$\Delta(1232)$ in nuclei and QCD sum rules

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We show how pion-nucleus scattering data can be used to constrain four-quark in-medium matrix elements, whose uncertainty is currently the most important problem inhibiting the use of QCD sum rules to study hadrons in nuclear matter, a problem related to early-universe phase transitions. We take an important step towards determining the matrix elements by extracting a value for a particular linear combination using the Isobar-Doorway model propagator of the in-medium $\Delta(1232)$ and a QCD sum rule analysis of both the free and in-medium Δ.

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I. INTRODUCTION

One of the important discoveries with meson facilities was the observation that it is possible to extract the propagator of a $\Delta(1232)$ from the analysis of pion-nucleus scattering. In the isobar doorway formalism [1], it was shown that the pion optical potential can be closely related to the Δ propagator if Δ-nucleus doorways are the dominant mechanism to entering the nucleus. An analysis of experimental data [2] showed that this is indeed a reasonable picture and the pole position of the in-medium Δ was extracted. In the region of the resonance, the real part M^*_Δ, the Δ mass in the nuclear medium, was found to be within about 10 MeV of the free Δ mass. The width was found to be broadened by about 10%. More recent studies [3] that take into account the theoretical and experimental advances of the last decade confirm the qualitative description of the Δ found in the isobar doorway analysis. In the present work we do not attempt to calculate the in-medium width, but use the phenomenological value of M^*_Δ to estimate certain four-quark matrix elements, which we discuss below.

The method of QCD sum rules [4] has been successful in fitting masses of the nucleon, the $\Delta(1232)$ [5], and other hadrons, as well as a number of other properties. Recently, there has been a great deal of interest in studying the properties of hadrons in nuclei. The masses are particularly interesting since hadronic masses come primarily from nonperturbative QCD (NPQCD) effects, mainly the quark condensates. Therefore, the sum rule method is the natural one for treating hadronic masses in nuclear matter, and in recent years there have been several studies [6–8]. In the pioneering work of Drukarev and Levin [6], an estimate of the quark condensate in nuclear matter was given. However, it has been found that the four-quark condensates (matrix elements in the nuclear medium of products of four-quark fields) are crucial for these studies. Four-quark condensates in the vacuum are generally treated by factorization [4]. In nuclear matter, however, there is no justification for factorization. Moreover, it has been shown [7] that for the nucleon in nuclear matter a factorization of the in-medium four-quark condensate as the product of two in the medium two-quark condensates leads to an increase of the scalar nucleon mass with increasing density for low density, while a factorization to vacuum values gives a reduction of this mass to approximately $0.7M_N$, about the value used in most relativistic nuclear mean-field-theory calculations. Since the former factorization is *a priori* most natural, this is in conflict with the expectation of many nuclear theorists. Recently, an estimate of four-quark condensates needed for nuclear matter has been made [9] using the Nambu-Jona-Lasino model.

In the present work we reverse the procedure: We use the known value of the position of the pole of the Δ propagator in the nucleus and carry out the operator product expansion of the correlator in the nuclear medium using the two-quark operators which are approximately known, and extract the value of the four-quark matrix element. In Sec. II we briefly review the sum rules for the free Δ and derive the sum rules for the Δ in nuclear matter in Sec. III. In Sec. IV we discuss the structure of the four-quark condensates. The results are discussed in Sec. V.

II. FREE Δ SUM RULES

The starting point of the analysis is the two-point function, usually called the correlator:

$$\Pi(p)^\Delta_{\mu\nu} = i\int \langle 0|T[\eta^\Delta_\mu(x)\bar\eta^\Delta_\nu(0)]|0\rangle d^4x\, e^{ix\cdot p}, \qquad (1)$$

where the quantity $\eta^\Delta_\mu(x)$ is a composite field operator, the $\Delta(x)$ current,

$$\eta^\Delta_\mu(x) = \epsilon^{abc}[u^a(x)^T C\gamma^\mu u^b(x)]u^c(x),$$
$$\langle 0|\eta^\Delta_\mu(x)|\Delta\rangle = \lambda_\Delta v_\mu, \qquad (2)$$

where C is the charge conjugation operator and the $u(x)$ are u-quark fields labeled by color, λ_Δ is a structure parameter, and v_μ is a Rarita-Schwinger (spin $\frac{3}{2}$) spinor. Note that [5] the current for the Δ is unique except for derivative terms.

The QCD sum rule method, in brief, involves three steps: (1) deriving a dispersion relation for $\Pi(p)_{\mu\nu}$ to express it in terms of physical quantities to be evaluated [called the right-hand side (RHS)]; (2) carrying out an operator product expansion (OPE) for $\Pi_{\mu\nu}(p)$, including operators of high enough dimensions to ensure that the expression is naively convergent [called the left-hand side (LHS)]; and (3) equating the RHS and LHS after a Borel transform to enhance the convergence of the LHS and to weight the desired states in the RHS expansion, usually pole terms or double-pole terms.

Using the Rarita-Schwinger spinors for the Δ,

$$\Pi(p)^{\Delta\text{RHS}}_{\mu\nu} = \lambda_\Delta^2[g_{\mu\nu} - \gamma^\mu\gamma^\nu/3]\frac{\hat{p} + M_\Delta}{p^2 - M_\Delta^2} + \text{NU} + \text{continuum}, \qquad (3)$$

where $\hat{p} = \gamma^\mu p_\mu$. The first term in the equation is the pole term of the RHS, arising from the one-Δ intermediate state, and by the notation NU we mean pole terms that are not used in the present analysis. For Eq. (3) the term NU$= \lambda_\Delta^2[2p_\mu p_\nu/9M_\Delta^2 + (\gamma^\mu p_\nu - \gamma^\nu p_\mu)/3M_\Delta]$, which follows from the use of the v_μ spinors.

The LHS OPE has been carried out through dimension 9 in Ref. [5]. Errors have been pointed out in Ref. [10], but in a term which is to small to change the numerical results significantly. We find it convenient to use the correlators defined by

$$\Pi_1^\Delta g_{\mu\nu} = \tfrac{1}{4}\text{Tr}[\Pi_{\mu\nu}^\Delta],$$

$$\Pi_p^\Delta g_{\mu\nu} = \frac{1}{4p^2}\text{Tr}[\hat{p}\Pi_{\mu\nu}^\Delta]. \qquad (4)$$

After the Borel transform we find from the Π_p^Δ correlator, with the LHS terms shown in Fig. 1 (with all propagators and condensates defined in the vacuum), the sum rule

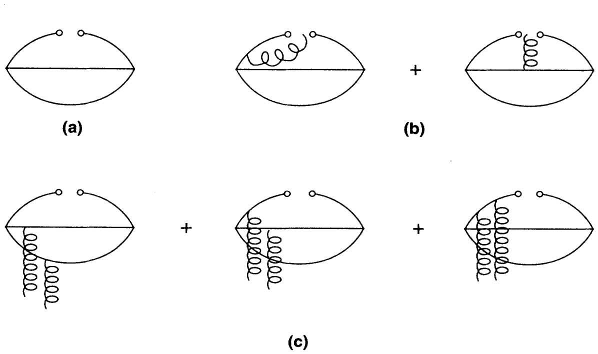

FIG. 1. Processes for the free $\Pi(x)_p^\Delta$ correlator.

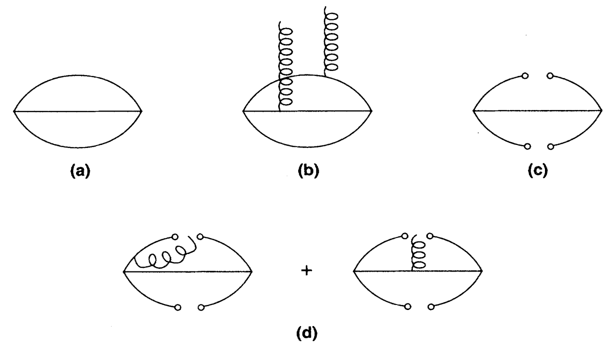

FIG. 2. Processes for the free $\Pi(x)_1^\Delta$ correlator.

$$\tfrac{2}{3}\hat{\lambda}_\Delta^2\exp(-M_\Delta^2/M_B^2) = \tfrac{11}{80}M_B^6 E_2 L^{4/27} - \tfrac{25}{576}bM_B^2E_0L^{4/27} + \tfrac{5}{6}a^2L^{28/27} - \tfrac{35}{72}m_0^2a^2L^{16/27}/M_B^2, \qquad (5)$$

and from the Π_1^Δ correlator, with processes shown in Fig. 2, the sum rule

$$\tfrac{2}{3}M_\Delta\hat{\lambda}_\Delta^2\exp(-M_\Delta^2/M_B^2) = \tfrac{11}{12}aM_B^4E_1L^{16/27} - m_0^2aM_B^2E_0L^{4/27}/2 - \tfrac{7}{288}abL^{16/27}. \qquad (6)$$

The quantities in Eqs. (5) and (6) are $\hat{\lambda}_\Delta = (2\pi)^2\lambda_\Delta$, $a = -(2\pi)^2\langle\bar{q}q\rangle$, $b = \langle g^2G^2\rangle$ (the gluon condensate), and $L = 0.621\ln(10M_B)$, corresponding to a 1 GeV renormalization point and a 100 MeV QCD parameter. The functions $E_0 = 1 - \exp(-x)$, $E_1 = E_0 - x\exp(-x)$, and $E_2 = E_1 - x^2\exp(-x)$, with $x = s_0/M_B^2$, are introduced to regulate the large M_B behavior of the continuum, s_0 being the continuum parameter [4]. We use standard values of $a = 0.55$ GeV^3 and $b = 0.474$ GeV^4 for the vacuum condensates, and take $s_0 = 2.6$ GeV^2. The last term in Eq. (6) differs from the results of Refs. [5,10], but the effect on our numerical results is very small.

In our analysis we take the ratio of Eqs. (6) and (5) and introduce a continuum factor of $C(M_B) = c_1 + c_2M_B^2 + c_3M_B^4$, with the condition that $C(M_B) \approx 1.0$ at the value of M_B at which the plateau is reached. We find that $M_\Delta \approx (1.35 \pm 10\%)$ GeV with $M_B \approx 1.4$ GeV.

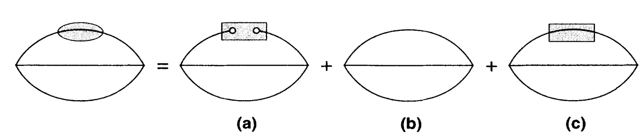

FIG. 3. Processes with one quark propagator in the medium.

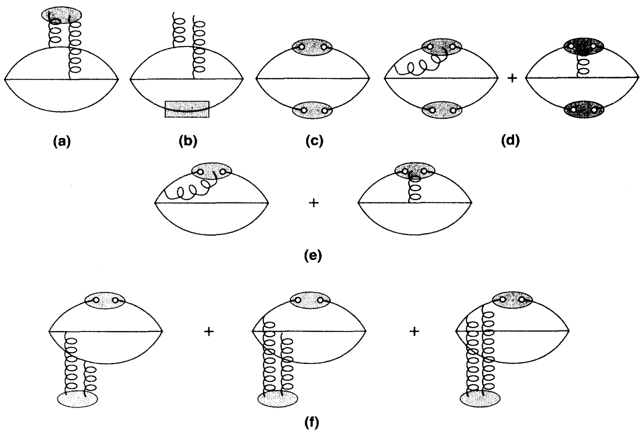

FIG. 4. Processes included in the $\Pi(x)^{\Delta^* 2q}_{\mu\nu}$ calculation in addition to those depicted in Fig. 3.

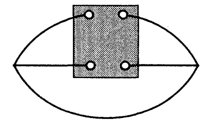

FIG. 5. Diagram for the $\Pi(x)^{\Delta^* 2q}_{\mu\nu}$ correlator.

III. IN-MEDIUM M^*_Δ SUM RULES

We now treat the propagation of the Δ in the nuclei. The starting point is the correlator in nuclear matter, which is the two-point function

$$\Pi(p)^{\Delta*}_{\mu\nu} = i\int \langle A|T[\eta^\Delta_\mu(x)\bar{\eta}^\Delta_\nu(0)]|A\rangle d^4x\, e^{ix\cdot p} = i\int d^4x\, e^{ix\cdot p}\Pi(x)^{\Delta*}_{\mu\nu}, \quad (7)$$

where $|A\rangle$ is the nuclear ground state. The advantage of using a two-point function is that our phenomenological expression has a pole term which is clearly identifiable with the in-medium baryon propagator, which we can evaluate from the results of the isobar doorway analysis. One can attempt to extract the in-medium mass using a three-point function [6,8], in which the alteration of hadronic properties is treated as arising from scattering by the medium, extracted from a double-pole term. However, by using a two-point function one has an immediate identification with the hadronic propagator.

We can write $\Pi(x)^{\Delta*}_{\mu\nu}$ as

$$\Pi(x)^{\Delta^*}_{\mu\nu} = \Pi(x)^{\Delta^* 2q}_{\mu\nu} + \Pi(x)^{\Delta^* 4q}_{\mu\nu} + \Pi(x)^{\Delta^* 6q}_{\mu\nu}, \quad (8)$$

where the three terms contain only two-quark matrix elements, four-quark matrix elements, and six-quark matrix elements, respectively. Using the current given by Eq. (2), we find

$$-i\Pi(x)^{\Delta^* 2q}_{\mu\nu} = 2\epsilon^{abc}\epsilon^{a'b'c'}[S(x)^{cc'}\mathrm{Tr}[S(x)^{bb'}\gamma^\nu CS(x)^{aa'T}C\gamma^\mu] + 2S(x)^{cc'}\gamma^\nu CS(x)^{bb'T}C\gamma^\mu S(x)^{aa'}], \quad (9)$$

with $S(x)$ a quark propagator, and

$$\begin{aligned}-i\Pi(x)^{\Delta^* 4q}_{\mu\nu} = \epsilon^{abc}\epsilon^{a'b'c'}\Big[&-\langle A|:S(x)^{cc'}\mathrm{Tr}[u(x)^b\bar{u}(0)^{b'}\gamma^\nu C\bar{u}(0)^{a'}u(x)^aC\gamma^\mu]:|A\rangle\\ &-2\langle A|:S(x)^{cc'}\gamma^\nu C\bar{u}(0)^{b'}u(x)^bC\gamma^\mu u(x)^a\bar{u}(0)^{a'}:|A\rangle\\ &+4\langle A|:u(x)^c\bar{u}(0)^{c'}\gamma^\nu CS(x)^{bb'T}C\gamma^\mu u(x)^a\bar{u}(0)^{a'}:|A\rangle\\ &-2\langle A|:u(x)^c\bar{u}(0)^{c'}\gamma^\nu C\bar{u}(0)^{b'}u(x)^bC\gamma^\mu S(x)^{aa'}:|A\rangle\Big]. \end{aligned} \quad (10)$$

We do not consider $\Pi(x)^{\Delta^* 6q}_{\mu\nu}$, corresponding to six-quark correlations in this work. The processes involving only two-quark operators considered here are shown in Figs. 3 and 4, and those involving four-quark operators are shown in Fig. 5.

We first carry out the microscopic QCD evaluation of $\Pi(x)^{\Delta^*}_{\mu\nu 2q}$ since the OPE for this part of the correlator is closely related to the free Δ calculation of Sec. II. The nonperturbative part of the quark propagator in nuclear matter has five lowest-dimensional terms in the OPE in contrast to the quark condensate for the vacuum. Following previous authors, we use only the scalar and vector terms, which seem to be most important. This gives [6]

$$\begin{aligned}\langle A:\bar{q}^a q^{a'}:|A\rangle &= -(\delta_{aa'}/12)[\langle\bar{q}q\rangle_\rho + \langle\bar{q}\hat{u}q\rangle_\rho\hat{u}],\\ \langle\bar{q}q\rangle_\rho &= \langle\bar{q}q\rangle + \sigma_N\rho/(2m_q),\\ \langle\bar{q}\hat{u}q\rangle_\rho &= 3\rho/2,\end{aligned} \quad (11)$$

where σ_N is the pion-nucleon sigma term, m_q is the quark mass, and ρ is the nuclear density at which the propagator is evaluated.

It is convenient to define three correlators to be used to obtain sum rules:

$$\Pi_1^{\Delta^*} g_{\mu\nu} = \mathrm{Tr}[\Pi_{\mu\nu}^{\Delta^*}]/4,$$
$$\Pi_p^{\Delta^*} g_{\mu\nu} = \left[\mathrm{Tr}[\hat{p}\Pi_{\mu\nu}^{\Delta^*}] - u\cdot p\mathrm{Tr}[\hat{u}\Pi_{\mu\nu}^{\Delta^*}]\right]/4[p^2-(u\cdot p)^2],$$
$$\Pi_u^{\Delta^*} g_{\mu\nu} = \left[p^2\mathrm{Tr}[\hat{u}\Pi_{\mu\nu}^{\Delta^*}] - u\cdot p\,\mathrm{Tr}[\hat{p}\Pi_{\mu\nu}^{\Delta^*}]\right]/4[p^2-(u\cdot p)^2]. \quad (12)$$

The diagrams of Fig. 3 correspond to one quark propagating in the medium. They are quite different than those of Fig. 1(a). We find

$$\Pi_1^{\Delta^* 3} = -\frac{11}{48\pi^2}\langle\bar{q}q\rangle_\rho p^2 \ln(-p^2),$$
$$\Pi_p^{\Delta^* 3} = \frac{11}{160(2\pi)^4} p^4 \ln(-p^2),$$
$$\Pi_u^{\Delta^* 3} = -\frac{1}{8\pi^2}\langle\bar{q}\hat{u}q\rangle_\rho p^2 \ln(-p^2). \quad (13)$$

There diagrams also include vacuum and in-medium terms corresponding to the diagram in Fig. 2(a) for the free Δ. The ellipses in Fig. 3 stand for obvious permutations of quark lines or other combinations of vacuum and in-medium condensates.

The remaining processes shown by the diagrams in Fig. 4 can be obtained fairly easily by appropriate changes in the calculations corresponding to Figs. 1 and 2. The results for nonvanishing terms needed to obtain the sum rules are

$$\Pi_p^{\Delta^* 4(a)} = -\frac{25}{576(2\pi)^4}[\langle g^2G^2\rangle + \delta\langle g^2G^2\rangle_\rho]\ln(-p^2),$$
$$\Pi_u^{\Delta^* 4(b)} = \frac{5}{576\pi^2}\langle g^2G^2\rangle\langle\bar{q}\hat{u}q\rangle_\rho/p^2,$$
$$\Pi_p^{\Delta^* 4(c)} = \tfrac{5}{3}\langle\bar{q}q\rangle[\langle\bar{q}q\rangle_\rho - \langle\bar{q}q\rangle/2]/p^2,$$
$$\Pi_p^{\Delta^* 4(d)} = \tfrac{35}{72}[\langle\bar{q}q\rangle_\rho\langle\bar{q}g\sigma\cdot Gq\rangle + \langle\bar{q}q\rangle\delta\langle\bar{q}g\sigma\cdot Gq\rangle_\rho]/p^4,$$
$$\Pi_1^{\Delta^* 4(e)} = -\frac{1}{8\pi^2}\langle\bar{q}g\sigma\cdot Gq\rangle_\rho\ln(-p^2),$$
$$\Pi_1^{\Delta^* 4(f)} = \frac{7}{1152\pi^2}[\langle\bar{q}q\rangle_\rho\langle g^2G^2\rangle + \langle\bar{q}q\rangle\delta\langle g^2G^2\rangle_\rho]/p^2. \quad (14)$$

We give our values of $a_\rho = -(2\pi)^2\langle\bar{q}q\rangle_\rho$ and $b_\rho = \langle g^2G^2\rangle + \delta\langle g^2G^2\rangle_\rho$ below. The density variation of the mixed condensate will be taken to be negligible, i.e., $\delta\langle\bar{q}g\sigma\cdot Gq\rangle_\rho = 0$.

The four-quark contribution to the correlator, $\Pi(x)_{\mu\nu}^{\Delta*4q}$ defined in Eqs. (8) and (10) is depicted in Fig. 5. Let us call the contributions of the three projections of $\Pi(x)_{\mu\nu}^{\Delta^* 4q}$ defined in Eq. (12), Π_1^{4q}, Π_p^{4q}, and Π_u^{4q}, with obvious notation. These are unknowns in our sum rules. We discuss them in detail in the next section.

The phenomenological expression for $\Pi_{\mu\nu}^{\Delta^*}$, the RHS, has a pole at the in-medium Δ mass. It can be obtained from the pole term in Eq. (3) by replacing M_Δ with M_{Δ^*}, with

$$M_{\Delta^*} = M_\Delta + \delta M_\Delta^s + \delta M_\Delta^u \hat{u}. \quad (15)$$

This gives, for the phenomenological side,

$$\Pi(p)_{\mu\nu}^{\Delta^*\mathrm{RHS}} = \lambda_{\Delta^*}^2[g_{\mu\nu} - \gamma^\mu\gamma^\nu/3]\frac{(\hat{p}+M_{\Delta^*})}{(p^2-M_{\Delta^*}^2)} + \mathrm{NU} + \mathrm{continuum}, \quad (16)$$

Including the projections of $\Pi(x)_{\mu\nu}^{\Delta^* 4q}$ in our sum rules here as unknown terms and carrying out the Borel transformation, we obtain sum rules for three correlators. The $\Pi_1^{\Delta^*}$ correlator leads to the sum rule

$$\tfrac{2}{3}\bar{\lambda}_{\Delta^*}^2 M_\Delta^* \exp(-\bar{M}_\Delta^{*2}/M_B^2) = \tfrac{11}{12}a_\rho M_B^4 E_1 L^{16/27} - \tfrac{1}{2}\bar{m}_0^2 a M_B^2 E_0 L^{4/27} - \tfrac{7}{288}a_\rho b_\rho L^{16/27} - \delta\Pi_1^{4q}. \quad (17)$$

The sum rule obtained from the $\Pi_p^{\Delta^*}$ correlator is

$$\tfrac{2}{3}\bar{\lambda}_{\Delta^*}^2 \exp(-\bar{M}_\Delta^{*2}/M_B^2) = \tfrac{11}{80}M_B^6 E_2 L^{4/27} - \tfrac{25}{576}b_\rho M_B^2 E_0 L^{4/27} + \tfrac{5}{3}a[a_\rho - a/2]L^{28/27} - \tfrac{35}{72}m_0^2 a a_\rho L^{16/27}/M_B^2 - \delta\Pi_p^{4q}. \quad (18)$$

Finally, the $\Pi_u^{\Delta^*}$ correlator leads to the sum rule

$$\tfrac{2}{3}\bar{\lambda}^2_{\Delta^*}\delta M^u_\Delta \exp(-\bar{M}^{*2}_\Delta/M_B^2) = 2\pi^2\langle\bar{q}\hat{u}q\rangle_\rho M_B^4 E_1 L^{4/27} - \frac{5\pi^2}{36}b\langle\bar{q}\hat{u}q\rangle_\rho L^{4/27} + \delta\Pi^{4q}_u, \quad (19)$$

where $\delta\Pi^{4q}$ is the value of the four-quark condensate relative to its factorized in-medium value Π^{4q} (fac, in-med). In the next section, we examine $\Pi(x)^{\Delta^* 4q}_{\mu\nu}$.

IV. FOUR-QUARK CONDENSATE CONTRIBUTIONS

Before we consider the four-quark matrix elements in nuclear states, let us review the two-quark matrix elements. For the vacuum, the familiar result

$$\langle 0| : q^b_k\bar{q}^a_j : |0\rangle = -\tfrac{1}{12}\delta_{ab}\delta_{jk}\langle\bar{q}q\rangle \quad (20)$$

follows from the fact that only scalar condensates are present in the vacuum. In a nuclear state, matrix elements of all five independent operators obtained from the Dirac matrices can be present:

$$\langle A| : q^a_j\bar{q}^b_k : |A\rangle = \tfrac{1}{4}\langle A|\Big[\bar{q}^a q^b\langle j|k\rangle + \bar{q}^a\gamma_5 q^b\langle j|\gamma_5|k\rangle + \bar{q}^a\gamma^\mu q^b\langle j|\gamma^\mu|k\rangle$$
$$-\bar{q}^a\gamma^\mu\gamma_5 q^b\langle j|\gamma^\mu\gamma_5|k\rangle + \bar{q}^a\sigma_{\mu\nu}q^b\langle j|\sigma^{\mu\nu}|k\rangle\Big]|A\rangle. \quad (21)$$

Extending Eq. (21), the expansion of four-quark fields needed for $\Pi(x)^{\Delta^* 4q}_{\mu\nu}$, defined in Eq. (10), has the form

$$\epsilon^{abc}\epsilon^{a'b'c'}\langle A| : \bar{q}^{a'}_j q^a_k\bar{q}^{b'}_l q^b_m : |A\rangle = \sum_{pq} C_{pq}\langle j|O^1_p|k\rangle\langle l|O^2_q|m\rangle, \quad (22)$$

where the C_{pq} are condensates in the nuclear medium and the $\langle j|O^1_p|k\rangle$ are matrix elements of various combinations of Dirac matrices taken in spinor states $|j\rangle$ and $|k\rangle$. From this general form we calculate the contribution from Fig. 5 to the correlator $\Pi(x)^{\Delta^* 4q}_{\mu\nu}$ and find, for the three projections of Eq. (12),

$$\Pi^{4q}_1 = 0,$$
$$\Pi^{4q}_u = 0,$$
$$\Pi^{4q}_p = \frac{1}{2p^2}[c_1 - c_5 - 2c_4/9 + 2c_6/3 + c_9/9 - c_{10}/3 + 4c_{11}/9 - 16c_{12}/9], \quad (23)$$

where

$$c_1 = \langle A| : \bar{q}^a q^a\bar{q}^b q^b : |A\rangle - 3\langle A| : \bar{q}^a\Lambda^{ac}_\xi q^c\bar{q}^b\Lambda^{bd}_\xi q^d : |A\rangle/4,$$

$$c_4 = \langle A| : \bar{q}^a\hat{u}q^a\bar{q}^b\hat{u}q^b : |A\rangle - 3\langle A| : \bar{q}^a\hat{u}\Lambda^{ac}_\xi q^c\bar{q}^b\hat{u}\Lambda^{bd}_\xi q^d : |A\rangle/4,$$

$$c_5 = \langle A| : \bar{q}^a\gamma^5 q^a\bar{q}^b\gamma^5 q^b : |A\rangle - 3\langle A| : \bar{q}^a\gamma^5\Lambda^{ac}_\xi q^c\bar{q}^b\gamma^5\Lambda^{bd}_\xi q^d : |A\rangle/4,$$

$$c_6 = \langle A| : \bar{q}^a\gamma^5\hat{u}q^a\bar{q}^b\gamma^5\hat{u}q^b : |A\rangle - 3\langle A| : \bar{q}^a\gamma^5\hat{u}\Lambda^{ac}_\xi q^c\bar{q}^b\gamma^5\hat{u}\Lambda^{bd}_\xi q^d : |A\rangle/4,$$

$$c_9 = \langle A| : \bar{q}^a\gamma^\mu q^a\bar{q}^b\gamma_\mu q^b : |A\rangle - 3\langle A| : \bar{q}^a\gamma^\mu\Lambda^{ac}_\xi q^c\bar{q}^b\gamma_\mu\Lambda^{bd}_\xi q^d : |A\rangle/4\ ,$$

$$c_{10} = \langle A| : \bar{q}^a\gamma^5\gamma^\mu q^a\bar{q}^b\gamma^5\gamma_\mu q^b : |A\rangle - 3\langle A| : \bar{q}^a\gamma^5\gamma^\mu\Lambda^{ac}_\xi q^c\bar{q}^b\gamma^5\gamma_\mu\Lambda^{bd}_\xi q^d : |A\rangle/4,$$

$$c_{11} = \langle A| : \bar{q}^a\sigma^{\alpha,\beta}q^a\bar{q}^b\sigma_{\alpha,\beta}q^b : |A\rangle - 3\langle A| : \bar{q}^a\sigma^{\alpha,\beta}\Lambda^{ac}_\xi q^c\bar{q}^b\sigma_{\alpha,\beta}\Lambda^{bd}_\xi q^d : |A\rangle/4,$$

$$c_{12} = \langle A| : \bar{q}^a\sigma^{\mu,\alpha}u_\alpha q^a\bar{q}^b\sigma_{\mu,\beta}u^\beta q^b : |A\rangle - 3\langle A| : \bar{q}^a\sigma^{\mu,\alpha}u_\alpha\Lambda^{ac}_\xi q^c\bar{q}^b\sigma_{\mu,\beta}u^\beta\Lambda^{bd}_\xi q^d : |A\rangle/4, \quad (24)$$

with Λ_ξ the color matrices. The second term in each case is closely related to the first term with permuted color indices.

In the limit that nucleons are completely separated (the density goes to zero), the ratios of the values of the eight four-quark condensates in Eq. (24) are 5:1:1:−1:4:−4:12:3, respectively, and the four-quark contribution in Eq. (23) approaches the value on the third line in Eq. (14). For nonzero nuclear density, the absolute as well as relative values of the c_i will change. These values are currently not known, but they are needed in order to apply QCD sum rules to study hadrons in nuclei. Determining them is thus an important problem in nuclear physics.

Note that the contribution are of the form of scalar, vector, pseudoscalar, pseudovector, and tensor. In a conventional meson exchange picture, there would correspond to interactions arising from mesons with those characteristics. In the sum rule approach that we are using, with an expansion in the nuclear density, these contributions will be closely related to terms in the microscopic optical potential for pion scattering and reactions at the region of the Δ [11]. In the present exploratory paper, we do not attempt this, but use only the isobar doorway fit to the mass of the Δ in the medium. This will determine a linear combination of the various coefficients at half nuclear matter density, which we express relative to the conventional factorized expression.

V. RESULTS AND DISCUSSION

The sum rules used in the present work are obtained by using Eqs. (23) and (24) in Eqs. (17)–(19). Note that only the p-type sum rule, Eq. (18), has a four-quark contribution. For the two-quark in-medium condensates, we use the values that have been found in Refs. [6–8], except that we do not use them at central nuclear density. Theoretical studies of pion-nucleus scattering [1,2,11] at resonance energy, where the isobar doorway model has been applied, have shown that the Δ is formed rather far into the nuclear surface. With a nuclear density $\frac{1}{2}$ of its central value, this gives us $a_\rho = 0.46$ GeV^3 and $b_\rho = 0.43$ GeV^4.

In our analysis of the sum rules, we follow the analysis of the free Δ as much as possible. We take the ratio of Eqs. (17) and (18) as one sum rule and the ratio of Eqs. (19) and (18) for the second sum rule. In each case we introduce the continuum factor of $C(M_B) = c_1 + c_2 M_B^2 + c_3 M_B^4$, with the condition that $C(M_B) \approx 1.0$ at the value of M_B at which the plateau is reached, using the identical values as in the free case in order to minimize the sensitivity of the changes in the mass of the Δ to the continuum.

Our results are as follows. (1) From the ratio of $\Pi_1^{\Delta^*}/\Pi_p^{\Delta^*}$, using the result that the pole position is approximately the same as its free value, which we take as $M_\Delta^* = 1.35$ GeV from the results of Sec. II, we find that $\delta\Pi_p^{4q} = -0.026$ GeV^6 at $\frac{1}{2}$ central nuclear density ($\delta\Pi_p^{4q}$ is presumably linear in the density). For purposes of comparison, we find that in the factorized approximation $\Pi_p^{4q}(\text{fac}, \rho = 0) = -0.42$ GeV^6 and $\Pi_p^{4q}(\text{fac}, \rho = \rho_0/2) = -0.27$ GeV^6. (2) From the ratio of $\Pi_u^{\Delta^*}/\Pi_p^{\Delta^*}$, we find that $\Delta M_\Delta^u = 97$ MeV. This result for the vector mass shift is about $\frac{1}{4}$ of that found in the calculation of Ref. [7] for the nucleon at nuclear density.

Without $\delta\Pi_p^{4q}$, M_Δ^* would be about 200 MeV higher, and so it is seen that the effect of $\delta\Pi_p^{4q}$ is quite significant. Because δM_Δ^u is quite large, it should be taken into account explicitly in the theory. When this is done, we estimate that it will increase the value of $\delta\Pi_p^{4q}$ by about 30%.

We conclude that it is possible to use the experimental data on pion nucleus scattering at the energies in the Δ(1232) region to extract a value for the four-quark condensate term that enters for that resonance in nuclear matter. To apply this result to the determination of the nucleon mass and other baryon masses in nuclei will require further study. The properties of the Δ(1232) in nuclei using QCD sum rules has also been considered recently by Jin [12] from a different point of view.

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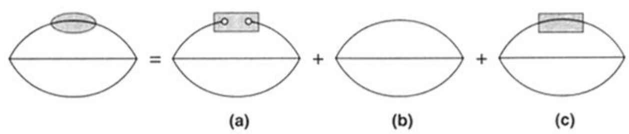

FIG. 3. Processes with one quark propagator in the medium.

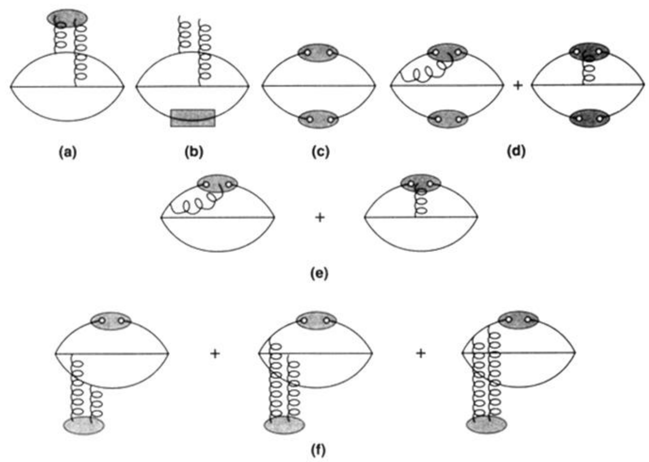

FIG. 4. Processes included in the $\Pi(x)_{\mu\nu}^{\Delta^* 2q}$ calculation in addition to those depicted in Fig. 3.