Influence of resonant channels on subbarrier heavy-ion fusion

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(Received 11 August 1994)

The subbarrier fusion of heavy ions is discussed when one of the nuclei is excited to a resonant state. The effect of the width of the resonant state on the barrier penetration is calculated within a schematic model. It is concluded that the width could either enhance or hinder the fusion probability, depending on the relative importance of the spreading to escape parts of the width. Application of the theory to the fusion of ^{11}Li with ^{208}Pb at near-barrier energies is made. The resulting fusion cross section calculated with coupling to the soft giant dipole state in ^{11}Li was found to be more than an order of magnitude *smaller*, in the barrier region, and *larger*, at subbarrier energies, than the uncoupled one.

PACS number(s): 25.70.Jj, 24.30.−v

I. INTRODUCTION

In recent years it has become a common practice to treat the subbarrier fusion of heavy ions as a multidimensional barrier tunneling problem. When cast into a reaction theory language one speaks of coupled-channels (CC) effects. These effects have been widely discussed in the recent literature as the cause of the enhancement, over the one-dimensional (one-channel) model prediction, of the fusion cross section clearly exhibited by a large body of data [1]. The overall picture that has emerged from these studies that the enhancement ensues as long as the coupling is restricted to normal channels. By normal we are referring to excited and particle transfer channels [1].

The lifetime of the excited states is always taken to be infinite. Thus the CC treatment so far developed precludes the study of the coupling to resonant states [1(g)]. Further, the effect of the coupling to breakup channels (which could be the final fate of the resonant states) is also not considered. A few attempts in this direction have been made recently but have only addressed part of the problem. A fully consistent way of taking into account the coupling to a resonance in the presence of breakup effects has recently been proposed by Hussein and de Toledo Piza (HP) [2]. The work of HP was published as a short Letter and accordingly little space was available to include several important details. The purpose of the present paper is to supply these details. We mention here that Balantekin and Takigawa [1(g)] have considered a model similar to the one we develop in Sec. III, though they address a different issue.

We should mention here that breakup coupling effects become important in cases involving low Q values, usually encountered in loosely bound neutron-rich projectiles such as ^{11}Li [3]. In this radioactive nucleus the Q value for the breakup into ^{9}Li+$2n$ is only 0.3 MeV. It has been lately debated whether this breakup proceeds through a two-step process involving first the excitation of a soft giant dipole (SGD) state followed by its decay, or through a direct, one-step process. At energies in the vicinity of the Coulomb barrier of, say, ^{11}Li + ^{208}Pb (26 MeV) one expects the excitation of the SGD to be relevant to the fusion process [4,5].

The theory developed by HP [2] has been applied to the fusion of ^{11}Li + ^{208}Pb. It was found that the finite width of the SGD state, being entirely due to breakup coupling, results in a reduction of the fusion cross section. In the general case the resonance width Γ is composed of a damping width $\Gamma^{\downarrow}$ and an escape width $\Gamma^{\uparrow}$ [6]. HP found that $\Gamma^{\downarrow}$ enhances the fusion while $\Gamma^{\uparrow}$ hinders it. Full details of the developments are given in this paper.

The paper is organized as follows. In Sec. II, a short review of the theory of coupled channels fusion (CCF) is presented. In Sec. III the exit doorway model [7] (EDM) of the excitation of a resonant state is described, and its application to fusion is presented. In Sec. IV, a schematic calculation within the EDM of the fusion cross section is presented and applied to ^{11}Li + ^{208}Pb. In Sec. V the effect of the escape width is discussed. Finally, in Sec. VI a general discussion and conclusions are given.

II. MULTICHANNEL FUSION CROSS SECTION

In this section we present a summary of the most pertinent aspects of coupled channels effect on the fusion cross section. We consider first "normal" channels in the sense we defined them in the Introduction.

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We take for the Hamiltonian of the nucleus-nucleus system $H = H_0+V$, where H_0 is diagonal in open channel space. Here

$$H_0 = h_0 + K + U \; , \tag{1}$$

where h_0 is the sum of the two intrinsic Hamiltonians, K is the kinetic energy operator, and U is the optical potential which contains the complex nuclear plus the Coulomb parts. The coupling among the channels is represented by V. The spectrum of h_0 is represented by

$$\begin{aligned} h_0|\varphi_0\rangle &= E_0|\varphi_0\rangle \; , \\ h_0|\varphi_i\rangle &= E_i|\varphi_i\rangle \; , \end{aligned} \tag{2}$$

where φ_0 is the ground state and $\{\varphi_i\}$ are the excited states (for simplicity we take one of the partners of the reaction to be inert).

The full Schrödinger equation of the system reads

$$[E - (H_0 + V)] \mid \Phi\rangle = 0 \; , \tag{3}$$

which upon projection onto the different channels represented by (2) yields the usual set of coupled channel equations:

$$\begin{aligned} (E - H_0)\Phi_0^{(+)} &= \sum_i V_{0i}\Phi_i^{(+)} \; , \\ (E - H_0)\Phi_i^{(+)} &= V_{i0}\Phi_0^{(+)} \; . \end{aligned} \tag{4}$$

A conspicuous feature of Eq. (4) is the absence of channel reorientation; namely, we have ignored the coupling among the excited channels. This restriction can be easily removed. In the discussion to follow, however, we shall use Eq. (4).

Equation (4) can be solved for $\Phi_0^{(+)}$, the *exact* wave function in the elastic channel (we set $E_0 = 0$ and emphasize the absence of channel reorientations, $V_{ii'} = 0, i, i' \neq 0$):

$$\Big(E - H_0 - \sum_i V_{0i} G_i^{(+)} V_{i0}\Big)\Phi_0^{(+)} = 0 \; , \tag{5}$$

where

$$G_i^{(+)} = \frac{1}{E - E_i - K_i - U_i + i\epsilon} \; . \tag{6}$$

We now derive the formula for the fusion cross section, σ_F, as done in Ref. [8]. We first write down the total reaction cross section using unitarity arguments in Eq. (5):

$$\sigma_R = \frac{k}{E}\langle\Phi_0^{(+)} \Big| -\text{Im}\left(U_0 + \sum_i V_{0i}G_i^{(+)}V_{i0}\right) \Big| \Phi_0^{(+)}\rangle \; , \tag{7}$$

where $E = \frac{\hbar^2 k^2}{2\mu}$.

We now use the identity

$$\begin{aligned} \text{Im}G_i^{(+)} &= G_i^{(+)\dagger}\text{Im}U_i G_i^{(+)} \\ &\quad -\pi\Omega_i^{(-)}\delta(E - E_i - K_i)\Omega_i^{(-)\dagger} \; , \end{aligned} \tag{8}$$

where $\Omega_i^{(-)\dagger}$ is the optical Möller operator:

$$\Omega_i^{(-)\dagger} = 1 + G_i^{(+)} U_i \; . \tag{9}$$

Taking V_{0i} and V_{i0} to be real, and using Eq. (8), we find for σ_R

$$\begin{aligned} \sigma_R = \frac{k}{E}\Big\langle \Phi_0^{(+)} \Big| & (-\text{Im}U_0) \\ & + \sum_i V_{0i} G_i^{(+)\dagger}(-\text{Im}U_i)G_i^{(+)}V_{i0} \Big| \Phi_0^{(+)}\Big\rangle \\ & + \frac{k^i}{E}\pi \sum_i \int \Big|\Big\langle \phi_i^{(-)}|V_{i0}|\Phi_0^{(+)}\Big\rangle\Big|^2 \\ & \times \delta\left(E - E_i - \frac{\hbar^2 k_i^2}{2\mu}\right) d\vec{k}_i/(2\pi)^3 \; . \end{aligned} \tag{10}$$

The second term in Eq. (10) which arises from the second term of the right-hand side (RHS) of Eq. (8) represents the total inelastic (direct) cross section. If we assume that $\text{Im}U_0$ and $\text{Im}U_i$ represent absorption due to fusion in the elastic and the ith inelastic channel, respectively, we can identify the first term in Eq. (10) with the total fusion cross section:

$$\begin{aligned} \sigma_F = \frac{k}{E}\Big\langle \Phi_0^{(+)} \mid & -(\text{Im}U_0) \\ & \times \sum_i V_{0i} G_i^{(+)\dagger}(-\text{Im}U_i) G_i^{(+)} V_{i0} \mid \Phi_0^{(+)}\Big\rangle \; . \end{aligned} \tag{11}$$

Since the exact wave function of the ith inelastic channel is given by [see Eq. (4)]

$$\mid \Phi_i^{(+)}\rangle = G_i^{(+)} V_{i0} \mid \Phi_0^{(+)}\rangle \; , \tag{12}$$

we can rewrite Eq. (11) in the simple form

$$\sigma_F = \frac{k}{E} \sum_{j=0,1,2,\ldots} \langle\Phi_j^{(+)} \mid -(\text{Im}U_j) \mid \Phi_j^{(+)}\rangle \; . \tag{13}$$

Equation (12) clearly shows the influence of coupled channels on σ_F. The two nuclei fuse in the elastic and the inelastic channels and the total fusion is just the sum of these individual channel fusion cross sections. If the two nuclei remain intact in the inelastic channels (no breakup), σ_F of Eq. (12) is in general larger than the fusion cross section in the limit of zero channel coupling, $\mathring{\sigma}_F$ (obtained by setting $V_{0i} = V_{i0} = 0$). The enhancement factor

$$E \equiv \frac{\sigma_F}{\mathring{\sigma}_F} \tag{14}$$

could become very large (several orders of magnitude) at subbarrier energies, where quantum tunneling dominates. This is easily seen if we consider only one inelastic channel, which we call 1. Then

$$\sigma_F = \frac{k}{E}\left[\langle\Phi_0^{(+)}|-\text{Im}U_0|\Phi_0^{(+)}\rangle + \langle\Phi_1^{(+)}|-\text{Im}U_1|\Phi_1^{(+)}\rangle\right] \; . \tag{15}$$

The coupling matrix in the two coupled equations is $\left(\begin{smallmatrix} 0 & V_{01} \\ V_{10} & Q \end{smallmatrix}\right)$, where Q is the Q value of the reaction. If we take $V_{01} = V_{10} = v(R_B) = \text{const}$, where R_B is the position of the Coulomb barrier, then the two equations can be diagonalized by a unitarity transformation [9] that diagonalizes the coupling matrix C,

$$C \equiv \begin{pmatrix} 0 & v \\ v & Q \end{pmatrix} = (x_+ x_-) \begin{pmatrix} \lambda_+ & 0 \\ 0 & \lambda_- \end{pmatrix} \begin{pmatrix} x_+^\dagger \\ x_-^\dagger \end{pmatrix} , \quad (16)$$

where

$$\lambda_\pm = \frac{1}{2}\left[Q \pm \sqrt{(Q^2 + 4v^2)}\right] \quad (17)$$

and

$$(x_+, x_+^\dagger) = (x_-, x_-^\dagger) = 1 ,$$
$$(x_+, x_-^\dagger) = (x_-, x_+^\dagger) = 0. \quad (18)$$

Calling the eigenchannel wave functions Ψ_+ and Ψ_-, we obtain two uncoupled equations:

$$(E - K_0 - U_0 - \lambda_+)\Psi_+ = 0 ,$$
$$(E - K_1 - U_1 - \lambda_-)\Psi_- = 0. \quad (19)$$

The transformation from (Ψ_0, Ψ_1) to (Ψ_+, Ψ_-) reads

$$\begin{pmatrix} \Psi_0 \\ \Psi_1 \end{pmatrix} = \begin{pmatrix} \frac{v\lambda_+}{\lambda_+^2+v^2} & \frac{v^2}{\lambda_-^2+v^2} \\ \frac{\lambda_+^2}{\lambda_+^2+v^2} & \frac{v\lambda_-}{\lambda_-^2+v^2} \end{pmatrix} \begin{pmatrix} \Psi_+ \\ \Psi_- \end{pmatrix} \quad (20)$$

$$\equiv M \begin{pmatrix} \Psi_+ \\ \Psi_- \end{pmatrix}. \quad (21)$$

With (21), the fusion cross section, Eq. (16), can be written as, after setting $U_0 = U_1$, $K_0 = K_1$,

$$\sigma_F = \frac{k}{E}(\Psi_+ \Psi_-) M^+ \begin{pmatrix} \text{Im}U & 0 \\ 0 & \text{Im}U \end{pmatrix} M \begin{pmatrix} \Psi_+ \\ \Psi_- \end{pmatrix} \quad (22)$$

or

$$\sigma_F = (M^+M)_{++}\left[\frac{k}{E}\langle \Psi_+ \mid \text{Im}U \mid \Psi_+\rangle\right]$$
$$+(M^+M)_{--}\left[\frac{k}{E}\langle \Psi_- \mid \text{Im}U \mid \Psi_-\rangle\right]$$
$$+2\text{Re}\left[\frac{k}{E}(M^+M)_{+-}\langle \Psi_+ \mid \text{Im}U \mid \Psi_-\rangle\right] . \quad (23)$$

In Eq. (23), $\frac{k}{E}\langle\Psi_\pm|\text{Im}U|\Psi_\pm\rangle$ is the fusion cross section and $\overset{\circ}{\sigma}{}^{\pm}_F$, is the eigenchannel ($\pm$), while the third term is an interference one.

The matrix elements in (23) are evaluated using the incoming wave boundary condition model to represent absorption. The full details of ImU are not needed. Only the penetrabilities of the real eigenbarriers $\text{Re}U(r) + \frac{\hbar^2 l(l+1)}{2\mu r^2} + \lambda_\pm$ are needed (once the flux penetrates the eigenbarrier, it is fully absorbed). Thus we can write

$$\overset{\circ}{\sigma}{}^{\pm}_F = \frac{\pi}{k^2}\sum_{l=0}^{\infty}(2l+1)\left[1 + \exp\left(2\int_{r_0^\pm}^{r_1^\pm} k_\pm^l(r)\,dr\right)\right]^{-1}$$
$$\equiv \frac{\pi}{k^2}\sum_{l=0}^{\infty}(2l+1)T_l^\pm , \quad (24)$$

where the l-dependent turning points $r_0^\pm$ and $r_1^\pm$ are the inner and outer solutions of $k_\pm^l(r) = 0$, and $k_\pm^l(r) = \left\{\frac{2\mu}{\hbar^2}\left[\text{Re}U(r) + \frac{\hbar^2 l(l+1)}{2\mu r^2} + \lambda_\pm - E\right]\right\}^{1/2}$. It is easy to understand the physics that $T_l^\pm$ describes. Taking the tunneling action $I_l^\pm \equiv 2\int_{r_0^\pm}^{r_1^\pm} k_\pm^l(r)\,dr$ to be large, we can write

$$T_l^\pm \equiv \frac{e^{-I_l^\pm}}{1 + e^{-I_l^\pm}} \approx e^{-I_l^\pm}\sum_{n=0}^{\infty}(-)^n e^{-nI_l^\pm} . \quad (25)$$

The series, Eq. (25), describes tunneling with an infinite number of internal reflections (within the barrier) [10] between the inner and outer turning points.

The interference term in Eq. (23) containing the matrix element $\langle\Psi_+|\text{Im}U|\Psi_-\rangle$ is negligible compared to the first two terms. This is so since Ψ_+ and Ψ_- contain phases and the product $\Psi_+^*(r)\Psi_-(r)$ will have a large overall phase:

$$\Psi_+^*(r)\Psi_-(r) = |\Psi_+(r)\Psi_-(r)|e^{i\Phi(r)}. \quad (26)$$

The presence of this phase, absent in the diagonal matrix elements, renders the integral $\langle\Psi_+|\text{Im}|\Psi_-\rangle$ very small. Thus we neglect the interference term. Accordingly, we find for the fusion cross section the following simple form:

$$\sigma_F(E) = A(\lambda_+)\,\overset{\circ}{\sigma}{}^{+}_F(E) + A(\lambda_-)\,\overset{\circ}{\sigma}{}^{-}_F(E) ,$$
$$A(\lambda) = \frac{v^2}{\lambda^2 + v^2} . \quad (27)$$

If the Q value is zero $\lambda\pm = \pm v$ [Eq. (17)] and we obtain the well-known result

$$\sigma_F(E) = \frac{1}{2}\left[\overset{\circ}{\sigma}{}^{+}_F(E) + \overset{\circ}{\sigma}{}^{-}_F(E)\right] . \quad (28)$$

At very low energies we find

$$\sigma_F(E) \underset{E \ll V_B}{\sim} A(\lambda_-)\,\overset{\circ}{\sigma}{}^{-}_F, \quad (29)$$

valid for both positive and negative Q values.

We can now make an estimate for the enhancement factor E, Eq. (14), by employing the Hill-Wheeler (parabolic) approximation for T_l, which gives the fermion cross section, according to Wong [11],

$$\overset{\circ}{\sigma}{}^{\pm}_F = \frac{\hbar w R_B^2}{2E}\ln\left[1 + \exp\frac{2\pi}{\hbar w}\left[E - V_B - \lambda_\pm\right]\right] , \quad (30)$$

where V_B is the height of the Coulomb barrier and R_B its radius, and $\hbar w$ is related to the barrier curvature, $\hbar w = \frac{\hbar}{\mu}\frac{d^2V_l}{dr^2}\Big|_{r=R_B}$, with $V_l = \text{Re}U(r) + \frac{\hbar^2 l(l+1)}{2\mu r^2}$. For

$E \ll V_B$, we obtain

$$E = \frac{v^2}{\lambda_-^2 + v^2} \exp\left[-\frac{2\pi}{\hbar w}\lambda_-\right] \tag{31}$$

or

$$E = \frac{2v^2}{\left(Q - \sqrt{Q^2+4v^2}\right)^2 + v^2} \times \exp\left[+\frac{2\pi}{\hbar w}\left|Q - \sqrt{Q^2+4v^2}\right|\right] . \tag{32}$$

Clearly the enhancement is largest if $Q < 0$, which may happen in some transfer channels. We may ask now, for a fixed value of v, what is the optimum Q value that gives the largest enhancement. For the low energy estimate, Eq. (31), we find

$$|\lambda_-| = \frac{\hbar w}{2\pi} + \frac{1}{2}\sqrt{\left(\frac{\hbar w}{\pi}\right)^2 - 4v^2} \tag{33}$$

and, accordingly,

$$E \sim \exp\left[1 + \frac{2\pi}{\hbar w}\sqrt{\left(\frac{\hbar w}{\pi}\right)^2 - 4v^2}\right] . \tag{34}$$

In the opposite limit, $E > E_B$, Eq. (27), with Eq. (30) yields

$$\sigma_F = [A(\lambda_+) + A(\lambda_-)]\,\pi R_B^2\left[1 - \frac{V_B}{E}\right] - \frac{\pi R_B^2}{E}\left[\lambda_+ A(\lambda_-) + \lambda_- A(\lambda_-)\right] , \tag{35}$$

which, using the forms of $A(\lambda)$ and λ, gives us the fusion cross section with no channel-coupling effect:

$$\sigma_F = \pi R_B^2\left[1 - \frac{V_B}{E}\right] , \qquad E > V_B . \tag{36}$$

Equation (36) is a consequence of using two channels. When many channels are involved, one expects a loss of energy (friction) that renders σ_F smaller.

In this section we have reviewed already known facts of multichannel fusion. We have used, however, a different framework, Eq. (12), to discuss the effect. Further, the two-channel case discussed in Ref. [9] is worked out here in a way that allows its extension to coupling to resonant channels which we turn to in the following section.

III. EXIT DOORWAY MODEL OF RESONANT CHANNELS (GIANT RESONANCES)

We develop in this section the exit doorway model [7] to treat first the spreading of an excited, collective state, on the fusion cross section. The entrance channel Ψ_0 couples to the compound nucleus (fusion) either directly, or, as in the previous section, via a bunch of excited channels. In this section we assume that these excited states are modulated by a doorway, which could be a giant resonance. To reach these "fine structure" channels the system has to pass through the doorway $|d\rangle$ [12,13]. This can be formalized by writing the intrinsic states $|\varphi_i\rangle$, Eq. (2), as

$$|\varphi_i\rangle = \alpha^{(i)}|d\rangle + \sum_j \beta_j^{(i)}|j\rangle ,$$

where the states $|j\rangle$ form an orthonormal set which spans the intrinsic subspace orthogonal to $|d\rangle$ and introducing the assumption

$$V_{0i} \equiv \langle 0|V|\varphi_i\rangle = \alpha^{(i)}\langle 0|V|d\rangle \equiv \alpha^{(i)}V_{0d} . \tag{37}$$

Our next task is to obtain the doorway amplitudes $\alpha^{(i)}$ associated with the various intrinsic states $|\varphi_i\rangle$. For this purpose we implement the intrinsic Hamiltonian h_0 in the form

$$h_0 = |\varphi_0\rangle E_0\langle\varphi_0| + \sum_j |j\rangle e_j\langle j\,| + |\,d\rangle E_d\langle d| + \sum_j \left[|j\rangle\Delta_j\langle d\,| + |\,d\rangle\Delta_j^*\langle j\,|\right] , \tag{38}$$

where, without loss of generality, it has been assumed that h_0 does not couple different states $|j\rangle$ (i.e., these states are taken to be eigenstates of the projection of h_0 onto the intrinsic subspace orthogonal to $|d\rangle$). The last term in Eq. (37) represents the interaction responsible for the spreading of $|d\rangle$. Note that, while $\langle d|j\rangle = 0$, $|d\rangle$ and the $|j\rangle$ are not eigenstates of h_0. Using Eq. (37) in the second of Eqs. (2) one finds

$$E_i = E_d + \sum_j \frac{|\Delta_j|^2}{E_i - e_j} \tag{39}$$

and

$$\left|\alpha^{(i)}\right|^2 = \frac{1}{1 + \sum_j \frac{|\Delta_j|^2}{(E_i - e_j)^2}} . \tag{40}$$

We need an expression for $|\alpha^{(i)}|^2$ in terms of the eigenvalues E_i and of the mean doorway energy E_d. This involves eliminating the e_j between Eqs. (39) and (40). The result will clearly depend on the values of the coupling matrix elements Δ_j and on the distribution of energies e_j. In fact any given distribution of the $|\alpha^{(i)}|^2$ can be produced by adjusting these quantities. A well-known special case [7] is that of a long, uniformly spaced sequence of energies e_j and state-independent coupling matrix elements Δ. This leads eventually to $|\alpha^{(i)}|^2$ which are Breit-Wigner distributed according to

$$\left|\alpha^{(i)}\right|^2 \simeq \frac{1}{2\pi\rho}\frac{\Gamma_d^{\downarrow}}{(E_i - E_d - \Delta E_d)^2 + \frac{\Gamma_d^{\downarrow 2}}{4}} , \tag{41}$$

where ρ is the density of states $|j\rangle$, $\Gamma_d^{\downarrow} \equiv 2\pi|\Delta|^2\rho$ is the spreading width of the doorway, and ΔE_d is an energy shift of the order of $\Gamma_d^{\downarrow}$. The Breit-Wigner distribution, Eq. (41), is normalized as

$$\sum_i |\alpha^{(i)}|^2 \to \int dE_i \rho |\alpha^{(i)}|^2 = 1 \, .$$

Note in particular that $|\alpha^{(i)}|^2$ decreases as E_i^{-2} both for very large and very small values of E_i. This underlines the need for a *long* sequence of background states $|j\rangle$.

Deviations from the assumptions involved in obtaining Eq. (41) will imply of course different distributions for the $|\alpha^{(i)}|^2$. In the case of wide doorway structures such as one finds notably in the case of dipole giant resonances, it is well known that a Lorentzian distribution

$$|\alpha^{(i)}|_L^2 = \frac{2}{\pi\rho} \frac{\Gamma_d E_i^2}{(E_i^2 - e_d^2)^2 + \Gamma_d^2 E_i^2}, \quad E_i > 0 \, , \tag{42}$$

reproduces very accurately the observed peak shapes. Here the sequence of background levels terminates at zero energy, so that the negative energy tail of the distribution disappears and the normalization condition reads accordingly

$$\int_0^\infty dE_i \rho |\alpha^{(i)}|_L = 1 \, . \tag{43}$$

The parameters e_d and Γ_d are usually adjusted to reproduce the position and width of the doorway peak.

In order to proceed with the discussion of the multichannel fusion problem under the exit doorway hypothesis, Eq. (37), a realistic strength distribution is given by Eq. (42). However, since this distribution has a more complicated analytical structure than the Breit-Wigner (BW) distribution, Eq. (41), for the sake of simplicity we base the following presentation on the latter, and defer a discussion of changes involved when one considers a Lorentzian line shape to the Appendix. Using Eq. (7) for the total reaction cross section and approximating $K_i + U_i$ by $K_d + U_d$ in the Green's function $G_i^{(+)}$ we have

$$\sigma_R = \frac{k}{E} \left\langle \Phi_0^{(+)} \left| -\text{Im} \left(U_0 + \sum_i |\alpha^{(i)}|^2 V_{0d} \frac{1}{E - E_i - K_d - U_d + i\epsilon} V_{0d} \right) \right| \Phi_0^{(+)} \right\rangle \tag{44}$$

or

$$\sigma_R = \frac{k}{E} \left\langle \Phi_0^{(+)} \right| -\text{Im} \left(U_0 + V_{0d} G_d^{(+)} V_{0d} \right) \left| \Phi_0^{(+)} \right\rangle \, , \tag{45}$$

where we have introduced the exit doorway propagator $G_d^{(+)}$,

$$G_d^{(+)}(E) \equiv \sum_i |\alpha^{(i)}|^2 \frac{1}{E - E_i - K_d - U_d + i\epsilon} \, . \tag{46}$$

Taking for $|\alpha^{(i)}|^2$ the BW form, Eq. (41), and changing the sum into integral, the resulting integration yields immediately

$$G_d^{(+)}(E) = \frac{1}{E - E_d + \frac{i\Gamma_d^\downarrow}{2} - K_d - U_d + i\epsilon} \, . \tag{47}$$

All reference to the fine structure states is contained in the spreading width $\Gamma_d^\downarrow$. Otherwise, $G_d^{(+)}(E)$ describes the propagation of the two-nucleus system, with one of the nuclei excited to the doorway state $|d\rangle$. The Q value associated with this excitation is complex and is given by

$$Q_d = E_d - i\frac{\Gamma_d^\downarrow}{2} \, . \tag{48}$$

Before we proceed further, we mention that so far we have not considered the escape width of the doorway that describes its coupling to open channels. The treatment of the escape will be developed later.

Since the escape width measures the actual fragmentation of the excited nucleus (except for the γ-emission contribution which we do not consider), whereas $\Gamma_d^\downarrow$ measures the degree of damping of the doorway due to its coupling to more complicated states in the same nucleus, it is natural to expect the effect of the coupling on σ_F to depend on the ratio $\mu \equiv \Gamma_d^\downarrow / \Gamma_d$. If this ratio is close to 1, we expect an *enhanced* fusion probability, since effectively (through $\Gamma_d^\downarrow$) there are *many routes* (excited states in the same nucleus) for fusion to occur. The other limit $\mu \ll 1$ should result in a smaller fusion probability, since the resonance could "break up" before fusion takes place. Of course, the degree of enhancement in σ_F when $\mu \sim 1$ is dictated by the value of E_d. As seen in the previous section, large values of E_d lead to smaller enhancement. We now proceed to the analysis of Eq. (45) with $G_d^{(+)}$given by Eq. (47). In order to extract σ_F from Eq. (45), we first need to calculate $\text{Im} G_d^{(+)}$, just as was done for $G_i^{(+)}$, Eq. (8). We accomplish this by operator manipulation. First we observe the following simple fact about the inverse of $G_d^{(+)}$:

$$G_d^{(+)^{-1}}(E) - G_d^{(+)\dagger^{-1}}(E) = i\Gamma_d^\downarrow - (U_d - U_d^\dagger) \, . \tag{49}$$

Multiplying the above from the left by $G_d^{(+)\dagger}$ and from the right by $G_d^{(+)}$, we find

$$\begin{aligned} G_d^{(+)} - G_d^{(+)\dagger} = & -G_d^{(+)\dagger}(i\Gamma_d^\downarrow)G_d^{(+)} \\ & + G_d^{(+)\dagger}(2i\,\text{Im}U_d)G_d^{(+)}. \end{aligned} \tag{50}$$

Define now the "free" exit doorway propagator $\overset{\circ}{G}_d$ as

$$\overset{\circ}{G}_d \equiv \left(E - E_d + i\frac{\Gamma_d^\downarrow}{2} - K_d + i\epsilon \right)^{-1} \, . \tag{51}$$

Then we may write

$$G_d^{(+)} \equiv \overset{\circ}{G}{}_d^{(+)} \, \Omega_d^{(+)} = \overset{\circ}{G}{}_d^{(+)} \, \Omega_d^{(-)\dagger} \, , \tag{52}$$

where we have introduced the exit doorway optical Möller operator $\Omega_d^{(+)}$,

$$\Omega_d^{(+)} \equiv 1 + U_d G_d^{(+)} = \Omega_d^{(-)\dagger} \ . \tag{53}$$

Thus we have finally

$$\mathrm{Im}G_d^{(+)} = -\pi\Omega_d^{(-)}\left[\overset{\circ}{G}{}_d^{(+)\dagger}\,\frac{\Gamma_d^{\downarrow}}{2\pi}\,\overset{\circ}{G}{}_d^{(+)}\right]\Omega_d^{(-)\dagger} + G_d^{(+)\dagger}\mathrm{Im}U_d G_d^{(+)} \ . \tag{54}$$

The quantity inside the square brackets in the first term on the right-hand side of Eq. (54) can be written in a symbolic form as

$$\overset{\circ}{G}{}_d^{(+)\dagger}\left(\Gamma_d^{\downarrow}/2\pi\right)\overset{\circ}{G}{}_d^{(+)} = \frac{\left(\frac{\Gamma_d^{\downarrow}}{2}\right)/\pi}{(E - E_d - K_d)^2 + \left(\frac{\Gamma_d^{\downarrow}}{2}\right)^2} \ . \tag{55}$$

Equation (55) represents a finite width version of the usual delta function, which describes on-shell processes. Accordingly, the first term in (54) accounts for the direct excitation of the doorway state. When Eq. (54) is inserted into Eq. (45), we find

$$\begin{aligned}\sigma_R = {} & \frac{k}{E}\left[\left\langle\Phi_0^{(+)}|\mathrm{Im}U_0|\Phi_0^{(+)}\right\rangle + \left\langle\Phi_d^{(+)}|\mathrm{Im}U_d|\Phi_d^{(+)}\right\rangle\right] \\ & + \pi\frac{k}{E}\left\langle\Phi_0^{(+)}|V_{0d}\Omega_d^{(-)}\frac{\left(\frac{\Gamma_d^{\downarrow}}{2}\right)/\pi}{(E - E_d - K_d)^2 + \left(\frac{\Gamma_d^{\downarrow}}{2}\right)^2} \right. \\ & \left. \times\Omega_d^{(-)\dagger}V_{0d}|\Phi_0^{(+)}\right\rangle ,\end{aligned} \tag{56}$$

where we have used

$$|\,\Phi_d^{(+)}\rangle = G_d^{(+)}V_{0d}\,|\,\Phi_0^{(+)}\rangle \ . \tag{57}$$

Equation (57) follows immediately from the recognition that (45) represents the total reaction cross section of a two-coupled-channels set of equations:

$$\begin{aligned}(E - K_0 - U_0)\Phi_0^{(+)} &= V_{0d}\Phi_d^{(+)} \ , \\ \left(E - E_d + i\frac{\Gamma_d^{\downarrow}}{2} - K_d - U_d\right)\Phi_d^{(+)} &= V_{0d}\Phi_0^{(+)} \ ,\end{aligned} \tag{58}$$

where we have set $E_0 = 0$.

Clearly, the second term on the right-hand side of Eq. (56) represents the total angle-integrated inelastic cross section, σ_{in},

$$\begin{aligned}\sigma_{\mathrm{in}}(E) = \pi\frac{k}{E}\left\langle\Phi_0^{(+)}\,|\,V_{0d}\Omega_d^{(-)}\frac{\Gamma_d^{\downarrow}/2\pi}{(E - E_d - K_d)^2 + \left(\Gamma_d^{\downarrow}/2\right)^2}\right. \\ \left.\times\Omega_d^{(-)\dagger}V_{0d}\,|\,\Phi_0^{(+)}\right\rangle ,\end{aligned} \tag{59}$$

while the first term of that equation is identified with the total fusion cross section which includes the coupling to the exit doorway:

$$\sigma_F = \frac{k}{E}\left[\left\langle\Phi_0^{(+)}\,|\,\mathrm{Im}U_0\,|\,\Phi_0^{(+)}\right\rangle + \left\langle\Phi_d^{(+)}\,|\,\mathrm{Im}U_d\,|\,\Phi_d^{(+)}\right\rangle\right] . \tag{60}$$

Equation (60) is the principal result of this section. It shows that the influence of, e.g., a giant resonance, treated as an exit doorway, on the fusion of two nuclei is the same as that of a normal excited state, except that the Q value is complex, $Q_d = E_d - \frac{i\Gamma_d}{2}$. Thus the two-channel model treated in the previous section can be applied here as well with an appropriate change in the diagonalization procedure.

IV. SCHEMATIC MODEL FOR A GIANT RESONANCE EFFECT ON σ_F

In this section we analyze Eq. (60) following the procedure used in Sec. II. We take $V_{0d} = V_{d0} = v =$ const. We also take $K_d = K_0$ and $U_d = U_0$ in Eq. (58) which can be rewritten as

$$\begin{aligned}(E - K_0 - U_0)\Phi_0^{(+)} &= v\Phi_d^{(+)} \ , \\ (E - K_0 - U_0)\Phi_d^{(+)} &= v\Phi_0^{(+)} + \left(E_d - \frac{i\Gamma_d^{\downarrow}}{2}\right)\Phi_d^{(+)} \ .\end{aligned} \tag{61}$$

The coupling matrix C that has to be diagonalized is now non-Hermitian,

$$C = \begin{pmatrix} 0 & v \\ v & E_d - \frac{i\Gamma_d^{\downarrow}}{2} \end{pmatrix} . \tag{62}$$

We can perform the diagonalization by using a biorthogonal basis [14] which is a generalized version of the (χ_+, χ_-) basis employed in Eq. (16). Thus

$$\begin{pmatrix} 0 & v \\ v & Q_d \end{pmatrix} = (\chi_+\chi_-)\begin{pmatrix} \lambda_+ & 0 \\ 0 & \lambda_- \end{pmatrix}\begin{pmatrix} \tilde{\chi}_+^{\dagger} \\ \tilde{\chi}_-^{\dagger} \end{pmatrix} , \tag{63}$$

with

$$\begin{aligned}(\chi_+, \tilde{\chi}_+^{\dagger}) = 1 &= (\chi_-, \tilde{\chi}_-^{\dagger}) \ , \\ (\chi_+, \tilde{\chi}_-^{\dagger}) = 0 &= (\chi_-, \tilde{\chi}_+^{\dagger}) \ ,\end{aligned} \tag{64}$$

and

$$\lambda\pm = \frac{1}{2}\left[Q_d \pm \sqrt{Q_d^2 + 4v^2}\right] , \tag{65}$$

where

$$Q_d = E_d - i\frac{\Gamma_d^{\downarrow}}{2}.$$

The rest of the discussion is exactly the same as in Sec. II; the matrix M, Eq. (21), has exactly the same structure, with $\lambda\pm$ given now by Eq. (65). The tunneling action that enters in the definition of the eigentransmission coefficient is now given by

$$I = 2\mathrm{Re}\int_{r_0^{\pm}}^{r_1^{\pm}} k_{\pm}^{l}(r)\,dr \ . \tag{66}$$

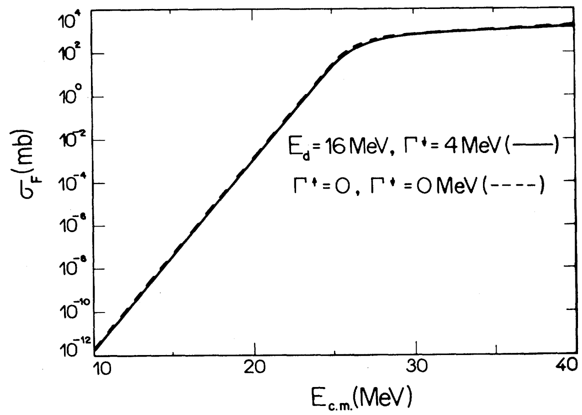

FIG. 1. Fusion cross section vs $E^{c.m.}$ for ^{11}Li + ^{208}Pb for $E_d = 16$ MeV and $\Gamma_d^{\downarrow} = 4$ MeV (solid curve) and $\Gamma_d^{\downarrow} = 0$ MeV (dashed curve).

The final formula for σ_F becomes

$$\sigma_F = A(\lambda_+)\ \overset{\circ}{\sigma}{}_F^{+} + A(\lambda_-)\ \overset{\circ}{\sigma}{}_F^{-}\ ,$$
$$A(\lambda) = \left[\frac{v^4}{|\lambda^2 + v^2|^2} + \frac{|\lambda v|^2}{|\lambda^2 + v^2|^2}\right]\ . \tag{67}$$

The eigenchannel fusion cross sections, in the Wong approximation, are given by [11]

$$\overset{\circ}{\sigma}{}_F^{\pm} = \frac{\hbar\omega R_B^2}{2E} \ln\left[1 + \exp\frac{2\pi}{\hbar\omega}\left[E - \mathrm{Re}\lambda_{\pm} - V_B(R_R)\right]\right]\ . \tag{68}$$

Equation (67) is the generalization of Eq. (27) to the case of coupling to a resonant state. The finite width of the resonance effectively reduces the Q value effect and thus the ratio

$$\frac{\sigma_F(\Gamma_d^{\downarrow})}{\sigma_F(\Gamma_d^{\downarrow} = 0)} = E(\Gamma_d^{\downarrow})$$

should be larger than unity, for a fixed value of the position of the resonance, E_d, and the strength of the coupling, v. To be specific, we consider the system ^{11}Li + ^{208}Pb, which has been recently discussed in several papers [4,5], the barrier height and curvature were taken to be 26.0 MeV and 3.0 MeV, respectively [4]. We consider the doorway to be the normal giant dipole resonance of the core (^{9}Li), whose excitation energy is $E_d \simeq 16$ MeV. We take for $v = 3$ MeV [4]. Because of the very high Q value, the effect of the coupling on σ_F is expected to be very small, and accordingly the effect of $\Gamma_d^{\downarrow}$ to be negligible. In Fig. 1 we show σ_F, Eq. (67), calculated with $\Gamma_d^{\downarrow} = 0$ and $\Gamma_d^{\downarrow} = 4$ MeV. Both results almost coincide with each other and with the no-coupling case (not shown in the figure). To exhibit the effect of $\Gamma_d^{\downarrow}$ more clearly we show in Fig. 2 the ratio $E(\Gamma_d^{\downarrow})$ for these cases plotted versus $E_{c.m.}$. As E_d is lowered σ_F is increased when $\Gamma_d^{\downarrow}$ is taken into account. This is expected on physical grounds since the resonance is reached even if the energy transfer is smaller than E_d. As we see clearly in the figure, the effect is basically restricted to $E_{c.m.} < V_B$.

V. EFFECT OF THE ESCAPE WIDTH

In our discussion so far we have considered only the spreading width of the doorway. The approximation $\Gamma_{GR} \sim \Gamma_{GR}^{\downarrow}$ is quite reasonable in heavy nuclei such as ^{208}Pb. For light nuclei the opposite limit is usually

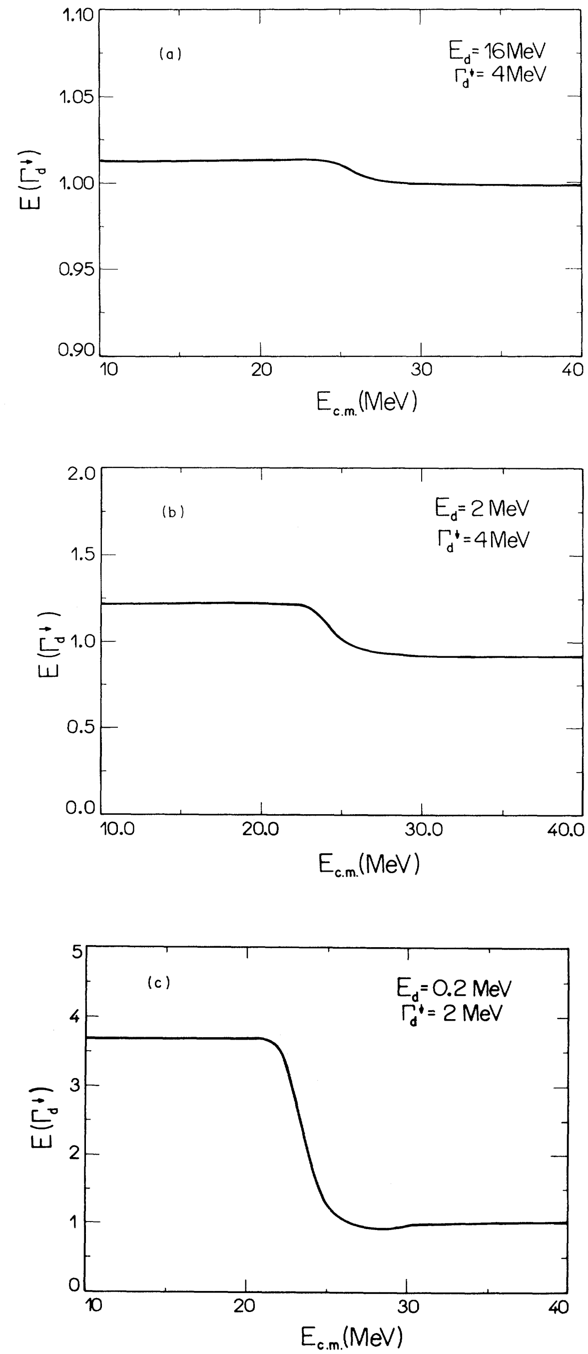

FIG. 2. Ratio $E(\Gamma_d^{\downarrow}) \equiv \frac{\sigma_F(\Gamma_d^{\downarrow})}{\sigma_F(\Gamma_d^{\downarrow}=0)}$ vs $E_{c.m.}$ for the system ^{11}Li + ^{208}Pb, for $\Gamma_d^{\uparrow} = 0$ and for different values of E_d. See text for details.

attained, $\Gamma_{GR} \sim \Gamma_{GR}^{\uparrow}$. In fact the soft giant dipole resonance in ^{11}Li has a 100% escape width since complex excited states in the vicinity of the resonance do not exist. It is of importance therefore to consider the effect of $\Gamma_{GR}^{\uparrow}$ on the fusion cross section. For simplicity we assume the giant resonance escapes by coupling to one channel which we call "breakup" channel. The wave function of this three-body channel (e.g., ^{9}Li+2n+^{208}Pb) is denoted by $\Psi_b^{(+)}$.

We assume that this channel is reached directly from the ground state and indirectly via the doorway. The set of equations (9), is now modified to read

$$[E - K_0 - U_0 - V_0^{\rm pol}(b)]\Psi_0^{(+)} = V_{0d}\Psi_d^{(+)} \ , \qquad (69)$$

$$[E - K_d - U_d - V_d^{\rm pol}(b)]\Psi_d^{(+)} = V_{d0}\Psi_0^{(+)} + \left(E_d - \frac{i\Gamma_d^{\downarrow}}{2}\right)\Psi_d^{(+)} \ , \qquad (70)$$

where we have introduced the usual dynamic polarization potential that accounts for the coupling of $\Psi_0^{(+)}$ to $\Psi_b^{(+)}$ and $\Psi_d^{(+)}$ to $\Psi_b^{(+)}$. In deriving Eq. (24) we have employed the approximation $V_0^{\rm pol}(b) \equiv V_{0b}G_b^{(+)}V_{b0}$ and $V_d^{\rm pol}(b) = V_{db}G_b^{(+)}V_{bd}$, where $G_b^{(+)}$ represents the propagation in the breakup channel. The polarization potential $V_0^{\rm pol}(b)$ has been calculated in Refs. [3,15] for ^{11}Li + ^{208}Pb. It was concluded that Re$V_0^{\rm pol}(b)$ is repulsive and Im$V_0^{\rm pol}(b)$ is absorptive and of long-range nature for a Q value of $\sim$ 0.2 MeV. Both of these properties would tend to reduce the amplitude in $\Phi_0^{(+)}$ and accordingly the fusion cross section. Similar conclusions may be reached concerning $V_0^{\rm pol}(b)$ except for the Q value. If the breakup channels are in the vicinity of E_d, the Q value that enters in $V_d^{\rm pol}(b)$ would be roughly related to $\Gamma_d^{\downarrow}$ alone. In contrast, $V_0^{\rm pol}(b)$ would contain a hindrance due to a large Q value roughly equal to E_d itself. Therefore, depending on the value of $E_d \approx E_b$, the roles of $V_0^{\rm pol}(b)$ and $V_d^{\rm pol}(b)$ will be different.

In cases involving large Q values, such as those related to the normal giant resonance excitation and its subsequent fragmentation, the polarization potential $V_0^{\rm pol}(b)$ at subbarrier energies and for the dipole case at hand, and ignoring nuclear excitation, can be written in closed form [16]

$$V_0^{\rm pol}(b) = -6.7\times 10^{-3}\left[\frac{N_p}{Z_pA_p^{1/3}} + \frac{N_T}{Z_TA_T^{1/3}}\right] Z_p^2Z_T^2/r^4 \ [\text{MeV}] \ , \qquad (71)$$

where p and T refer to projectile and target, respectively. Thus $V_0^{\rm pol}(b)$ contributes very little attraction due to the virtual excitation of the isovector giant dipole resonance in both target and projectile. In contrast $V_d^{\rm pol}(b)$, with the doorway state sitting close to the b channel, the result of Ref. [15] is applicable and one finds a repulsive, absorptive polarization potential. This implies that, effectively, there is a very small increase in the fusion from the entrance channel and a more significant decrease in the fusion from the doorway. However, since the Q value is large, these details will be hardly detected.

In the other extreme of very small Q value such as the case encountered in the breakup of ^{11}Li, both $V_0^{\rm pol}(b)$ and $V_d^{\rm pol}(b)$ should be repulsive, absorptive, and long ranged. In principle, Im$V_d^{\rm pol}(b)$ is related to $\Gamma_d^{\downarrow}$ and, naively speaking, this latter should be added to $\Gamma_d^{\downarrow}$ to obtain the total width of the doorway resonance that appears in Eq. (69). However, this is completely misleading since Im$V_d^{\rm pol}(b)$ and thus $\Gamma_d^{\uparrow}$ describe the actual *loss* of the projectile (or target), whereas $\Gamma_d^{\downarrow}$ describes its survival. In the fusion process the effect of the breakup of one of the partners naturally leads to a reduction of the cross section [4]. This is so, since, as said above, the breakup couplings lead to a repulsive real part and an absorptive imaging part of $V_d^{\rm pol}(b)$. Both of these lead to lower penetrabilities at energies in the vicinity of the Coulomb barrier.

Since $V^{\rm pol}$ is generally small compared to other potentials in the problem and is of longer range, its effect can be expressed as a damping factor. In Refs. [4,5] it was shown that σ_F can be written as (after approximating $V^{\rm pol}$ by its local equivalent version; see Ref. [15] for details)

$$\sigma_F = \frac{\pi}{k^2}\sum_\ell (2\ell+1)T_\ell\,(V_{\rm pol}=0) \times \exp\left[\frac{-2}{\hbar}\int_0^\infty {\rm Im}V_{\rm pol}\,dt\right] \ . \qquad (72)$$

It should be easy to convince oneself that the breakup survival probability $\exp\left[\frac{-2}{\hbar}\int_0^\infty {\rm Im}V_{\rm pol}\,dt\right]$ involves an appropriate energy scale $\Gamma_d^{\uparrow}$ and an appropriate time scale, the effective collision time τ_c. Thus we write $2\int_0^\infty {\rm Im}V_d^{\rm pol}\,dt = \Gamma_d^{\uparrow}\tau_c(\ell)$.

The treatment of $V_0^{\rm pol}(b)$ follows exactly similar steps as above (the Q values in both cases are roughly equal), the difference residing in higher-order effects in $V_d^{\rm pol}(b)$.

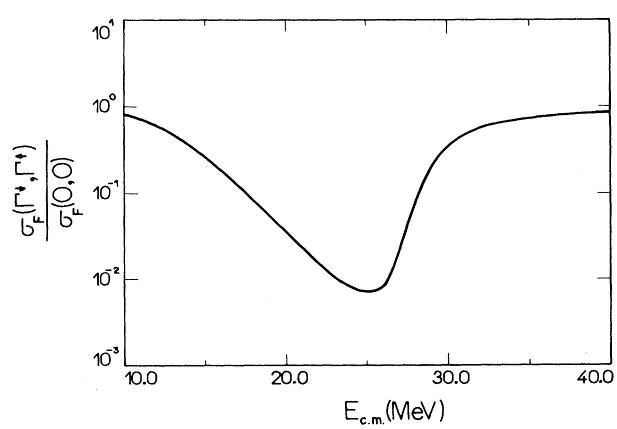

FIG. 3. Ratio $\frac{\sigma_F(\Gamma_d^{\downarrow},\Gamma_d^{\uparrow})}{\sigma_F(\Gamma^{\downarrow}_d=0,\Gamma_d^{\uparrow}=0)}$ for the system ^{11}Li + ^{208}Pb, $\Gamma^{\downarrow}$ = 2 MeV, $\Gamma^{\uparrow}$ = 1 MeV, and E_d = 0.2 MeV. See text for details.

Thus we also write $2\int_0^\infty \mathrm{Im}V_0^{\mathrm{pol}}\,dt \equiv \Gamma_0^\uparrow \tau_c(\ell)$, where $\Gamma_0^\uparrow$ may be called the "channel escape width." For simplicity we set $\Gamma_0^\uparrow = \Gamma_d^\uparrow$. We now introduce the mixing parameter considered earlier in the study of the decay of giant resonances [17], $\mu \equiv \frac{\Gamma_d^\downarrow}{\Gamma_d^\uparrow+\Gamma_d^\downarrow}$. Thus for a fixed Γ_d, $\Gamma_d^\uparrow = (1-\mu)\Gamma_d$, $\Gamma_d^\downarrow = \mu\Gamma_d$, we have for the fusion cross section

$$\sigma_F(\mu) = \frac{\pi}{k^2}\sum_{\ell=0}^{\infty}(2\ell+1)\left\{\frac{A(\lambda_+)e^{-(1-\mu)\frac{\Gamma_d}{\hbar}\tau_c(\ell)}}{1+\exp\left\{\frac{2\pi}{\hbar\omega}\left[V_B+\mathrm{Re}\lambda_+(\mu)+\frac{\hbar^2\ell(\ell+1)}{2\mu R_B^2}-E\right]\right\}} + \frac{A(\lambda_-)e^{-(1-\mu)\frac{\Gamma_d}{\hbar}\tau_c(\ell)}}{1+\exp\left\{\frac{2\pi}{\hbar\omega}\left[V_B+\mathrm{Re}\lambda_-(\mu)+\frac{\hbar^2\ell(\ell+1)}{2\mu R_B^2}-E\right]\right\}}\right\}. \quad (73)$$

The collision time $\tau_c(l)$ can be calculated using the result of Ref. [16]; namely, if we write for the equivalent l-independent $\mathrm{Im}V^{\mathrm{pol}}(r) \cong W_0 e^{-2r/\alpha}$, where α is related to the Q value of breakup and expand $r(+)$ around the classical turning point $r(0)$, we find $r(t) \simeq r_l(0) + \frac{1}{2}a_l(0)t^2$, where $a_l(0)$ is the radial acceleration at $r_l(0)$. We then find

$$\tau_c(l) \simeq \sqrt{\frac{\pi\alpha}{a_l(0)}}\, e^{-2(r_l(0)-r_{l=0}(0))/\alpha} \quad (74)$$

and therefore

$$\Gamma^\downarrow = W_0 e^{-2r_{l=0}(0)/\alpha}\ . \quad (75)$$

In deriving Eqs. (74) and (75) we have assumed a pure Rutherford trajectory for the relative motion of the colliding nuclei. Then $r_l(0) = \frac{Z_1Z_2e^2}{2E}\left[1+\frac{l(l+1)}{\eta^2}\right]^{1/2}$ and $a_l(0) = \frac{2}{\mu}\left[\frac{Z_1Z_2e^2}{r_l(0)} + \frac{\hbar^2 l(l+1)}{2\mu r(0)}\right]$, where η is the Sommerfeld parameter, $\eta = \frac{Z_1Z_2e^2}{2E}k$.

In Fig. 3, we show the ratio $\frac{\sigma(\Gamma_d^\downarrow,\Gamma_d^\uparrow)}{\sigma(\Gamma_d^\downarrow=0,\Gamma_d^\uparrow=0)}$ for ^{11}Li + ^{208}Pb taking for $E_d = 0.2$ MeV. We took $\Gamma_d^\downarrow = 2$ MeV and $\Gamma_d^\uparrow = 1$ MeV. It is clear that now the fusion is strongly hindered, by a factor of 100 in the barrier region. Thus the effect of $\Gamma_d^\uparrow$ is much more important than that of $\Gamma_d^\downarrow$. Considering now the realistic version of the soft dipole mode in ^{11}Li, its width is totally escape (to the $2n$ + ^{9}Li channel) and thus the fusion of ^{11}Li is hindered [4,18]. Finally, we mention that the formal manipulation used on Eq. (69) to reach the final general result, Eq. (73), is based on the observation that by defining the reduced wave functions

$$\Phi_0 = C_0\Psi_0\ ,\qquad \Phi_d = C_d\Psi_d\ , \quad (76)$$

one has the freedom in choosing the functions C_0 and C_d to be such that the following equations are satisfied [see Eq. (58)]:

$$(E-K_0-U_0)\Phi_0 = V_{0d}\Phi_d\ ,\qquad (E-K_d-U_d)\Phi_d = V_{d0}\Phi_0 + \left(E_d - i\frac{\Gamma_d^\downarrow}{2}\right)\Phi_d\ . \quad (77)$$

The simplest version of the WKB approximation (assume a predominance of Coulomb repulsion) would give, asymptotically,

$$C_0 = \exp\left[-\frac{i}{\hbar}\int_0^\infty V_0^{\mathrm{pol}}(r(t))\,dt\right],\qquad C_d = \exp\left[-\frac{i}{\hbar}\int_0^\infty V_d^{\mathrm{pol}}(r(t))\,dt\right]. \quad (78)$$

Thus one first diagonalizes Eq. (77) and then inserts (76) with (78) in the formula for σ_F to obtain the desired relation. Clearly, a lot of room is available for improvements.

VI. CONCLUSION

In conclusion, we have developed in this paper a reaction theory that enables one to study the influence of the coupling to a resonant channel on the heavy-ion fusion cross section σ_F. In particular, the effect of the finite width of the resonance, which is excited in one of the partners, on the Coulomb barrier penetrability is comprehensively investigated. It is found that the damping width only mildly enhances σ_F at subbarrier energies, whereas the escape width strongly hinders it, when the Q value is small. Applications were made to the system ^{11}Li + ^{208}Pb. The hindrance in σ_F of this system was found to be as large as a factor of 100 at $E \sim V_B$. It would be of great interest to verify this finding experimentally. Further, a more detailed numerical calculation that solves Eq. (69), without the approximations used in our schematic model, is called for. Work in this direction is in progress.

This work was supported in part by funds provided by the U.S. Department of Energy (D.O.E.) under cooperative research agreement DE-FC02-94ER40818, and by the CNPq and FAPESP (Brazil). M.P.P. was partly supported by ECT*-Trento.

APPENDIX A: DOORWAY PROPAGATOR FOR LORENTZIAN LINE SHAPE

When a Lorentzian line shape, Eq (42), is used to define the doorway propagator $G_d^{(+)}$ [see Eq. (46)], one has to evaluate the convolution integral

$$G_d^{(+)}(E) = \int_0^\infty dE' \frac{1}{E - E' - K_d - U_d + i\epsilon} \frac{2}{\pi} \times \frac{\Gamma_d E'^2}{(E'^2 - e_d^2)^2 + \Gamma_d^2 E'^2} . \quad \text{(A1)}$$

The Lorentzian weight factor in the integral has in general four simple poles in the complex E' plane. In the case of "narrow" Lorentzians, in the sense that $\Gamma_d < 2e_d$, the poles are located at $\text{Re}E' = \pm E_d = \pm \left[e_d^2 - \frac{\Gamma_d^2}{4}\right]^{1/2}$, $\text{Im}E' = \pm\frac{\Gamma_d}{2}$; when $\Gamma_d = 2e_d$, they coalesce to one pair of double poles on the imaginary axis; and for "wide" Lorentzians, $\Gamma_d > 2e_d$, there are again four simple poles but now on the imaginary axis. Two of these poles coalesce at the origin when $e_d \to 0$.

Consider first the "narrow" case. Here a contour encircling the pole at $E_d - i\frac{\Gamma_d}{2}$ clockwise can be deformed so as to include the positive real axis [as required for (A1)], in addition to the negative imaginary axis and a quarter circle at infinity. The latter part of the deformed contour does not contribute to the integral, so that (A1) can be expressed in terms of the pole contribution and of an integral along the negative imaginary axis. The result is

$$G_d^{(+)}(E)= \left(1 - \frac{i\Gamma_d}{2E_d}\right) \frac{1}{E - K_d - U_d - E_d + i\frac{\Gamma_d}{2}} + \Delta G_d^{(+)}(E) , \quad \text{(A2)}$$

with

$$\Delta G_d^{(+)}(E)= -\frac{2\Gamma_d}{\pi} \int_0^\infty dy \frac{1}{y + \epsilon - i(E - K_d - U_d)} \times \frac{y^2}{(y^2 + e_d^2)^2 - \Gamma_d^2 y^2} . \quad \text{(A3)}$$

The quantitative importance of (A3) grows with the proximity of the poles to the imaginary axis. Qualitatively, this term accounts for a nonexponential correction to the time decay of the doorway. This can be seen by evaluating the time correlation amplitude $\langle d(0)|d(t)\rangle$ with

$$|d(t)\rangle = \sum_i e^{\frac{-i}{\hbar} E_i t} |i\rangle \alpha_i .$$

For the case of a Breit-Wigner line shape this gives the usual exponential decay law

$$\langle d(0)|d(t)\rangle \xrightarrow{\text{BW}} e^{\frac{-i}{\hbar} E_d t - \frac{\Gamma_d^\downarrow t}{2\hbar}} ,$$

while for the Lorentzian line shape a procedure analogous to that leading to (A2) gives

$$\langle d(0)|d(t)\rangle \xrightarrow{L} \left(1 - \frac{i\Gamma_d}{2E_d}\right) e^{\frac{-i}{\hbar} E_d t - \frac{\Gamma_d^\downarrow t}{2\hbar}} + \Delta C(t) , \quad \text{(A4)}$$

with

$$\Delta C(t) = -\frac{2i}{\pi E_d} \text{Im} \left[e^* \text{ci} \frac{e^* t}{\hbar} \sin \frac{e^* t}{\hbar} - e^* \text{si} \frac{e^* t}{\hbar} \cos \frac{e^* t}{\hbar} \right] , \quad \text{(A5)}$$

where $e^* = E_d - i\frac{\Gamma_d}{2}$, and ci(si)($x$) are the cosine and sine integrals:

$$\text{ci(si)}(x) = -\int_x^\infty du \frac{\cos(\sin)(u)}{u} .$$

When $t \to 0$ the quantity $\Delta C(t)$ approaches the limit $i\frac{\Gamma_d}{2E_d}$ as it should. Furthermore, a simple formal relationship exists between Eqs. (A2) and (A4). It can be expressed as

$$G_d^{(+)}(E) = \frac{-i}{\hbar} \int_0^\infty dt \, e^{\frac{i}{\hbar}(E + i\epsilon)t} \langle d(0)|d(t)\rangle e^{\frac{-i}{\hbar}(K_d + U_d)t} , \quad \text{(A6)}$$

showing in particular that (A6) is in fact related to $\Delta C(t)$, Eq. (A5).

The case of "wide" (i.e., $\Gamma_d > 2e_d$) Lorentzians is best handled in terms of an extension of Eq. (A6) to this case. Equation (A4) is now replaced by

$$\langle d(0)|d(t)\rangle \xrightarrow[\Gamma_d > 2e_d]{L} \frac{\Gamma_d}{\pi} \sum_{n=1}^{2} \frac{A_n}{\lambda_n} \left\{ e^{\frac{-\lambda_n t}{\hbar}} \left[\pi - i\,\text{li}(e^{\frac{\lambda_n t}{\hbar}})\right] + i e^{\frac{\lambda_n t}{\hbar}} \text{li}(e^{\frac{-\lambda_n t}{\hbar}}) \right\} , \quad \text{(A7)}$$

where

$$\lambda_n = \frac{\Gamma_d}{2} - (-1)^n \sqrt{\frac{\Gamma_d^2}{4} - e_d^2} , \qquad A_n = -(-1)^n \frac{\lambda_n^2}{\Gamma_d \sqrt{\Gamma_d^2 - 4e_d^2}} , \quad \text{(A8)}$$

and li(x) is the logarithmic integral $\int_0^x \frac{dt}{\ln t}$. Now there are two exponential decay constants λ_n which correspond to the distances of the poles (which lie on the imaginary axis) to the real axis in addition to nonexponential terms involving the logarithmic integral.

We turn next to the consequences of the above changes for the fusion calculation of Secs. III and IV. The situation considered there corresponds to the "narrow" case, so that the relevant Green's function is that given by Eq. (A2). First write $\Delta G_d^{(+)}$ as

$$\Delta G_d^{(+)}(E)= f_d(E) \frac{1}{E - E_d + i\frac{\Gamma_d}{2} - K_d - U_d} \equiv f_d(E) G_{d\text{BW}}^{(+)}(E) , \quad \text{(A9)}$$

where the operator $f_d(E)$ is

$$f_d(E) = \frac{2\Gamma_d}{\pi} \int_0^\infty dy \frac{E_d - \frac{i\Gamma_d}{2} - (E - K_d - U_d)}{y + \epsilon - i(E - K_d - U_d)} \times \frac{y^2}{(y^2 + e_d^2)^2 - \Gamma_d^2 y^2} . \quad \text{(A10)}$$

This operator commutes with $G^{(+)}_{d\mathrm{BW}}(E)$, since both objects are functions of K_d+U_d. We are allowed to express $G^{(+)}_d(E)$ as

$$G^{(+)}_d(E)=\left[1-\frac{i\Gamma_d}{2E_d}+f_d(E)\right]^{1/2}\times G^{(+)}_{d\mathrm{BW}}(E)\left[1-\frac{i\Gamma_d}{2E_d}+f_d(E)\right]^{1/2},$$

so that Eq. (48) becomes, in the case of a "narrow" Lorentzian doorway,

$$\sigma_R=\frac{k}{E}\langle\Phi_0^{(+)}|-\mathrm{Im}[U_0+\tilde{V}_{0d}G^{(+)}_{d\mathrm{BW}}(E)\tilde{V}_{d0}]|\Phi_0^{(+)}\rangle, \qquad \text{(A11)}$$

where we have introduced the modified couplings

$$\tilde{V}_{0d}=V_{0d}\left[1-\frac{i\Gamma_d}{2E_d}+f_d(E)\right]^{1/2},$$

$$\tilde{V}_{d0}\equiv\left[1-\frac{i\Gamma_d}{2E_d}+f_d(E)\right]^{1/2}V_{d0}. \qquad \text{(A12)}$$

Equation (A11) may now be recognized as the total reaction cross section of the two-coupled-channels equations (58) with the coupling potentials replaced by Eqs. (A12). Note that $\tilde{V}^+_{0d}\neq\tilde{V}_{d0}$ in this case, on account of the non-Hermitean character of the square root factor.

A numerical evaluation of Eq. (A4) for $E_d=16$ MeV, $\Gamma_d=4$ MeV shows that the time scale for the decay of $\Delta C(t)$ is $\sim 0.027\hbar$ MeV$^{-1}\ll 2\hbar/\Gamma_d$. This suggests that $f_d(E)$ may in this case be ignored as an approximation, so that the non-Hermitean character of the coupling reduces essentially to the c-number factor $[1-i\Gamma_d/2E_d]^{1/2}\simeq 1-0.0625i$.

[1] (a) See, e.g., M. Beckerman, Rep. Prog. Phys. **51**, 1047 (1988); (b) R. G. Stokstad and E. E. Gross, Phys. Rev. C **23**, 281 (1981); (c) H. Esbensen, Nucl. Phys. **A352**, 147 (1981); (d) R. A. Broglia, C. H. Dasso, S. Landowne, and A. Winther, Phys. Rev. C **27**, 2433 (1983); (e) H. J. Krappe, K. Mohring, M. C. Nemes, and H. Rossner, Z. Phys. A **314**, 23 (1983); (f) A. B. Balantekin, S. E. Koonin, and J. W. Negele, Phys. Rev. C **28**, 1565 (1983); (g) A. B. Balantekin and N. Takigawa, Ann. Phys. (N.Y.) **160**, 441 (1985).

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[18] We define fusion as that process which involves the absorption of the whole projectile. In Ref. [5], the fusion cross section was presented as the sum of the fusion of ^{11}Li + ^{208}Pb plus that involving the breakup channel, ^{9}Li + ^{208}Pb. If this is done, the hindrance is obviously removed.