of the Pauli spin matrices [8],

$$M(\theta) = \sum_j a_j(\theta)\sigma_j, \quad j = 0, x, y, z, \quad \sigma_0 = 1. \qquad (1)$$

Choosing the center-of-mass helicity frame [9,10], in which the conditions imposed by T symmetry on the scattering or reaction amplitudes are most naturally expressed, unit vectors along the coordinate axes are

$$\mathbf{z}_i\ (\mathbf{z}_f) = \mathbf{k}_i\ (\mathbf{k}_f), \qquad (2)$$
$$\mathbf{y} = \mathbf{k}_i \times \mathbf{k}_f, \quad \mathbf{x}_i(\mathbf{x}_f) = \mathbf{y} \times \mathbf{z}_i(\mathbf{z}_f),$$

where $\mathbf{k}_i$ ($\mathbf{k}_f$) is the c.m. momentum of particle a (c). Then with the P and T transformations $\mathbf{k}_{i,f} \to -\mathbf{k}_{i,f}$, $\sigma \to \sigma$ and $\mathbf{k}_i \leftrightarrow -\mathbf{k}_f$, $\sigma \to -\sigma$, respectively, and noting that $\sigma_x \equiv \sigma \cdot \mathbf{x}$, etc., one has the following transformations under the P,T symmetry operations:

$$P: \quad \sigma_x, \sigma_y, \sigma_z \to -\sigma_x, \sigma_y, -\sigma_z; \qquad (3)$$
$$T: \quad \sigma_x, \sigma_y, \sigma_z \to -\sigma_x, \sigma_y, \sigma_z.$$

Then the corresponding M-matrix amplitudes in (1) and in $M^t = \sum_j a_j^t \sigma_j$, the M matrix for the inverse reaction, can be classified according to their P and/or T symmetries as follows:

$$P: \quad a_j = (-1)^{(n_x+n_z)} a_j; \qquad (4a)$$
$$T: \quad a_j = (-1)^{n_x} a_j^t; \qquad (4b)$$

and an amplitude a_j is P odd (T odd) if $n_x + n_z(n_x)$ is odd, where n_x (n_z) is the number of x (z) subscripts [11]. Thus PC requires the P-odd (P-even) amplitudes to vanish when the product of the particles' intrinsic parities is even (odd), but T symmetry imposes no such condition on the T-odd amplitudes. It requires only that the T-odd amplitudes satisfy the condition (4b) that $a_x^t = -a_x$ [12]. Only in the case of elastic scattering, which is its own inverse reaction, does this condition force the amplitude to vanish.

Another form of this 2×2 matrix is that in which the elements $M(if)$ are the actual transition amplitudes between initial and final helicity states,

$$M(\theta) = \begin{pmatrix} M(++) & M(-+) \\ M(+-) & M(--) \end{pmatrix} . \qquad (5)$$

A comparison with (1), expressed in terms of the (invariant) amplitudes a_j,

$$M(\theta) = \sum_j a_j(\theta)\sigma_j = \begin{pmatrix} a_0 + a_z & a_x - ia_y \\ a_x + ia_y & a_0 - a_z \end{pmatrix}, \qquad (6)$$

shows the connection between the two sets of amplitudes. For the general case of all four particles with helicities $\alpha + \beta \to \gamma + \delta$, $M(if) = M(\alpha\beta, \gamma\delta)$, and the conditions imposed by the P and T symmetries directly on these helicity amplitudes,$M(\alpha\beta,\gamma\delta)$ and $M^t(\gamma\delta,\alpha\beta)$ have been established [10], and these conditions are correspondingly satisfied in (6) by the conditions (4) imposed on the amplitudes a_j and a_j^t (a_{jk} and a_{jk}^t in the general case).

Consider, now, the experimental observables for reactions with this particular spin structure [13],

$$X(j,k) = \mathrm{Tr} M\sigma_j M^\dagger \sigma_k / \mathrm{Tr} MM^\dagger, \quad j,k = 0,x,y,z, \qquad (7)$$

where j labels the polarization component of the initial-state particle, k labels the observed final-state polarization component, and $j(k) = 0$ for unpolarized incident particles (unobserved final polarization). Since, by definition, the P transformation of the M matrix is $M \to M$, the combination $M, M^\dagger$ contributes no change of sign in the P transformation of an observable, and so its P symmetry is determined by the explicit spin operators σ_j and σ_k in (7). Its T symmetry is determined in the same manner. Thus, with (3), it follows from (7) that these observables can be classified according to their P and T symmetries in exactly the same way as was found for the amplitudes in (4) [15]

$$P: \quad X(j,k) = (-1)^{n_x+n_z)} X(j,k); \qquad (8a)$$
$$T: \quad X(j,k) = (-1)^{n_x} X^t(k,j). \qquad (8b)$$

So, now, PC requires a P-odd *observable* to be zero, but the T-symmetry condition is that an observable is equal to $(+/-)$, a *different* observable in the inverse process (k,j), which proves the "no-null-test" theorem that there can be no single vanishing T-odd observable.

Even in the case of elastic scattering, where the T-odd amplitude a_x vanishes, the theorem applies. This circumstance can be understood from the specific expressions for appropriate pairs of observables in terms of the amplitudes, for example, analyzing powers and polarizing powers. From (1) and (7), with $I = \frac{1}{2}\mathrm{Tr}MM^\dagger$, one obtains

$$IA_j \equiv IX(j,0) = 2(\mathrm{Re}a_0 a_j^* + \mathrm{Im}a_k a_l^*), \qquad (9a)$$
$$IP_j \equiv IX(0,j) = 2(\mathrm{Re}a_0 a_j^* - \mathrm{Im}a_k a_l^*),$$
$$j,k,l \text{ cyclic in } x,y,z. \qquad (9b)$$

It is clear from (9), with the T-odd amplitude $a_x = 0$, that none of these observables goes to zero [16] and that

$$A_x = -P_x, \quad A_y = P_y, \quad A_z = P_Z, \qquad (10)$$

all in accordance with (8b). Since there are *two* either P-odd or P-even amplitudes which vanish when parity is conserved, the P-odd observables vanish, now in accordance with (8a).

In the more general case of a reaction, consider (9b) for the inverse reaction,

$$I^t P_j^t = 2(\mathrm{Re}a_0^t a_j^{t*} - \mathrm{Im}a_k^t a_l^{t*}). \qquad (11)$$

Then, with the T-odd amplitudes $a_x^t = -a_x$, from (9a) and (11) one finds [17]

$$A_x = -P_x^t, \quad A_y = P_y^t, \quad A_z = P_z^t, \qquad (12)$$

just as required by (8b).

The entire foregoing discussion is easily generalized to reactions and/or scattering of more complex spin structures. Consider, for example, the case of particular interest in particle physics, $a + b \to c + d$, with four spin-$\frac{1}{2}$

particles. The required 4×4 M matrix can now be expanded in terms of direct products of the 2×2 (a, c) and (b, d) matrices σ_j and σ_k, respectively [18],

$$M(\theta) = \sum_{j,k} a_{jk}(\theta)\sigma_j \otimes \sigma_k, \quad j,k = 0,x,y,z, \quad \sigma_0 = 1. \tag{13}$$

In a more compact form, with the 4×4 matrix $\sigma_{jk} \equiv \sigma_j \otimes \sigma_k$,

$$M = \sum_{j,k} a_{jk}\sigma_{jk}, \tag{14}$$

and the 16 M-matrix amplitudes

$$a_{00},\ a_{0x},\ a_{0y},\ a_{0z},\ a_{x0},\ a_{xx},\ a_{xy},\ a_{xz},$$
$$a_{y0},\ a_{yx},\ a_{yy},\ a_{yz},\ a_{z0},\ a_{zx},\ a_{zy},\ a_{zz}, \tag{15}$$

can then be classified, as in (4), according to their P and/or T symmetries,

$$P: \quad a_{jk} = (-1)^{(n_x+n_z)}a_{jk}; \tag{16a}$$
$$T: \quad a_{jk} = (-1)^{n_x}a^t_{jk}. \tag{16b}$$

Also, again from (3), the experimental observables

$$X(jk,lm) = \mathrm{Tr}M\sigma_{jk}M^\dagger\sigma_{lm}/\mathrm{Tr}MM^\dagger, \quad j,k,l,m = 0,x,y,z, \tag{17}$$

have, as in (8a) and (8b), the symmetries

$$P: \quad X(jk,lm) = (-1)^{(n_x+n_z)}X(jk,lm); \tag{18a}$$
$$T: \quad X(jk,lm) = (-1)^{n_x}X^t(lm,jk). \tag{18b}$$

Here j,k designate the polarization components of particles a,b and l,m the observed polarization components of c,d.

Finally, since the components S_j of the spin operator for any spin **S** transform just as the σ_j in (3), the symmetries (18a) and (18b) apply to reactions of particles with arbitrary spins. This includes the second- (and higher-) rank tensor observables, since the corresponding spin operators are constructed from combinations of the rank-1 operators S_j [15]. The equivalent symmetries imposed on the observables in their spherical-tensor form, rather than the Cartesian form used here, have been established [10,19].

As stated above in connection with (4) and (8) and how with (16) and (18), unlike the PC requirement that a P-odd amplitude or observable vanish, T symmetry requires that a *pair* of amplitudes or observables satisfy the condition that

$$X - X^t = 0 \ (T \text{ even}) \quad \text{or} \quad X + X^t = 0 \ (T \text{ odd}). \tag{19}$$

Only in elastic scattering do the T-odd amplitudes vanish, but, in general, there is no single T-odd observable.

It is clear, then, that the valid argument, that a P-odd observable must vanish from P symmetry, has no counterpart with respect to T-odd observables and T symmetry. In fact, both (pairs of) T-even and T-odd observables, e.g., (12), are available for tests of T symmetry when P is not conserved, and it is only when the appropriate condition (19) is not satisfied that T-symmetry violation is demonstrated. Since PC requires that $A_x = P_x = A_z = P_z = 0$, it is interesting to note that the rather standard nuclear physics test of $A_y - P^t_y = 0$ is a T-even test of T symmetry.

B. Inclusive reactions $a + b \to c + X$

In view of the fact that many inclusive experiments are pursued, especially in particle physics, it is of obvious interest to know whether or not there are P- and/or T-imposed symmetries on the available experimental observables in such reactions $a + b \to c + X$, where only particle c is detected in the final state [20]. From energy and momentum conservation, X can be treated as a composite "particle" of known mass and momentum, with, however, unobservable spin. This latter fact has no effect on the observables involving particles a, b, and c, and it will be seen that these observables retain the same symmetries as in the $2 \to 2$ exclusive reactions, namely, (18a) and (18b).

Consider a reaction in which a, b, and c are spin-$\frac{1}{2}$ particles, i.e., fermions. Then from baryon and lepton conservation, "particle" X is also a "fermion" and, for the purpose of illustration, is taken to be spin $\frac{1}{2}$. Then the available observables are given as in (17) with $m = 0$, corresponding to the fact that the "polarization" of X is not observed [21],

$$X(jk,l0) = \mathrm{Tr}M\sigma_{jk}M^\dagger\sigma_{l0}/\mathrm{Tr}MM^\dagger. \tag{20}$$

Then, just as before, these observables have the symmetries given in (18). In order to better understand the specific details of these results, we consider again, for example, the expressions for the analyzing power A_{y0} and the inverse-reaction polarizing power P^t_{y0}, even though the latter cannot be determined experimentally. These are

$$IA_{y0} \equiv IX(y0,00) = \tfrac{1}{4}\mathrm{Tr}M\sigma_{y0}M^\dagger, \tag{21a}$$
$$I^tP^t_{y0} \equiv I^tX^t(00,y0) = \tfrac{1}{4}\mathrm{Tr}M^tM^{t\dagger}\sigma_{y0}, \tag{21b}$$

and with (14), Eq.(21a) becomes

$$IA_{y0} = \tfrac{1}{4}\mathrm{Tr}\left[\left(\sum_{j,k} a_{jk}\sigma_{jk}\right)\sigma_{y0}\left(\sum_{j'k'} a^*_{j'k'}\sigma_{j'k'}\right)\right]. \tag{22}$$

Then, noting that

$$\mathrm{Tr}\sigma_{jk}\sigma_{lm} = \mathrm{Tr}[(\sigma_j\sigma_l)\otimes(\sigma_k\sigma_m)] = \mathrm{Tr}\sigma_j\sigma_l\mathrm{Tr}\sigma_k\sigma_m, \tag{23}$$

we have

$$IA_{y0} = \tfrac{1}{4}\sum_{j,k}\sum_{j',k'} a_{jk}a^*_{j'k'}\mathrm{Tr}\sigma_j\sigma_y\sigma_{j'}\mathrm{Tr}\sigma_k\sigma_{k'}. \tag{24}$$

Here one sees that the matrix operations factor, as they must, into operations in the separate (a,c) and (b,X) spin spaces. Then using the properties $\sigma_j\sigma_k = i\sigma_l$, $\sigma_j^2 = \sigma_0$, $\mathrm{Tr}\sigma_j\sigma_{j'} = 2\delta_{jj'}$, one finds

$$\mathrm{Tr}\sigma_j\sigma_y\sigma_{j'} = \begin{cases} 2i\ [-2i] \text{ for } (j,j') = (x,z)\ [(z,x)], \\ 2 \text{ for } (j,j') = (o,y) \text{ or } (y,0), \\ 0 \text{ otherwise,} \end{cases} \tag{25}$$

and (24) becomes

$$IA_{y0} = \sum_k 2(\mathrm{Re}a_{0k}a_{yk}^* + \mathrm{Im}a_{zk}a_{xk}^*) \tag{26}$$

and, similarly,

$$I^tP_{y0}^t = \sum_k 2(\mathrm{Re}a_{0k}^t a_{yk}^{t*} - \mathrm{Im}a_{zk}^t a_{xk}^{t*}) \tag{27}$$

for the inverse reaction. Comparing these two equations with (9a) and (11), one sees that they have identical forms, with the additional summation over k coming from taking the *trace* over the (b,X) part of the spin space, which performs the sums over the spin projections of particles b and X. One then recovers the symmetries (12) and, more generally (18b) among these inclusive observables, and these are independent of the "spin" of "particle" X. The parity-imposed symmetries on the (spherical-tensor) observables in a reaction with a three-particle final-state have been discussed in a detailed treatment which uses the P symmetries of the amplitudes to deduce those of the observables [22]. The corresponding inclusive observables are also included.

III. LIMITED NULL TESTS OF TIME-REVERSAL INVARIANCE

A. T-odd operators and observables

The usual argument made to establish that a T-odd observable vanishes is that it is the expectation value of a T-odd operator, which (by definition) changes sign under time reversal, and so must vanish. This argument is wrong, simply because there are two different observables that are the expectation values of the same operator in a reaction and its inverse. For example, consider the T-odd operator $\sigma(\mathbf{k}_i \times \mathbf{k}_f) \times \mathbf{k}_i \equiv \sigma_x$ in a reaction with the initial-state polarization p_x and in the inverse reaction with its polarizing power P_x^t. In the inverse reaction, only the final-state expectation value of the operator $\langle\sigma_x\rangle$ is relevant, and this is the final-state observable P_x^t with *unpolarized* initial-state particles, thus with initial-state density matrix $\rho_i = \sigma_0/2$, as follows [14]:

$$\begin{aligned} \langle\sigma_x\rangle^t &= \mathrm{Tr}(\rho_f\sigma_x)/\mathrm{Tr}\rho_f \\ &= \mathrm{Tr}(M^t\rho_i M^{t\dagger}\sigma_x)/\mathrm{Tr}M^tM^{t\dagger}, \\ \langle\sigma_x\rangle^t &= \mathrm{Tr}M^tM^{t\dagger}\sigma_x/\mathrm{Tr}M^tM^{t\dagger} = P_x^t, \end{aligned} \tag{28}$$

as in (7). But in the forward reaction, $\langle\sigma_x\rangle$ is simply the prepared initial-state polarization p_x with density matrix $\rho_i = (1 + p_x\sigma_x)/2$:

$$\langle\sigma_x\rangle = \mathrm{Tr}\rho_i\sigma_x = \tfrac{1}{2}\mathrm{Tr}(1 + p_x\sigma_x)\sigma_x = p_x, \tag{29}$$

which is trivial, not an observable, and bears no relationship to the polarizing power P_x^t. The determination of the T-odd partner of P_x^t, the analyzing power A_x, involves only the final-state intensity asymmetry with an initial beam polarization p_x. It appears as a term in the expectation value of that intensity and as in (28),

$$\begin{aligned} I_x = \langle\sigma_0 \equiv 1\rangle = \mathrm{Tr}\rho_f &= \mathrm{Tr}M\rho_iM^\dagger \\ &= \tfrac{1}{2}\mathrm{Tr}M(1 + p_x\sigma_x)M^\dagger, \end{aligned} \tag{30}$$

$$I_x = \tfrac{1}{2}\mathrm{Tr}MM^\dagger\left(1 + p_x\frac{\mathrm{Tr}M\sigma_xM^\dagger}{\mathrm{Tr}MM^\dagger}\right) = I(1 + p_xA_x).$$

Here I (I_x) is the cross section with an unpolarized (polarized) beam. Thus the fact that the spin operator σ_x changes sign under the T transformation, as in (3), does not result in a corresponding condition on any observable.

B. Electron scattering and β and γ decay correlations

Even though the argument used above to identify a (T-odd) null-test observable is seen to be invalid, it does *not* follow, however, that such an observable is prohibited when the dynamical restrictions of the electromagnetic and/or the weak interactions are imposed. That is, to first order in these interactions, the M matrix is Hermitian. Then the *combined* requirements of T symmetry *and* hermiticity, $M \to M^t$ and $M \to M^\dagger$, impose conditions on the relative phases of the matrix elements $M(\alpha\beta,\gamma\delta)$, which result in the vanishing of some observables. These, then, are limited null-test observables, because nonvanishing contributions are provided by second-order terms of the interaction and/or final-state interactions. Thus the precision attainable in such a test is limited by that available in the calculation of these nonvanishing components and not by the experimental precision alone. To the present, excluding searches for nonvanishing (P-odd and T-odd) electric dipole moments, such tests have been made in electron scattering and in β-decay and γ-decay experiments. The level of precision attained in the decay-correlation experiments has been in the range 10^{-2}–10^{-3}, with the determination of the neutron β-decay analyzing power A_x (triple correlation coefficient D) reaching $(-0.5 \pm 1.4) \times 10^{-3}$ [23].

Electron scattering provides clear counterexamples to the argument connecting a T-odd operator to a single T-odd null-test observable. There have been inclusive experiments that searched for a nonvanishing analyzing power A_{0y} in the reaction $ep \to eX$ with a polarized proton target at energies from 4 to 18 GeV [24]. Even though A_{0y} is a T-even observable, it was shown to be a (limited) null-test observable, since $A_{0y} \propto \mathrm{Im}F_-F_z^*$, and the amplitude form factors F_- and F_z must be relatively real from T symmetry and hermiticity [25]. Then, on the other hand, there is a genuine T-odd, P-even observable that does not vanish in (parity-conserving) electron scattering. This observable is the spin-correlation coefficient $A_{zx} \equiv X(zx, 00)$, which in *ep* elastic scattering is proportional to G_M^2 [14], and the proton magnetic form factor G_M is certainly not zero.

IV. SUMMARY AND CONCLUSIONS

The theorem, that there can be no null test of T symmetry in a two-particle in and two-particle out reaction, has been extended to include inclusive reactions. Also, the often used general argument, that an experimental observable vanishes from T symmetry if a corresponding operator changes sign under time reversal, has been shown to be faulty.

It is clear, then, that there have been no nonlimited tests of T symmetry in the weak or electromagnetic interaction. It is somewhat discouraging to realize that the required comparison, between a reaction observable and its inverse-reaction counterpart, is essentially impossible in weak processes and is very difficult in electron scattering. The two (equivalent [14]) possibilities in elastic *ep* scattering, for example, are to test one of the P-even, T-odd conditions (18) that

$$K_{zx} \equiv X(z0, 0x) = -K_{xz} \equiv -X(0x, z0), \quad \text{(31a)}$$

$$A_{zx} \equiv X(zx, 00) = -C_{zx} \equiv -X(00, zx). \quad \text{(31b)}$$

The polarization-transfer coefficient comparison (31a) may be experimentally more accessible than the spin-correlation coefficient comparison (31b) since only one final-state polarization at a time needs to be determined. In any case, the most difficult part of either comparison would seem to be the precise determination of the polarization p_z of the scattered electron. However, in view of the fact that previous experiments have reached the level of only $\pm$ 1% in tests of T symmetry [24], it is perhaps worthwhile to consider how precise a comparison (31) can be achieved at a present-day continuous electron-beam facility with polarized beams and polarized targets.

Although T tests have been more readily available in strong-interaction processes, it is now important to improve the precision beyond the 10^{-2}–10^{-3} level of experimental error that has been achieved. Following from the fact that spin-dependent total cross-section observables are not included in the "no-null-test" theorem, a total cross-section spin-correlation coefficient has been identified as a genuine null-test observable [5]. It is important to pursue such tests because of the considerably higher precision that is inherently possible in null tests.

ACKNOWLEDGMENTS

I am grateful to M. Simonius for fruitful discussions. This work was partially supported by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Nuclear Physics Division of the U.S. Department of Energy under Contract No. DE-AC03-76SF00098.

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tion, Eq. (8a), from a different argument. The results of Eqs. (8), which follow directly from Eqs. (3) and (7), extend easily to arbitrary spins, and this is a simpler general proof.

[16] Somewhat before the establishment of the theorem of Ref. [4], M. J. Moravcsic [Phys. Rev. Lett. **48**, 718 (1982)] erred in his conclusion that the vanishing of this T-odd amplitude provided a corresponding vanishing observable in elastic scattering.

[17] The reaction cross sections are related by $k_i^2 I = k_f^2 I^t$ [10], and so I have absorbed the k^2 factors into the cross sections.

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