Target dependence of K^+-nucleus total cross sections

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We investigate the total cross section and its target dependence for K^+-nucleus scattering using a relativistic momentum-space optical potential model which incorporates relativistically normalized wave functions, invariant two-body amplitudes, covariant kinematics, and an exact full-Fermi averaging integral. The definition of the total cross section in the presence of a Coulomb interaction is reviewed and the total cross section is calculated in a way that is consistent with what is extracted from experiment. In addition, the total cross sections for a nucleus and for the deuteron are calculated utilizing the same theory. This minimizes the dependence of the ratio of these cross sections on the details of the theory. The model dependence of the first-order optical potential calculations is investigated. The theoretical results are found to be systematically below all existing data.

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I. INTRODUCTION

One of the basic tenets of classical nuclear physics is that a nucleon does not significantly change its properties when inside a nucleus. QCD implies that as the nuclear density increases, the underlying chiral symmetry could be partially restored. This would lead to the surmise that partial deconfinement would occur for nuclear matter of sufficient density. As such, the question arises as to whether this phenomenon can be seen at normal nuclear densities.

The K^+ meson has a distinct advantage as a probe to investigate this question. With a quark content of $u\bar{s}$, the K^+ cannot in a simple way form an s-channel resonance with a nucleon. Experimentally, no such resonances are seen for incident momentum below $\sim$ 800 MeV/c. Of all the hadrons, this makes the K^+ the weakest strongly interacting particle. This also means that the K^+ has a long mean free path and can penetrate into the interior of a nucleus [1, 2], thus probing the nucleus in a region where the density is high. Moreover, the weak K^+-nucleon interaction implies that the first-order optical potential should dominate [2–6] and conventional second-order effects should be small. This makes theoretical models more reliable and less dependent on the details of the theory.

Differential and total cross section measurements [7–12] show a significant discrepancy when compared to theory [2,4–6]. In order to help reduce possible systematic errors in the data, it was proposed [6] that the ratio of the total cross section for a nuclear target, $\sigma_t(A)$, to that of the deuteron, $\sigma_t(D)$, be measured. This ratio,

$$r = \frac{\sigma_t(A)/A}{\sigma_t(D)/2}, \tag{1}$$

would also have the advantage of helping to cancel theoretical uncertainties if the numerator and denominator were calculated consistently.

Since the experiment of Bugg *et al.* on ^{12}C [7], several recent measurements of this ratio, including its target dependence, have been carried out at Brookhaven National Laboratory [9–12]. Theoretical results [2,4–6] are 10–20% smaller than the data. As conventional nuclear physics mechanisms appear to be unable to account for this discrepancy, more exotic mechanisms have been proposed. These include partial deconfinement as exhibited by a larger nucleon in the nuclear medium [6] or partial restoration of chiral symmetry as exhibited by reduced meson masses [13] in the medium. Meson exchange currents [14, 15], meson exchange currents combined with long range (random phase approximation) correlations [16], and various other mechanisms [17] have been examined.

It is important to understand how much of the discrepancy might be due to approximations made in the theory. The purpose of this work is to apply the first-order momentum-space optical model approach [2, 18] to the cross section ratio, including its target dependence, and to examine the model dependence of the theoretical predictions for the ratio. There are many reasons to employ the momentum-space approach here. These include (1) fully covariant kinematics, normalizations, and phase-space factors [19], (2) invariant amplitudes [20], (3) the crossing symmetric Klein-Gordon propagator, and (4) an exact evaluation of the Fermi-averaging integration. The model, originally developed for pion-nucleus studies [18], was extended to the kaon-nucleus problem in [2]. The first-order potential is the leading term of a formal systematic expansion of the full optical model developed in [21].

In addition, we examine the model dependence of the results by comparing to a very simple semiclassical eikonal model originally developed for high-energy pion scattering [22, 23]. This eikonal model is computationally straightforward, whereas the momentum-space approach

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becomes computationally intensive at high energies, especially for heavy targets.

In this work, we also carefully examine the definition of the total cross section in the presence of the Coulomb potential. We note that the definition used in [2] does not correspond to the way that experimentalists extract total cross sections from the data and that the Coulomb-free cross section is significantly different from the experimentally measured quantity, particularly at low incident momentum.

The paper is organized as follows. Section II is a brief review of both the momentum-space optical model and the eikonal model. In Sec. III A we examine in detail K^+ scattering from ^{12}C and investigate the model dependence of our results. In Sec. III B we compare our results for the target dependence of the cross section ratio with the recent data. Summary, conclusions, and future prospects are presented in Sec. IV.

II. THEORETICAL MODELS

In this section, we discuss very briefly the models we use to calculate kaon-nucleus scattering: the covariant momentum-space optical potential model and the eikonal model. The details of these models can be found in Refs. [18, 19] and [23], respectively. We include here only a few necessary formulas. We also review the meaning of "total cross section" in the presence of the Coulomb field.

The first-order impulse approximation to the optical potential for meson-nucleus scattering is given by

$$\langle \mathbf{k}'_K \mathbf{k}'_A \,|\, U(E) \,|\, \mathbf{k}_K \mathbf{k}_A \rangle = \sum_\alpha \int \frac{d^3k_{A-1}}{2\bar{E}_{A-1}} \frac{d^3k'_N}{2\bar{E}'_N} \frac{d^3k_N}{2\bar{E}_N} \langle \Psi_{\mathbf{k}'_A} \,|\, \mathbf{k}'_N \mathbf{k}_{A-1} \alpha \rangle \langle \mathbf{k}'_K \mathbf{k}'_N \,|\, T(E) \,|\, \mathbf{k}_K \mathbf{k}_N \rangle \langle \mathbf{k}_N \mathbf{k}_{A-1} \alpha \,|\, \Psi_{\mathbf{k}_A} \rangle \ , \quad (2)$$

where $\mathbf{k}_K$, $\mathbf{k}_N$, $\mathbf{k}_{A-1}$, and $\mathbf{k}_N$ are the momenta of the kaon, the struck nucleon, the $A-1$ residual nucleus, and the target nucleus in an arbitrary frame; $\bar{E}_i$'s are the corresponding energies; E is the asymptotic energy of the kaon-nucleus system; and α represents a set of quantum numbers that specify the state of the struck nucleon.

The optical potential is calculated by changing variables to appropriately defined covariant relative momenta, utilizing relativistic three-body recoupling coefficients defined in Ref. [24], and performing the integration over the momentum of the struck nucleon numerically. The scattering of the kaon from the nucleus is then calculated by inserting the optical potential into the partial-wave Lippmann-Schwinger equation

$$T_J(W;q',q) = U_J(W;q',q) + \int \frac{q''^2 dq''}{4W''_1 W''} U_J(W;q',q'') \times \frac{1}{W - W'' + i\eta} T_J(W;q'',q) \ , \quad (3)$$

and solving this equation numerically.

The calculation of the optical potential requires a model of the target wave functions Ψ_A and a model of the kaon-nucleon t matrix $t_{j\ell}$. The target wave functions are taken from [25] and the deuteron wave function is generated from the Bonn potential [26]. By using an invariant normalization [27] of these wave functions, we extend the validity of this approach to light targets and high energies. This was found to be necessary in order for us to treat the deuteron at energies approaching 1 GeV. For the kaon-nucleon t matrix, we use a simple separable form to continue the amplitude off shell. The angular momentum decomposed t matrix is then

$$t_{j\ell}(\omega,p',p) = \left(\frac{pp'}{p_{\rm on}^2}\right)^\ell \left(\frac{v(p')v(p)}{v(p_{\rm on})^2}\right) t_{j\ell}(\omega), \quad (4)$$

where p and p' are momenta in the kaon-nucleon center-of-mass (c.m.) frame, j and ℓ are the total and orbital angular momentum, and $p_{\rm on}$ is the on-shell momentum corresponding to ω. We use a Gaussian $v(p) = \exp(p^2/\Lambda^2)$ for the form factor with a range given by $\Lambda = 1000$ MeV/c. The on-shell t matrix is taken from either the phase shift analysis of Arndt [28] or that of Martin [29]. These two analyses are very similar at low energies but have small differences at high energies. We include s, p, and d waves in the two-body amplitude. The momentum-space calculations are performed using a modified version on the computer code ROMPIN [18] called ROMKAN (relativistic optical model for kaon nucleus).

Particularly for pions in the region of the Δ_{33} or lower energies, care must be taken [30] in choosing the energy ω at which the two-body t matrix is to be evaluated. We follow [30] and choose

$$\omega^2 = (\sqrt{E^2 - \mathbf{P}^2} - \sqrt{\mathbf{q}^2 + m_{A-1}^2})^2 - \mathbf{q}^2 \ , \quad (5)$$

where $\mathbf{P}$ is the momentum of the pion-nucleon pair and $\mathbf{q}$ is the momentum of the recoiling $A-1$ nucleons. The mass of the $A-1$ system, m_{A-1}, is given by $m_{A-1} = m_A - m_N - E_b$, and E_b is the binding energy of the struck nucleon. In addition, ω is shifted [30] by an amount $E_{\rm ms}$ (the mean-spectral energy) to approximately account for the interaction of the intermediate kaon-nucleon system with the residual nucleus.

The eikonal model which we employ has been developed in Ref. [23] based on the earlier work [22]. The eikonal model can be derived from the momentum-space optical potential by factoring the Fermi integration [19], expanding the meson-nucleon amplitude about the forward direction (it is naturally on shell in the forward direction if the nucleon binding energy is cancelled against $E_{\rm ms}$), and utilizing the eikonal approximation for the propagator (we include the first Wallace [31] correction). The eikonal model is quite simple and can serve as a theoretical tool to understand the qualitative features of the dynamics of the scattering. At sufficiently high energies it is expected that it will become a quantitative substitute for the much more numerically intensive momentum-space approach. To calculate kaon-nucleus scattering in the eikonal approach we require a model of the nuclear density and the on-shell kaon-nucleon t matrix. We construct the density from the wave functions used in the momentum-space approach and use the on-shell amplitudes from [28].

In the presence of a Coulomb potential, the definition of a total cross section is not unique. The problem was first resolved in [32] and is discussed in detail in [33]. The main criterion is for the theorist to calculate the same quantity as is extracted from the data by the experimentalist. Since the r^{-1} long-range behavior of the Coulomb potential leads to an infinite differential cross section at zero degrees, one must subtract the point Coulomb cross section $\sigma_c(\Omega)$ from a measured cross section $\sigma_T(\Omega)$ before taking the limit as Ω approaches zero. As was pointed out in [33], this is not a suitable definition. Although the difference is finite, it oscillates rapidly at small angles and thus cannot be reliably continued to zero degrees. The oscillations arise from the Coulomb-nuclear interference term which comes from taking the square of f

$$f(q) = f_C(q) + f_{\rm CN}(q) \tag{6}$$

with $f_C(q)$ the point Coulomb amplitude and $f_{\rm CN}(q)$ the remainder. The approach adopted experimentally is to also subtract $\sigma_{\rm CN}$,

$$\sigma_t \equiv \sigma_T - \sigma_C - \sigma_{\rm CN} , \tag{7}$$

where $\sigma_{\rm CN}$ is the Coulomb-nuclear interference term [the integrated cross term which arises from squaring f; see Eq. (11) below] taken from a model. Since a model of the amplitude $f_{\rm CN}$ is needed, the result is not model independent. The reliability of this procedure for nuclei larger than ^{12}C has been questioned in [34]. The important point is that of the three different possible definitions of a total cross section, the Coulomb-free cross section (i.e., turning the Coulomb interaction off), $\sigma_T - \sigma_C$, and σ_t of Eq. (7), only σ_t corresponds to the experimentally measured quantity. To be consistent with the number extracted from experiment, the correct theoretical calculation must also correspond to Eq. (7) and is given explicitly by

$$\sigma_t = -\frac{\pi^2}{kW}\sum_J (2J+1)\,\mathrm{Im}\,[T_J\, S_J^{c*}], \tag{8}$$

where T_J is the result of the solution of Eq. (3), and $S_J^c = e^{2i\sigma_J}$ with $\sigma_J = \arg[\Gamma(J+1+i\eta)]$ (the pure Coulomb phase shift).

In this respect the eikonal model is very limited. In the eikonal model the scattering amplitude is given by

$$\begin{aligned} f_C(q) &= ik\int bdb\, J_0(qb)\,[1-e^{i\chi_c}] ,\\ f_{\rm CN}(q) &= ik\int bdb\, J_0(qb)\, e^{i\chi_c}\,[1-e^{i\chi_n}] , \end{aligned} \tag{9}$$

where χ_c and χ_n are eikonal phases [22, 23] for the Coulomb and nuclear potentials, respectively, b is the impact parameter, k is the kaon-nucleus c.m. momentum. The momentum transfer q is given by $q = 2k\,\sin(\theta/2)$ with θ the scattering angle in the kaon-nucleus c.m. frame.

In the eikonal model $\sigma_T - \sigma_C$ is given by

$$\sigma_T - \sigma_C = 4\pi\mathrm{Re}\left(\int bdb\, e^{i\chi_c}\,[1-e^{i\chi_n}]\right). \tag{10}$$

We need also to subtract $\sigma_{\rm CN}$, which is given by [33],

$$\sigma_{\rm CN} = 2\mathrm{Re}\left(\int d\Omega\, f_C f_{\rm CN}^*\right) = 4\pi\mathrm{Re}\left[\int qdq\left(\int bdb\, J_0(qb)\, e^{i\chi_c(b)}\,(1-e^{i\chi_n(b)})\right)\left(\int b'db'\, J_0(qb')\,(1-e^{-i\chi_c(b')})\right)\right]. \tag{11}$$

At sufficiently high energies ($k\to\infty$), the approximate orthogonality of the Bessel functions

$$\int_0^{2k} qdq\, J_0(qb)\, J_0(qb') \simeq \frac{1}{b}\,\delta(b-b') , \tag{12}$$

leads to

$$\sigma_{\rm CN} = 4\pi\mathrm{Re}\left(\int bdb\,[1-e^{i\chi_n(b)}]\,[e^{i\chi_c(b)}-1]\right) . \tag{13}$$

The total cross section in the eikonal model is thus

$$\sigma_t = \sigma_T - \sigma_C - \sigma_{\rm CN} = 4\pi\mathrm{Re}\,\left(\int bdb\,[1-e^{i\chi_n}]\right) . \tag{14}$$

This implies that σ_t in the eikonal model is nearly equal to $\sigma_t^{\rm cf}$, the Coulomb-free (nuclear only) total cross section. Because we have used a finite-range Coulomb interaction, the difference between the point Coulomb and finite-range Coulomb potential will be present in the calculation of χ_n. The main point is, however, that the eikonal approximation, because the long range part of the Coulomb interaction enters as a multiplicative factor in Eq. (9), it disappears from σ_t. As a result, in the eikonal model, σ_t is nearly equal to the Coulomb-free cross section and does not treat correctly the full complexity of nuclear-Coulomb interference.

III. RESULTS

A. Model dependence of theoretical results

The final ingredient required to calculate kaon-nucleus total cross sections is $E_{\rm ms}$. For pion-nucleus scattering in the resonance region, the mean spectral energy is calculated assuming the delta-nucleus shell model potential is equal to the nucleon shell model potential. We do not have values of E_{ms} for the high-energy kaons and these would be difficult to estimate, especially for the deuteron target. The mean-spectral energy accounts approximately for the interaction of the projectile and the struck nucleon with the remaining nucleons. Since the kaon-nucleon interaction is weaker than the nucleon-nucleon interaction, we would expect E_{ms} to be dominated by the nucleon-nucleon interaction. As such we know that E_{ms} should be negative (an attractive interaction) and must be less than the nucleon shell model potential at the center of the nucleus. We would expect it to be of the order of −20 MeV, to −30 MeV, as was calculated for resonance-energy pions.

In the absence of a microscopic calculation for E_{ms}, we have chosen to adjust E_{ms} at each incident momentum so that the calculated total cross section for scattering from the deuteron is equal to the experimentally measured cross section. This phenomenological adjustment has the advantage that, when we take the ratio of a total cross section to that of the deuteron, we make no error in the result for the deuteron, while we calculate the numerator and the denominator utilizing the same theory. The values for E_{ms} and the total cross section for scattering from the deuteron are given in Table I. The adjusted values are negative and of the order one would expect. In calculating the scattering from the deuteron, we ignore the spin of the deuteron and treat the deuteron as a spinless target. This is a reasonable approximation as rescattering is very small; utilizing the optical potential in the Born approximation makes only a fraction of a percent error for the deuteron. The correct treatment of the spin would produce a correction that involves two spin flips and would be negligibly small. One would expect similarly small corrections for the double isospin flip contribution. The more serious error made in treating the deuteron with an optical potential is in the neglect of the second-order correlation term in the optical potential. In Table I, we also present results for ^{12}C in which we use E_{ms} as determined from deuteron scattering. This implies that E_{ms} is target independent. For nuclei other than the deuteron this was approximately true for the values [30] calculated for resonance-energy pions. We would expect similar behavior here. The deuteron, however, is not a typical nucleus so that this approximation should be investigated further.

The results for the deuteron, for ^{12}C, and the ratio are pictured in Fig. 1. The discrepancy with the data is similar to that found in previous work [2, 4, 6]. The theory is of the order of 10–20% below the data. However, our results show a different energy dependence. We find a stronger dependence than Ref. [2] but weaker than that of Ref. [6]. We may trace this difference to the definition of the total cross section. We show in Fig. 2 the three definitions of the total cross section, the Coulomb free cross section, $\sigma_T - \sigma_C$, and the experimental cross section σ_t. In Ref. [2] $\sigma_T - \sigma_C$ was calculated. We see that this result is about 15% smaller than the others. Since that cross section does not correspond to the measured quantity, the results presented there should be replaced with the results presented here. The results from [6] appear to be the Coulomb-free results, which because of the larger cross section near 400 MeV/c, give a larger upturn to the ratio at the lower energies and lead to a stronger energy dependence.

The results pictured in Fig. 1 are thus typical of any calculation if the same quantity is calculated. An important question is just how sensitive these results are to the

TABLE I. The mean spectral energy E_{ms}, as adjusted to reproduce the experimental total cross section on the deuteron and the total cross sections of K^+-deuteron scattering for incident momentum P_{lab} from 400 MeV to 1 GeV. The deuteron total cross sections are calculated with the momentum-space optical potential code ROMKAN. Also given are the momentum-space results for K^+ scattering from ^{12}C and the Born approximation to these results.

P_{lab} (MeV/c)	E_{ms} (MeV)	$\sigma_t(D)$ (mb)	$\sigma_t(C)$ Full (mb)	$\sigma_t(C)$ Born (mb)
400	−10.0	23.84	134.2	120.3
500	−20.0	26.28	138.4	131.1
600	−30.0	27.96	145.3	140.6
700	−34.0	28.46	148.2	145.1
800	−36.5	29.48	153.0	150.9
900	−38.0	31.82	162.5	160.7
1000	−39.5	34.66	172.2	170.3

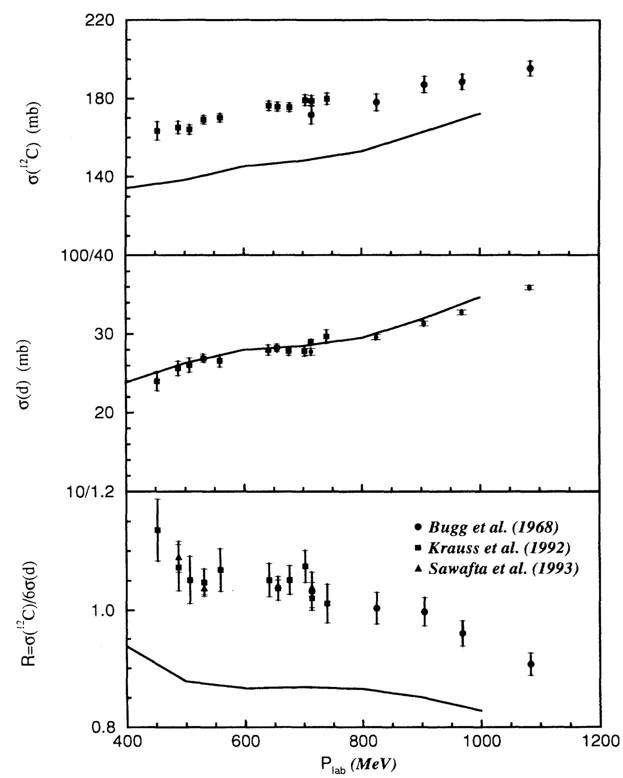

FIG. 1. Total cross sections for K^+ scattering on ^{12}C, the deuteron, and the ratio of these cross sections as a function of the kaon laboratory momentum P_{lab}. The calculations use the momentum-space optical model with E_{ms} adjusted as given in Table I. The data are from [7, 10, 11].

details of the theory. We first ask the question of how important is the exact calculation of the multiple scattering of the kaon. We can answer this by considering the use of the optical potential in Born approximation. The results of the Born approximation are also given in Table I [35]. We see that the rescattering is ten percent at the low energies and decreases to only one percent at 1 GeV/c. It is principally the difference between the exact answer and the Born approximation that the theorists are calculating. Since this difference is small, it is not surprising there is reasonable agreement among theoretical models.

A comparison between the momentum-space and eikonal results is shown in the bottom diagram of Fig. 3. We see that at low energies there are significant differences but that the two approaches appear to be converging as the energy is increased. The origin of the difference lies in the inadequate treatment of the Coulomb-nuclear interference in the eikonal model. This can be seen by noting that the two models give nearly the same results for the Coulomb-free total cross section. The similarity of the two models in the absence of the Coulomb potential strengthens the argument that the total cross sections are fairly independent of the theoretical model, but this statement can only be used for models which do not approximate the treatment of the Coulomb nuclear interference. The eikonal model can be used for qualitative studies, but we find that it should not be considered quantitative for the calculation of total cross sections, in particular, in the low incident momentum region.

The principal model dependences which enter the momentum-space calculations of the first-order optical potential are related to the kaon-nucleon t matrix used. These would include uncertainties in the on-shell two-body amplitude, the off-shell extrapolation of the amplitude, and in the choice of the mean spectral energy. We examine here the dependence of the calculated total cross sections on the features of the model of the two-body amplitude.

We first examine the importance of the choice of the on-shell two-body amplitude. In the top diagram of Fig. 3, we show the total cross sections which result from using the Arndt [28] and the Martin [29] phase shift analysis. The difference is small and is increasing with energy. In taking the ratio of the nuclear total cross section to that of the deuteron, the difference almost totally cancels out demonstrating that working with the ratio can cancel uncertainties in the theory as well as in the experiment. The difference in the ratio of cross sections using the two different on-shell amplitudes is less than one percent and would not be visible in Fig. 1.

We also vary the range parameter Λ in the off-shell extrapolation of the two-body amplitude. Results for Λ equal to 800–1200 MeV/c are given in Fig. 3. As was found in [4, 6], we find very little dependence on the off-shell behavior of the two-body amplitude.

The remaining parameter in the calculation is the mean spectral energy E_{ms}, which we have determined

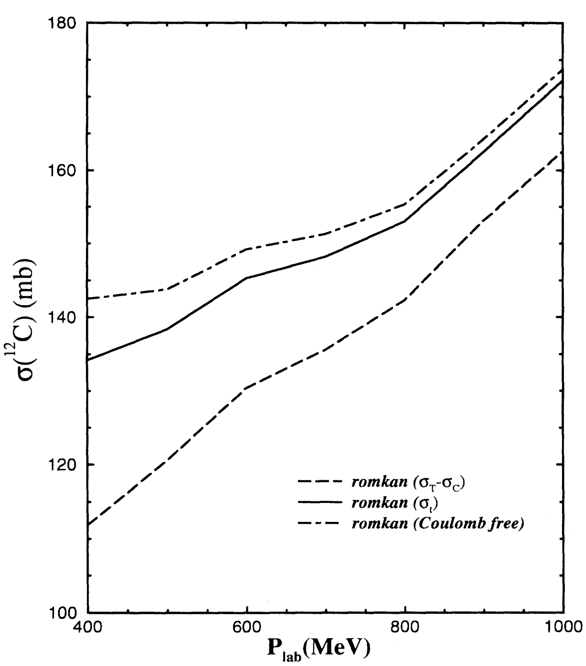

FIG. 2. Results for three possible definitions of the total cross section for K^+ scattering from ^{12}C as a function of the kaon laboratory momentum P_{lab}. The solid curve is σ_t of Eq. (7); the dash-dotted is Coulomb-free cross section; and the dashed is $\sigma_T - \sigma_C$. All are calculated using the momentum-space theory.

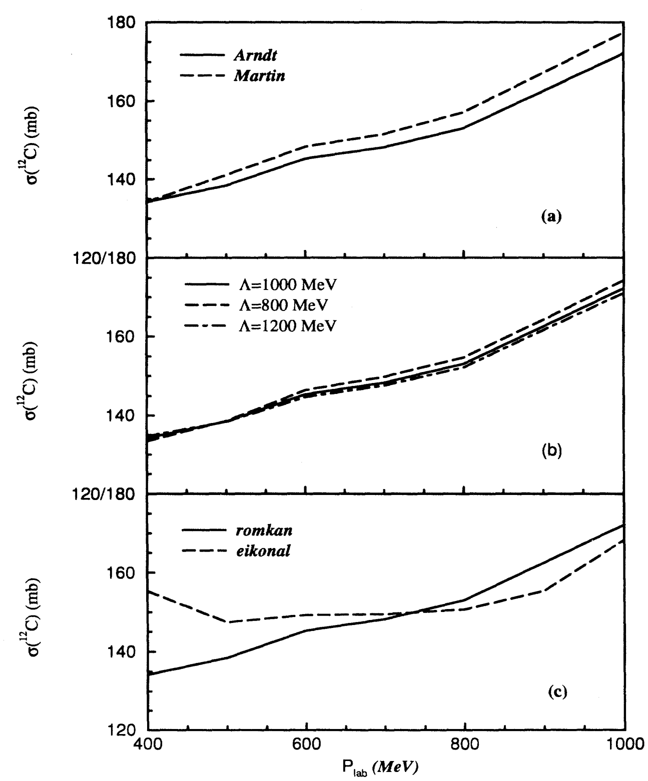

FIG. 3. Dependencies of the total cross section for the K^+ scattering from ^{12}C calculated with the momentum-space model: (a) using the two-body KN interaction of Arndt [28] (solid) and Martin [29] (dashed), (b) using various range parameters Λ, and (c) momentum-space and eikonal models.

by fitting to the deuteron data. In Fig. 4 we show two results, one from our full model and the other with $E_{ms} = 0$. Setting $E_{ms} = 0$ is an extreme and unphysical limit. A better approximation might be to cancel E_{ms} against the nuclear binding energy. From Fig. 4, we see that there is a dependence on the value of the energy at which the two-body amplitude is evaluated. However, the deuteron and ^{12}C total cross sections are affected in the same way, so that the ratio of the cross sections is stable.

B. Target dependence of K^+ nucleus total cross sections

We examine here the target dependence of the discrepancy between theory and experiment. Total cross sections are calculated for ^{4}He, ^{6}Li, ^{12}C, ^{16}O, ^{28}Si, and ^{40}Ca, using the momentum-space model. We neglect the spin of the ^{6}Li target and treat it as a spinless particle, as we have done for the deuteron. The theoretical cross sections are given in Table II, and a comparison between theory and experiment of the total cross section ratio is presented in Figs. 5 and 6. The data are from [7, 10, 11]. We see a universal discrepancy between the theory and the data with the discrepancy becoming larger for the heavier nuclei. The energy dependence of the predicted cross section ratios is very similar to that of the data. The eikonal result is consistently larger than the momentum-space result and thus exhibits a smaller discrepancy. This is reminiscent of the results found in [4], where momentum-space results (with a factorization of the fermi integration) were compared with earlier coordinate-space [36] results. There it was found that the momentum-space results were al-

TABLE II. The total cross section for K^+ scattering from ^{4}He, ^{6}Li, ^{12}C, ^{16}O, ^{28}Si, and ^{40}Ca for incident momentum P_{lab} from 400 MeV/c to 1 GeV/c. The results are calculated using the momentum-space model with mean spectral energies E_{ms} from Table I.

P_{lab} (MeV/c)	$\sigma_t(A)$ ^{4}He (mb)	^{6}Li (mb)	^{12}C (mb)	^{16}O (mb)	^{28}Si (mb)	^{40}Ca (mb)
400	44.96	68.57	134.2	179.3	301.1	420.3
500	47.63	74.14	138.4	185.2	306.5	425.3
600	50.20	79.37	145.3	194.7	320.2	444.4
700	51.11	81.32	148.2	198.7	326.6	453.4
800	52.50	84.33	153.0	204.8	337.4	467.2
900	55.62	90.19	162.5	217.7	357.4	494.6
1000	59.42	97.09	172.2	231.7	377.3	522.2

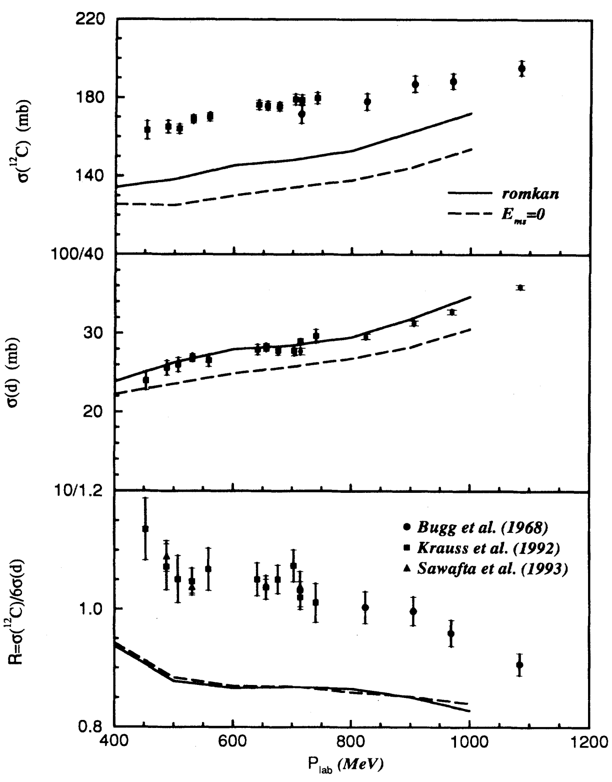

FIG. 4. Dependence of the total cross section on the mean spectral energy E_{ms} for K^+ scattering on ^{12}C, the deuteron, and the ratio of these cross sections. The solid curves are results of the momentum-space model with E_{ms} adjusted as given in Table I while the dashed curves are those with $E_{ms} = 0$. The data are from [7, 10, 11].

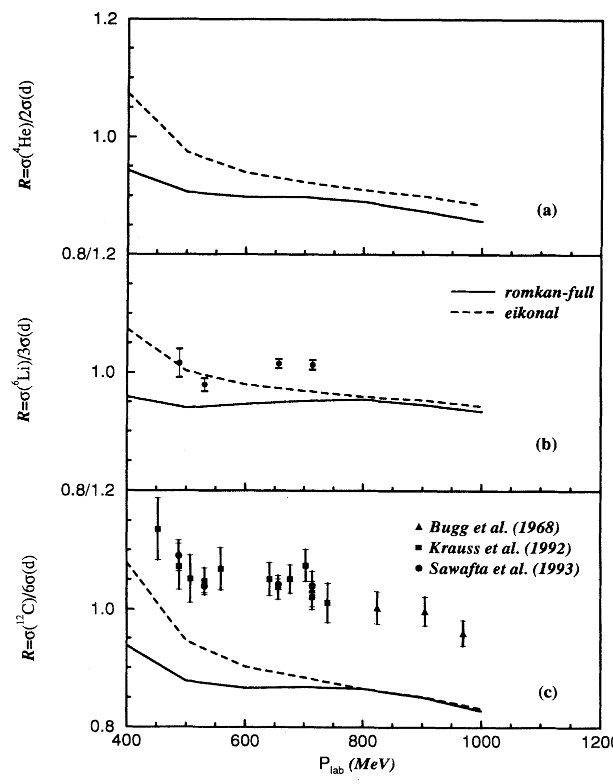

FIG. 5. Target dependence of the total cross section ratio defined in Eq. (1) for the K^+-nucleus scattering. Results from the romkan (solid) and eikonal (dashed) models are shown for ^{4}He (a), ^{6}Li (b), and ^{12}C (c), respectively. The experimental data are from [7, 10, 11].

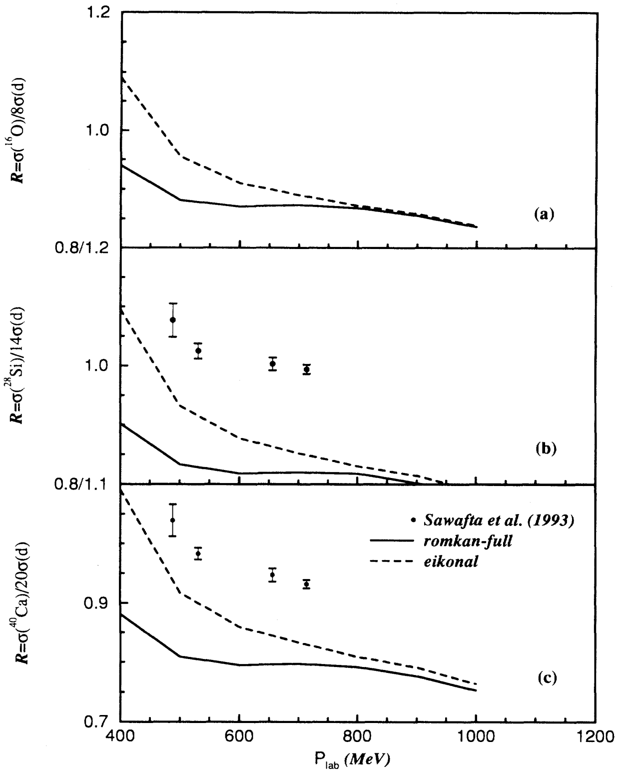

FIG. 6. Same as Fig. 5 except the targets are ^{16}O (a), ^{28}Si (b), and ^{40}Ca (c), respectively.

ways lower than the coordinate-space results. The reason given was the lack of a correct frame transformation of the two-body amplitude in the coordinate-space approaches. Our eikonal model, however, incorporates the transformation correctly [23] in the forward direction and is thus closer to the momentum-space results than are the earlier coordinate-space results. The remaining difference between the momentum-space calculation and the eikonal calculation is produced largely by the incorrect treatment of the Coulomb-nuclear interference in the eikonal model. The difference between the two calculations gives an indication of the importance of utilizing a theoretical approach that is capable of correctly treating this interference and emphasizes the possible model dependence [34] that is present in experimentally extracting total cross sections from data.

IV. SUMMARY, CONCLUSIONS, AND FUTURE PROSPECTS

The theoretical results from the first-order momentum-space optical potential lie consistently and significantly below the data. This is qualitatively similar to previous work. The energy dependence of the ratios of cross sections calculated here is less than was found in previous work [10–12] and generally follows more closely the energy dependence observed in the data. This could be due to the way that the cross sections are calculated. The Coulomb-free cross section ratios show a stronger dependence on energy at the lower energies. The correct treatment of the Coulomb-nuclear interference term which corresponds to the method used by experimentalists to extract these cross sections lessens this dependence.

The results here support models [6, 13] which increase the in-medium kaon-nucleon amplitude as these models provide an approximately energy independent increase in the nuclear total cross sections. Pion exchange-current contributions would produce [15] a strong increase with energy as the energy goes above the pion production threshold. The conclusion that the pion exchange-current contributions are small and are not capable of resolving the discrepancy [15] utilizing a reasonable number of excess pions in the nucleus is thus supported by the near energy independence of the difference between the theory and the data.

Future work should include a more careful theoretical treatment of conventional second order effects. These would include Pauli and short range correlations; these have been investigated in [5], but an independent check is probably merited. Data with smaller errors and particularly elastic differential cross sections with good absolute normalization would provide additional information. Data in the forward direction which would emphasize the Coulomb-nuclear interference term would be especially useful. These would provide information on both the phase and the amplitude of the strong interaction in addition to constraining any ambiguities which might remain in how the theorists and experimentalists are extracting total cross sections. Inelastic cross sections to specific final states, scattering from polarized targets, and charge-exchange scattering could also be used to learn the spin and isospin dependence of the missing piece of physics.

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[1] C. B. Dover and G. E. Walker, Phys. Rev. C **19**, 1393 (1979).
[2] C. M. Chen and D. J. Ernst, Phys. Rev. C **45**, 2011 (1992).
[3] D. J. Ernst, J. T. Londergan, G. A. Miller, and R. M. Thaler, Phys. Rev. C **16**, 537 (1977).
[4] M. J. Páez and R. H. Landau, Phys. Rev. C **24**, 1120 (1981).

[5] P. B. Siegel, W. B. Kaufmann, and W. R. Gibbs, Phys. Rev. C **30**, 1256 (1984).
[6] P. B. Siegel, W. B. Kaufmann, and W. R. Gibbs, Phys. Rev. C **31**, 2184 (1985).
[7] D. Bugg *et al.,* Phys. Rev. **168**, 1466 (1968).
[8] Y. Marlow *et al.,* Phys. Rev. C **25**, 2619 (1982).
[9] E. Mardor *et al.,* Phys. Rev. Lett. **65**, 2110 (1990).
[10] R.A. Krauss *et al.,* Phys. Rev. C **46**, 655 (1992).
[11] R. Sawafta *et al.,* Phys. Lett. B **307**, 293 (1993).
[12] R. Weiss *et al.,* Phys. Rev. C **49**, 2569 (1994).
[13] G. E. Brown, C. B. Dover, P. B. Siegel, and W. Weise, Phys. Rev. Lett. **26**, 2723 (1988).
[14] S. V. Akulinichev, Phys. Rev. Lett. **68**, 290 (1992).
[15] M. F. Jiang and D. S. Koltun, Phys. Rev. C **46**, 2462 (1992).
[16] C. Garcia-Recio, J. Nieves, and E. Oset, in *Proceedings of the International Conference on Meson-Nucleus Interaction*, Cracow, 1993 [Acta. Phys. Polonica (to be published)]; Phys. Rev. C (submitted).
[17] J. C. Caillon and J. Labarsouque, Phys. Rev. C **45**, 2503 (1992); Phys. Lett. B **295**, 21 (1992); J. Phys. G **19**, L117 (1993); Phys. Lett. B **311**, 19 (1993).
[18] D. R. Giebink and D. J. Ernst, Comput. Phys. Commun. **48**, 407 (1988).
[19] D. J. Ernst and G. A. Miller, Phys. Rev. C **21**, 1472 (1980); D. L. Weiss and D. J. Ernst, *ibid.* **26**, 605 (1982); D. R. Giebink, *ibid.* **25**, 2133 (1982).
[20] D. J. Ernst, G. E. Parnell, and C. Assad, Nucl. Phys. **A518**, 658 (1990).
[21] M. B. Johnson and D. J. Ernst, Phys. Rev. C **27**, 709 (1983); Ann. Phys. (N.Y.) **219**, 266 (1992); C. M. Chen, D. J. Ernst, and M. B. Johnson, Phys. Rev. C **47**, R9 (1993).
[22] J. Germond and M. B. Johnson, Phys. Rev. C **22**, 1622 (1980); J. Germond, M. B. Johnson, and J. A. Johnstone, *ibid.* **32**, 983 (1985).
[23] C. M. Chen, D. J. Ernst, and M. B. Johnson, Phys. Rev. C **48**, 841 (1993).
[24] D. R. Giebink, Phys. Rev. C **32**, 502 (1985).
[25] J. W. Negele, Phys. Rev. C **1**, 1260 (1970); M. Beiner, H. Flocard, N. Van Gai, and P. Quintin, Nucl. Phys. **A238**, 29 (1975).
[26] R. Machleidt, K. Holinde, and Ch. Elster, Phys. Rep. **140**, 1 (1987).
[27] M. F. Jiang and D. J. Ernst, this issue, Phys. Rev. C **51**, 1037 (1995).
[28] R. Arndt, computer code SAID, Phys. Rev. D **28**, 97 (1983).
[29] B. R. Martin, Nucl. Phys. **B94**, 413 (1975).
[30] D. J. Ernst and M. B. Johnson, Phys. Rev. C **32**, 940 (1985).
[31] S. J. Wallace, Ann. Phys. (N.Y.) **78**, 190 (1973); S. J. Wallace, Phys. Rev. D **8**, 1846 (1973); S. J. Wallace, Phys. Rev. C **8**, 2043 (1973).
[32] J. T. Holdeman and R. M. Thaler, Phys. Rev. **139**, 1186B (1965).
[33] W. B. Kaufmann and W. R. Gibbs, Phys. Rev. C **40**, 1729 (1989).
[34] M. Arima and K. Masutani, Phys. Rev. C **47**, 1325 (1993).
[35] The results for the Born approximation shown in Table I are calculated by approximating the reaction matrix R by the optical potential. The exact Coulomb amplitude is used and the T matrix, generated from the R matrix, is used in Eq. (8).
[36] C. B. Dover and P. J. Moffa, Phys. Rev. C **16**, 1087 (1977); C. B. Dover and G. E. Walker, *ibid.* **19**, 1393 (1979); S. R. Cotanch and F. Tabakin, *ibid.* **15**, 1379 (1977); S. R. Cotanch, *ibid.* **18**, 1941 (1978); Nucl. Phys. **A308**, 253 (1978); Phys. Rev. C **21**, 2115 (1980); **23**, 807 (1981).