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Low-energy theorem for a composite particle in mean scalar and vector fields

Michael C. Birse
Theoretical Physics Group, Department of Physics and Astronomy, The University of Manchester, Manchester, M13 9PL, United Kingdom

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For a relativistic particle moving in the presence of mean scalar and vector fields, the energy at second order in the scalar field is shown to contain two contributions in general. One is a momentum-dependent repulsive interaction satisfying a low-energy theorem pointed out by Wallace, Gross, and Tjon. The other does not vanish at zero momentum and involves a "polarizability" of the particle by the scalar field. The first of these contributions is independent of the details of the structure of the particle and the couplings of its constituents to the external fields. The appearance of such a piece in the central nucleon-nucleus potential thus would support the existence of strong scalar fields in nuclei, without requiring the use of a Dirac equation for the nucleon.

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Treatments of nuclei based on the Dirac equation have been very successful, particularly in describing intermediate-energy proton-nucleus scattering [1,2]. These typically involve large scalar and vector potentials of opposite sign. Although these cancel to leave a small net central potential, the significant reduction of the nucleon mass by the scalar potential gives rise to a number of interesting effects. Among these are a strong momentum dependence of the central potential and an enhanced spin-orbit coupling in nucleon-nucleus scattering [1,2], and an enhanced axial charge [3]. For pointlike Dirac nucleons all of these arise from "Z-graphs," which can be interpreted as excitations of virtual nucleon-antinucleon pairs. However Brodsky has argued that pair creation should be suppressed by form factors for composite systems [4] and the validity of such an explanation for these effects has therefore been questioned.

Recently Wallace, Gross, and Tjon [5] have pointed out that, when the Dirac equation is reduced to two-component form, the Z-graphs produce an interaction of second order in the scalar field that satisfies a low-energy theorem. This interaction provides an important momentum-dependent repulsive piece in the central nucleon-nucleus potential. This low-energy theorem was obtained from the classical energy of a relativistic particle and its validity was demonstrated in a simple model for a composite fermion consisting of a bound fermion and boson. This model was based on a zero-range force between the constituents, and external vector and scalar fields which coupled to the conserved vector current and scale anomaly, respectively.

The appearance of the second-order interaction in an expansion of the classical energy of a relativistic particle suggests that this result should be valid in general. However the arguments of Ref. [5] relied heavily on the chosen forms of scalar and vector couplings and left open the possibility that this result might not apply to QCD. The purpose of the present paper is to show that the term identified by Wallace *et al.* is indeed universal and does not depend on the details of the nucleon structure, or on how the external fields are coupled to the constituents. I also point out that for a composite particle in general there can be other second-order interactions, with different momentum dependence, which do depend on its structure through various "polarizabilities."

Consider a composite particle moving in the presence of a uniform medium which generates mean scalar and vector fields, σ and ω^μ. I will refer to the particle as a nucleon, although the results will be more general. The dispersion relation connecting the energy and the three-momentum can be written in the invariant form

$$G(p^2,\sigma,\omega^2,\omega\cdot p)=0. \tag{1}$$

This form relies only on the covariance of the dynamics and it holds for both composite and "elementary" particles, irrespective of their spin. It is convenient to re-express this as an equation for p^2 in terms of the other invariants,

$$p^2=F(\sigma,\omega^2,\omega\cdot p). \tag{2}$$

For simplicity, let me assume that the ω field is sufficiently weak that only terms to second order are needed:

$$p^2\simeq\tilde{M}^2(\sigma)+[\alpha_v(\sigma)-G_v(\sigma)^2]\omega^2+2G_v(\sigma)\omega\cdot p + \alpha_p(\sigma)(\omega\cdot p)^2. \tag{3}$$

Here $\tilde{M}(\sigma)$ is the mass of the nucleon in the absence of the vector field, and $G_v(\sigma)$ is its coupling to the vector field. In general these will depend nonlinearly on the scalar field σ. The quantities $\alpha_{v,p}$ are "monopole polarizabilities" of the nucleon by the uniform vector field. A similar scalar polarizability α_s can be defined by expanding $\tilde{M}$ in powers of the σ field:

$$\tilde{M}(\sigma)=M_0+g_s\sigma+\tfrac{1}{2}\alpha_s\sigma^2+\cdots, \tag{4}$$

and a mixed scalar-vector one α_{sv} by

$$G_v(\sigma)=g_v+\alpha_{sv}\sigma+\cdots. \tag{5}$$

Here M_0 denotes the mass of a nucleon in vacuum and $g_{s,v}$ are its couplings to the scalar and vector fields.

In the rest frame of the medium, where ω^μ has only a time component, Eq. (3) can be solved to give the energy of a nucleon of momentum $\mathbf{p}$:

$$E(\mathbf{p})\simeq G_v\omega+\sqrt{(1+\alpha_p\omega^2)\mathbf{p}^2+\tilde{M}^2+(\alpha_v+\alpha_p\tilde{M}^2)\omega^2}, \tag{6}$$

where terms beyond ω^2 have again been dropped. The rest energy of the nucleon is

$$E^*\equiv E(\mathbf{p}=0)\simeq \tilde{M}+G_v\omega+\frac{\alpha_v+\alpha_p\tilde{M}^2}{2\tilde{M}}\,\omega^2. \tag{7}$$

The energy for small $\mathbf{p}$ can be used to define an inertial mass M^* for the nucleon by

$$E(\mathbf{p})\equiv E^*+\frac{\mathbf{p}^2}{2M^*}+\cdots. \tag{8}$$

This inertial mass has the form

$$M^*\simeq \tilde{M}+\frac{\alpha_v-\alpha_p\tilde{M}^2}{2\tilde{M}}\,\omega^2. \tag{9}$$

Note that, because of the factor $1+\alpha_p\omega^2$ multiplying $\mathbf{p}^2$ in Eq. (4), the inertial mass is not, in general, equal to the Lorentz scalar piece of E^*,

$$M'\equiv E^*-G_v\omega\simeq \tilde{M}+\frac{\alpha_v+\alpha_p\tilde{M}^2}{2\tilde{M}}\,\omega^2. \tag{10}$$

Expanding the energy to second order in the scalar and vector fields (but keeping terms to all orders in the momentum) gives

$$E(\mathbf{p})\simeq \epsilon(\mathbf{p})+(g_v+\alpha_{sv}\sigma)\omega+\frac{M_0}{\epsilon(\mathbf{p})}\left[g_s\sigma+\tfrac{1}{2}\alpha_s\sigma^2+\frac{[\alpha_v+\alpha_p\epsilon(\mathbf{p})^2]\omega^2}{2M_0}\right]+\frac{\mathbf{p}^2}{2\epsilon(\mathbf{p})^3}\,g_s^2\sigma^2, \tag{11}$$

where $\epsilon(\mathbf{p})$ is the energy of a free nucleon of momentum $\mathbf{p}$. The second term in this expression is linear in ω and can be interpreted as the vector potential experienced by the nucleon in the presence of the fields. The third term can be expressed in terms of a scalar potential for the nucleon,

$$S\equiv M'-M\simeq g_s\sigma+\tfrac{1}{2}\alpha_s\sigma^2+(\alpha_v+\alpha_pM^2)\omega^2, \tag{12}$$

plus a momentum-dependent piece involving the polarizability α_p.

Finally there is the term of second order in σ pointed out by Wallace *et al.* [5]:

$$\mathscr{V}=\frac{\mathbf{p}^2}{2\epsilon(\mathbf{p})^3}\,g_s^2\sigma^2. \tag{13}$$

This is a repulsive interaction which increases rapidly with the momentum of the nucleon. Note that it involves only the scalar coupling of the nucleon as a whole. It is thus unlike the other second-order terms which depend on the details of the nucleon structure through the various polarizabilities. Such a term appears in any relativistic treatment and can be thought of as arising from the modification of the nucleon's mass by the scalar field. This is clearer if we replace $g_s\sigma$ in (13) by the scalar potential S, which we can do since we are considering only terms in the energy to second order in the fields.[1] Although the second-order term appears here as a momentum-dependent repulsion, it is equivalent to the more familiar energy dependence of the central potential that appears when the Dirac equation is reduced to a two-component Schrödinger equation [6].

In general the second-order dependence of the energy (11) on the scalar field also contains a piece which does not vanish at zero momentum. This arises from the scalar polarizability of the nucleon α_s and corresponds to a second-order dependence of the scalar potential (12) on the scalar field σ. In the model of Ref. [5] the scalar field is coupled to the scale anomaly, ensuring that the nucleon mass remains linear in σ and so no such term appears. (The vector polarizabilities $\alpha_{v,p}$ and α_{sv} are identically zero in that model, because the vector field is coupled to the conserved fermion current.) Similar models with more general scalar couplings do give rise to a term of this kind in the energy [7].

In models of the type studied by Wallace *et al.* the second-order interaction $\mathscr{V}$ is produced by composite-fermion Z-graphs. This is a consequence of the zero-range force between the constituents and so may not occur in realistic treatments of nucleon structure. More generally though, such graphs are not necessary. As has been noted in other contexts, quark excitations and quark Z-graphs can conspire to yield the same result as nucleon Z-graphs [8]. This is just what happens in the more familiar cases of the Thomson limit of Compton scattering and low-energy theorems for πN interactions [9], and is unsurprising given that these are all basically classical results.

The fact that the interaction (13) does not depend on nucleon Z-graphs is illustrated by nontopological soliton models for a nucleon embedded in mean scalar and vector fields [10], where such graphs do not appear. In these semiclassical models, "pushing" can be used to determine the inertial mass of the soliton [11]. For comparison with the results of Ref. [12], the effective coupling strength of the ω to a nucleon with zero momentum is defined by

$$g_v^*=\frac{\partial E^*}{\partial\omega}=G_v+\frac{\alpha_v+\alpha_p\tilde{M}^2}{\tilde{M}}\,\omega. \tag{14}$$

The inertial mass (9) can then be expressed in the form

$$M^*=E^*-g_v^*\omega+\frac{\alpha_v}{\tilde{M}}\,\omega^2, \tag{15}$$

to order ω^2 as usual. Comparing this with Eq. (8) of Ref. [12], one can see that both have the same form and that the polarizability α_v corresponds to the response of the nucleon's structure to pushing in the presence of the vector field. The energy of the nucleon in these models thus has the form discussed above and contains a piece of the form (13).

[1] In fact such a replacement holds to all orders in σ and to second order in ω for the special case of vanishing polarizability α_p.

Although such a momentum-dependent repulsion could provide good evidence for strong scalar fields in nuclei, it has proved difficult to identify unambiguously such a term in the nucleon-nucleus optical potential [2,13]. As well as the mean-field effects included in (1), the self-energy of a nucleon in matter should include Pauli-exchange (Fock) terms. These are nonlocal and can lead to a similar momentum dependence to that generated by the relativistic effect studied here. As discussed by Kleinmann *et al.* [13], it is possible that, at least at low energies, the large Fock terms required for nonrelativistic parametrizations of the *NN* interaction are mocking up the momentum dependence of a relativistic description.

Various other properties of a nucleon in the nuclear medium are sensitive to the reduction in mass caused by the scalar potential. Of particular interest is the axial charge whose observed enhancement [14] is too large to be explained by pion exchange effects and so provides strong evidence for scalar fields in nuclei [3]. Nuclear magnetic moments are also influenced by these fields but they do not provide an unambiguous signal. Spin-dependent couplings to external axial or magnetic fields can be included in (1) and used to extract effective coupling constants. However all of these involve polarizabilities of the nucleon and so do not satisfy low-energy theorems. This is illustrated by soliton and bag model calculations of axial couplings and magnetic moments in medium [10,12,15], where the results depend on the details of the nucleon structure. The only quantities which do satisfy such theorems are the orbital magnetic g factors. These arise from the part of the current that classically is proportional to the nucleon velocity, and hence is inversely related to its mass. The enhancement of these due to a mean scalar field is independent of the details of the nucleon structure. Unfortunately there are too many other exchange-current and configuration-mixing contributions to the orbital g factors to allow this enhancement to be identified in measured magnetic moments [16].

In summary: for a relativistic particle in the presence of scalar and vector fields the momentum-dependent repulsive interaction of second order in the scalar potential is universal. The appearance of such behavior in the central nucleon-nucleus potential could thus provide evidence for strong scalar fields in nuclei, independently of the use of a Dirac equation for the nucleon.

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