Use of the Nambu–Jona-Lasinio model in the calculation of the density dependence of four-quark condensates

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Recent works concerning QCD sum rules in nuclear matter have provided a new method for the calculation of the nucleon self-energy in matter. The results of that program depend strongly on assumptions made concerning the density dependence of four-quark condensates. If a factorization scheme is used to express the four-quark condensate values in terms of the two-quark condensates, the (Lorentz) scalar self-energy of the nucleon is small. However, if the four-quark condensates have only a weak density dependence, the nucleon scalar self-energy is large and attractive, and is in accordance with Dirac phenomenology. In this paper our goal is to show how the Nambu–Jona-Lasinio model may be used to calculate the density dependence of four-quark condensates. As an elementary example we calculate some of the contributions to a four-quark condensate containing scalar-isoscalar $\bar{q}q$ pairs. These calculations suggest only a very small modification of the value of the scalar-isoscalar condensate in matter relative to the value obtained in the factorization scheme. However, when we continue our study of the correlator of operators with the quantum numbers of the nucleon, we encounter some new and important terms among the four-quark condensates. These have their origin in an exchange process between diquarks in the nucleon and diquarks present in the nucleons of the nuclear medium (nuclear matter). These terms may be taken to represent the effects of "diquark condensates" that are present in nuclear matter. If we use the interpolating field advocated by Ioffe we obtain a correction that eliminates the problematic density dependence of the four-quark condensates described above, if nuclear matter contains a similar amount of scalar ($T = 0$) diquarks and axial-vector ($T = 1$) diquarks. We believe that our paper provides increased confidence in the use of QCD sum rules in the study of the properties of hadrons in matter.

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I. INTRODUCTION

Properties of hadrons in vacuum and in matter may be studied using QCD sum rules. The basic quantity of interest is a vacuum (or nuclear matter) matrix of the time-ordered product of two "currents." In the study of the nucleon, these currents are interpolating fields, $\bar{\eta}(x)$ and $\eta(y)$, that have the quantum numbers of the nucleon. For example, $\bar{\eta}(x)$ creates three quarks at the space-time point x, and $\eta(y)$ destroys three quarks at the point y. The object of interest is the Fourier transform of $i\langle\Psi_0|T[\eta(0)\bar{\eta}(x)]|\Psi_0\rangle$, where $|\Psi_0\rangle$ is either the vacuum or the ground state of nuclear matter. For the moment let us concentrate on quark degrees of freedom. The evaluation of the matrix element of the time-ordered product may be made using Wick's theorem. (Note that normal-ordered products are taken with respect to the perturbative vacuum.) If there were no condensates, we would only need to calculate the fully contracted version of the operator $T[\eta(0)\bar{\eta}(x)]$. However, in the presence of condensates, doubly contracted terms and singly contracted terms of $T[\eta(0)\bar{\eta}(x)]$ contribute. The doubly contracted terms contain the product of two quark propagators, $S(0,x)S(0,x)$. The third quark then goes into the condensate, yielding an expression proportional to $\langle\bar{q}q\rangle_0$ in vacuum, or $\langle\bar{q}q\rangle_{\rho_B}$ in matter of density ρ_B.

Proceeding in a similar fashion, we see that the singly contracted terms of the Wick expansion of the time-ordered product, $T[\eta(0)\bar{\eta}(x)]$, contains a single-quark propagator $S(0,x)$, while the remaining four-quark operators appear in condensates such as $\langle\Psi_0|\bar{u}_{i\alpha}u_{j\beta}\bar{u}_{k\gamma}u_{l\delta}|\Psi_0\rangle$. Here, the u's are up-quark fields, i, j, ... are color indices, and α, $\beta = 1,\ldots,4$ are Dirac-matrix indices. The present work is mainly concerned with the proper evaluation of such four-quark condensates as we pass from the study of the correlator in vacuum to the correlator evaluated in nuclear matter. [Note that specific forms of the interpolating fields, $\eta(x)$, will be described in Sec. IV.]

A very extensive study of QCD sum rules in matter has been carried out by the group associated with the University of Maryland [4,5]. We are particularly interested in their results for the nucleon self-energy in matter [5]. The results may be expressed in terms of a number of condensates. At the two-quark level (dimension-three condensates) one has $\langle\bar{q}q\rangle_\rho$ and $\langle q^\dagger q\rangle_\rho$, where the subscript denotes the fact that we are forming matrix elements between states of nuclear matter. [The first condensate has the value $\langle\bar{q}q\rangle_0 = \langle\bar{u}u\rangle_0 = \langle\bar{d}d\rangle_0 \simeq (-250\ \text{MeV})^3$ in vacuum.] Dealing only with the simplest condensates, it was found that the nucleon self-energy had a large, repulsive (Lorentz) vector part and a large attractive (Lorentz) scalar part [1], a result in correspondence with Dirac phenomenology [7] and with Dirac-Brueckner-Hartree-Fock theory [8]. The Maryland group then went on to study various quark-gluon condensates, gluon condensates and four-quark condensates. We are here mainly concerned with the four-quark condensates, since the results for the nucleon self-energy are strongly dependent on how these condensates are treated [5]. The analysis of Refs. [2–6] makes use of the factorization hypothesis. For calculations in vacuum, the factorization hypothesis is represented by the approximation

$$\langle \bar{q}_{i\alpha} q_{j\beta} \bar{q}_{k\gamma} q_{l\delta} \rangle_0 \simeq \langle \bar{q}_{i\alpha} q_{j\beta} \rangle_0 \langle \bar{q}_{k\gamma} q_{l\delta} \rangle_0 - \langle \bar{q}_{i\alpha} q_{l\delta} \rangle_0 \langle \bar{q}_{k\gamma} q_{j\beta} \rangle_0 \ , \tag{1.1}$$

where $i,\ j = 1,\ldots,3$ are color indices and $\alpha,\ \beta = 1,\ldots,4$ are Dirac indices. Here $q_{i\alpha}$ is either an up or a down quark. Using this scheme, values are given in Ref. [4] for the condensates $\langle \bar{q}\Gamma_1 q \bar{q}\Gamma_2 q\rangle$, $\langle \bar{q}\Gamma_1 \lambda^A q \bar{q}\Gamma_2 \lambda^A q\rangle$, $\langle \bar{u}\Gamma_1 d \bar{d}\Gamma_2 u\rangle$, $\langle \bar{u}\Gamma_1 \lambda^A d \bar{d}\Gamma_2 \lambda^A u\rangle$, $\langle \bar{u}\Gamma_1 u \bar{d}\Gamma_2 d\rangle$, and $\langle \bar{u}\Gamma_1 \lambda^A u \bar{d}\Gamma_2 \lambda^A d\rangle$, where Γ_1 and Γ_2 are Dirac matrices and the $\lambda^A (A = 1,\ldots,8)$ are the Gell-Mann matrices in the color space. The results for these condensates are expressed in terms of $\langle \bar{q}q\rangle_\rho$ and $\langle q^\dagger q\rangle_\rho$. For example, we present Eq. (A12) of Ref. [4]:

$$\begin{aligned}\langle \bar{q}\Gamma_1 q \bar{q}\Gamma_2 q\rangle_\rho = \frac{1}{16}\langle \bar{q}q\rangle_\rho^2 &\left[\mathrm{Tr}(\Gamma_1)\mathrm{Tr}(\Gamma_2) - \frac{1}{N_c}\mathrm{Tr}(\Gamma_1\Gamma_2)\right] \\ &+ \langle \bar{q}q\rangle_\rho \langle \bar{q}\gamma_\mu q\rangle_\rho \left[\mathrm{Tr}(\Gamma_1)\mathrm{Tr}(\gamma^\mu\Gamma_2) - \frac{1}{N_c}\mathrm{Tr}(\Gamma_1\gamma^\mu\Gamma_2) + \mathrm{Tr}(\gamma^\mu\Gamma_1)\mathrm{Tr}(\Gamma_2) - \frac{1}{N_c}\mathrm{Tr}(\gamma^\mu\Gamma_1\Gamma_2)\right] \\ &+ \langle \bar{q}\gamma_\mu q\rangle_\rho \langle \bar{q}\gamma_\nu q\rangle_\rho \left[\mathrm{Tr}(\gamma^\mu\Gamma_1)\mathrm{Tr}(\gamma^\nu\Gamma_2) - \frac{1}{N_c}\mathrm{Tr}(\gamma^\mu\Gamma_1\gamma^\nu\Gamma_2)\right] \ . \end{aligned} \tag{1.2}$$

Let us further consider the much simpler result that pertains when $\Gamma_1 = \Gamma_2 = 1$. If all the q's represent either up quarks or down quarks (that is, if $\langle \bar{q}q\bar{q}q\rangle_0 = \langle \bar{u}u\bar{u}u\rangle_0$, or $\langle \bar{q}q\bar{q}q\rangle_0 = \langle \bar{d}d\bar{d}d\rangle_0$), we have

$$\langle \bar{q}q\bar{q}q\rangle_\rho = \langle \bar{q}q\rangle_\rho^2 \left[1 - \frac{1}{4N_c}\right] \tag{1.3}$$

$$= \langle \bar{q}q\rangle_\rho^2 \left(\frac{11}{12}\right) \ , \tag{1.4}$$

for $N_c = 3$. Now, there exists a well-known, model-independent, relation [2],

$$\langle \bar{q}q\rangle_\rho = \langle \bar{q}q\rangle_0 \left(1 - \frac{\sigma_N}{m_\pi^2 f_\pi^2}\rho_B\right) \ , \tag{1.5}$$

valid to first order in the baryon density ρ_B. Here σ_N is the pion-nucleon sigma term which has the value $\sigma_N = 45 \pm 8$ MeV [9]. Therefore, Eq. (1.5) predicts a reduction of about 35 percent for $\langle \bar{q}q\rangle_\rho$ relative to $\langle \bar{q}q\rangle_0$. If we insert the relation given in Eq. (1.5) into Eq. (1.4), we find that $\langle \bar{q}q\bar{q}q\rangle_\rho$ is *reduced* by about 70 percent from its vacuum value, if we only keep terms linear in ρ_B. The vacuum value, as calculated using the factorization hypothesis, is

$$\langle \bar{q}q\bar{q}q\rangle_0 = (11/12)\langle \bar{q}q\rangle_0^2 \ . \tag{1.6}$$

There is an alternative way to perform these calculations. For any operator, O, we may write [4]

$$\langle O\rangle_\rho = \langle O\rangle_0 + \langle N|O|N\rangle \rho_B + \cdots \ , \tag{1.7}$$

where ρ_B is the density of symmetric nuclear matter and $\langle N|O|N\rangle$ is the spin and isospin-averaged *nucleon* matrix element of O. [Note that the nucleon states are here normalized such that $\langle \vec{P}' \mid \vec{P}\rangle = (2\pi)^3\delta(\vec{P} - \vec{P}')$.] For example,

$$\langle \bar{q}q\rangle_\rho = \langle \bar{q}q\rangle_0 + \langle N|\bar{q}q|N\rangle \rho_B + \cdots \ . \tag{1.8}$$

The use of the Gell-Mann–Oakes–Renner relation [10] allows one to rewrite Eq. (1.8) in the (model-independent) form of Eq. (1.5).

Now, consider the calculation of $\langle \bar{q}q\bar{q}q\rangle_\rho$ in this scheme [11]. We have

$$\langle \bar{q}q\bar{q}q\rangle_\rho = \langle \bar{q}q\bar{q}q\rangle_0 + \langle N|\bar{q}q\bar{q}q|N\rangle \rho_B + \cdots \ . \tag{1.9}$$

It is useful to write the second term in Eq. (1.9) as two terms so that we may identify corrections to the factorization approximation,

$$\begin{aligned}\langle \bar{q}q\bar{q}q\rangle_\rho = \langle \bar{q}q\bar{q}q\rangle_0 + 2\langle N|\bar{q}q|N\rangle \langle \bar{q}q\rangle_0 \rho_B \\ + \langle N|\bar{q}q\bar{q}q|N\rangle_C \rho_B + \cdots \ . \end{aligned} \tag{1.10}$$

If we put $\langle \bar{q}q\bar{q}q\rangle_0 = \langle \bar{q}q\rangle_0^2$, the first two terms of Eq. (1.10) are those that would appear in the factorization scheme. The third term in Eq. (1.10) is a new feature of our analysis and is defined such that it has its origin in the constituent quarks of the nucleon and does not give rise to any factors of $\langle \bar{q}q\rangle_0$. We suggest that these constituent quarks, and their associated meson cloud, can give rise to important condensatelike terms, if the momentum of the quarks of the nucleon is small compared to large momentum, Q^2, characteristic of the sum rule calculations.

In this work, we will concentrate on the calculation of terms such as $\langle N|\bar{q}q\bar{q}q|N\rangle_C$ and attempt to understand the size of corrections to the factorization scheme. Therefore, we outline a method for the calculation of nucleon matrix element of various four-quark operators, making use of the Nambu–Jona-Lasinio model [12,13], generalized to include a description of confinement. To this end, we make use of a quark-diquark model of the nucleon, that is motivated by the dynamics of the NJL model [14,15]. The organization of our work is as follows. In Sec. II we calculate the contribution to $\langle N|\bar{q}q\bar{q}q|N\rangle_C$ from the meson cloud of the nucleon, considering both sigma and pion fields. In Sec. III we consider the contribution to $\langle N|\bar{q}q\bar{q}q|N\rangle_C$ from the three constituent quarks of the nucleon. We calculate only a direct term that has a very large statistical factor. The calculation of the corresponding exchange term is quite complicated. In Secs. IV and V we discuss the correlator for colorless interpolating fields, $\eta(x)$ and $\eta_1(x)$, which have the quantum numbers of a nucleon. Section VI contains some further discussion and conclusions, while various technical details are given in the appendices. (The reader may wish to proceed directly to Sec. IV, if he is only interested in our most significant results.)

In our discussion, a problem arises with respect to notation. In the published work on QCD sum rules in matter, one has $\langle\bar{q}q\rangle_0 = \langle\bar{u}u\rangle_0 = \langle\bar{d}d\rangle_0$, for example. However, in discussions of the NJL model, the notation used is $\bar{q}q = \bar{u}u + \bar{d}d$. Rather than adopt still another notation, we attempt to make clear, at each point in our discussion, which notation is being used.

II. THE SCALAR-ISOSCALAR CONDENSATE IN NUCLEAR MATTER: MESONIC FIELDS

In the previous section, upon use of the factorization approximation, we found

$$\langle\bar{u}u\bar{u}u\rangle_0 = \langle\bar{u}u\rangle_0^2\left[1-\frac{1}{4N_c}\right] \tag{2.1}$$

and

$$\langle\bar{d}d\bar{d}d\rangle_0 = \langle\bar{d}d\rangle_0^2\left[1-\frac{1}{4N_c}\right] . \tag{2.2}$$

It is easy to see that $\langle\bar{u}u\bar{d}d\rangle_0 = \langle\bar{d}d\bar{u}u\rangle_0$ and that $\langle\bar{u}u\bar{d}d\rangle_0 = \langle\bar{u}u\rangle_0\langle\bar{d}d\rangle_0$. Thus, with $\langle\bar{u}u\rangle_0 = \langle\bar{d}d\rangle_0$,

$$\langle(\bar{u}u+\bar{d}d)(\bar{u}u+\bar{d}d)\rangle_0 = 4\langle\bar{u}u\rangle_0^2\left(1-\frac{1}{2(4N_c)}\right) . \tag{2.3}$$

At this point it is useful to change our notation and introduce

$$\bar{q}q \equiv \bar{u}u + \bar{d}d , \tag{2.4}$$

so that Eq. (2.3) becomes

$$\langle\bar{q}q\bar{q}q\rangle_0 = \langle\bar{q}q\rangle_0^2\left(1-\frac{1}{8N_c}\right) . \tag{2.5}$$

We now write $\langle\bar{q}q\bar{q}q\rangle_\rho = \langle\bar{q}q\bar{q}q\rangle_0 + 2\langle N|\bar{q}q|N\rangle\langle\bar{q}q\rangle_0\rho_B + \langle N|\bar{q}q\bar{q}q|N\rangle_C\rho_B$ and concentrate on the evaluation of $\langle N|\bar{q}q\bar{q}q|N\rangle_C$, with the *new definition* of Eq. (2.4). The form of the operator is such that it is relatively easy to calculate the contribution arising from the presence of the sigma meson in the nucleon "meson cloud."

In Fig. 1(a) we represent the operator $\bar{q}q\bar{q}q$ acting first to destroy and then create a $\bar{q}q$ pair. In Fig. 1(b) we show a nucleon composed of three quarks. A string of "bubbles," appropriate to the NJL model, is shown. The black dots again indicate the annihilation and creation of a $\bar{q}q$ pair by the operator $\bar{q}q\bar{q}q$. In Fig. 1(c), the string of quark-antiquark bubbles has been replaced by a sigma meson *via* a bosonization procedure [14,16,17]. We remark that these two pictures may be related by noting

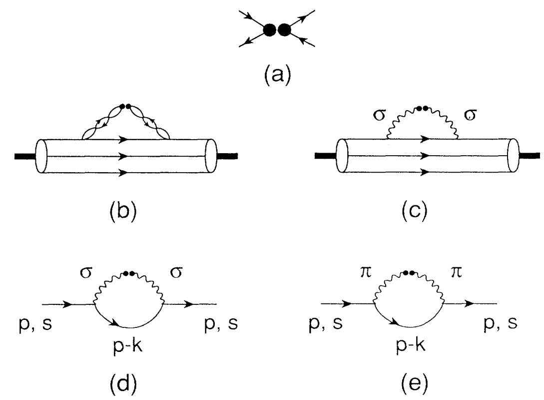

FIG. 1. (a) Schematic representation of the operator $\bar{q}q\bar{q}q$, which annihilates and then creates a $\bar{q}q$ pair. (b) A nucleon (heavy line) is composed of three constituent quarks (light lines). The string of $q\bar{q}$ "bubbles" may be summed as in the NJL model. (c) Alternately, a bosonization scheme allows one to work with meson fields that are represented by a wavy line. (d) The calculation implied by figure (c) may be simplified by considering the contribution of a single quark (see text). (e) Similar caption to (d), except that we here consider the contribution of the pion field. That contribution to the scalar-isoscalar condensate is nonzero only due to exchange matrix elements.

the relation

$$\sigma = -\frac{G_s}{g}\bar{q}q , \tag{2.6}$$

which is used in the bosonization scheme [16,17]. (Recall that $\bar{q}q = \bar{u}u + \bar{d}d$ here.) The coupling constant, G_s, appears in the NJL Lagrangian

$$\mathcal{L}(x) = \bar{q}(x)(i\,\partial\!\!\!/ - \hat{m}_q)q(x) + \frac{G_s}{2}[(\bar{q}q)^2 + (\bar{q}i\gamma_5\,\vec{\tau}\,q)^2] , \tag{2.7}$$

where $\hat{m}_q$ is the current quark mass. In Eq. (2.6), g is the sigma-quark (or pion-quark) coupling constant that arises upon bosonization. For the work reported here, we use $m_q = 0.30$ GeV, $G_s = 8.40$ GeV^{-2}, $g = 3.05$, and $m_\sigma = 0.50$ GeV. Here m_q is the constituent quark mass and m_σ is an (effective) mass of the sigma meson. (The sigma meson is a useful degree of freedom if the meson momentum is spacelike [17].)

For simplicity, we will approximate the calculation implied by Fig. 1(c) by taking the result to be three times the values obtained for a single (on-mass-shell) quark, calculated as indicated in Fig. 1(d). (Since the contribution of the meson cloud to $\langle N|\bar{q}q\bar{q}q|N\rangle_C$ is not large, this approximation is adequate for our purposes. Similar approximations have been used in Ref. [13] in another context.) We consider the single quark to emit a sigma meson. That meson is annihilated by the factor $\bar{q}q$ on the right of $\bar{q}q\bar{q}q$. The factor $\bar{q}q$ on the left then recreates the sigma meson which then is absorbed by the quark, as depicted in Fig. 1(d). With the labeling as in Fig. 1(d), we define the contribution to $\langle N|\bar{q}q\bar{q}q|N\rangle_C$ as

$$A_\sigma = 3g^2\left(\frac{g}{G_s}\right)^2 i\int\frac{d^4k}{(2\pi)^4}\frac{(1/2)\text{Tr}[(\not p - \not k + m_q)\Lambda^{(+)}(\vec{p})]}{(k^2-m_\sigma^2)^2[(p-k)^2-m_q^2]} . \tag{2.8}$$

In this calculation the quark is taken to be on-mass-shell. Here, $\Lambda^{(+)}(p) = (\not p + m_q)/2m_q$ appears since we have averaged over the direction of the quark spin. Recall that $\Lambda^{(+)}(p) = \sum_s u(\vec{p},s)\bar{u}(\vec{p},s)$, where the $u(\vec{p},s)$ are positive-energy spinors for the quark. The factor of $(g/G_s)^2$ appears when relating the $\bar{q}q$ operator to the sigma field, as in Eq. (2.6), and the factor of 3 arises because the nucleon contains 3 quarks. Note that Eq. (2.8) contains two sigma propagators and a single quark propagator in accordance with Fig. 1(d). We have, for $\vec{p}=0$, $p^0 = m_q$

$$A_\sigma = 3\times 2g^4G_s^{-2}i\int_0^1 x dx\int\frac{d^4k}{(2\pi)^4}\frac{m_q+p^0-k^0}{[xk^2-xm_\sigma^2+(1-x)(p-k)^2-(1-x)m_q^2]^3} \tag{2.9}$$

$$= 3\times 2g^4G_s^{-2}i\int_0^1 x dx\int\frac{d^4k}{(2\pi)^4}\frac{m_q+p^0-k^0}{\{[k-(1-x)p]^2-xm_\sigma^2-(1-x)m_q^2+x(1-x)p^2\}^3} \tag{2.10}$$

$$= 6g^4G_s^{-2}i\int_0^1 x dx\int\frac{d^4k'}{(2\pi)^4}\frac{m_q+xp^0}{[k'^2-B_\sigma]^3} , \tag{2.11}$$

with $B_\sigma = xm_\sigma^2+(1-x)m_q^2-x(1-x)p^2$. Finally

$$A_\sigma = 6g^4G_s^{-2}i\int_0^1 x dx(m_q+xp^0)I_3(B_\sigma) , \tag{2.12}$$

where

$$I_3(B_\sigma) = \int\frac{d^4k'}{(2\pi)^4}\frac{1}{[k'^2-B_\sigma]^3} \tag{2.13}$$

$$= \frac{-i}{32\pi^2}\frac{1}{B_\sigma} . \tag{2.14}$$

We find $A_\sigma = 0.0317$ GeV^3 upon making use of the various parameters listed above. To estimate the contribution to $\langle\bar{q}q\bar{q}q\rangle_\rho$ we put

$$\langle\bar{q}q\bar{q}q\rangle_\rho = \langle\bar{q}q\rangle_0^2\left(1-\frac{1}{8N_c}\right)\left[1+\frac{A_\sigma\rho_B}{\langle\bar{q}q\rangle_0^2\left(1-\frac{1}{8N_c}\right)}\right] . \tag{2.15}$$

With $\rho_B = (0.108 \text{ GeV})^3$ and $\langle\bar{q}q\rangle_0 = \langle\bar{u}u+\bar{d}d\rangle_0 = -2(0.250 \text{ GeV})^3$, we find that the second term in the large bracket is equal to 0.0427. That is, we have a four percent increase of $\langle\bar{q}q\bar{q}q\rangle_\rho$ over its vacuum value due to the sigma mesons of the nucleon's meson cloud. That is, of course, a quite small correction compared to that arising from the second term of Eq. (1.10).

In order to calculate the contribution of the pion, as shown in Fig. 1(d), it is useful to perform a Fierz rearrangement of the operator $\bar{q}q\bar{q}q$. [See Appendix A.] The relevant (rearranged) operator is $\frac{1}{24}[\bar{q}i\gamma_5\vec{\tau}q\cdot\bar{q}i\gamma_5\vec{\tau}q]$, where the factor $\frac{1}{24}$ arises from the Fierz rearrangement. With respect to Fig. 1(e), we have for the contribution to $\langle N|\bar{q}q\bar{q}q|N\rangle_C$ of the pion cloud of the nucleon,

$$A_\pi = 3\times 3g^2\left(\frac{g}{G_s}\right)^2\left(\frac{1}{24}\right)i\int\frac{d^4k}{(2\pi)^4}\frac{(1/2)}{(k^2-m_\pi^2)^2}\frac{\text{Tr}[\gamma_5(\not p-\not k+m_q)\gamma_5\Lambda^{(+)}(\vec{p})]}{[(p-k)^2-m_q^2]} , \tag{2.16}$$

where the new factor of 3 has an origin in the isospin trace and the factor $\frac{1}{24}$ is that arising from the Fierz rearrangement. Further, with $p^0 = m_q$ and $\vec{p}=0$,

$$A_\pi = \tfrac{3}{8}g^4G_s^{-2}i\int\frac{d^4k}{(2\pi)^4}\frac{m_q+k^0-p^0}{(k^2-m_\pi^2)^2[(p-k)^2-m_q^2]} \tag{2.17}$$

$$= \tfrac{3}{8}g^4G_s^{-2}i\int_0^1 x dx\int\frac{d^4k}{(2\pi)^4}\frac{m_q+k^0-p^0}{\{[k-(1-x)p]^2-xm_\pi^2-(1-x)m_q^2+x(1-x)p^2\}^3} \tag{2.18}$$

$$= \tfrac{3}{8}g^2G_s^{-2}i\int_0^1 x dx(m_q-xp^0)I_3(B_\pi) , \tag{2.19}$$

where B_π is the quantity defined after Eq. (2.11) with m_σ replaced by m_π. We find $A_\pi = 0.0045$ GeV^3, which is an order of magnitude smaller than A_σ and may also be neglected.

III. CALCULATION OF THE SCALAR-ISOSCALAR CONDENSATE IN NUCLEAR MATTER: QUARK FIELDS IN THE NUCLEON

In the last section we have seen how the use of a Fierz rearrangement simplifies the calculation of processes that proceed through exchange terms of the scalar-isoscalar operator $\bar{q}q\bar{q}q$. To study the contribution of the three (constituent) quarks of the nucleon to the evaluation of $\langle N|\bar{q}q\bar{q}q|N\rangle_C$, it is useful to perform a Fierz rearrangement. This transformation is then the same as that used when studying the diquark sector of the NJL model [13]. (See the Appendices for further details.)

In order to evaluate $\langle N|\bar{q}q\bar{q}q|N\rangle_C$, one needs a relativistic model of the nucleon. One model of nucleon structure that is relatively easy to use is based upon the NJL model. In the study of the diquark sector of that model, one finds a strong attraction in the case of $J=0$ and $T=0$ (scalar-isoscalar) diquarks [13]. The energy of the scalar diquark is calculated to be about 400 MeV in our work. In addition there is a $J=1$, $T=1$ (axial-vector, isovector) diquark whose energy is about 800–1000 MeV. While a satisfactory description of nucleon magnetic moments will require a significant amount of the axial-vector diquark in the nucleon, in this section we will study a simple model where the nucleon is composed of a constituent quark bound to a scalar diquark. For quark masses of about 300 MeV, and a diquark mass of 400 MeV, one needs to provide a model of confinement. We have carried out a study of this quark-diquark model of the nucleon; however, we will not discuss the details of our calculations here.

In Fig. 2(a) we show the vertex for a nucleon to go into a quark and a scalar-isoscalar diquark (double line). In Fig. 2(b) we show the same vertex with the quark on mass shell (indicated by a cross on the quark line). In general, for an on-mass-shell nucleon of momentum P, we may write the vertex as $\hat{\Gamma}(P,Q)u(\vec{P},s) = [A + B\not{Q}]u(\vec{P},s)$, where A and B are functions of two scalar variables. However, since we consider the quark to be on-mass-shell in our model, we only need matrix elements of the form $\Lambda^{(+)}(\vec{Q})\hat{\Gamma}(P,Q)$ with $\Lambda^{(+)}(\vec{Q}) = (\not{Q}+m_q)/2m_q$. With that in mind, we can parametrize the vertex by a single function. We introduce

$$\Gamma(P,Q)u(\vec{P},s) = \left[\frac{2E_q(\vec{Q})}{E_q(\vec{Q})+m_q}\right]^{1/2} [(P-Q)^2 - m_d^2] \times \Psi^{(+)}(P,Q)u(\vec{P},s)\ , \quad (3.1)$$

where $\Psi^{(+)}(P,Q)$ is a wave function. Equation (3.1) is valid in the space defined by the projection operator $\Lambda^{(+)}(\vec{Q})$. In Fig. 3 we show the values of $\Psi^{(+)}(|\vec{Q}|)$ calculated in the nucleon rest frame ($\vec{P}=0$, $P^0=m_N$).

Now, consider the evaluation of the diagram shown in Fig. 4(a). Through a generalized Fierz rearrangement we relate the operator $\bar{q}q\bar{q}q$ to the operator $(\bar{q}\gamma_5 t_c\tau_2 C\bar{q}^T)(q^T C^{-1}\tau_2\gamma_5 t_c q)$ where $t^c_{ab} = i\sqrt{\frac{3}{2}}\epsilon_{abc}$. Here ϵ_{abc} is the completely antisymmetric symbol, with $\epsilon_{123}=1$, and $C = i\gamma^0\gamma^2$ is the charge conjugation operator [13]. That operator connects the two quarks in the first diquark to the vacuum, as in Fig. 4(a), and then creates the quarks of the second diquark. We are motivated to study that operator, since we are here using a quark and scalar-diquark model of the nucleon. Therefore, the matrix element $\langle N|(\bar{q}\gamma_5 t_C\tau_2 C\bar{q}^T)(q^T C^{-1}\tau_2\gamma_5 t_c q)|N\rangle$ will be large. We find upon rearrangement that

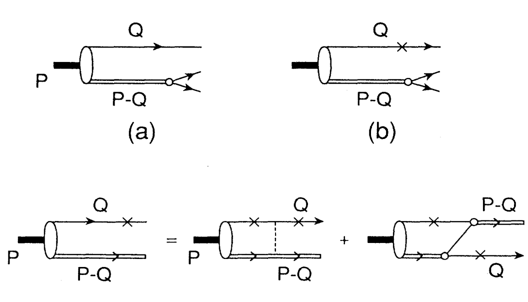

FIG. 2. (a) The vertex for a nucleon of momentum P to decay (virtually) into a quark of momentum Q and a diquark of momentum P-Q. (b) The cross denotes an on-mass-shell quark with $Q^0 = [\vec{Q}^2 + m_q^2]^{1/2}$. (c) An equation to determine the vertex shown in (b). Here V_c is a confining potential and the last term is the exchange interaction extensively studied in the literature [15]. (Here the wavy line denotes a photon and crosses again denote on-mass-shell quarks.)

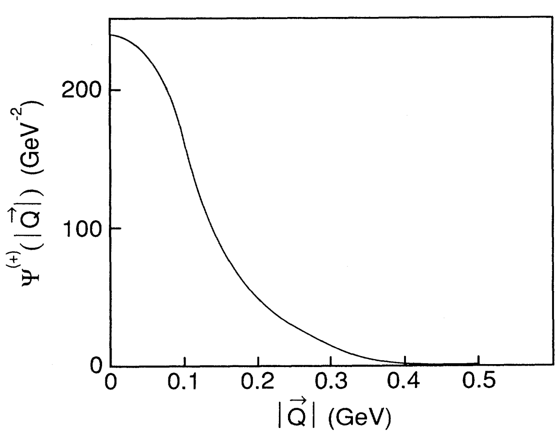

FIG. 3. The wave function $\Psi^{(+)}(|\vec{Q}|)$ that parametrizes the nucleon-quark-diquark vertex when the quark is on-mass-shell [see Fig. 2(b)]:

$$\Lambda^{(+)}(\vec{Q})\Gamma(P,Q)u(\vec{P},s) = \Lambda^{(+)}(Q)\left[\frac{2E_q(\vec{Q})}{E_q(\vec{Q})+m_q}\right]^{1/2} \times [(P-Q)^2 - m_d^2]\Psi^{(+)}(P,Q)u(\vec{P},s)\ ,$$

where $\Lambda^{(+)}(\vec{Q}) = (\not{Q}+m_q)/(2m_q)$. In the nucleon rest frame we write $\Psi^{(+)}(P,Q)$ as $\Psi^{(+)}(|\vec{Q}|)$.

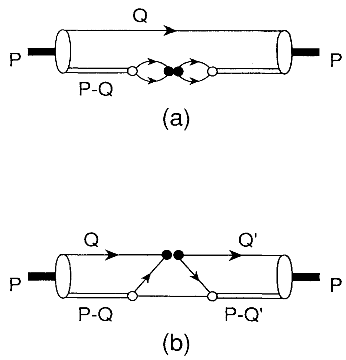

FIG. 4. (a) Evaluation of the direct term for the operator of Eq. (3.2). Here the double line is a scalar-isoscalar diquark and the heavy line is a nucleon. The filled circles were defined in the caption to Fig. 1. (b) An exchange term that appears when evaluating the matrix element $\langle N|\bar{q}q\bar{q}q|N\rangle$.

$$(\bar{q}q)(\bar{q}q) = \tfrac{1}{24}(\bar{q}\gamma_5 t_c\tau_2 C\bar{q}^T)(q^T C^{-1}\tau_2\gamma_5 t_c q) + \cdots \ . \quad (3.2)$$

Using Eq. (3.2) in the evaluation of the diagram of Fig. 4(a) yields the expression

$$\begin{aligned} A_D = &-4g^2_{Dqq}\int \frac{d\vec{Q}}{(2\pi)^3}\left[\frac{J_\pi((P-Q)^2)}{i}\right]^2 \frac{1}{24} \\ &\times \frac{1}{2}\mathrm{Tr}\left(\frac{1+\gamma^0}{2}\Lambda^{(+)}(\vec{Q})\right)\frac{m_q}{E_q(\vec{Q})}\frac{2E_q(\vec{Q})}{E_q(\vec{Q})+m_q} \\ &\times[\Psi^{(+)}(Q)]^2 \end{aligned} \quad (3.3)$$

in the nucleon rest frame. Here, the factor of 4 is a statistical factor and the factor of (1/24) is that appearing in Eq. (3.2). We have used the fact that the quark of momentum $\vec{Q}$ is on-mass-shell so that we may make the replacement

$$\frac{1}{\not{Q} - m_q + i\epsilon} \to \frac{m_q}{E_q(\vec{Q})}(-2\pi i)\delta(Q^0 - E_q(\vec{Q}))\Lambda^{(+)}(\vec{Q}) \ . \quad (3.4)$$

Further, $J_\pi((P-Q)^2)$ is a basic quark-loop integral of the NJL model [13],

$$\begin{aligned} &J_\pi((P-Q)^2) \\ &\quad = N_c N_f \mathrm{Tr} i\int \frac{d^4k}{(2\pi)^4} S(P-Q+k)\gamma_5 S(k)\gamma_5 \ . \end{aligned} \quad (3.5)$$

Here $N_f = 2$ and $N_c = 3$. In the nucleon rest frame, we have the contribution to $\langle N|\bar{q}q\bar{q}q|N\rangle_C$,

$$A_D = \frac{g^2_{Dqq}}{12\pi^2}\int Q^2 dQ [J_\pi((P-Q)^2)]^2[\Psi^{(+)}(Q)]^2 \ , \quad (3.6)$$

where $Q = |\vec{Q}|$. Here the factors $(m_q/E_q)[2E_q/(E_q + m_q)]$ have been canceled by the factor arising from the evaluation of the trace that appears in Eq. (3.3). We note that for small q^2, $J_\pi(q^2) \simeq 0.118 + 0.132q^2$ (GeV^2) with q^2 in units of GeV^2. From our other studies, we found that $g^2_{Dqq} = 7.57\ \mathrm{GeV}^{-4}$. With $m_q = 0.305$ GeV, we find $A_D = 0.0276\ \mathrm{GeV}^3$.

We may now generalize Eq. (2.15) to read

$$\begin{aligned} \langle\bar{q}q\bar{q}q\rangle_\rho = &\langle\bar{q}q\rangle_0^2\left(1 - \frac{1}{8N_c}\right) \\ &\times\left[1 + \frac{(A_\sigma + A_D)\rho_B}{\langle\bar{q}q\rangle_0^2\left(1-\frac{1}{8N_c}\right)} + \cdots\right] \ . \end{aligned} \quad (3.7)$$

(Recall that here $\bar{q}q = \bar{u}u + \bar{d}d$.) The second term in the large bracket is small and may be neglected.

For future applications, it is useful to introduce $C_{\mathrm{SD}} = 24A_D$, where

$$\begin{aligned} C_{\mathrm{SD}} &= \langle N|[\bar{q}t_c\tau_2\gamma_5 C\bar{q}^T][q^T C^{-1}\gamma_5\tau_2 t_c q]|N\rangle \quad (3.8) \\ &= 0.662\ \mathrm{GeV}^3 \ . \quad (3.9) \end{aligned}$$

As we will see, $C_{\mathrm{SD}}\rho_B$ plays the role of scalar-isoscalar diquark condensate of dimension 6 and is to be classed with other four-quark condensate terms.

IV. NUCLEON CORRELATORS FOR NUCLEONS IN NUCLEAR MATTER

As usual, we define the Fourier transform of a time-ordered correlation function of nucleon interpolating fields, $\eta(x)$ and $\bar{\eta}(0)$, to be a nucleon correlator [1–6],

$$\Pi(q,u) = i\int d^4x e^{iq\cdot x}\langle\Psi_0|T[\eta(x)\bar{\eta}(0)]|\Psi_0\rangle \ . \quad (4.1)$$

A four-vector u^μ is needed if $|\Psi_0\rangle$ represents the ground state of nuclear matter. This vector describes the flow of the matter. In this case there are two Lorentz-invariant quantities, q^2 and $q\cdot u$. If $|\Psi_0\rangle$ denotes the vacuum, only q^2 appears as a Lorentz-invariant. Note that $\Pi(q,u)$ has two Dirac indices corresponding to the Dirac indices of the operators $\eta(x)$ and $\bar{\eta}(0)$. Various forms for $\eta(x)$ may be used. Some found in the literature include

$$\eta_1(x) = \epsilon_{abc}[u_a^T(x)C\gamma_5 d_b(x)]u_c(x) \ , \quad (4.2)$$

$$\eta_2(x) = \epsilon_{abc}[u_a^T(x)C d_b(x)]\gamma_5 u_c(x) \ , \quad (4.3)$$

and

$$\eta(x) = \epsilon_{abc}[u_a^T(x)C\gamma_\mu u_b(x)]\gamma_5\gamma^\mu d_c(x) \ . \quad (4.4)$$

Here $a, b, c\ldots$ are color indices, $u(x)$ is the up-quark field, $d(x)$ is the down-quark field, and $C = i\gamma^0\gamma^2$ is the charge conjugation operator. We will call $\eta(x)$ the Ioffe current [18]. (Note that $\eta(x) = 2[\eta_2(x) - \eta_1(x)]$.)

In this work we will study the nucleon correlators for the fields $\eta_1(x)$ and $\eta(x)$. First, we note that in the study of nucleons in nuclear matter we can write

$$\Pi(q,u) = \Pi_s(q^2, q\cdot u) + \Pi_q(q^2, q\cdot u)\ \not{q} + \Pi_u(q^2, q\cdot u)\ \not{u} \quad (4.5)$$

where, as above, u_μ is the four-vector describing the flow of nuclear matter. Following, Ref. [4], we may also divide

$\Pi(q,u)$ into parts that are even or odd in $q\cdot u$:

$$\Pi_s(q^2,q\cdot u)\equiv \Pi_s^E[q^2,(q\cdot u)^2]+(q\cdot u)\Pi_s^O[q^2,(q\cdot u)^2]\ , \tag{4.6}$$

$$\Pi_q(q^2,q\cdot u)\equiv \Pi_q^E[q^2,(q\cdot u)^2]+(q\cdot u)\Pi_q^O[q^2,(q\cdot u)^2]\ , \tag{4.7}$$

$$\Pi_u(q^2,q\cdot u)\equiv \Pi_u^E[q^2,(q\cdot u)^2]+(q\cdot u)\Pi_u^O[q^2,(q\cdot u)^2]\ . \tag{4.8}$$

For ease of reference, we reproduce some results of Ref. [4] for the even functions of $(q\cdot u)$. (We present only the most important quark condensate terms and a familiar gluon condensate term. The complete expressions are given in Ref. [4].) We have [4]

$$\Pi_s^E[q^2,(q\cdot u)^2]=\frac{1}{4\pi^2}q^2\ln(-q^2)\langle\bar q q\rangle_{\rho_B}+\cdots\ , \tag{4.9}$$

$$\Pi_q^E[q^2,(q\cdot u)^2]=-\frac{1}{64\pi^2}q^4\ln(-q^2)-\frac{1}{32\pi^2}\ln(-q^2)\langle\frac{\alpha_S}{\pi}G^2\rangle_{\rho_B}-\frac{2}{3q^2}\langle\bar q q\rangle^2_{\rho_B}-\frac{4}{3q^2}\langle\bar q\ \not{u} q\rangle^2_{\rho_B}+\cdots\ , \tag{4.10}$$

$$\Pi_u^E[q^2,(q\cdot u)^2]=\frac{2}{3\pi^2}q^2\ln(-q^2)\langle\bar q\ \not{u} q\rangle_{\rho_B}+\cdots\ . \tag{4.11}$$

Note that here $\langle\bar q q\rangle_{\rho_B}=\langle\bar u u\rangle_{\rho_B}=\langle\bar d d\rangle_{\rho_B}$, etc. Also, all polynomial terms that vanish under a Borel transformation have been neglected in these expressions. The four-quark condensates have been obtained using the factorization approximation. The problematic term is the third term of Π_q^E. It is seen that if $\langle\bar q q\rangle^2_{\rho_B}$ in Eq. (4.10) is replaced by $\langle\bar q q\rangle_0^2$ the satisfactory results of Ref. [1] are preserved [5]. In this work we will discuss corrections to Eqs. (4.9)–(4.11) that involve "condensates" that have not been considered previously.

Let us now consider the evaluation of Eq. (4.1) in the case that the Ioffe interpolating field of Eq. (4.4) is used. We now calculate the four-quark condensate terms that do not appear in Eqs. (4.9)–(4.11). One such term is obtained if we consider a contraction between the two down-quark fields, $\bar d(0)$ and $d(x)$, with all the up-quark fields appearing in the nucleon matrix element. To carry out this calculation it is useful to write the Ioffe current as

$$\eta(x)=-\sqrt{\frac{2}{3}}\left(q^T(x)(C\gamma_\mu)\frac{1+\tau_3}{2}(it_c)q(x)\right)[\gamma_5\gamma^\mu d_c(x)]\ . \tag{4.12}$$

We insert this expression into Eq. (4.1) and consider a single contraction between the down-quark fields to obtain

$$\Pi(q,u)=\frac{1}{3}S(q)\Bigg\{\Big\langle\left(\bar q(0)(\gamma^\mu C)\frac{1+\tau_3}{2}(it_c)\bar q^T(0)\right)\left(q^T(0)(C^{-1}\gamma_\mu)\frac{1+\tau_3}{2}(it_c)q(0)\right)\Big\rangle_{\rho_B}$$
$$+\langle N|\left(\bar q(0)(\gamma^\mu C)\frac{1+\tau_3}{2}(it_c)\bar q^T(0)\right)\left(q^T(0)(C^{-1}\gamma_\mu)\frac{1+\tau_3}{2}(it_c)q(0)\right)|N\rangle_C\rho_B\Bigg\}\ , \tag{4.13}$$

with $S(q)=\not{q}/q^2$. The first term of Eq. (4.13) is evaluated using the factorization approximation and the result is

$$\Pi_1(q,u)=-\frac{2}{3}\frac{\not{q}}{q^2}\langle\bar u u\rangle^2_{\rho_B} \tag{4.14}$$

$$\simeq -\frac{2}{3}\frac{\not{q}}{q^2}(\langle\bar u u\rangle_0^2+2\langle N|\bar u u|N\rangle\langle\bar u u\rangle_0\rho_B+\cdots)\ . \tag{4.15}$$

Consideration of other singly-contracted terms does not change this answer. The expression given in Eq. (4.14) appears in Π_q^E of Eq. (4.10), where the notation $\langle\bar q q\rangle_0=\langle\bar u u\rangle_0=\langle\bar d d\rangle_0$ is used.

It is useful to define

$$C_{\rm VD}=\langle N|[\bar q(it_c)(\vec\tau\ \tau_2)(\gamma^\mu C)\bar q^T]\cdot[q^T(C^{-1}\gamma_\mu)(\tau_2\ \vec\tau)(it_c)q]|N\rangle\ . \tag{4.16}$$

In a calculation to be reported elsewhere, we found $C_{\rm VD}=0.380\ {\rm GeV}^3$. (Note that $C_{\rm VD}\rho_B$ plays the role of an axial-vector diquark condensate.)

The evaluation of the second term of Eq. (4.13) yields a contribution to $\Pi(q,u)$ that we call $\Pi_2(q,u)$. Both $\Pi_1(q,u)$ and $\Pi_2(q,u)$ contribute to Π_q^E of Eq. (4.10). We note that the nucleon matrix element in Eq. (4.13) may be shown to equal $C_{\rm VD}/6$. Therefore,

$$\Pi_2(q,u)=\frac{1}{18}\frac{\not{q}}{q^2}C_{\rm VD}\rho_B \tag{4.17}$$

$$=0.0211\frac{\not{q}}{q^2}\rho_B\ . \tag{4.18}$$

Inclusion of all other singly contracted terms adds a correction to Eq. (4.17), so that Eq. (4.17) is replaced by

$$\Pi_2(q,u)=\frac{1}{36}\frac{\not{q}}{q^2}C_{\rm VD}\rho_B\ . \tag{4.19}$$

Now let us continue to use the Ioffe interpolating field, but allow for the presence of scalar-isoscalar diquarks in nuclear matter. After some calculation, we find

$$\Pi_2(q,u)=-\left[\frac{2}{9}\frac{\not{q}}{q^2}C_{\rm SD}\rho_B\right]\alpha^2+\left[\frac{1}{36}\frac{\not{q}}{q^2}C_{\rm VD}\rho_B\right]\beta^2\ , \tag{4.20}$$

with $\alpha^2 + \beta^2 = 1$. Here, α^2 represents the probability of finding a nucleon composed of a scalar diquark and a quark, while β^2 is the probability that the nucleon contains a quark coupled to an axial-vector ($T = 1$) diquark. A study of nucleon magnetic moments in a quark-diquark model leads us to believe that $\alpha^2 \simeq \beta^2 \simeq 0.5$. With that in mind let us write $\Pi_q^E(q^2, q\cdot u)$ of Eq. (4.10) as

$$\Pi_q^E[q^2,(q\cdot u)^2] = -\frac{1}{64\pi^4}(q^2)^2\ln(-q^2) - \frac{1}{32\pi^2}\ln(-q^2)\left\langle \frac{\alpha_S}{\pi}G^2\right\rangle_{\rho_B} - \frac{2}{3q^2}\{\langle\bar{u}u\rangle_0^2 + 2\langle N|\bar{u}u|N\rangle\langle\bar{u}u\rangle_0\rho_B\}$$
$$-\left(\frac{2}{9q^2}C_{\rm SD}\rho_B\right)\alpha^2 + \left(\frac{1}{36q^2}C_{\rm VD}\rho_B\right)\beta^2 - \frac{4}{3q^2}\langle\bar{q}\ \not{u}q\rangle_{\rho_B}^2\ . \tag{4.21}$$

Upon using $C_{\rm SD} = 0.662\ \text{GeV}^3$, $C_{\rm VD} = 0.380\ \text{GeV}^3$, $\langle N|\bar{u}u|N\rangle = 4.08$, $\langle\bar{u}u\rangle_0 = -(0.25\ \text{GeV})^3$, and $\alpha^2 = \beta^2 = 0.5$, we find that about 75 percent of the problematic term in Eq. (4.21) is canceled. Thus, we can write

$$\Pi_q^E[q^2,(q\cdot u)^2] \simeq -\frac{1}{64\pi^4}(q^2)^2\ln(-q^2) - \frac{1}{32\pi^2}\ln(-q^2)\left\langle\frac{\alpha_S}{\pi}G^2\right\rangle_{\rho_B} - \frac{2}{3q^2}\langle\bar{u}u\rangle_0^2 - \frac{4}{3q^2}\langle\bar{q}\ \not{u}q\rangle_{\rho_B}^2 \tag{4.22}$$

to a good approximation if the parameters are as we have indicated.

This is a most satisfactory result, since it implies that, while $\langle\bar{q}q\rangle_0$ goes over to $\langle\bar{q}q\rangle_{\rho_B}$ in matter, the four-quark condensates effectively remain at their vacuum value. This is the situation in which the properties of the nucleon in matter found by QCD sum-rule techniques agree with the results of Dirac phenomenology. (See the discussion of Sec. III of Ref. [5].)

V. INTERPOLATING FIELDS CONTAINING SCALAR-ISOSCALAR DIQUARKS

In this section we will use some of our previous results to investigate the dynamics of four-quark condensates when we use a different interpolating field. To this end, let us recall $\eta_1(x)$ of Eq. (4.2) and write

$$\eta_1(x) = -[u_a^T(x)C^{-1}\gamma_5 d_b(x)]u_c(x)\epsilon_{abc} \tag{5.1a}$$

and

$$\bar{\eta}_1(x) = -\bar{u}_{c'}(x)[\bar{d}_{b'}(x)\gamma_5 C\bar{u}_{a'}^T(x)]\epsilon_{a'b'c'} \tag{5.1b}$$

as a proton interpolating field. (Note that $C^{-1} = -C = -i\gamma^0\gamma^2$ here. Further $C^\dagger = -C$.) With our definition of $(t_c)_{ab} = i\sqrt{3/2}\epsilon_{abc}$, and noting that $C^T = -C$, we may also write

$$\eta_1(x) = -\frac{1}{2}\sqrt{\frac{2}{3}}[q^T(x)C^{-1}\gamma_5\tau_2 t_c q(x)]u_c(x) \tag{5.2a}$$

and

$$\bar{\eta}_1(x) = -\frac{1}{2}\sqrt{\frac{2}{3}}\bar{u}_c(x)[\bar{q}(x)t_c\tau_2\gamma_5 C\bar{q}^T(x)]\ . \tag{5.2b}$$

We see that Eq. (5.2) describes a scalar ($T = 0$) diquark coupled to an up quark. Now define the correlator for the fields $\eta_1(x)$:

$$\Pi(q,u) = i\int d^4x e^{iq\cdot x}\langle\Psi_0|T[\eta_1(x)\bar{\eta}_1(0)]|\Psi_0\rangle\ . \tag{5.3}$$

Again, $|\Psi_0\rangle$ may be either the vacuum or the ground state of nuclear matter. We will first concentrate on the term where the large momentum, q^μ, is carried by the up quark on the far right of Eq. (5.2a) and by the up quark on the far left of Eq. (5.2b). Thus, we have a contribution to $\Pi(q,u)$:

$$\Pi(q,u) = i\int d^4x e^{iq\cdot x}\langle T[u(x)\bar{u}(0)]\rangle\frac{1}{6}\langle\Psi_0|[\bar{q}(0)C\gamma_5\tau_2 t_c\bar{q}^T(0)][q^T(0)C^{-1}\gamma_5 t_c\tau_2 q(0)]|\Psi_0\rangle \tag{5.4}$$

$$= -\frac{1}{6}S(q)\{\langle[\bar{q}(0)C\gamma_5\tau_2 t_c\bar{q}^T(0)][q^T(0)C^{-1}\gamma_5 t_c\tau_2 q(0)]\rangle_{\rho_B} + \langle N|[\bar{q}(0)C\gamma_5\tau_2 t_c\bar{q}^T(0)][q^T(0)C^{-1}\gamma_5 t_c\tau_2 q(0)]|N\rangle_C\rho_B\}\ , \tag{5.5}$$

where $S(q) = \not{q}/q^2$ is the Feynman propagator. The first term of Eq. (5.5) will contribute to $\Pi_1(q,u)$, as in Sec. IV. That term is calculated in terms of $\langle\bar{q}(0)q(0)\rangle_{\rho_B}$ and $\langle q^\dagger(0)q(0)\rangle_{\rho_B}$ by using the factorization scheme, while the second term is defined such that it does not contain a factor of $\langle\bar{q}(0)q(0)\rangle_0$. The second term, which contributes to $\Pi_2(q,u)$, may be easily evaluated, since we have given a value for the quantity $C_{\rm SD}$ in Eq. (3.8). Thus, with $\Pi(q,u) = \Pi_1(q,u) + \Pi_2(q,u)$, we have the second term of Eq. (5.5)

$$\Pi_2(q,u) = -\frac{1}{6}\frac{\not{q}}{q^2}C_{\rm SD}\rho_B \tag{5.6}$$

$$= -0.11\frac{\not{q}}{q^2}\rho_B\ , \tag{5.7}$$

upon using the value $C_{\rm SD} = 0.662$ GeV^3 found previously. As we will see, this term is large compared to the vacuum value of $\Pi_1(q,u)$ to be given in Eq. (5.13). [Other singly-contracted terms significantly modify this result, see Eqs. (5.14) and (5.15).]

Now let us use the factorization scheme to calculate $\langle[\bar{d}(0)C\gamma_5 t_c\bar{u}^T(0)][u^T(0)C^{-1}\gamma_5 t_c d(0)]\rangle_{\rho_B}$. [If we multiply the result of this calculation by four, we obtain the first term in the bracket in Eq. (5.5).]

We note that, with $\bar{q}q$ either $\bar{u}u$ or $\bar{d}d$,

$$\langle\bar{q}_{\alpha a}q_{\beta b}\rangle_{\rho_B} = \frac{\delta_{ab}}{12}\left\{\langle\bar{q}q\rangle_{\rho_B}\delta_{\beta\alpha} + \langle\bar{q}\gamma_\mu q\rangle_{\rho_B}(\gamma^\mu)_{\beta\alpha}\right\} \quad (5.8)$$

when $N_c = 3$. For simplicity, let us keep only the first term on the right-hand side of Eq. (5.8). Then

$$\langle[\bar{d}(0)C\gamma_5 t_c\bar{u}^T(0)][u^T(0)C^{-1}\gamma_5 t_c d(0)]\rangle_{\rho_B}$$

$$= (C\gamma_5)_{\alpha\beta}(t_c)_{ab}(C^{-1}\gamma_5)_{\gamma\delta}(t_c)_{a'b'}$$

$$\times\langle\bar{d}_{\alpha a}(0)\bar{u}^T_{\beta b}(0)u^T_{\gamma a'}(0)d_{\delta b'}(0)\rangle_{\rho_B} \quad (5.9)$$

$$= (C\gamma_5)_{\alpha\beta}(t_c)_{ab}(C^{-1}\gamma_5)_{\gamma\delta}(t_c)_{a'b'}[\delta_{\alpha\delta}\delta_{ab'}\delta_{\beta\gamma}\delta_{ba'}]$$

$$\times\frac{1}{(12)(12)}\langle\bar{d}d\rangle_{\rho_B}\langle\bar{u}u\rangle_{\rho_B} \quad (5.10)$$

$$= \frac{1}{4}[\langle\bar{u}u\rangle_0^2 + 2\langle\bar{u}u\rangle_0\langle N|\bar{u}u|N\rangle_{\rho_B} + \cdots]\ , \quad (5.11)$$

where we have used $\langle\bar{d}d\rangle_0 = \langle\bar{u}u\rangle_0$. Thus, to first order in ρ_B,

$$\Pi_1(q,u) = -\frac{1}{6}\frac{\not{q}}{q^2}\{\langle\bar{u}u\rangle_0^2 + 2\langle\bar{u}u\rangle_0\langle N|\bar{u}u|N\rangle\rho_B\} \quad (5.12)$$

$$= -\frac{1}{6}\frac{\not{q}}{q^2}\langle\bar{u}u\rangle_0^2 + 0.021\frac{\not{q}}{q^2}\rho_B\ . \quad (5.13)$$

Again, other singly-contracted terms modify this result. If we include all possible singly-contracted terms in the calculation of Eq. (5.3), we find that Eq. (5.6) is replaced by

$$\Pi_2(q,u) = -\frac{25}{96}\frac{\not{q}}{q^2}C_{\rm SD}\rho_B \quad (5.14)$$

$$= -0.172\frac{\not{q}}{q^2}\rho_B\ . \quad (5.15)$$

The contribution of an axial-vector condensate is very small if the interpolating field $\eta_1(x)$ is used and we drop that contribution from consideration. [We found $\Pi_2(q,u) = (1/576)C_{\rm VD}(\not{q}/q^2)\rho_B$.]

If we include all possible singly-contracted terms we also find that Eq. (5.12) is replaced by

$$\Pi_1(q,u) = -\frac{7}{24}\frac{\not{q}}{q^2}\{\langle\bar{u}u\rangle_0^2 + 2\langle\bar{u}u\rangle_0\langle N|\bar{u}u|N\rangle\rho_B\} \quad (5.16)$$

$$= -\frac{7}{24}\frac{\not{q}}{q^2}\langle\bar{u}u\rangle_0^2 + 0.037\frac{\not{q}}{q^2}\rho_B\ , \quad (5.17)$$

to first order in ρ_B. The result for $\Pi_1(q,u)$ given in Eq. (5.16) agrees with the corresponding term that appears in Eq. (2.18) of Ref. [5]. There the value of the correlator is presented for the current

$$\eta_t(x) = 2[t\eta_1(x) + \eta_2(x)]\ . \quad (5.18)$$

By taking $t = -1$, one obtains the result for the Ioffe current. One can also obtain the result for the current $\eta_1(x)$ by isolating the terms of order t^2 in Eqs. (2.16)–(2.21) of Ref. [5].

The result given in Eq. (5.7) is quite large when compared to the second term in Eq. (5.17). If we were to put $\alpha^2 = 0.20$ and $\beta^2 = 0.80$ we could eliminate the density-dependent terms from $\Pi(q,u) = \Pi_1(q,u) + \Pi_2(q,u)$. However, we have argued that $\alpha^2 = 0.5$ and $\beta^2 = 0.5$ is probably close to the actual situation. With the latter choice, we would still have a large density-dependent term of sign opposite to that in Eq. (5.17). It may be that the coupling of the interpolating field $\eta_1(x)$ to the scalar condensate is so large as to preclude the use of that field in these calculations. For example, $\Pi_2(q,u)$ of Eq. (5.15) is about three times the vacuum value of the four-quark condensate. [Note that $-\frac{7}{24}\langle\bar{u}u\rangle_0^2 = 0.71\times10^{-4}$ GeV^6, while from Eq. (5.15) we have $-0.172\rho_B = 2.17\times10^{-4}$.] Therefore, it is not possible to assume that we are calculating relatively small corrections to the vacuum value of the four-quark condensate in this case.

VI. DISCUSSION

Of the various results presented in this work, that of most interest was given in Sec. IV. There we saw that, if we use $\eta(x)$ of Eq. (4.4) as the interpolating field, and we also use a model of the nucleon in which a quark is coupled to both a scalar and an axial-vector diquark with equal probability, we obtained a density-dependent term that canceled the density dependence of the four-quark condensate that arose in the factorization (or mean-field) approximation. This was a particularly satisfactory result in that it corresponded to the situation where the QCD sum rule studies reproduced the results of Dirac phenomenology [5].

In this work we have stressed the importance of the proper calculation of four-quark condensates in studies of the nucleon self-energy in matter. Another example where one can see the importance of the four-quark condensates is in the calculation of the properties of the rho meson in nuclear matter [19]. Jin has recently provided an expression for the change in the longitudinal part of the rho polarization operator in matter [20]. If one includes condensates up to dimension six, the terms linear in ρ_B are (with $Q^2 = -q^2$),

$$\Delta\Pi_\ell(Q^2) \equiv \Pi_\ell(Q^2,\rho_B) - \Pi_\ell(Q^2,\rho_B=0) \tag{6.1}$$

$$= \frac{\hat{m}_q}{Q^4}\langle N|\bar{q}q|N\rangle\rho_B + \frac{1}{24Q^4}\left\langle N\left|\frac{\alpha_s}{\pi}G_{\mu\nu}G^{\mu\nu}\right|N\right\rangle\rho_B + \frac{m_N}{4Q^4}A_2^{u+d}\rho_B - \frac{224}{81Q^6}\pi\alpha_s\langle\bar{q}q\rangle_0\langle N|\bar{q}q|N\rangle\rho_B - \frac{5m_N^3}{24Q^6}A_4^{u+d}\rho_B\ . \tag{6.2}$$

In Eq. (6.2) and in Ref. [20], the notation $\langle\bar{q}q\rangle_0 = \langle\bar{u}u\rangle_0 = \langle\bar{d}d\rangle_0$ is used, as well as $\langle N|\bar{q}q|N\rangle = \langle N|\bar{u}u + \bar{d}d|N\rangle/2$. In Eq. (6.2) A_2^{u+d} and A_4^{u+d} are moments of structure functions that may be obtained in the study of deep-inelastic scattering, and $\hat{m}_q$ is the current quark mass. The fourth term in the above expression arises from the approximation $\langle\bar{q}q\rangle_\rho^2 - \langle\bar{q}q\rangle_0^2 \simeq 2\langle\bar{q}q\rangle_0\langle N|\bar{q}q|N\rangle\rho_B$. The values for the various quantities appearing in Eq. (6.2) are given in Ref. [20]. One has $\langle N|\bar{q}q|N\rangle = \sigma_N/(2\hat{m}_q)$ with $\sigma_N \simeq 45$ MeV and $\hat{m}_q = 5.5$ MeV. Further $\langle N|(\alpha_s/\pi)G_{\mu\nu}G^{\mu\nu}|N\rangle \simeq -0.650$ GeV, $A_2^{u+d} \simeq 0.938$, and $A_4^{u+d} \simeq 0.121$ (at a scale $\mu^2 = 1$ GeV^2). The value $\langle\bar{q}q\rangle_0 = (-0.245 \text{ GeV})^3$ is used, as well as $\alpha_s \simeq 0.3$.

If we evaluate the right-hand side of Eq. (6.2) using the numbers given above, one finds that the fourth term, which is calculated in the factorization approximation, is at least five times larger than any other term, if we take $Q^2 = 1$ GeV^2. This observation again points to the need for a proper calculation of the four-quark condensate terms. Some discussion of four-quark condensates and their importance in the calculation of a vector-isovector current correlator is given in Appendix C.

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APPENDIX A

In this Appendix we indicate how the Fierz rearrangement of various operators of the form $\bar{q}\Gamma_1 q\bar{q}\Gamma_2 q$ may be made. Here we adopt the notation of Ref. [21] and define

$$s_{\alpha\beta;\alpha'\beta'} = 1_{\alpha\beta}1_{\alpha'\beta'}\ , \tag{A1}$$

$$p_{\alpha\beta;\alpha'\beta'} = (i\gamma_5)_{\alpha\beta}(i\gamma_5)_{\alpha'\beta'}\ , \tag{A2}$$

$$v_{\alpha\beta;\alpha'\beta'} = (\gamma_\mu)_{\alpha\beta}(\gamma^\mu)_{\alpha'\beta'}\ , \tag{A3}$$

$$a_{\alpha\beta;\alpha'\beta'} = (\gamma_\mu\gamma_5)_{\alpha\beta}(\gamma^\mu\gamma_5)_{\alpha'\beta'}\ , \tag{A4}$$

$$t_{\alpha\beta;\alpha'\beta'} = (\sigma^{\mu\nu})_{\alpha\beta}(\sigma_{\mu\nu})_{\alpha'\beta'}\ . \tag{A5}$$

One has [21]

$$[s]_{\alpha\beta';\alpha'\beta} = \tfrac{1}{4}[s + v + \tfrac{1}{2}t - a - p]_{\alpha\beta,\alpha'\beta'}\ , \tag{A6}$$

$$[p]_{\alpha\beta';\alpha'\beta} = -\tfrac{1}{4}[s - v + \tfrac{1}{2}t + a - p]_{\alpha\beta,\alpha'\beta'}\ , \tag{A7}$$

$$[v]_{\alpha\beta';\alpha'\beta} = \tfrac{1}{4}[4s - 2v - 2a + 4p]_{\alpha\beta,\alpha'\beta'}\ , \tag{A8}$$

$$[a]_{\alpha\beta';\alpha'\beta} = -\tfrac{1}{4}[4s + 2v + 2a + 4p]_{\alpha\beta,\alpha'\beta'}\ . \tag{A9}$$

We also need the relation for SU(2) isospin:

$$1_{fg'}1_{gf'} = \tfrac{1}{2}1_{ff'}1_{gg'} + \tfrac{1}{2}\,\vec{\tau}_{ff'}\cdot\vec{\tau}_{gg'}\ , \tag{A10}$$

and the relations for SU(3) color:

$$1_{fg'}1_{gf'} = \tfrac{1}{3}1_{ff'}1_{gg'} + \tfrac{1}{2}\sum_{i=1}^{8}(\lambda^i)_{ff'}(\lambda^i)_{gg'}\ , \tag{A11}$$

$$\sum_{i=1}^{8}(\lambda^i)_{fg'}(\lambda^i)_{gf'} = \tfrac{16}{9}1_{ff'}1_{gg'} - \tfrac{1}{3}\sum_{i=1}^{8}(\lambda^i)_{ff'}(\lambda^i)_{gg'}\ . \tag{A12}$$

As an example, we now consider the operator $\bar{q}q\bar{q}q$ and obtain the part of the Fierz rearranged form that is proportional to $\bar{q}i\gamma_5\vec{\tau}q\cdot\bar{q}i\gamma_5\vec{\tau}q$. Using Eqs. (A6), (A10), and (A11)

$$\bar{q}q\bar{q}q \overset{\text{F.R.}}{=} -\left(-\frac{1}{4}\right)\left(\frac{1}{2}\right)\left(\frac{1}{3}\right)(\bar{q}i\gamma_5\vec{\tau}q)\cdot(\bar{q}i\gamma_5\vec{\tau}q) + \cdots \tag{A13}$$

$$= \frac{1}{24}(\bar{q}i\gamma_5\vec{\tau}q)\cdot(\bar{q}i\gamma_5\vec{\tau}q) + \cdots\ , \tag{A14}$$

where a minus sign appears due to the change in order of the fields.

Now consider the rearrangement of the operator $(\bar{q}q)(\bar{q}q)$ into a diquark-diquark structure. Keeping only the term proportional to $(\bar{q}\tau_2 t_c\gamma_5 C\bar{q}^T)(q^T C^{-1}\gamma_5 t_c\tau_2 q)$ we find

$$\langle N|(\bar{q}q)(\bar{q}q)|N\rangle = \frac{1}{24}C_{\rm SD} + \cdots \tag{A15}$$

$$= 0.023 \text{ GeV}^3\ , \tag{A16}$$

using the value $C_{\rm SD} = 0.662$ GeV^3 given previously.

APPENDIX B

The interaction Lagrangian,

$$\mathcal{L}_I(x) = \frac{G_s}{2}[(\bar{q}q)^2 + (\bar{q}i\gamma_5\vec{\tau}q)^2], \tag{B1}$$

may be rearranged to exhibit the interaction in the scalar diquark and axial-vector diquark channels. (Here $\bar{q}q = \bar{u}u + \bar{d}d$.) One finds [22,23]

$$\mathcal{L}_{I,S}(x) = -\frac{\tilde{G}_s}{2}[\bar{q}((\gamma_5 C)\tau_2 t_c)\bar{q}^T][q^T((C^{-1}\gamma_5)\tau_2 t_c)q]\ . \tag{B2}$$

and

$$\mathcal{L}_{I,A}(x) = -\frac{\tilde{G}_A}{2}[\bar{q}((\gamma_\mu C)(\vec{\tau}\,\tau_2)t_c)\bar{q}^T] \cdot[q^T((C^{-1}\gamma^\mu)(\tau_2\,\vec{\tau})t_c)q(x)] \ . \tag{B3}$$

The T matrix in the scalar diquark channel is [22]

$$T(q) = [(\gamma_5 C)\tau_2 t_c]t(q)[(C^{-1}\gamma_5)\tau_2 t_c] \ , \tag{B4}$$

with

$$t(q) = 2\frac{i\tilde{G}_s}{1-\tilde{G}_s J_\pi(q^2)} \tag{B5}$$

and

$$J_\pi(q^2) = -6i\int\frac{d^4k}{(2\pi)^4}\mathrm{tr}[\gamma_5 S(k-q)\gamma_5 S(k)] \ . \tag{B6}$$

[See Eq. (3.5).]

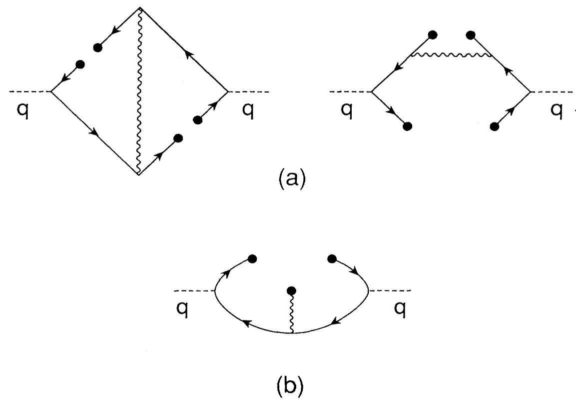

FIG. 5. (a) Two diagrams that contribute to the rho polarization tensor in vacuum. (See Ref. [24], page 177.) There are two additional diagrams with the fermion lines reversed, (b) a mixed condensate term that contributes to the rho polarization tensor. (See page 184 of Ref. [24].)

APPENDIX C

As an example of the modification in the calculation of four-quark condensates implied by our formalism, let us consider some contributions to the vector-isovector polarization tensor defined in Ref. [24]. In this case the current is $J_\mu(x) = \bar{q}(x)\gamma_\mu\tau_3 q(x)$. We also put

$$\Pi^{\mu\nu}_{(\rho)}(q) = (q^\mu q^\nu - q^2 g^{\mu\nu})\Pi_{(\rho)}(q^2) \tag{C1}$$

and consider the calculation of $\Pi_{(\rho)}(q^2)$. The contribution of the two diagrams of Fig. 5(a) (and the ones with the fermion lines reversed) is denoted as $\Pi^{(1)}_{(\rho)}(q^2)$ in Ref. [24]. For ease of reference, we use the notation of Ref. [24] in this Appendix. There, A, B, ..., are flavor indices, α, β, ..., are color indices, and i, j, ..., are Dirac indices.

Central to the calculation of Ref. [24] is the factorization of the matrix element of a general four-quark operator. The vacuum matrix elements are approximated as

$$\langle\bar{q}^A_{\alpha a}\bar{q}^B_{\beta b}q^C_{\gamma c}q^D_{\delta d}\rangle_0 = \frac{1}{144}[\delta_{AD}\delta_{BC}\delta_{\alpha\delta}\delta_{\beta\gamma}\delta_{ad}\delta_{bc} - \delta_{AC}\delta_{BD}\delta_{\alpha\gamma}\delta_{\beta\delta}\delta_{ac}\delta_{bd}]\langle\bar{q}^A q^A\rangle_0\langle\bar{q}^B q^B\rangle_0 \ , \tag{C2}$$

where $\langle\bar{q}^A q^A\rangle$ contains an implicit sum on color indices.

With the approximation of Eq. (C2), it is found that

$$\Pi^{(1)}_{(\rho)}(q^2) = -\frac{g^2}{(144)(12)q^8}\langle\bar{u}u\rangle_0^2\{16\mathrm{Tr}[\gamma^\lambda\,\not{q}\gamma^\mu\gamma_\lambda\,\not{q}\gamma_\mu] - 16\mathrm{Tr}[\gamma_\mu\,\not{q}\gamma_\lambda\gamma^\lambda\,\not{q}\gamma^\mu]\} \tag{C3}$$

$$= \frac{16\pi}{9}\alpha_S\langle\bar{u}u\rangle_0^2\frac{1}{q^6} \ . \tag{C4}$$

On the other hand, our analysis requires that we expand the operator on the left-hand side of Eq. (C2) in such a manner that one can sum over the color, flavor, and Dirac indices. In that general expansion, let us pick up only the term proportional to the operator $\bar{q}q\bar{q}q$. We write

$$\bar{q}^A_{\alpha a}\bar{q}^B_{\beta b}q^C_{\gamma c}q^D_{\delta d} = \frac{1}{N}\delta_{AD}\delta_{BC}\delta_{\alpha\delta}\delta_{\beta\gamma}\delta_{ad}\delta_{bc}(\bar{q}q\bar{q}q) + \cdots \ . \tag{C5}$$

To obtain the factor $1/N$, we put $A = D$, $B = C$, $\alpha = \delta$, $\beta = \gamma$, $a = d$, $b = c$, and sum over A, B, α, β, a, b. Thus, we find that $N = 4(144)$. In Eq. (C5) the dots indicate that there are a large number of operators in the expansion that are not shown.

Now we use Eq. (C5) in Eq. VI.52 of Ref. [24], with the result that

$$\Pi^{(1)}_{(\rho)}(q^2) = -\frac{g^2}{(4)(144)(12)q^8}\langle\bar{q}q\bar{q}q\rangle_\rho\{16\mathrm{Tr}[\gamma^\lambda\,\not{q}\gamma^\mu\gamma_\lambda\,\not{q}\gamma_\mu] - 16\mathrm{Tr}[\gamma_\mu\,\not{q}\gamma_\lambda\gamma^\lambda\,\not{q}\gamma^\mu]\} + \cdots \tag{C6}$$

$$= \frac{16\pi}{9(4)}\alpha_S\langle\bar{q}q\bar{q}q\rangle_\rho\frac{1}{q^6} + \cdots \ . \tag{C7}$$

We write, for nuclear matter,

$$\langle\bar{q}q\bar{q}q\rangle_{\rho_B} = \langle\bar{q}q\bar{q}q\rangle_0 + 2\langle N|\bar{q}q|N\rangle\langle\bar{q}q\rangle_0\rho_B + \langle N|\bar{q}q\bar{q}q|N\rangle_C\rho_B + \cdots , \quad \text{(C8)}$$

so that, keeping the term linear in the density, we find

$$\Pi^{(1)}_{(\rho)}(q^2) = \frac{16}{9}\frac{\pi\alpha_s}{q^6}\left(\langle\bar{u}u\rangle_0^2 + 2\langle\bar{u}u\rangle_0\langle N|\bar{u}u|N\rangle\rho_B + \frac{1}{4}\langle N|\bar{q}q\bar{q}q|N\rangle_C\rho_B\right) . \quad \text{(C9)}$$

The expression given in Eq. (C9) is to be compared to the result for nuclear matter based upon the factorization scheme,

$$\Pi^{(1)}_{(\rho)}(q^2) = \frac{16}{9}\frac{\pi\alpha_s}{q^6}\langle\bar{u}u\rangle_\rho^2 \quad \text{(C10)}$$

$$\simeq \frac{16}{9}\frac{\pi\alpha_s}{q^6}\left(\langle\bar{u}u\rangle_0^2 + 2\langle\bar{u}u\rangle_0\langle N|\bar{u}u|N\rangle\rho_B + \cdots\right) . \quad \text{(C11)}$$

[Equation (C10) follows from Eq. (C4) upon replacing $\langle\bar{u}u\rangle_0$ by $\langle\bar{u}u\rangle_\rho$.] We note that for a complete calculation one should estimate the contribution of the various operators (other than $\bar{q}q\bar{q}q$) that contribute to the expansion indicated in Eq. (C5).

As another example, consider the result for the mixed condensate shown in Fig. 5(b). The result given in Ref. [24] is

$$\Pi^{(2)}_{(\rho)}(q^2) = -\frac{32\pi}{81}\frac{\alpha_s}{q^6}\langle\bar{u}u\rangle_0^2 , \quad \text{(C12)}$$

for a calculation made in vacuum. This result arises from the evaluation of $\langle\bar{q}\gamma^\mu\frac{\lambda^a}{2}q\bar{q}\gamma_\mu\frac{\lambda^a}{2}q\rangle_0$ using the factorization scheme.

We now consider the matrix element of that operator taken between states of the nucleon. Upon use of Eq. (A15), we may obtain the value of that matrix element as

$$\left\langle N\left|\bar{q}\gamma^\mu\frac{\lambda^a}{2}q\bar{q}\gamma_\mu\frac{\lambda^a}{2}q\right|N\right\rangle_C = -\frac{1}{9}C_{\rm SD} + \cdots , \quad \text{(C13)}$$

where $C_{\rm SD}$ was defined in Eq. (3.8). Upon making use of our previous result given in Eq. (3.9), $C_{\rm SD} = 0.662$ GeV^3, we have

$$\left\langle N\left|\bar{q}\gamma^\mu\frac{\lambda^a}{2}q\bar{q}\gamma_\mu\frac{\lambda^a}{2}q\right|N\right\rangle_C = -0.074 \text{ GeV}^3 . \quad \text{(C14)}$$

This result may be used in the evaluation of the mixed condensate term of Fig. 5(b). (See pages 184–185 of Ref. [24].)

As another example, we discuss another contribution to the polarization $\Pi^{(1)}_{(\rho)}(q^2)$. Consider the term in the expansion of the general four-quark operator of Eq. (C5) that is of the form

$$\bar{q}^A_{\alpha a}\bar{q}^B_{\beta b}q^C_{\gamma c}q^D_{\delta d} = \frac{1}{N}(i\gamma_5)_{da}(i\gamma_5)_{cb}(\tau_j)_{DA}(\tau_k)_{CB}\delta_{\alpha\delta}\delta_{\beta\gamma}(\bar{q}i\gamma_5\tau_j q)(\bar{q}i\gamma_5\tau_k q) + \cdots , \quad \text{(C15)}$$

with $N = 4(144)$.

We use this term in Eq. (VI.52) of Ref. [24]. Since the calculation of Ref. [24] is made for the polarization tensor of the ρ^0 meson, we need to evaluate

$$\text{Tr}[\tau_3\tau_j\tau_3\tau_k] = -2\delta_{jk} + 4\delta_{3j}\delta_{3k} . \quad \text{(C16)}$$

Further, in the case of symmetric nuclear matter we can write

$$\langle N|(\bar{q}i\gamma_5\tau_3 q)(\bar{q}i\gamma_5\tau_3 q)|N\rangle_C = \frac{1}{3}\langle N|(\bar{q}i\gamma_5\vec{\tau}q)\cdot(\bar{q}i\gamma_5\vec{\tau}q)|N\rangle_C . \quad \text{(C17)}$$

Then using Eqs. (C16) and (C17) we have

$$\Pi^{(1)}_\rho(q^2) = -\frac{g^2(-2/3)(8)}{4(144)(12)q^8}\{\text{Tr}[\gamma^\lambda \not{q}\gamma^\mu(i\gamma_5)\gamma_\lambda \not{q}\gamma_\mu(i\gamma_5)] - \text{Tr}[\gamma_\mu \not{q}\gamma_\lambda(i\gamma_5)\gamma^\lambda \not{q}\gamma^\mu(i\gamma_5)]\}$$

$$\times\langle N|(\bar{q}i\gamma_5\vec{\tau}q)\cdot(\bar{q}i\gamma_5\vec{\tau}q)|N\rangle_C\rho_B . \quad \text{(C18)}$$

Here the factor $(-2/3)$ is an isospin factor and 8 is a color factor. Now use

$$\text{Tr}[\gamma_\lambda \not{q}\gamma^\mu\gamma^\lambda \not{q}\gamma_\mu] = 16q^2 \quad \text{(C19)}$$

and

$$\mathrm{Tr}[\gamma_\mu \not{q}\gamma_\lambda\gamma^\lambda \not{q}\gamma^\mu] = 64q^2 \ , \quad \text{(C20)}$$

to obtain

$$\Pi^{(1)}_{(\rho)}(q^2) = -\frac{4\pi}{27}\frac{\alpha_S}{q^6}C_\pi\rho_B \quad \text{(C21)}$$

$$= \frac{4\pi}{27}\frac{\alpha_S}{Q^6}C_\pi\rho_B \ , \quad \text{(C22)}$$

where $C_\pi = \langle N|(\bar{q}i\gamma_5 \vec{\tau}\, q)\cdot(\bar{q}i\gamma_5 \vec{\tau}\, q)|N\rangle_C$. From our previous work, we had found $C_\pi = 0.108$ GeV^3. Then, we see that the contribution of Eq. (C21) is quite small compared to the fourth term on the right-hand side of Eq. (6.2) and, therefore, may be dropped from consideration.

A more complete discussion of QCD sum rules for the vector-isovector current correlator, $\Pi^{\mu\nu}_{(\rho)}$, will be presented in Ref. [25].

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