Nucleation of quark-gluon plasma from hadronic matter

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The energy densities achieved during central collisions of large nuclei at Brookhaven's AGS may be high enough to allow the formation of quark-gluon plasma. Calculations based on relativistic nucleation theory suggest that rare events, perhaps one in every 10^2 or 10^3, undergo the phase transition. Experimental ramifications may include an enhancement in the ratio of pions to baryons, a reduction in the ratio of deuterons to protons, and a larger source size as seen by hadron interferometry.

I. INTRODUCTION

Experiments at Brookhaven's AGS with beams of oxygen, sulphur, and gold at laboratory energies of 10 to 15 GeV per nucleon have indicated a nearly complete stopping of the nuclei during central collisions [1]. This massive pile-up of nuclear matter is also seen in numerical simulations which approximate the nuclear collisions as a sequence of elementary hadron-hadron collisions, such as relativistic quantum molecular dynamics (RQMD) [2] and a relativistic cascade (ARC) [3]. Energy densities of up to 2 GeV/fm^3 may be realized in the laboratory. One may legitimately ask the question: Is quark-gluon plasma produced during these collisions? Despite the fact that most experimental data so far are consistent with the hadron-based cascade simulations we suggest that the answer may be yes, at least in rare events.

The basic picture we have in mind is as follows. During the initial stage of the collision the nuclei stop each other and get heated due to elementary nucleon-nucleon collisions and the associated production of mesons. Occasionally the local energy density may reach a very high value due to fluctuations. In this small region of space the matter is more readily described as a plasma droplet of quarks and gluons rather than as a gas of hadrons. If the average energy density in the space surrounding this plasma droplet is above a certain critical value then the plasma droplet will grow, converting more hadrons to quark-gluon plasma. Since there are no containment walls the matter, whatever phase it is in, will eventually expand and cool. In the end all quarks and gluons must be rehadronized and will be detected as such.

Quantitative questions now arise. Is the energy density achieved at the AGS high enough? How big must a plasma droplet be to grow? What is the time scale for producing such a critical size droplet? How much of the total volume is converted to plasma, and how long does it last? Of course, in order to ask these questions we must assume the existence of a deconfinement/chiral symmetry restoring phase transition, or at least a rapid crossover.

The picture at the AGS is different than that expected at Brookhaven's RHIC which is now under construction. At RHIC, where the energy is to be 100 GeV per nucleon per beam, the nuclei are expected to be transparent to each other. Hard collisions between and among the quarks and gluons in the nuclear structure functions will produce a hot, nearly baryon-free, plasma in the central rapidity region, the so-called inside-outside cascade [4,5]. The receding nuclei will be compressed and heated [6]. As the matter expands and cools it will undergo a hadronization phase transition as bubbles of hadronic matter are nucleated in the preexisting quark-gluon plasma [7]. The picture is the same for lead on lead collisions at CERN's anticipated LHC. The situation at CERN's existing SPS is not clear. At its lower energies it may be like the AGS, and at its maximum energies of 100 to 200 GeV per nucleon in the laboratory frame it may be more similar to RHIC.

The approach we follow is analogous to that taken under the assumption that the transition begins in the quark-gluon plasma phase, as appropriate for RHIC [7]. First, we describe a very simple model parametrizing the time evolution of the hadronic matter in a central collision assuming complete stopping. Second, we convert the baryon and energy densities into temperatures and chemical potentials via the use of a hadronic equation of state. We also need an equation of state describing the quark-gluon plasma phase. Third, we determine the rate of nucleation of plasma droplets in superheated hadronic matter and their subsequent growth velocities. Then we put it all together and solve the resulting equations numerically. Those interested only in the results may turn directly to Sec. V.

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We discuss and propose some experimental signatures in the conclusion.

II. DYNAMICS OF NUCLEAR COLLISIONS

The dynamics of a central nucleus-nucleus collision at the AGS is extremely complicated. We shall be satisfied with a simple model for an exploratory excursion into the problem of nucleation of plasma. To first approximation this model is consistent with the ARC cascade simulations and with direct experimental measurements.

Imagine the colliding nuclei as two Lorentz contracted disks in the center-of-momentum frame. At time $t = 0$ they touch. They interpenetrate between $0 \leq t \leq t_0$ where $t_0 = R/\gamma$, R is the nuclear radius, and γ is the Lorentz factor in the center-of-momentum frame. At the end of this time the nuclei are completely stopped. The volume of overlap as a function of time is

$$V(t) = \frac{V_0 t}{t_0} \qquad 0 \leq t \leq t_0, \tag{1}$$

where $V_0 = 4\pi R^3/3$. The matter within this overlap volume is assumed to be thermalized with constant baryon density $2\gamma n_0$ and energy density $2\gamma^2 m_N n_0$, where n_0 is normal nuclear matter density and m_N is the nucleon mass.

After the time t_0 the hot fireball expands radially. At late times we would expect its radius to grow linearly with time. Therefore we parametrize the volume as $V(t) = A(t + a)^3$. The constants A and a are determined by matching the volume and its first derivative at t_0. This gives for the expansion volume

$$V(t) = V_0 \left(\frac{t + 2t_0}{3t_0} \right)^3, \qquad t_0 \leq t. \tag{2}$$

Eventually the particles will begin free-streaming, but we shall not be interested in what happens at such low densities.

The time dependence of the baryon density is

$$n_B(t) = \begin{cases} 2\gamma n_0, & t \leq t_0, \\ 2\gamma n_0 \left[3t_0/(t + 2t_0) \right]^3, & t_0 \leq t. \end{cases} \tag{3}$$

We assume an entropy-conserving hydrodynamic expansion. Hence the entropy density is

$$s(t) = \begin{cases} s(t_0), & t \leq t_0, \\ s(t_0) \left[3t_0/(t + 2t_0) \right]^3, & t_0 \leq t. \end{cases} \tag{4}$$

In other words, the entropy per baryon is constant during the expansion. The initial entropy density $s(t_0)$ must be determined from the initial baryon and energy densities via an equation of state.

To get some typical numbers consider gold on gold collisions at a beam energy of 11.6 GeV per nucleon. Then $R = 7$ fm and $\gamma = 2.7$ resulting in a characteristic time of 2.6 fm/c, an initial baryon density of 0.78 fm^{-3} and an initial energy density of 1.95 GeV/fm^3. These numbers are very similar to those obtained from ARC with the

caveat that the matter is not completely thermalized in the cascade simulation.

III. EQUATION OF STATE FOR BARYON RICH MATTER

In this section we discuss the equation of state for the quark-gluon plasma and the hadron gas. Despite much progress in lattice QCD studies there is much uncertainty in the equation-of-state results when dynamical quarks are included. In addition, current lattice results
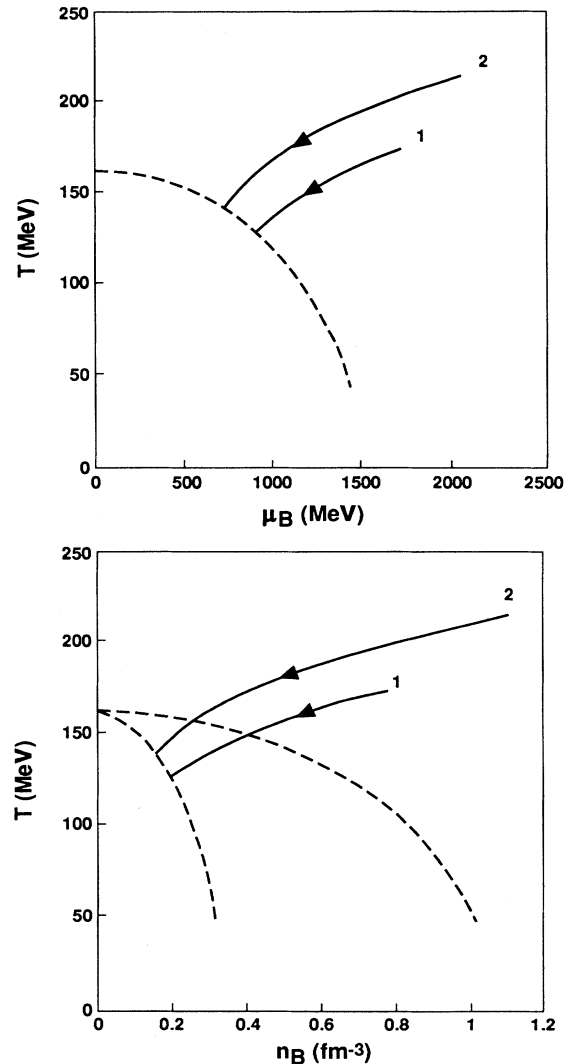

FIG. 1. Phase diagram of strongly interacting matter in the temperature-baryon chemical potential plane (top panel) and in the temperature-baryon density plane (bottom panel). The dashed curve represents phase coexistence between hadronic and quark-gluon matter. It does not extend to zero temperature because our description is too crude there. The solid curves represent the trajectories followed by heavy-ion collisions (neglecting nucleation of plasma) for parameter sets 1 and 2 in our simplified model.

provide little insight into the equation of state for the large baryon chemical potentials relevant to this work. We shall therefore (i) assume that the hadron to quark-gluon phase transition is first order, (ii) use simple models to describe the equation of state in each of the two phases, and (iii) perform a Maxwell construction to join the two phases along their common boundary.

For simplicity we will assume that the quark-gluon plasma consists of a free gas of quarks and gluons with a bag constant to represent confinement. For a plasma with up, down, and strange quarks we choose the independent variables to be the temperature T, the chemical potential for up and down quarks $\mu_u = \mu_d = \mu_q$ under the assumption of charge symmetric matter [8], and the chemical potential for strange quarks μ_s. Since the strong interactions conserve strangeness, and there is insufficient time for the weak interactions to be operative, the plasma has no net strangeness; this requirement implies that $\mu_s = 0$. We collect below the expressions for the pressure, the baryon density, the entropy density, and the energy density in the quark-gluon phase:

$$P_{\mathrm{qg}} = \frac{32 + 42 + 21 f_1(m_s/T)}{180} \pi^2 T^4 + \mu_q^2 T^2$$
$$+ \frac{1}{2\pi^2}\mu_q^4 - B ,$$

$$n_B^{\mathrm{qg}} = \frac{2}{3}\mu_q \left(T^2 + \frac{1}{\pi^2}\mu_q^2 \right) ,$$

$$s_{\mathrm{qg}} = \frac{32 + 42 + 21 f_2(m_s/T)}{45} \pi^2 T^3 + 2T\mu_q^2 ,$$

$$\varepsilon_{\mathrm{qg}} = -P_{\mathrm{qg}} + Ts_{\mathrm{qg}} + 3\mu_q n_B^{\mathrm{qg}} . \tag{5}$$

The baryon chemical potential is $\mu_B = 3\mu_q$, the electric charge chemical potential is $\mu_Q = 0$, and the strangeness chemical potential is $\mu_S = -\mu_q$. The bag constant B is chosen to be $(220 \text{ MeV})^4$. The up and down quark

masses are set to zero. The strange quark mass, m_s, is somewhere in the range of 150 to 280 MeV. The functions $f_1(m_s/T)$ and $f_2(m_s/T)$ involve a momentum integration over the strange quark Fermi distribution function. Their limits are $f_1(0) = f_2(0) = 1$ and $f_1(\infty) = f_2(\infty) = 0$. Rather than doing the integrals numerically, we will ignore the strange quark in the plasma phase altogether. The matter at the AGS is quite baryon rich, and the chemical potential μ_q is typically of order 500 MeV. Since the temperature is typically of order 200 MeV, the strange quark contributes very little. As an example, if the strange quark is included with zero mass then the critical temperature at zero baryon density is about 150 MeV. If the strange quark is not included, but nothing else is changed, then the critical temperature is 161 MeV. A realistic quark mass would give something in between. For increasing baryon density the difference gets even smaller.

For the hadronic equation of state we consider a gas of mesons $(\pi, K, K^*, \eta, \eta', \rho, \omega, \phi, \text{ and } a_1)$, and baryons (nucleons, Δ, Λ, and Σ), and the corresponding antibaryons. The only interaction we directly account for is the repulsive mean field in the baryon sector. This is done in the usual way by adding a term proportional to the baryon density to the baryon chemical potential [9].

$$\mu_B^* = \mu_B - Kn_B . \tag{6}$$

Here K is the strength of the repulsive mean field. The chemical potential for a hadron of type i is expressed in terms of the baryon, electric charge, and strangeness chemical potentials as

$$\mu_i = B_i\,\mu_B^* + Q_i\,\mu_Q + S_i\,\mu_S , \tag{7}$$

where B_i, Q_i, and S_i are the corresponding quantum numbers of the hadron.

The pressure in the hadronic phase is [9]

$$P_{\mathrm{had}} = \frac{1}{2}Kn_B^2 + T\sum_i g_i \int \frac{d^3q}{(2\pi)^3}\left[\ln\left(1 \pm e^{-\beta(\epsilon_i-\mu_i)}\right)^{\pm 1} + \ln\left(1 \pm e^{-\beta(\epsilon_i+\mu_i)}\right)^{\pm 1} \right] , \tag{8}$$

where $\epsilon_i = \sqrt{q^2 + m_i^2}$ and the $\pm$ refer to fermions or bosons. The baryon density, electric charge density, and strangeness density are all determined in the usual way by differentiating the pressure with respect to μ_B, μ_Q, and μ_S, respectively.

The baryon density must be determined self-consistently by solving the nonlinear equation

$$n_B = \sum_{i=N,\Delta,\Lambda,\Sigma} g_i \int \frac{d^3q}{(2\pi)^3}\left\{ \frac{1}{\exp\left[\beta(\epsilon_i - \mu_i)\right] + 1} - \frac{1}{\exp\left[\beta(\epsilon_i + \mu_i)\right] + 1} \right\} . \tag{9}$$

Since we require the net strangeness of the hadrons to be zero and the charge to baryon ratio to be $1/2$ these additional constraints must be implemented simultaneously with the one for the baryon density.

The final step is to perform the Maxwell construction joining the two phases. This is done by equating the temperatures, baryon chemical potentials, and pres-

sures in the two phases. In the top panel of Fig. 1 we plot the coexistence curve in the T-μ_B plane for a bag constant $B = (220 \text{ MeV})^4$ and a mean field parameter $K = 1500 \text{ MeV fm}^3$. For each temperature/chemical potential point on the coexistence curve, there is a particular value of the pressure. Since the energy density, number density, and entropy density are all discontinu-

ous across the coexistence curve, there is a mixed phase consisting of different regions of space which are in either the quark-gluon or the hadron phase. This is illustrated in the bottom panel of Fig. 1 for the T-n_B plane. To interpolate across the boundary one introduces the quark-gluon fraction q which ranges from 0 to 1. The hadron fraction is then $1 - q$.

$$\varepsilon_{\mathrm{mix}} = (1 - q)\,\varepsilon_{\mathrm{had}} + q\,\varepsilon_{\mathrm{qg}}\,,$$
$$n_B^{\mathrm{mix}} = (1 - q)\,n_B^{\mathrm{had}} + q\,n_B^{\mathrm{qg}}\,,$$
$$s_{\mathrm{mix}} = (1 - q)\,s_{\mathrm{had}} + q\,s_{\mathrm{qg}}\,. \tag{10}$$

It is worth remarking that without the incorporation of a bag constant B or a repulsive baryon mean field K one generally does not get a sensible transition from hadrons to quarks and gluons in the whole T-μ_B plane.

IV. NUCLEATION RATE FOR BARYON RICH MATTER

The rate I to nucleate droplets of quark-gluon plasma in a hadronic gas per unit time per unit volume is given by [10–12]

$$I = \frac{\kappa}{2\pi}\Omega_0 \exp(-\Delta F_*/T)\,. \tag{11}$$

Here κ is the dynamical prefactor, Ω_0 is the statistical prefactor, and ΔF_* is the change in free energy of the system due to the formation of a single critical size droplet of plasma. Each of the three factors will be discussed in turn.

The nucleation process is driven by statistical fluctuations which produce droplets of quark-gluon plasma in the hadronic phase. The size of these fluctuations is determined by the free energy difference of the hadronic phase with and without the plasma droplet. This energy difference can be approximated by a liquid-drop expansion [13]

$$\Delta F = \frac{4\pi}{3}R^3\left[P_{\mathrm{had}}(T,\mu_B) - P_{\mathrm{qg}}(T,\mu_B)\right] + 4\pi R^2\sigma$$
$$+\tau_{\mathrm{crit}}T\ln\left[1 + \frac{4\pi}{3}R^3 s_{\mathrm{qg}}\right]\,. \tag{12}$$

The first term represents the usual volume or pressure contribution, the second term is the surface contribution which is proportional to the surface tension σ, and the third term is the so-called shape contribution. Close to the phase transition the volume contribution approaches zero. The shape contribution is an entropy term which takes into account small fluctuations in the shape of the droplet which conserve both the volume and the surface area (Fisher's magic carpet effect). It is proportional to the logarithm of the entropy of the quark-gluon droplet $\frac{4\pi}{3}R^3 s_{\mathrm{qg}}$. The 1 under the logarithm is added to ensure regular behavior at $R \to 0$. The critical exponent τ_{crit} would determine the behavior of the distribution close to a critical point where the surface tension vanishes and the phase transition is second order. Such a critical point

exists for liquid-gas type of phase transitions, but does not exist in the scenario of the hadron to quark-gluon phase transition assumed here. Generally τ_{crit} is slightly larger than 2. Figure 2 shows a sketch of ΔF as a function of R.

The system under discussion is in a superheated state so that the pressure difference in Eq. (12) is negative. Minimizing ΔF with respect to the droplet radius R yields the critical radius $R_*(T,\mu_B)$. Droplets with a radius larger than R_* will expand into the hadronic phase, while droplets with a radius smaller than R_* will collapse. ΔF_* is the activation energy needed to create a droplet of critical size R_*.

The dynamical prefactor κ determines the exponential growth rate of critical-size droplets. For the droplets to grow beyond the critical radius, latent heat must be carried to the surface of the droplet from the surrounding hadronic matter. This is achieved through thermal dissipation and/or viscous damping. The general result for the dynamical prefactor is [14]

$$\kappa = \frac{2\sigma}{(\Delta w)^2 R_*^3}\left[\lambda T + 2\left(\frac{4}{3}\eta + \zeta\right)\right]\,. \tag{13}$$

Here Δw is the difference in enthalpy densities of the two phases. λ is the thermal conductivity and η and ζ are the viscosities of the hadronic phase. Notice that κ is linearly proportional to the dissipative coefficients, as expected for linear viscous fluid dynamics.

For the dissipative coefficients we use the parametrization of Danielewicz [15], extrapolated to the region of temperatures and baryon densities we are interested in.

$$\eta = \left(\frac{1700}{T}\right)^2\left(\frac{n}{n_0}\right)^2 + \frac{22}{1 + T^2/1000}\left(\frac{n}{n_0}\right)^{0.7}$$
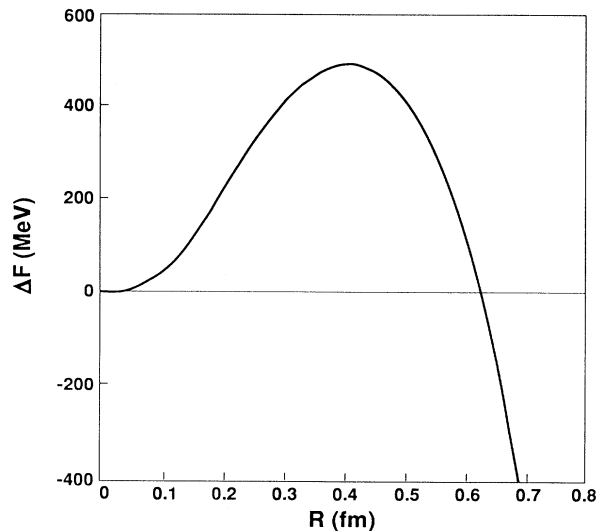$$+\frac{5.8\,T^{1/2}}{1 + 160/T^2}\,, \tag{14}$$

FIG. 2. The free energy difference $\Delta F(R)$ between a hadronic phase with and without a quark-gluon plasma droplet. This corresponds to the starting point for a collision at the AGS, parameter set 1.

$$\lambda = \frac{0.15}{T}\left(\frac{n}{n_0}\right)^{1.4} + \frac{0.02}{1 + T^4/7 \times 10^6}\left(\frac{n}{n_0}\right)^{0.4}$$

$$+ \frac{0.0225\, T^{1/2}}{1 + 160/T^2}. \tag{15}$$

Here T is given in MeV, η in Mev/fm^{2}c, and λ in c/fm^2. The bulk viscosity ζ is neglected since it is a lot smaller than the shear viscosity η.

The free energy ΔF given in (12) is a functional of a set of collective variables, chosen here to be the local temperature T and the chemical potential μ_B. Using the equations of state we could have chosen instead the local energy density ε and baryon density n_B. Figure 2 is a one-dimensional projection of this space of collective variables. The statistical prefactor Ω_0 is a measure of the phase space volume of the saddle point region of the free energy functional, and ΔF_* is the change in the free energy required to cross the saddle. To first approximation the statistical prefactor is [10,16,12,14]

$$\Omega_0 = \frac{2}{3\sqrt{3}}\left(\frac{\sigma}{T}\right)^{3/2}\left(\frac{R_*}{\xi_{\text{had}}}\right)^4. \tag{16}$$

The correlation length in the hadronic phase, ξ_{had}, is expected to be on the order of 0.5 to 1.0 fm at the relevant energy densities. Higher-order corrections to Ω_0, arising from fluctuations, are already included phenomenologically in ΔF when we evaluate it with the *measured* values of the surface tension, equation of state, and shape contribution. See Langer and Turski [17].

There are several crucial assumptions inherent in this expression for the nucleation rate. First, it is assumed that the phase transition is of first order. Second, it is assumed that the temperature and chemical potentials are well defined, and vary more or less smoothly and slowly throughout the system. Third, it is assumed that when nucleation takes place the critical-size droplet has a radius which is no smaller than the correlation length, otherwise the validity of statistical averaging becomes dubious. We do not necessarily believe that these assumptions are correct in detail; rather, we use them as a basis to present interesting possibilities.

To be concrete in what follows, we take the surface tension to be $\sigma = 50$ MeV/fm^2, the correlation length in the hadronic phase to be $\xi_{\text{had}} = 0.7$ fm, and the critical coefficient to be $\tau_{\text{crit}} = 2.2$. In principle σ cannot be varied completely independently of the equation of state. If the latent heat goes to zero so that the first-order phase transition goes over into a second-order one, σ must go to zero also. Conversely, as the latent heat increases, one might expect σ to become larger. Lattice gauge theory calculations so far give us no information on its magnitude at large baryon densities.

The expressions given here for the various components of the nucleation prefactor are relevant for one dense phase of matter (quark-gluon plasma) immersed in another (hadronic matter). The basic physics is that initial droplet (bubble) growth is limited by the ability of viscosity and heat conduction to carry latent heat to (away) from the surface. One may consider a different scenario where a dense droplet of one phase (quark-gluon plasma)

is surrounded by a dilute gas of the other phase (hadronic matter). The plasma droplet then would grow by accretion of individual hadrons. Although we don't think that this is the relevant situation, since in the superheated hadronic phase the mean free path and correlation length are of order 1 fm or less, we include an Appendix outlining the theoretical expression for the prefactor if it were.

Nucleation begins when the two nuclei first collide with each other and a superheated overlap region is created. It ends when the expanding system reaches the phase coexistence curve. The fraction of space which is converted into quark-gluon matter q is computed from the expression [7]

$$q(t) = \frac{1}{V}\int_0^t dt'\, I(t')V(t')[1 - q(t')]V_{\text{drop}}(t, t'). \tag{17}$$

The total volume of the system at time t is $V(t)$. The volume already occupied by quark-gluon plasma is not available for nucleation. The volume which is available for nucleation is $V(t)[1 - q(t)]$. Once a drop has been nucleated, with radius $R_*(T, \mu_B)$, it will grow radially with speed $v(T, \mu_B)$. The volume of the droplet is therefore a nonlocal function of time. It can be written as

$$V_{\text{drop}}(t, t') = \frac{4\pi}{3}\left[R_*(t') + \int_{t'}^t dt''\, v(t'')\right]^3. \tag{18}$$

I, R_* and v all depend on time because they depend on the (time dependent) temperature and chemical potential.

The speed $v(t)$ with which the droplet expands into the hadronic matter is relatively unknown. To determine this speed would require a detailed microscopic study of the system. Instead we make the plausible assumption that the expansion into the new phase is driven by the pressure difference $\Delta P(t) = P_{\text{qg}}(t) - P_{\text{had}}(t)$ between them [18]. The greater the pressure difference, the faster the plasma droplet expands. As the critical curve is approached the pressure difference goes to zero, and so should the droplet expansion velocity, since on the critical curve neither of the phases is thermodynamically preferred over the other. Thus we write

$$\gamma v = v_0\frac{\Delta P}{P_{\text{qg}}}, \tag{19}$$

where $\gamma = 1/\sqrt{1 - v^2}$; v_0 is a free phenomenological parameter which we expect to be on the order of 1.

V. NUMERICAL RESULTS

We consider two sets of initial conditions. Set 1 corresponds to a central gold-gold collision with a lab kinetic energy of $E_{\text{kin}} = 11.6$ GeV/nucleon and $\gamma = 2.68$, as achieved at Brookhaven's AGS. Set 2 corresponds to a central lead-lead collision with a lab kinetic energy of $E_{\text{kin}} = 25$ GeV/nucleon and $\gamma = 3.78$, as could be achieved at CERN's SPS. The average baryon density in a nucleus is taken as $n_0 = 0.145$ fm^{-3} so that the radii of the nuclei are given by $R = r_0A^{1/3}$ with $r_0 = 1.18$ fm.

The interpenetration time $t_0 = R/\gamma$, defined in (1), is 2.56 fm (1.85 fm), and the initial baryon density in the overlap region, defined in (3), is 0.78 fm^{-3} (1.10 fm^{-3}) for set 1 (2), respectively. The energy density $2\gamma^2 m_N n_0$ reached in this first stage of the collision is thus 1.95 GeV/fm^3 (3.90 GeV/fm^3) for set 1 (2). Table I gives a summary of the initial conditions for the two parameter sets. Some of the quantites in this Table, like the temperature and baryon chemical potential, are dependent on the equation of state, while others are not.

The assumed volume of kinetically equilibrated matter $V(t)$ is determined by Eqs. (1) and (2). Figure 3 displays the volume as a function of time for the two parameter sets. We see the linear increase of the overlap volume up to the interpenetration time t_0 after which the nuclei are completely stopped and start to expand spherically, leading to a cubic increase with time.

Knowledge of the time evolution of the volume allows us to evaluate the time dependence of the baryon and entropy densites, as given in Eqs. (3) and (4). With the help of the equations of state discussed in Sec. III we can then evaluate the chemical potential $\mu_B(t)$ and the temperature $T(t)$ of the hadronic phase as functions of time. Figure 4 shows them, as well as the baryon density and the energy density for both parameter sets. The ordinates are normalized to their initial values as displayed in Table I.

Next we plot the path of the collision in the μ_B-T plane of Fig. 1. The system starts out as an extremely superheated hadron gas deep within what ought to be the quark-gluon phase at a temperature of 173 MeV (214 MeV) and a chemical potential of 1724 MeV (2064 MeV). It stays at this point up to the interpenetration time t_0, then expands and cools, reaching the phase coexistence curve at a time $t_f = 7$ fm (6.95 fm) later.

It is important to recognize that we are neglecting the feedback of quark-gluon plasma nucleation on the temporal evolution of the temperature and chemical potential. We shall return to this point later.

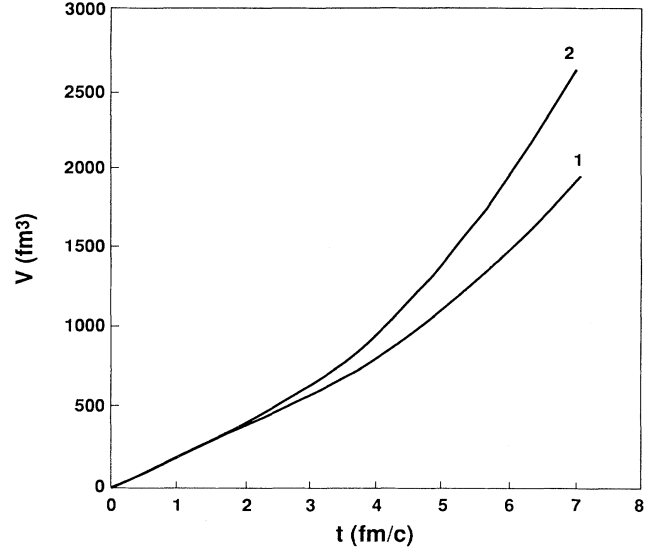In Fig. 5 we plot the nucleation rate $I(t)$ along the

FIG. 3. Time evolution of the volume $V(t)$ of the collision region.

path in the T-μ_B plane for the two parameter sets. During interpenetration both temperature and chemical potential are constant; the nucleation rate is therefore also constant. After t_0 the rate first increases, reaches a maximum, then decreases to zero as the coexistence curve is approached. We would expect the rate of nucleation of plasma to *increase* as the initial state of the system gets further from the phase coexistence curve, and therefore the rate should be a monotonically *decreasing* function of time after t_0. Why isn't it? The reason is rather fundamental. The usual analytic expression for the nucleation rate, Eq. (11), is derived under the assumption that the system has been either superheated or supercooled just a small amount from the phase coexistence curve. This means that the trajectory in phase space which goes from a metastable point to a stable point is dominated by a saddle, and the saddle configuration is a spherical droplet or bubble. All other configurations have a $\Delta F/T$ which is significantly larger, and therefore exponentially suppressed in comparison. When the system is superheated as dramatically as it is in our examples, $\Delta F_*/T \approx 2$. Then the dominant contribution to ΔF_* is from the shape term. More extreme configurations with shapes like lasagna and spaghetti ought to be contributing too. However, these are difficult to take into account in any simple manner, especially concerning the preexponential factors.

Mathematically, the reason the rate turns over in this figure can be explained this way: Going away from the phase coexistence curve, the Boltzmann exponential increases. The preexponential factor is proportional to

$$\frac{R_*}{T^{3/2}(\Delta w)^2}\left[\lambda T + \frac{8}{3}\eta\right] .$$

The dissipation coefficients are relatively slowly varying functions, and increase by only 20 or 30%. But R_*
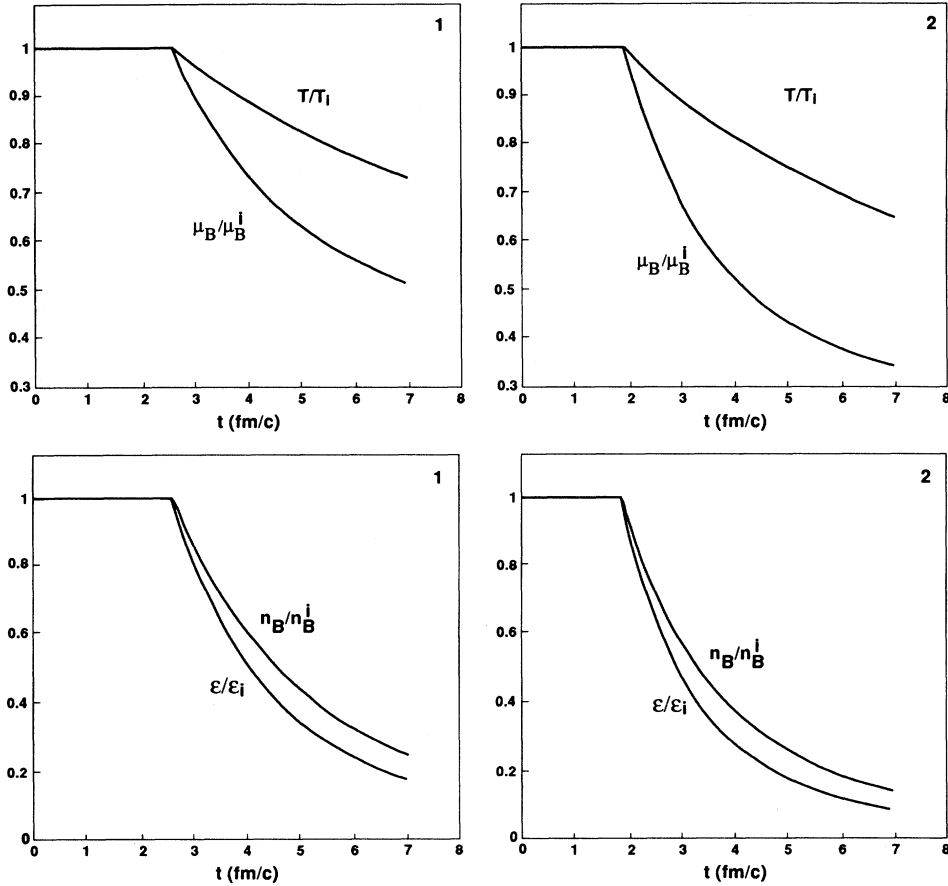
TABLE I. Initial conditions for the two chosen parameter sets and some resulting characteristic scales.

	Set 1	Set 2
E_{beam} (GeV)	11.6	25.0
A	197	208
γ	2.68	3.78
t_0 (fm/c)	2.56	1.85
t_f (fm/c)	7.0	6.95
T (MeV)	173	214
μ_B (GeV)	1.72	2.06
P_{had} (GeV/fm^3)	0.67	1.41
P_{qg} (GeV/fm^3)	2.19	5.07
ε_{had} (GeV/fm^3)	1.95	3.90
ε_{qg} (GeV/fm^3)	7.78	16.44
n_B^{had} (fm^{-3})	0.78	1.10
n_B^{qg} (fm^{-3})	3.17	5.58
s_{had} (fm^{-3})	7.42	14.27
s_{qg} (fm^{-3})	25.93	46.94

FIG. 4. Upper row: Time evolution of the baryon chemical potential μ_B and the temperature T for parameter set 1 (2) on the left (right). Lower row: Time evolution of the baryon number density n_B^{had} and the energy density ε_{had} for parameter set 1 (2) on the left (right).

gets smaller, T gets bigger, and the enthalpy difference squared increases by about an order of magnitude. Overall, the preexponential factor decreases. Multiplication of a decreasing function by an increasing function results, in this case, in a product with a maximum, as seen in Fig. 5. Actually, the dominant effect comes from Δw, and this arises from the extreme superheating of hadronic matter in the collisions. It is quite likely that we are *underestimating* the initial nucleation rate by an order of magnitude. In reality, for small droplets σ effectively depends on R [19,20]. With increasing superheating one eventually reaches a point where spinodal decomposition sets in, and the phase transition will be extremely rapid [19]. This situation is not very well understood. Still, an uncertainty even this large is acceptable in a first study of this nature.

In Fig. 6 we plot the average droplet density, defined by

$$n_{\text{drop}}(t) = \int_0^t dt'\, I(t'), \qquad (20)$$

as a function of time. The average droplet density is independent of the droplet growth speed as long as $q(t)$ remains small, which it does in these examples. The maximum possible value reached by the droplet density is $n_{\text{drop}}(t_f) = 2\ (1)\ \times 10^{-5}\ \text{fm}^{-3}$ for parameter set 1 (2). The volume of the expanding system, on the other hand,

is on the order of $2\ \times 10^3\ \text{fm}^3$, see Fig. 3. The average number of droplets nucleated is rather small, roughly 1/50 and 1/100 at the AGS and SPS, respectively. The reason for the smaller number for set 2, SPS, is that the superheating is even more extreme, and contribu-
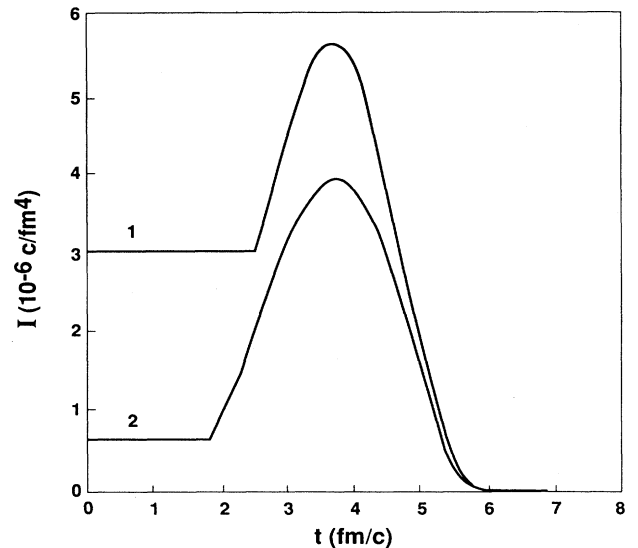

FIG. 5. Time evolution of the nucleation rate $I(t)$ along the dynamical trajectories for both parameter sets.
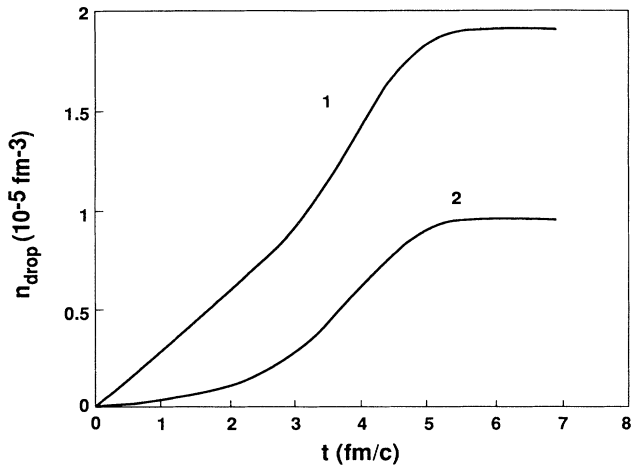
FIG. 6. Time evolution of the average plasma droplet number density n_{drop} for both parameter sets.

tions from configurations other than spherical are even more likely to be significant.

In Fig. 7 we plot the volume fraction converted to quark-gluon plasma, $q(t)$, and the average droplet radius, $\bar{R}(t)$, defined as

$$\bar{R}(t) = \left[\frac{3}{4\pi} \frac{q(t)}{n_{\mathrm{drop}}(t)} \right]^{1/3}. \tag{21}$$

Results for several values of the parameter v_0 are shown. For $v_0 > 10$ we are in an asymptotic regime where both $q(t)$ and $\bar{R}(t)$ hardly change anymore from their values obtained with $v_0 = 10$. The maximum value for $q(t_f)$ is 9.2×10^{-3} and 3.2×10^{-3} for parameter set 1 and 2, respectively. These values are somewhat disappointingly small. On the other hand, the corresponding maximum average bubble radii are 5.0 (4.3) fm for set 1 (2), which are interestingly large.

How are we to interpret these results? If we were dealing with an expansion chamber with a volume of 1 m^3 the answer would be clear. Droplets of quark-gluon plasma would be nucleated randomly throughout the system. Since the droplet density is so small they would be widely separated. They would grow to a size of perhaps 3 to 5 fm and therefore would hardly ever touch each other. They would be scattered like stars in the night sky. Only about 10^{-3} to 10^{-2} of the volume would be occupied by plasma before the system cooled below the phase coexistence curve. The interpretation for heavy-ion collisions at the AGS, we claim, is different. By the end of the cooling period, at t_f, the distribution of plasma droplets should be a Poisson function in the variable $\bar{N}_{\mathrm{drop}} = V n_{\mathrm{drop}}$. The likelihood that more than one droplet nucleates in a given collision is very small. Therefore, either one droplet nucleates or none. If one nucleates, it will have eaten a good fraction of the hadronic matter, converting it to plasma. Although we have neglected feedback of plasma formation on the dynamical evolution of the system, we
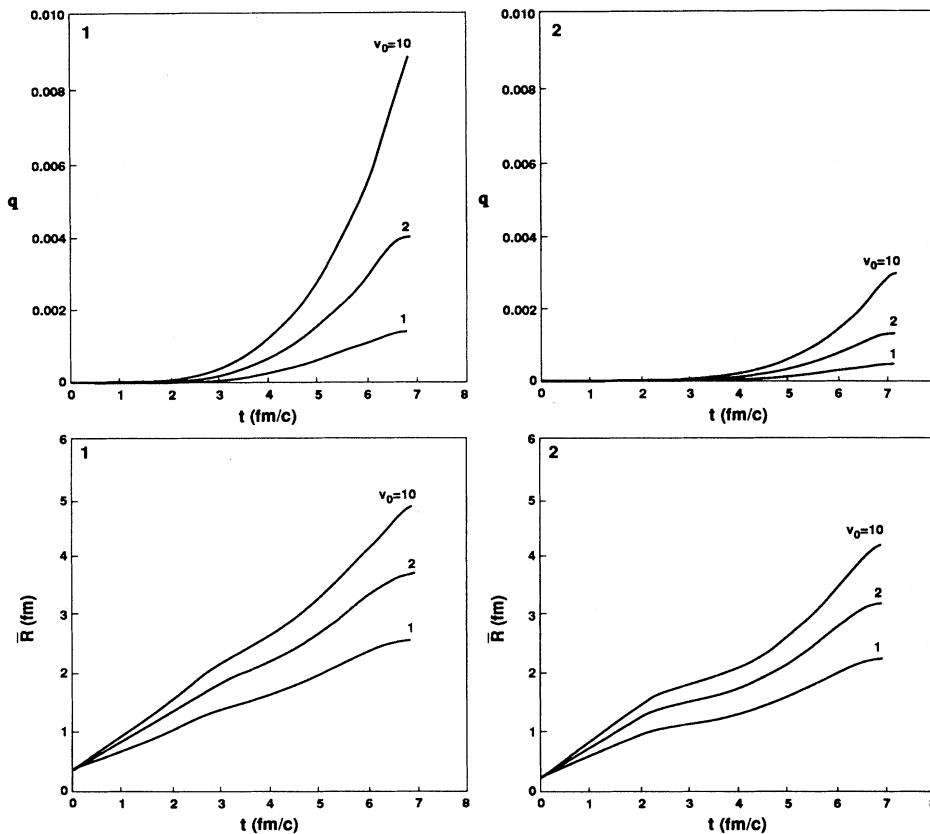

FIG. 7. Upper row: Time evolution of the quark-gluon fraction q for different values of v_0 for parameter set 1 (2) on the left (right). Lower row: Time evolution of the average droplet radius $\bar{R}$ for different values of v_0 for parameter set 1 (2) on the left (right).

can confidently say that plasma formation would *slow* the expansion of the system. This is due to the fact that the pressure is much reduced when compared at the same energy density; that is, the equation of state is softened by the phase transition, and it is the pressure which drives the expansion. Therefore, the plasma has more time to eat the rest of the hadronic matter. Taking into account also the fact that we have probably underestimated the nucleation rate, we conclude that perhaps one in every 100 or 1000 central collisions at the AGS will have undergone an almost complete phase transition by the time the matter has expanded to the phase coexistence curve.

VI. DISCUSSION AND CONCLUSIONS

In this paper we have applied relativistic nucleation theory to address the issue of quark-gluon plasma formation at the AGS and at the lower end of the SPS energy range. Our simple modeling of the magnitude and time dependence of the baryon and energy densities seem to be in reasonable agreement with those obtained in hadronic cascade simulations. The numerical results suggest that perhaps one out of every 100 or 1000 central collisions will exhibit a significant phase conversion from hadronic matter to quark-gluon plasma.

Standard homogeneous nucleation theory assumes that the matter undergoing the phase transition is not superheated or supercooled too much from phase coexistence. Central collisions of the most massive nuclei at the AGS and the SPS apparently lead to *very* superheated hadronic matter. As a consequence, the critical size plasma droplets have radii which are comparable to or even smaller than the expected correlation length. Standard homogeneous nucleation theory also underestimates the nucleation rate because configurations which are far from spherical become important. It may be that the superheating is so extreme that one is approaching spinodal decomposition.

It is interesting to think about the transition from nucleation of plasma at these relatively modest beam energies to the almost instantaneous formation of plasma expected at RHIC and LHC energies. In free space, hard nucleon-nucleon collisions produce a significant number of secondary mesons. It takes a finite time for the produced quarks and gluons actually to hadronize as asymptotic meson states. In a central nucleus-nucleus collisions these little star bursts may overlap before hadronization can occur, thereby providing seeds or nucleation sites for quark-gluon plasma.

Neither of the effects discussed above are taken into account in our calculations. Therefore, we have most likely *underestimated* the formation of plasma at AGS and lower SPS energies, perhaps significantly so.

What about experimental implications? Since the phase transition is occurring so far out of equilibrium we would expect a significant increase in the entropy of the final state. This could be seen in the ratio of pions to baryons, for example, or in the ratio of deuterons to protons [21]. Along with the increased entropy should come a slowing down of the radial expansion due to a

softening in the matter, that is, a reduction in pressure for the same energy density. Together, these would imply a larger source size and a longer lifetime as seen by hadron interferometry [22]. Of course, these and other signals would have to be investigated experimentally on an event by event basis. It should be straightforward to develop models of the probability distributions of the entropy to baryon ratio, source size and lifetime, and so on. Parameters could be adjusted to learn about the probability of phase conversion in a given central collision, the latent heat release, and so forth.

APPENDIX

The classical theory of nucleation culminated in the work of Becker and Döring [23]. It is nicely reviewed by McDonald [24]. It was developed to describe the nucleation of a liquid droplet in a dilute yet supersaturated vapor. Langer's formalism, as used in this paper, is meant to apply when neither phase may be considered dilute. We do not believe that the superheated hadronic matter is dilute enough to apply the classical theory. Nevertheless, for comparison we would like to summarize the nucleation rate in the classical regime.

The classical expression for the nucleation of a droplet of dense liquid in a dilute gas is

$$I = a(i_*) \left(\frac{|\Delta E''(i_*)|}{2\pi T} \right)^{1/2} n_1 \exp\left(-\Delta E(i_*)/T \right),$$

where $\Delta E(i_*)$ is the formation energy of a critical sized droplet consisting of i_* molecules, prime denotes differentiation with respect to the number of molecules i, T is the temperature, n_1 is the density of single molecules, and $a(i_*)$ is the accretion rate of single molecules on a critical droplet. Usually the accretion rate is taken to be

$$a(i_*) = \frac{1}{2} n_1 \bar{v} 4\pi R_*^2 s,$$

which is the flux of particles ($\bar{v}$ is the mean speed of gas molecules) striking the surface of the critical droplet times a "sticking fraction" s less than one. The first term in the nucleation rate is a dynamical factor influencing the growth rate, the second term characterizes fluctuations about the critical droplet, and the product of the third and fourth terms gives the quasiequilibrium number density of critical sized droplets. The energy is measured with respect to the gas molecules so that $\Delta E(1) = 0$.

To attempt to apply the classical expression to the nucleation of a plasma droplet, the first thing we do is to multiply the Boltzmann factor by the number of states available to the hot droplet,

$$e^{-\Delta E/T} \rightarrow e^{-\Delta E/T} e^{\Delta S} .$$

Due to the thermodynamic identities $S = -dF/dT$ and $F = E - TS$ this modifies the Boltzmann factor to $e^{-\Delta F/T}$.

We prefer to characterize the size of the droplet not by the number of molecules it contains, but by its radius. Then integration over quadratic fluctuations about the mean size will give the prefactor

$$\left(\frac{|\Delta F''(R_*)|}{2\pi T}\right)^{1/2} .$$

The accretion rate must be multiplied by the increase in radius per particle absorbed to compensate for this change of variable. Upon absorption of one more particle the droplet free energy changes by

$$\delta \Delta F = \Delta F'(R_*)\delta R + \frac{1}{2}\Delta F''(R_*)(\delta R)^2 .$$

The derivatives are evaluated at R_* whereupon the first derivative vanishes. The (Gibbs) free energy added by one gas molecule is just minus the pressure of the gas molecules divided by their number density. Therefore the accretion rate is multiplied by the factor

$$\delta R = \left(-\frac{P_1}{n_1 \Delta F''(R_*)}\right)^{1/2} .$$

Putting everything together we arrive at

$$I = 2\pi s \bar{v} R_*^2 n_1^2 \left(\frac{P_1}{n_1 \pi T}\right)^{1/2} \exp\left(-\Delta F_*/T\right) .$$

Generalizing to different species of molecules (hadrons) we write

$$I = 2\pi R_*^2 n_0 \exp\left(-\Delta F_*/T\right) \Sigma_j s_j \bar{v}_j n_j \left(\frac{P_j}{n_j \pi T}\right)^{1/2} ,$$

where P_j is the partial pressure of the jth species, n_j is their density, etc. The quasiequilibrium density of critical droplets is normalized to the density of the lightest species of hadrons, n_0.

Note especially the appearance of R_*^2 in the prefactor. This arises from the fact that the absorption rate is proportional to the surface area. In contrast, when the growth rate is dominated by dissipation the prefactor has only one power of R_*. Over most of the cooling curve it turns out that the prefactor estimated in this classical approach is about the same order of magnitude as the prefactor used in the text.

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