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Limits of proton stability near ^{100}Sn

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The two-proton stability in even-even nuclei around the doubly magic ^{100}Sn is examined by the self-consistent Skyrme-Hartree-Fock-Bogoliubov theory. According to our analysis, the nuclei ^{98}Sn and ^{106}Te are stable to two-proton decay while ^{96}Sn and ^{104}Te lie beyond the two-proton drip line.

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A study of properties of heavy doubly magic nuclei and neighboring systems is a very important testing ground for the ability of the existing models to describe and predict the underlying shell structure. Until recent years, the nucleus ^{208}Pb was the only heavy doubly magic system with the experimentally known binding energies of the valence single-particle states. On the neutron-rich side, a considerable amount of data have recently been collected for the doubly magic ^{132}Sn [1]. The heaviest self-conjugate system (doubly magic ^{100}Sn) has been found very recently in multifragmentation studies [2,3]. This nucleus is expected to lie very close to the border of the proton stability. Indeed, already in the early work by Leander *et al.* [4], based on a structural analogy between ^{100}Sn and ^{56}Ni [5], the binding energy of the proton $g_{9/2}$ shell in ^{100}Sn was predicted to be only about 3 MeV. Undoubtedly, the many-faceted spectroscopy of nuclei around ^{100}Sn will soon become the major part of the research program using radioactive beam facilities [6]. Theoretically, considerable effort has been devoted to explore the single-particle and collective aspects of ^{100}Sn and its neighbors [4,7–16].

The main objective of the present study is to investigate the position of the two-proton drip line around ^{100}Sn. We present calculations which were performed using the spherical Hartree-Fock-Bogoliubov (HFB) method in spatial coordinates introduced in Ref. [7]. All the details of calculations closely follow Ref. [7]. The effective interaction used in our study is the Skyrme parametrization SkP obtained in Ref. [7] by a fit to properties of several magic nuclei and to the chain of the tin isotopes. In addition, a set of HFB calculations is presented with the forces SIII$^{\delta\rho}$ and SkM$^{\delta\rho}$ of Ref. [17]. In the p-h channel, these forces are the standard SkM* [18] and SIII [19] Skyrme parametrizations, respectively. The pairing components of these forces are given by the density-dependent contact interaction. (For details of these parametrizations, see Ref. [17].)

Figure 1 shows the calculated two-proton separation energies, S_{2p}, for the proton-rich even-even Sn and Te isotopes. Together with the HFB predictions, are shown results of two macroscopic-microscopic models, the extended Thomas-Fermi-Strutinsky-integral (ETFSI) model [20] and the finite-range droplet model (FRDM) [21]. Both of these approaches explicitly take into account deformation effects, and they treat pairing by means of the BCS method.

For the tin isotopes, the results of HFB+SkP, ETFSI, and FRDM are very similar. Namely, the nucleus ^{98}Sn is two-proton bound ($S_{2p}^{(\mathrm{SkP})}=1.63$ MeV) and the lighter isotope, ^{96}Sn, is weakly unbound ($S_{2p}^{(\mathrm{SkP})}=-0.46$ MeV). For ^{100,102}Sn, all three models give a fairly good agreement with the masses of Ref. [22] deduced from systematic trends. In addition, the calculated values of Q_α for ^{104}Sn (-251 keV, HFB+SkP; -275 keV, ETFSI; 65 keV, FRDM) are close to the experimental value of 290 ± 190 keV. According to the ETFSI and FRDM calculations, the Cd isotopes with $44\leqslant N\leqslant 52$ are nearly spherical; the equilibrium quadrupole deformations, β_2, vary between 0 and 0.05, and the deformation energies do not exceed 30 keV [20]. Consequently, the assumption of spherical shape in our HFB calculations seems to be well justified.

According to the HFB+SkP, ETFSI, and FRDM, the nucleus ^{106}Te is on the border of particle stability (see Fig. 1). All three models underestimate the experimental values

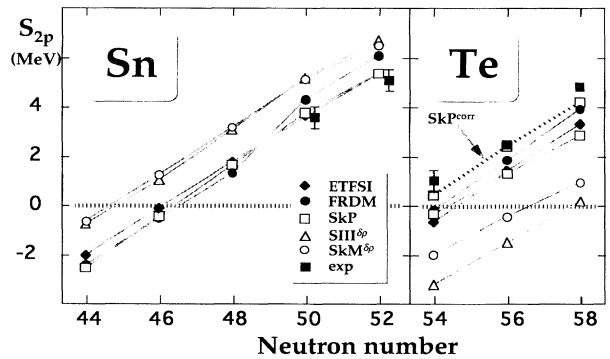

FIG. 1. Two-proton separation energies for the even-even Sn and Te isotopes around ^{100}Sn predicted in the HFB calculations with the SkP, SIII$^{\delta\rho}$, and SkM$^{\delta\rho}$ effective interactions (this work), the ETFSI model [20], and the FRDM model [21]. Experimental data are taken from Ref. [22]. (The points with error bars indicate values estimated from systematic trends.) The deformation-corrected HFB+SkP values for the Te isotopes are denoted as SkP$^{\mathrm{corr}}$ (see text).

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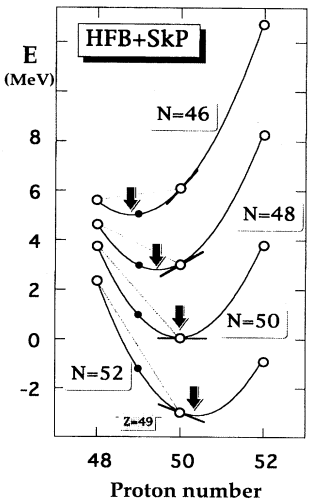

FIG. 2. Total energy, $E(N,Z)$, for the $N=46$, 48, 50, and 52 isotones of Cd, Sn, and Te. The smooth curves were obtained by making a fit to calculated values for fixed N and varying Z. For each N, the arrows indicate the minima of $E(N,Z)$. The tangent $\lambda_p=dE/dZ$ at $Z=50$ is shown schematically. The energy of the HFB vacuum at $Z=49$ is indicated by a black dot. The curves are shifted vertically with respect to one another in order not to expand the energy scale too much; the calculated binding energies of ^{96,98,100,102}Sn are given in the text.

of S_{2p} in ^{108}Te (2.48 ± 0.16 MeV) and ^{110}Te (4.84 ± 0.06 MeV) [22]. In a recent study of ^{106}Te, its alpha decay of 4128 ± 9 keV and a half-life of 60^{+40}_{-20} μs were identified. As expected from Fig. 1, the HFB+SkP, ETFSI, and FRDM calculations overestimate this experimental value of Q_α giving 5.45, 5.75, and 6.02 MeV, respectively. The assumption of spherical shape, valid for the Sn-Cd pair, is less justified for the Te-Sn pair. Indeed, in the ETFSI model the values of β_2 for the ^{106,108,110}Te isotopes are 0.09, 0.125, and 0.14, and the corresponding deformation energies are 0.65, 0.99, and 1.20 MeV, respectively. By adding these deformation energies to the HFB+SkP binding energies, one obtains values denoted by SkP$^{\rm corr}$, which present a more satisfactory agreement with the data. In particular, the nucleus ^{106}Te becomes proton stable, $S_{2p}=0.27$ MeV, and the value of Q_α in ^{106}Te is now $Q_\alpha(\mathrm{SkP}^{\rm corr})=4.82$ MeV. The nucleus ^{104}Te is predicted to lie beyond the two-proton drip line in all models ($S_{2p}=-1.84$ MeV in SkP$^{\rm corr}$).

In the SIII$^{\delta\rho}$ and SkM$^{\delta\rho}$ calculations, the Sn isotopes are too strongly bound with respect to the Cd and Te isotopes, thus yielding too large (small) values of S_{2p} in Sn (Te). (For a recent analysis of shell-gap sizes in the SkP, SIII$^{\delta\rho}$, and SkM$^{\delta\rho}$ models, see Ref. [17].) We checked that even by substantially increasing the strength of the pairing interaction in the SIII$^{\delta\rho}$ model, one does not change the pattern shown in Fig. 1.

Total energies, $E(N,Z)$, calculated in the HFB+SkP model for the $N=46$, 48, 50, and 52 isotones of Cd, Sn, and Te are shown in Fig. 2. The curves representing different isotonic chains are shifted with respect to one another in order not to expand the vertical scale too much. The actual binding energies of ^{96,98,100,102}Sn in the HFB+SkP model are 760.7, 792.7, 823.5 [23], and 847.9 MeV, respectively. Our value of binding energy for ^{100}Sn agrees well with other calculations, namely 824.4 MeV (ETFSI), 825.9 MeV (FDSM), 831.0 MeV (relativistic HF model [10]), 832.3–835.8 MeV (relativistic mean field approach (RMF) [16]), and with 824.5 MeV estimated from systematic trends [22].

In the HFB theory with Skyrme interaction, there exists an exact relation between the proton Fermi energy, λ_p, and the derivative of the total energy with respect to the proton number

$$\left.\frac{dE}{dZ}\right|_N=\lambda_p=\lambda_p'+\frac{\langle T\rangle}{A^2}, \tag{1}$$

where λ_p' is the Lagrange multiplier fixing the number of protons, and the term containing the average value of the kinetic energy, $\langle T\rangle$, comes from the standard center-of-mass correction (see the Appendix in Ref. [19]). On the other hand, an approximate relation giving the two-proton separation energy reads [24,25]:

$$S_{2p}\approx-2\left.\frac{dE}{dZ}\right|_N. \tag{2}$$

Consequently, the system is stable when $\lambda_p<0$, the two-proton drip line ($S_{2p}=0$) is met when $\lambda_p=0$, and for $\lambda_p>0$ the nucleus is expected to be unstable to two-proton decay.

As seen in Fig. 2, for the considered isotopes of Cd ($Z=48$), $\lambda_p<0$, and they are expected to be particle-stable. For the Te isotopes, there is also a consistency between results presented in Figs. 1 and 2. Namely, all Te isotopes with $N\leqslant52$ are predicted to be proton-unstable and $\lambda_p>0$. An interesting situation is predicted for the Sn isotopes. As discussed above, ^{98}Sn is calculated to be particle bound. However, according to Fig. 2, its value of λ_p is positive. This apparent contradiction can be easily explained by recalling that Eq. (2) is based on the assumption that the finite difference can be well approximated by a local derivative. This assumption breaks down around the drip line where λ_p changes quickly with Z. As discussed in Ref. [24] in such a situation, Eqs. (1) and (2) can be expressed as $S_{2p}(N,Z)\approx-2\lambda_p(N,Z-1)$, where $\lambda_p(N,Z-1)$ is the Fermi level corresponding to the HFB vacuum at an *odd* proton number. (In Fig. 2, the energy of the HFB vacuum at $Z=49$ is indicated by a black dot; $\lambda_p(N,Z-1)$ is a tangent at this point.) With this definition of the two-proton drip line, the results for the tin isotopes shown in Figs. 1 and 2 are consistent. Namely, for ^{98}Sn the value of $\lambda_p(N=48,Z=49)$ is still negative [it lies on the left-hand side of the minimum in $E(Z)$ indicated by the arrow], and for the proton-unstable ^{96}Sn the value of $\lambda_p(N=46,Z=49)$ is positive. However, the asymptotic properties of the HFB solutions [7] are still governed by the values of $\lambda_p'(N,Z)$ of Eq. (1). The differences discussed above are a clear illustration of applicability limits encountered when particle-number-violating theories are used near drip lines.

The particle stability of the tin isotopes can be related to the neutron-number dependence of the binding energy of the $g_{9/2}$ proton shell, $e_p(g_{9/2})$. The single-proton Hartree-Fock (HF) SkP energies for $Z=50$ are displayed in Fig. 3 as a function of N. The binding energy of the proton $g_{9/2}$ shell in

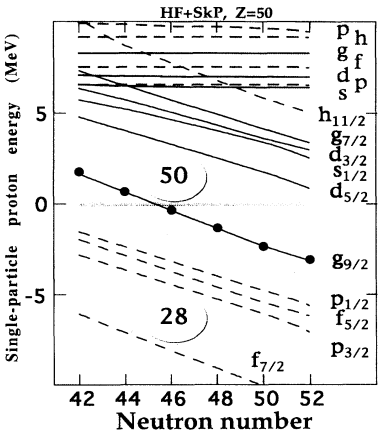

FIG. 3. Single-particle proton HF+SkP energies for the even-even tin isotopes with $N=42$–52. The nonlocalized, spin-degenerate orbitals with $j=\ell\pm 1/2$ do not depend on N; they are indicated by the orbital quantum number ℓ. The energies of the proton $g_{9/2}$ orbitals obtained within the HFB approach (vanishing proton pairing and nonzero neutron pairing for $N\neq 50$) are indicated by dots.

^{100}Sn is predicted to be 3.17 MeV in SkP. This is consistent with other calculations: 3.17 MeV (HF+SIII [4]), 3.47–3.88 MeV (RMF [16]), and 2.99 MeV (shell model [15]). In the HFB calculations, in tin isotopes only the neutron pairing is present, and its influence on proton single-particle energies is rather weak. For the $g_{9/2}$ orbital, these HFB values are indicated by dots in Fig. 3. For ^{98}Sn, one obtains $e_p(g_{9/2})=1.4$ MeV, and for ^{96}Sn $e_p(g_{9/2})$ becomes very small (0.41 MeV); however, this orbital is still bound. Nevertheless, ^{96}Sn is unbound with respect to two-proton emission. This is so, because the total energy of ^{94}Cd is lowered due to nonvanishing proton pairing correlations. Without pairing (pure HF calculations), ^{96}Sn would have been bound with respect to the two-proton emission by 1.15 MeV.

The first particle-type proton orbital, $d_{5/2}$, appears in the positive-energy part of the spectrum. As discussed in Refs. [14,26], positive-energy quasibound HF states are usually unstable against the size of the basis (size of the box; here 20 fm), and they do not represent physical resonances. Exceptions are the low-energy quasibound proton states which are localized inside the nuclear volume by the Coulomb barrier (8.3 MeV in ^{100}Sn). As seen in Fig. 3, the energies of positive-energy orbitals below $\sim$6 MeV decrease smoothly with the neutron number. This is not true for higher-lying states whose single-particle energies do not change with N. The corresponding wave functions are not localized inside the nuclear volume and, consequently, these states are not affected by the nuclear mean field. For instance, since these orbitals are not affected by the spin-dependent part of the interaction, the states with $j=\ell\pm 1/2$ are *degenerate*. (Hence, in Fig 3, they are labeled by the orbital quantum number ℓ only.) Of course, with the increasing size of the basis, the low-lying (especially low-ℓ) quasibound states become unphysical as well, and their use in the continuum shell model calculations is questionable [27].

According to the HF+SkP model, in ^{100}Sn the $g_{7/2}$ shell appears *above* the $d_{5/2}$ shell in both protons and neutrons. This result appears systematically in microscopic calculations of single-particle states in this mass region. For instance, the neutron binding energies of the $d_{5/2}$ and $g_{7/2}$ shells, crucial for properties of Gamow-Teller transitions around ^{100}Sn, are 11.8 and 9.5 MeV (HF+SkP), 10.1 and 9.6 MeV (HF+SIII [4]), 11.3 MeV and 10.2 MeV (folded-Yukawa potential [4]), and 12.7 and 10.7 MeV (Woods-Saxon potential [11]). In contrast, in recent shell-model calculations of Ref. [15], the authors assumed the inverted order of $d_{5/2}$ and $g_{7/2}$ shells, namely 10.15 MeV ($g_{7/2}$) and 10.1 ($d_{5/2}$). We agree with the authors of Ref. [15] that an experimental confirmation of the level structure of ^{101}Sn is crucial.

In summary, the self-consistent HFB+SkP model predicts the nuclei ^{98}Sn and ^{106}Te to be stable to two-proton decay. The stability of the first one is obtained within the spherical approximation, which is probably well justified here. On the other hand, the stability of the second one requires additional binding energy coming from the deformation effects at $Z=52$, and is an important indication of the deformability of very neutron-deficient nuclei.

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