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Inversion potential analysis of the nuclear dynamics in the triton

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We report ^{3}H binding energy calculations using inversion potentials generated from phase shifts corresponding to contemporary nucleon-nucleon potentials as well as modern phase shift analyses. We place limits upon the calculated triton binding energy based on a local potential due to the underlying uncertainties in the potential model generated directly from the nucleon-nucleon phase shifts. We explore the role of the nonlocality of momentum-dependent potentials in the triton binding energy. In particular, we find the additional binding energy in the case of the Bonn-B potential is due to a long range nonlocality.

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Triton model calculations, utilizing new Nijmegen local potentials [1] that fit the nucleon-nucleon scattering data in the range 0–350 MeV almost as well as the Nijmegen phase shift analysis, were recently reported [2]. The results for the three local potential models, for which a one-pion-exchange (OPE) tail was enforced, were summarized as yielding a binding energy of 7.62 ± 0.01 MeV, some 0.86 MeV smaller than the experimental value of 8.48 MeV. We examine here (1) the limits placed upon local potential triton calculations by the uncertainties in potentials generated directly from the nucleon-nucleon phase shifts, (2) the role of the nonlocality of momentum dependent potentials in the triton binding energy, and (3) the effects of long range nonlocality exhibited by certain contemporary potential models.

Realistic potential models fitted to the nucleon-nucleon scattering observables have been generated by several groups: for example, Paris [3], Nijmegen [4], and Bonn [5]. The fits are at least semiquantitative. The limited number of parameters in these potential models implies that the χ^2/N fit to the observables will necessarily be larger than that obtained in a precision phase-shift analysis. By constructing partial-wave local potentials, the Nijmegen group [1] have succeeded in obtaining models whose fits to the data are comparable to those obtained in phase-shift analyses. Alternatively, the technique for constructing inversion potentials from phase-shift data has been developed to the point that one can generate equally precise partial-wave local potentials [6–8]. Thus, one can compare triton binding energy results for potentials constructed from (1) a theoretical, meson-exchange approach and (2) an inversion prescription that generates an equivalent partial-wave local function. Therefore, the effect of nonlocality in the potential models can be investigated quantitatively, both for short range nonlocality as one finds in the Paris potential and for long range nonlocality as appears in the Bonn-B potential.

There is no claim for a particular interaction model dynamics in the inversion prescription. (In particular, no specific radial form for the potential is enforced.) The physics resides in the assumption about the applicabilty of a differential form of the Blankenbecler-Sugar (relativistic Schrödinger) equation with a local potential for each partial wave. What we gain, by dint of its construction with the Gelfand-Levitan-Marchenko integral equations, is that the resulting partial-wave local potentials reproduce the input phase shifts along with the deuteron spectroscopic data; that is, the physical observables and phase shifts from which the potentials were generated are exactly reproduced. If nonlocality in the nucleon-nucleon interaction can be shown to be mandated then, in the future, it shall be included in the inversion prescription.

The discrepancy between the experimental value for the triton binding energy and results for various potential models has been attributed by some to the need to include a three-body force (3BF) in the Hamiltonian [9]. Carlson [10] has shown that a phenomenological 3BF adjusted to reproduce the triton binding energy will also yield a correct value for the ground-state binding energy of the alpha particle. Sauer and collaborators [11] have argued that the three-nucleon force, which results when the Δ is eliminated from an NN-$N\Delta$ coupled-channel model of the nucleon-nucleon interaction, contributes little to the triton binding. Further model calculations by Picklesimer and collaborators [12] support this claim. However, when the full Tucson-Melbourne (TM) three-nucleon force [13] ($\pi\pi$, $\pi\rho$, and $\rho\rho$ terms as recently published by Coon and Peña [14]), which was designed to be used with nucleon-nucleon potentials that incorporate only nucleon-nucleon degrees of freedom, was combined with the Reid soft-core [15], Paris [3], and Nijmegen [4] potentials, Stadler *et al.* [16] found that the model ^{3}H binding energies were close to 8.48 MeV. Critics of this approach point out that the meson-nucleon form factor cutoffs in the TM 3BF are soft whereas those in the nucleon-nucleon potential models are hard. Nonetheless, the calculations demonstrate that a 3BF can play a role in the triton. Furthermore, Polyzou and Glöckle have shown [17] that there is a unitary transform relationship between specific classes of Hamiltonian comprised of local NN plus NNN potentials and Hamiltonians comprised of nonlocal potentials.

Therefore, we wish to study the bounds on the triton binding energy that result from the assumption of a local poten-

TABLE I. ^{2}H properties for the NN potential models.

Quantity	Nijm-II	Bonn-B	Paris	Experiment
B_2	2.22458	2.22461	2.2249	2.22459 [18]
P_D	5.66	4.99	5.77	
Q	0.2707	0.278	0.279	0.286 [19]
A_s	0.8847	0.8860	0.8866	0.88 [20]
η	0.0252	0.0264	0.0261	0.026 [21]
$r_{\rm rms}$	1.9671	1.9688	1.9716	1.96 [20]

tial derived directly from our current knowledge of the nucleon-nucleon observables. In addition, we explore the consequences of the long range (in r-space) nonlocality that exists in the Bonn-B potential model. We choose Bonn-B because the signature of its long range nonlocality is clear, even though a detailed exploration of the r-space versions of the Bonn one-boson-exchange potentials does not exist. It is only the role of the long range r-space nonlocality in the deuteron D-state probability and the triton binding energy that is of concern here.

To establish the reliability of the inversion potential prescription, we consider first the new Nijm-II model. The ^{2}H properties for this potential as well as those of the Bonn-B and Paris momentum-dependent potentials are summarized in Table I. Of the three potentials, only the Nijm-II model contains explicit charge dependence. While each provides at least a semiquantitative fit to the on-shell nucleon-nucleon data below 300 MeV, we are interested in comparing ^{3}H binding energy results for each model with those of the corresponding inversion potential, obtained by using the pion subthreshold model phases for $j\leq 2$ plus those potentials from OPE for $2<j\leq 4$ as input.

The ^{2}H properties for the inversion potentials are listed in Table II, along with those resulting from the phase-shift analysis of Arndt [22]. The input for the inversion prescription, in addition to the model or experimental phase shifts within 0–300 MeV, include B_2, A_s, and η, respectively. Above 300 MeV the phase shifts are smoothly extrapolated to infinity with $\lim_{k\to\infty}\delta(k)\sim 1/k$. Comparing the remaining entries in the two tables, it should be clear that the physical observables for the model deuterons are all reasonably reproduced by the inversion potentials. Even the model-dependent D-state probability P_D from the Nijm-II and Paris potentials show quite similar entries in the two tables. However, we emphasize that P_D for Bonn-B differs noticeably between model and inversion potential. That is, the local inversion potential that is phase-shift equivalent to Bonn-B has a significantly larger P_D, one much closer to that of Paris and Nijm-II. It is the integrand

TABLE II. ^{2}H properties for the inversion potential models investigated.

Quantity	Nijm-II	Bonn-B	Paris	Arndt
B_2	2.2245(8)	2.2246(5)	2.2249(0)	2.2246(0)
P_D	5.53	5.81	5.69	6.27
Q	0.2705	0.2877	0.2788	0.2870
A_s	0.8848	0.8861	0.8869	0.8860
η	0.0252	0.0264	0.0261	0.0264
$r_{\rm rms}$	1.9672	1.9709	1.9716	1.9748

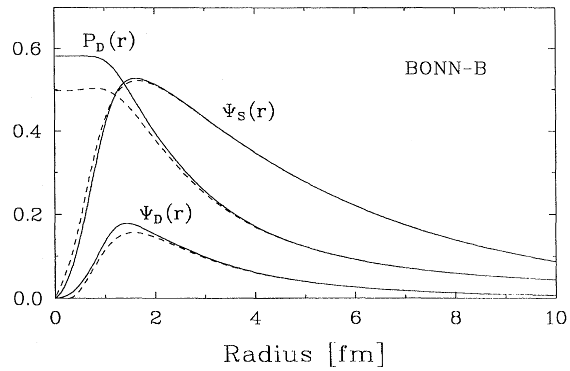

FIG. 1. Comparison of the ^{2}H wave function components from the Bonn-B model and its inversion potential counterpart along with the integral for P_D (scaled by 10). The solid line corresponds to the inversion potential, the dashed line to the original model.

$$P_D(r)=\frac{\int_r^\infty \Psi_D^2(x)\ dx}{\int_r^\infty[\Psi_S^2(x)+\Psi_D^2(x)]\ dx},$$

which demonstrates clearly that the difference in the deuteron wave functions arises at long range in r space. This is illustrated in Figs. 1 and 2, for the Bonn-B and Paris potentials. The difference in the case of Paris occurs essentially inside of 1 fm, whereas for the Bonn-B model differences persist out to at least 3 fm. In Ref. [23], the tensor component of the Bonn-B model was studied using a unitary transformation to convert it to a local form; significant differences were found well inside 2 fm which do not account for those we find at longer range. There is in the Bonn-B potential a long range r-space nonlocality of significant magnitude whose origin requires further investigation.

How do the model ^{3}H binding energies compare with the ^{3}H binding energies generated by their inversion potential counterparts? The inversion potentials are generated on a grid in r space. Triton binding energies for the partial-wave local potential, using a spline interpolation on a mesh, agreed exactly with previously obtained Reid soft core model results

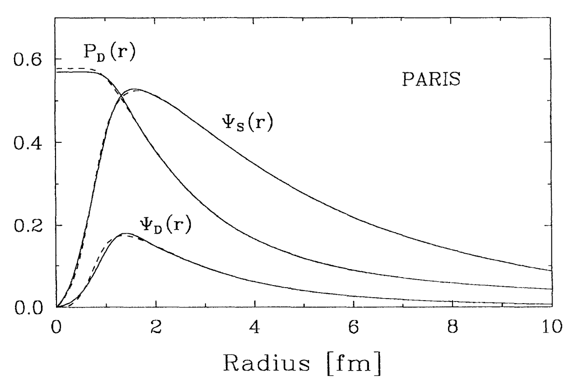

FIG. 2. Comparison of the ^{2}H wave function components from the Paris model and its inversion potential counterpart along with the integral for P_D (scaled by 10). The solid line corresponds to the inversion potential, the dashed line to the original model.

TABLE III. Comparison of ^{3}H binding energies in MeV for model and inversion potentials.

Source	BE (model) [MeV]	BE (inversion) [MeV]
Nijm-II	7.62	7.60
Nijm-II*		7.67
Paris	7.467	7.472
Bonn-B	8.14	7.84
Arndt FA91		7.36

[24]. Results for the Nijm-II model are listed in the first line of Table III. (For Nijm-II we have used the effective charge-symmetric interaction [25] given by

$$V(^1S_0)=\tfrac{2}{3}V_{nn}+\tfrac{1}{3}V_{np}$$

to account for the charge dependence of the nuclear force.) The difference between 7.62 MeV for the Nijm-II model [2] and 7.60 MeV for its inversion potential is attributed to the different phase shifts between 300 MeV and infinity, and to the use of OPE for the higher partial waves ($3\leqslant j\leqslant 4$) in the inversion potential calculations. We do consider this as a measure of agreement (a null signal) in view of the high energy phase uncertainty. That is, starting from pion subthreshold phase shifts coming from a partial-wave local potential, we have generated a corresponding inversion potential which is also partial-wave local and subthreshold phase equivalent, and we have demonstrated that the two yield essentially the same ^{3}H binding energy.

We turn next to the Paris potential. The fact that the ^{2}H D state for the model and its inversion potential are very similar implies that the momentum-dependent Paris potential behaves almost like the equivalent local inversion partner in this context. This is confirmed by the triton binding energies listed in Table III. The 7.47 MeV from the model calculation [26,27] is close to the 7.47 MeV from the inversion potential. Comparing the results for the Paris potential with those from the Paris inversion potential, one can see that there is no significant nonlocality effect noticeable for the Paris model, even though 40% of the potential energy in the triton calculation comes from nonlocal operators [26]. The difference between the Paris and Nijm-II results can be understood in terms of the lack of charge dependence in the Paris $^1S_0$ potential model [9,27].

The Bonn-B potential is considerably different, as we emphasized above. The ^{2}H D-state probability difference between the model and its local inversion potential signals this. The triton binding energies in Table III confirm it. (We note that the difference in the value of B_2 in Tables I and II is a measure of the numerical precision in this calculation; the calculated numbers are of 5 digit accuracy at the 2-body level.) The long range r-space nonlocality in the Bonn-B potential is a nontrivial aspect of the model. If one also takes into account the subthreshold model phase-shift differences, such as those due to the charge dependence omitted in the Bonn-B model, then the result from the inversion Bonn-B potential is quite close to that for the Nijm-II model. That is, the ^{3}H binding energy from the Bonn-B local inversion potential is very close to that obtained from other local potentials which fit the two nucleon data. Therefore, we conclude that phase differences between the Bonn-B and Nijm-II models do not play a key role in determining the binding energy of the triton. In contrast, the remaining difference between the 7.84 MeV from the inversion potential and the 8.14 MeV [27] from the original Bonn-B model is a clear measure of the effect that can be produced by a long range r-space nonlocality ($\geqslant$ 1 fm) in the nucleon-nucleon interaction.

Finally we turn to the question of calculating the binding energy of the triton using inversion potentials for the Arndt phase-shift analysis FA91. The result is listed in Table III and the noticeable difference is explained primarily by the very different ϵ_1 phase shift as compared with all other data. The 0–200 MeV energy interval is of particular concern [22]. As a result, the ^{3}H binding energy for the Arndt inversion potential does not agree with that of the other inversion potentials. In other words, the exceptionally high value for P_D of 6.27% combined with the known correlation between P_D and the triton binding energy implies a lack of binding for the Arndt inversion potentials, as we find. This indicates an urgent need for reconsideration of the phase-shift analysis and the experimental data employed in that analysis.

Let us now consider the question of the uncertainty in the triton binding energy due to uncertainties in the fits to the underlying high energy nucleon-nucleon phase shifts. The first element of this analysis can be found in Table III. The Nijm-II potential is said to reproduce as precise a fit to the NN observables as does the Nijmegen phase shift analysis for 0 to 300 MeV laboratory incident energies [2]. However, the phases at higher energies are not strongly constrained. The Nijmegen group are careful to state that the phases from their potentials are not to be taken as definitive outside of the region in which they were fitted. In fact, the Nijm-II phases correspond to attractive and repulsive core potentials at short distances. We also observe a short range attraction in the $^1F_3$, $^1P_1$, $^3P_1$, and $^3F_3$ channels. For that reason we modified the Nijm-II phase-shift extrapolation towards the Arndt phase shifts [22] at higher energies or simply took a smooth rational function extrapolation to enforce soft repulsive core potentials for all channels.

Implementing an alternative choice of a smooth extrapolation is straightforward with inversion techniques. Thus the influence of the high energy phases upon the ^{3}H binding energy can be investigated without changing the low energy phase shifts. A different choice of high energy phase extrapolation, which aims toward even softer repulsive core potentials for the Nijm-II inversion potential [7], yields the result for the triton identified in Table III as Nijm-II*. The 70 keV difference between these two inversion potential triton binding energies represents a minimum of uncertainty in local potential model calculations. This degree of uncertainty is further exemplified by the triton results summarized in Table IV. We use the Bonn-B $^1S_0$ phase [7] as a reference, and we smoothly continue above 300 MeV such that the phase passes through the series of quoted values at 800 MeV. The corresponding resulting inversion potentials change from having a softly repulsive to a strongly replusive core potential producing the triton binding energies listed in Table IV. Such a modification of the high energy phase shifts affects most clearly the repulsive core region but concomitantly adjusts the potential out to distances of several fm [7]. Clearly, the high energy phase shifts do play a role in the triton binding energy. As expected, however, this is not as large an

TABLE IV. Comparison of the Bonn inversion potential 34-channel triton binding energy results in MeV for the $^1S_0$ phase shifts fixed at 800 MeV at the values shown.

δ [degr]	Triton BE [MeV]	Comment
−35.62	7.87	
−41.15	7.85	
−43.04	7.84	~ Arndt [22]
−44.28	7.83	
−45.01	7.82	
−47.97	7.77	
−54.73	7.62	

effect as that coming from the low energy phase shifts, illustrated by the Arndt model result.

Based upon these studies of triton binding energy variations due to uncertain constraints upon higher energy phase shift extrapolation, we estimate the ^{3}H binding energy to be $B(^3\mathrm{H}) = 7.6 \pm 0.1$ MeV for a local potential model having phases identical to Nijm-II in the range 0–300 MeV. The difference between this estimate and the experimental value of 8.48 MeV may be ameliorated at the two-body interaction level by new phase-shift analyses and/or the inclusion of nonlocality in the potentials. For the FA91 potential the difference between model and experiment is 1.1 MeV, 250 keV more than for the Nijm-II potential. A specific example of how long range r-space nonlocality can shift the triton binding energy is provided by the Bonn-B potential. However, for a complete picture, dynamically consistent three-body force effects must also be included.

During the course of this work we had repeated communication with R. Machleidt and W. Glöckle concerning use of the Bonn-B potential. The work of the B.F.G. was performed under the auspices of the U.S. DOE. He thanks G.L. Payne for assistance in modifying the codes to handle numerical input. B.F.G. gratefully acknowledges the financial support of the Alexander von Humboldt-Stiftung. H.K. was supported in part by COSY, KFA-Jülich, Grant No. 41126865.

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