Quasielastic knockout of clusters from p-shell nuclei by 1 GeV protons: Spectroscopic amplitudes of virtually excited clusters and the eikonal approximation

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A calculation technique of spectroscopic amplitudes of the lightest virtually excited clusters d, t, and α in p-shell nuclei is presented. These amplitudes are incorporated into a cluster quasielastic knockout theory based on the Glauber-Sitenko multiple scattering formalism where deexcitation of virtual clusters is taken into account and the outgoing cluster distorted wave is described by means of an eikonal approximation. Numerical calculations for a few target nuclei and knocked-out clusters complement our previous results and show that the strong influence of deexcitation effects is a general property of the reactions investigated. It is exemplified by angular anisotropy of the recoil momentum distributions, etc.

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I. INTRODUCTION

The reaction of cluster quasielastic knockout from light nuclei by various projectiles remains to an area of active experimental and theoretical research [1–3]. It was demonstrated in our previous paper [3] by means of generalized distorted-wave impulse approximation (DWIA) calculations in terms of Glauber-Sitenko multiple scattering theory that the process of cluster quasielastic knockout by 1 GeV protons is expected to show a series of significant original features reflecting the large influence of cluster deexcitation amplitudes when a virtual internally excited cluster changes its internal state to the ground one during the collision process. These features, predicted in our preliminary qualitative plane-wave considerations [4,5], are as follows: (i) strong Θ_q anisotropies of recoil momentum distributions (form factors) depending on the scattering angle of the fast proton (Θ_q and φ_q are angles which characterize the orientation of the recoil momentum $\mathbf{q}$ with respect to incident beam direction (z axis) and to the scattering plane $[\mathbf{p}_0;\mathbf{p}_0']$ of the fast proton, respectively), (ii) strong φ_q anisotropies also depending on the scattering angle of the fast proton, and (iii) recoil momentum distributions measured along some direction $\mathbf{q}$ defined by the values of Θ_q and φ_q depending both on the orientation of this "ray" and on the fast proton scattering angle.

The formal origin of all of the above features consists of the change of internal orbital momentum of the knocked-out cluster due to the collision which results in the rich interference of many recoil momentum amplitudes $\Psi_{nlm}(\mathbf{q})$ [4,5]. Of course, the general background here is that the sum of spectroscopic factors of the internally excited clusters dominate the ground-state one [6,5] and that the reaction process is of a very surface localized character [3].

A formal connection can be noted here [5] with the Treiman-Yang anisotropy in the quasielastic knockout reaction ^{2}H$(\pi^+,\pi^+p)n$, etc. [7,8], where this anisotropy observed at large recoil momenta $q \geq 300$ MeV/c originates from the influence of the third body n in the above kinematical region (strong three-body rescatterings in the final state obscuring the quasifree mechanism of proton knockout). However, our case is physically very different. We remain within the quasifree cluster knockout kinematical region (q values are small, $q \sim 100$ MeV/c), but our knocked-out particle is a composite one and can be internally rearranged with the accompanying change of internal orbital momentum.

It is clear from the aforesaid that our theory consists of three principal parts. The first part deals with cluster rearrangement (deexcitation) amplitudes calculated within the Glauber-Sitenko formalism. It is illuminated in detail in Refs. [3–5]. The algebra of spectroscopic amplitudes of virtually excited clusters in nuclei is just the subject of the second part, its general formulation being given in Refs. [5,6]. In the present paper, having in mind the lightest knocked-out clusters and ground-state shell-model configurations of both initial and final nuclei, we show that there exists a far-reaching simplification of the above complicated algebra which can be exposed in a very practical form. The third part is the treatment of cluster distorted waves. In Ref. [3], the numerical calculations were based on the partial-wave expansion of the knocked-out cluster as an outgoing distorted wave in combination with the use of proton plane waves, which appears to be a good approximation. However, this approach is still rather cumbersome practically, and in the present paper we formulate a more economical eikonal approximation version of the DWIA. It was successfully verified in Ref. [3] but was not presented here.

Finally, the last aim of our paper is to extend our previous DWIA calculations of the ^{12}C$(p,p\alpha)^8$Be$(0^+;2^+;4^+)$ reaction [3] to the reactions ^{12}C$(p,pt)^9$B, ^{14}N$(p,pd)^{12}$C, ^{16}O$(p,pt)^{13}$N, and ^{16}O$(p,p\alpha)^{12}$C, enabling us, jointly with the results of Ref. [3], to see some general features of the investigated phenomena in p-shell nuclei.

II. EIKONAL APPROXIMATION FORMALISM

Using the eikonal analytic expression of the knocked-out cluster distorted wave [9],

$$\chi^{(-)}(\mathbf{k},\mathbf{R}) = e^{-\gamma k R_N} e^{i(1+\beta-i\gamma)\mathbf{k}\cdot\mathbf{R}}, \qquad (1)$$

and the partial wave expansions of plane waves,

$$\exp[-i(1+\beta)\mathbf{k}\cdot\mathbf{R}] = 4\pi\sum_{lm}(-i)^l j_l((1+\beta)kR)Y_{lm}(\hat{\mathbf{R}})Y^*_{lm}(\hat{\mathbf{k}}),$$

side by side with the Taylor series,

$$\exp(\gamma\mathbf{k}\cdot\mathbf{R}) = \sum_{t=0}^{\infty}\frac{\gamma^t}{t!}(\mathbf{k}\cdot\mathbf{R})^t,$$

$$(\mathbf{k}\cdot\mathbf{R})^t = (kR)^t t!\sum_{l=1}^{t}N_{tl}\sum_{m=-l}^{l}Y_{lm}(\hat{\mathbf{k}})Y^*_{lm}(\hat{\mathbf{R}}),$$

where the value N_{tl} is presented below in Eq. (20), we can write the generalized Fourier transform of the bound-state cluster wave function [3],

$$\varphi_{n\Lambda M}(\mathbf{k},\mathbf{p}) = \int d\mathbf{R}\exp\left(-i\frac{m_{A-b}}{m_A}\mathbf{p}\cdot\mathbf{r}\right)\chi^{(+)}(\mathbf{k},\mathbf{R}) \mid n\Lambda M\rangle, \qquad (2)$$

where $\mathbf{p}\equiv\mathbf{p}_0-\mathbf{p}'_0$ is the momentum transferred by the fast proton, $\mathbf{k}=(m_{A-b}/m_A)\mathbf{p}+\mathbf{q}$ ($-\mathbf{q}$ being the recoil momentum of the A-b spectator nucleus) as

$$\begin{aligned}\varphi_{n\Lambda M}(\mathbf{k},\mathbf{p}) = {} & (4\pi)^{\frac{3}{2}}e^{-\gamma kR_N}\sum_{t=0}^{t}(\gamma k)^t\sum_{l=1}^{t}N_{tl}\sum(-i)^{l_1+l_2}F_{tn\Lambda l_1 l_2}(p,k)(2l_2+1)(2l+1)\sqrt{\frac{2l_1+1}{(2\Lambda+1)(2l_4+1)}}\\ & \times(l0,l_20\mid l_30)(l0,l_20\mid l_40)(l_30,l_10\mid\Lambda0)(lm,l_2m_2\mid l_3m_3)\\ & \times(lm,l_2m_2\mid l_4m_4)(l_3m_3,l_1m_1\mid\Lambda M)Y_{l_4m_4}(\hat{\mathbf{k}})Y_{l_1m_1}(\hat{\mathbf{p}}),\end{aligned} \qquad (3)$$

with the last summation running over l_1, l_2, l_3, l_4, m_1, m_2, m_3, m_4 and

$$F_{tn\Lambda l_1l_2}(p,k) = \int_0^\infty R^{t+2}\varphi_{n\Lambda}(R)j_{l_1}\left(\frac{m_{A-b}}{m_A}pR\right)j_{l_2}((1+\beta)kR)dR.$$

Next, in terms of Refs [4,5] the reaction amplitude looks like

$$M_{if}(\mathbf{q},\mathbf{p}) = \frac{ip'_0}{2\pi}\int d\rho\, e^{ip\rho}\int d\tau\,\Psi^*_{f,\mathbf{k}}\Omega\Psi_i, \qquad (4)$$

where $\boldsymbol{\rho}$ is the impact parameter (two-dimensional vector) and Ω the multiple scattering operator of Glauber-Sitenko theory. Using the detailed formulas of Refs. [3–5] and Eq. (3) above we obtain the square of the reaction amplitude averaged and summed over the proper angular momenta projections in the form [10]

$$\begin{aligned}\frac{1}{2J+1}\sum_1\mid M_{if}\mid^2 = {} & 4(p'_0)^2\binom{A}{b}e^{-2\gamma kR_N}\sum_2\alpha^{A,JT}_{[f]LS}\alpha^{A,JT}_{[f']L'S'}\\ & \times\sum_3\alpha^{A-b,J_1T_1}_{[f_1]L_1S_1}\alpha^{A-b,J_1T_1}_{[f'_1]L'_1S'_1}\sum_4\langle A\alpha[f]NLST\mid A-b\alpha_1[f_1]N_1L_1S_1T_1;n\Lambda;b\alpha_0[f_0]N_0L_0S_0T_0\{\mathcal{L}\}\rangle\\ & \times\langle A\alpha[f']NL'S'T\mid A-b\alpha_1[f'_1]N_1L'_1S'_1T_1;n'\Lambda';b\alpha'_0[f'_0]N'_0L'_0S_0T_0\{\mathcal{L}'\}\rangle\\ & \times(2J_1+1)(2j+1)(2J'_0+1)(2J_0+1)\sqrt{\frac{(2L+1)(2S+1)(2\mathcal{L}+1)(2L'+1)(2S'+1)(2\mathcal{L}'+1)}{2L'_0+1}}\\ & \times(-1)^{\Lambda+L'_0-J'_0-J_0-S_0-j}\begin{Bmatrix}\Lambda & L_0 & \mathcal{L}\\ S_0 & j & J_0\end{Bmatrix}\begin{Bmatrix}\Lambda' & L'_0 & \mathcal{L}'\\ S_0 & j & J'_0\end{Bmatrix}\begin{Bmatrix}L_1 & S_1 & J_1\\ \mathcal{L} & S_0 & j\\ L & S & J\end{Bmatrix}\\ & \times\begin{Bmatrix}L'_1 & S'_1 & J'_1\\ \mathcal{L}' & S_0 & j\\ L' & S' & J'\end{Bmatrix}(T_1M_{T_1},T_0M_{T_0}\mid TM_T)^2\sum_{t,t'=0}^{\infty}(\gamma k)^{t+t'}\\ & \times\sum_{l=1}^{t}\sum_{l'=1}^{t'}\frac{1}{(t-l)!!(t+l+1)!!(t'-l')!(t'+l'+1)!!}\sum_5 i^{l_1+l_2+l'_1+l'_2}F_{tn\Lambda l_1l_2}(p,k)F_{t'n'\Lambda'l'_1l'_2}(p,k)\\ & \times(-1)^{l'_2+l'_3}(2l+1)(2l_2+1)(2l'+1)(2l'_2+1)(2l_1+1)(2l'_1)\\ & \times(l0,l_20\mid l_30)^2(l_30,l_10\mid\Lambda0)(l'0,l'_20\mid l'_30)^2(l'_30,l'_10\mid\Lambda'0)\sum_6 B^{b,000[b]}_{N_0L_0M_0[f_0]}(p)B^{b,000[b]}_{N'_0L'_0M'_0[f_0]}(p)\\ & \times(l_3,l'_30\mid\tilde{l}0)(l_10,l'_10\mid\tilde{l}_10)\{Y_{\tilde{l}}(\hat{\mathbf{k}})\times Y_{\tilde{l}_1}(\hat{\mathbf{p}})\}_{\tilde{L}\tilde{M}}(-1)^{\tilde{L}}\sqrt{2\tilde{L}+1}\\ & \times\begin{Bmatrix}l_3 & l'_3 & \tilde{l}\\ l_1 & l_1 & l_1\\ \Lambda & \Lambda' & \tilde{L}\end{Bmatrix}\begin{Bmatrix}J'_0 & J_0 & \tilde{L}\\ \Lambda & \Lambda' & j\end{Bmatrix}\begin{Bmatrix}L_0 & L'_0 & \tilde{L}\\ J'_0 & J_0 & S_0\end{Bmatrix}(\tilde{L}\tilde{M},L_0M_0\mid L'_0M'_0)\end{aligned} \qquad (5)$$

and the reaction cross section

$$\frac{d^3\sigma}{d\Omega_p d\Omega_b E_p} = \frac{m_p}{\hbar^2}\frac{1}{2J+1}\sum_1 \mid M_{if}(\mathbf{q},\mathbf{p}) \mid^2 = \frac{m_p}{\hbar^2}\frac{1}{2J+1} \mid F_{if}(\mathbf{q},\mathbf{p}) \mid^2 \left(\frac{d\sigma_{pb}}{d\Omega_p}\right)_{\rm free} . \quad (6)$$

The first through sixth summations in formula (5) run over the indices M_J, M_{S_0}, M_{J_1}, $[f]LS$, $[f']L'S'$, $[f_1]L_1S_1$, $[f_1']L_1'S_1'$; $N_0\alpha_0[f_0]l_0J_0\Lambda n\mathcal{L}$, $N_0'\alpha_0'[f_0']L_0'J_0'\Lambda' n'\mathcal{L}'$, $l_1l_2l_3$, $l_1'l_2'l_3'$, and $\tilde{l}l_1\tilde{\hat{L}}\tilde{M}M_0M_0'$, respectively. The coefficients $\alpha^{A,JT}_{[f]LS}$ here characterize the intermediate (between LS and jj) coupling in p-shell nuclei. Spectroscopic amplitudes of internally excited clusters $\langle A\alpha \mid A-b\alpha_1,n\Lambda;b\alpha_0\rangle$ (see below) jointly with the cluster internal rearrangement amplitudes $B^{b,000[b]}_{N_0L_0M_0[f_0]}(p)$ (see Ref. [3]) define the contribution of various cluster deexcitation processes.

The first application of the above eikonal technique was exposed briefly in Ref. [11] on the example of the ^{12}C$(p,p\alpha)^8$Be(g.s.) reaction but a detailed independent verification confirming its good accuracy was given later [3].

III. SPECTROSCOPIC AMPLITUDES OF VIRTUALLY EXCITED d-, $t(h)$-, AND α-CLUSTER IN p-SHELL NUCLEI

The spectroscopic amplitudes of arbitrary virtually excited clusters in p-shell nuclei can be calculated by exploiting the method of Refs. [5,6,12] based on the use of creation and annihilation operators of the oscillator shell model. However, the general formalism is rather complicated. It will be shown below that if we have in mind shell-model configurations with a minimal possible number of oscillator quanta, $N = N_{\rm min}$, in both the initial and final nucleus and if we also deal with the lightest knocked-out clusters d, $t(h)$, and α, then the above formalism can be simplified very much and can be presented in a rather practical form.

Let us introduce the subsidiary integral [5,12]

$$I \equiv \langle \varphi_{(000)}(\mathbf{R}_A), AN^{\rm min}[f](\lambda\mu)LST \mid \varphi_{(000)}(\mathbf{R}_{A-b}), A-bN_1^{\rm min}[f_1](\lambda_1\mu_1) \times L_1S_1T_1 : \varphi_{n\Lambda}(\mathbf{R}_b)bN_2[f_2](\lambda_2\mu_2)L_2S_2T_2\mathcal{L}LST\rangle, \quad (7)$$

written in terms of the translationally invariant shell model (TISHM).

Integrating over $\mathbf{R}_A$ we obtain

$$I = (-1)^n\left(\frac{A}{A-b}\right)^{-n/2}\langle AN^{\rm min}[f](\lambda\mu)LST \mid A-bN_1^{\rm min}[f_1](\lambda_1\mu_1)L_1S_1T_1; n\Lambda, bN_2[f_2](\lambda_2\mu_2)L_2S_2T_2\{\mathcal{L}\}LST\rangle, \quad (8)$$

where the value of the simplest Talmi-Moshinsky-Smirnov (TMS) coefficient [5,12] is substituted. Alternatively, expressing the initial and final state wave functions via shell-model configurations by means of the Bethe-Rose-Elliott-Skyrme (BRES) theorem [5,12] we can write

$$I = \sum_{(\lambda_2'\mu_2')}\langle s^4p^{A-4}[f](\lambda\mu)LST \mid s^4p^{N_1}[f_1](\lambda_1\mu_1)L_1S_1T_1; p^{N_2+n}[f_2](\lambda_2'\mu_2')\mathcal{L}S_2T_2\rangle \times\langle p^{N_2+n}[f_2](\lambda_2'\mu_2')\mathcal{L}S_2T_2 \mid n\Lambda, bN_2[f_2](\lambda_2\mu_2)S_2T_2\rangle. \quad (9)$$

For the lightest clusters under consideration the Young scheme $[f_2]$ has only one row, $(\lambda_2\mu_2)$=$(N_2,0)$, and the summation in Eq. (9) is absent.

By comparison of Eqs. (8) and (9) we conclude that

$$\langle AN^{\rm min}[f](\lambda\mu)LST \mid A-bN_1^{\rm min}[f_1](\lambda_1\mu_1)L_1S_1T_1; n\Lambda bN_2[f_2](\lambda_2\mu_2)L_2S_2T_2\{\mathcal{L}\}\rangle = (-1)^n\left(\frac{A}{A-b}\right)^{\frac{n}{2}}\binom{A-4}{b}^{\frac{1}{2}}\binom{A}{b}^{-\frac{1}{2}}\langle p^{A-4}[f](\lambda\mu)LST \mid p^{A-b-4}[f_1](\lambda_1\mu_1)L_1S_1T_1, p^b[f_2](\lambda_2'\mu_2')\mathcal{L}S_2T_2\rangle \times\langle p^b[f_2](\lambda_2'\mu_2')\mathcal{L}S_2T_2 \mid n\Lambda, bN_2(\lambda_2\mu_2)L_2S_2T_2\rangle; \quad (10)$$

i.e., we can express the cluster spectroscopic amplitude on the left side (written down in terms of the TISHM) via the multiparticle fractional parentage coefficient (FPC) of the usual shell model and a specific cluster coefficient K_b [the last factor on the right-hand side (rhs) of Eq. (10)] which, in fact, does not depend on the quantum numbers S_2T_2. To be complete, the usual definition of the cluster spectroscopic amplitude is as follows [5,12]:

$$
\begin{aligned}
C^b_\Lambda(J_0,J_2) &= \begin{pmatrix} A \\ b \end{pmatrix}^{\frac{1}{2}} \langle \Psi_A \mid \Psi_{A-b}; n\Lambda\Psi_b\rangle \\
&= \begin{pmatrix} A \\ b \end{pmatrix}^{\frac{1}{2}} \sum_{\mathcal{L}} U(\Lambda L_2 J_0 S_2 : \mathcal{L} J_2)\langle TM_T \mid T_1 M_{T_1}, T_2 M_{T_2}\rangle \\
&\times \begin{Bmatrix} L_1 & S_1 & J_1 \\ \mathcal{L} & S_2 & j \\ L & S & J \end{Bmatrix} \sqrt{(2J_1+1)(2J_0+1)(2L+1)(2S+1)} \\
&\times \langle AN[f](\lambda\mu)\alpha LST \mid A-bN_1[f_1](\lambda_1\mu_1)\alpha_1 L_1 S_1 T_1; n\Lambda, bN_2[f_2](\lambda_2\mu_2)\alpha_2 L_2 S_2 T_2\{\mathcal{L}\}\rangle. \qquad (11)
\end{aligned}
$$

Returning to Eq. (10), our problem is to find the expression for the coefficient K_b. As the first step, its SU(3) scalar part can be factorized out:

$$
\begin{aligned}
K_b &= \langle p^{N_2+n}[f_2](\lambda_2'\mu_2')\mathcal{L} \mid n\Lambda(\mathbf{R}_b)bN_2[f_2](\lambda_2\mu_2)L_2\rangle \\
&= \langle (n0)\Lambda(\lambda_2\mu_2)L_2 \mid (\lambda_2'\mu_2')\mathcal{L}\rangle\langle p^{N_2+n}[f_2](\lambda_2'\mu_2') \mid \\
&\quad \times (n0), bN_2[f_2](\lambda_2\mu_2)\rangle, \qquad (12)
\end{aligned}
$$

the first factor being a scalar factor of the SU(3) Clebsh-Gordan coefficient. Here we have omitted the SU(3)⊃O(3) scheme projections K_2 and K in all expressions. They become useful when more than one wave function with the same $(\lambda\mu)$ and L values exists. For p-shell nuclei only one example of type $(\lambda\mu)=(22)$ $L=0$ takes place. It is not a subject of the investigation.

As the second step, we introduce projection operators. The hole states of the usual shell model,

$$\Psi = \mid s^\nu p^{A-\nu}[f](\lambda\mu)LST\rangle, \qquad (13)$$

called sp states (as far as they contain only s and p nucleons), have both true components with the zero c.m. oscillations and ghost states,

$$
\begin{aligned}
&\mid s^\nu p^{b-\nu}[f](\lambda_2\mu_2)LST\rangle = \\
&\qquad \Omega_A \varphi_{(000)}(\mathbf{R}_b) \mid bN = b-\nu[f](\lambda_2\mu_2)LST\rangle \\
&\qquad + \text{ghost states}, \qquad (14)
\end{aligned}
$$

where the $\mid bN = b-\nu[f](\lambda_2\mu_2)LST\rangle$ state entering the first term of Eq. (14) is the sp state of the TISHM.

The projection operator $\hat{P}_{000}$ eliminates the ghost states,

$$
\begin{aligned}
&\varphi_{(000)}(\mathbf{R}_b) \mid bN = b-\nu[f](\lambda_2\mu_2)LST\rangle \\
&\qquad = \frac{1}{\Omega_b}\hat{P}_{000} \mid s^\nu p^{b-\nu}[f](\lambda_2\mu_2)LST\rangle, \qquad (15)
\end{aligned}
$$

where the amplitude Ω_b can be calculated by means of the formula

$$\Omega_b^2 = \langle s^\nu p^{b-\nu}[f](\lambda_2\mu_2)LST \mid \hat{P}_{000} \mid s^\nu p^{b-\nu}[f](\lambda_2\mu_2)LST\rangle. \qquad (16)$$

It is helpful in the expression

$$
\begin{aligned}
&\varphi_{(000)}(\mathbf{R}_b) \mid bN = b-\nu[f](\lambda_2\mu_2)LST\rangle \\
&\qquad = \Omega_b \mid s^\nu p^{b-\nu}[f](\lambda_2\mu_2)LST\rangle + C_1 \mid s^{\nu+1}p^{b-\nu-2}(2s-2d):[f](\lambda_2\mu_2)LST\rangle + \cdots, \qquad (17)
\end{aligned}
$$

connecting the TISM wave function with the usual shell-model basis. Here, dots mean the highest shell-model configurations,

$$C_1 = \frac{1}{\Omega_b}\langle s^{\nu+1}p^{b-\nu-2}(2s-2d)[f](\lambda_2\mu_2)LST \mid \hat{P}_{000} \mid s^\nu p^{b-\nu}[f](\lambda_2\mu_2)LST\rangle, \qquad (18)$$

etc.

Defining the creation operator $\mathbf{U}_b^+$ as

$$\mathbf{U}_b^+ = \frac{1}{\sqrt{2}}\left(\frac{\mathbf{R}_b}{R_{0b}} - i\frac{\mathbf{P}_b}{P_{0b}}\right), \qquad (19)$$

$$
\begin{aligned}
\varphi_{nlm}(\mathbf{R}_b) &= (-1)^{(n-l)/2}\sqrt{4\pi}\frac{1}{[(n-l)!!(n+l+1)!!]^{\frac{1}{2}}}(\mathbf{U}_b^+\cdot\mathbf{U}_b^+)^{(n-l)/2}Y_{lm}(\mathbf{U}_b^+)\varphi_{000}(\mathbf{R}_b) \\
&\equiv N_{nl}(\mathbf{U}_b^+\cdot\mathbf{U}_b^+)^{(n-l)/2}Y_{lm}(\mathbf{U}_b^+)\varphi_{000}(\mathbf{R}_b), \qquad (20)
\end{aligned}
$$

we can write down the cluster coefficient K_b as

$$K_b = \Omega_b\langle p^{N_2+n}[f_2](\lambda_2'\mu_2')\mathcal{L}ST \mid N_{n\Lambda}(\mathbf{U}^+\cdot\mathbf{U}^+)^{(n-\Lambda)/2}Y_{\Lambda M}(\mathbf{U}^+) \mid s^n p^{N_2}[f_2](\lambda_2\mu_2)L_2ST\rangle, \qquad (21)$$

where $\mathbf{\Lambda} + \mathbf{L}_2 = \mathcal{L}$. The second and next terms of the expansion (17) give no contribution; only $0s \to 1p$ transitions contribute here. We can write the wave function entering the rhs of the matrix element as

$$| s^n; p^{N_2}(\lambda_2\mu_2)L_2 : [f_2](\lambda_2\mu_2)L_2ST\rangle = \sum_{S_1T_1S_2T_2} \langle[\tilde{n}]S_1T_1[\tilde{f}_{p_2}]S_2T_2 \,||\, [\tilde{f}_2]ST\rangle \times \left(\begin{array}{c} b \\ n_2 \end{array}\right)^{-\frac{1}{2}} \hat{A} \mid s^n[n]0S_1T_1; p^{N_2}[f_{p_2}]L_2S_2T_2 : LST\rangle, \quad (22)$$

with the operator $\hat{A}$ meaning antisymmetrization between separated shells and the first factor in the rhs being the SU(4) Clebsch-Gordan coefficient [i.e., the spin-isospin fractional parentage (FP) coefficient]. Subsituting expression (22) into (21) and having in mind that the operator entering (21) is permutationally symmetric and is commuting with $\hat{A}$ we obtain

$$K_b = \Omega_b \left(\begin{array}{c} b \\ N_2 \end{array}\right)^{\frac{1}{2}} \sum_{S_1T_1S_2T_2} \langle[\tilde{n}]S_1T_1[\tilde{f}_{p_2}]S_2T_2 \mid [\tilde{f}_2]ST\rangle\langle p^{N_2+n}[f_2](\lambda_2'\mu_2')\mathcal{L}ST \mid N_{n\Lambda}(\mathbf{U}^+\cdot\mathbf{U}^+)^{(n-\Lambda)/2}Y_{\Lambda M}(\mathbf{U}^+) \mid \times \mid s^n[n]0S_1T_1; p^{N_2}[f_{p_2}](\lambda_2\mu_2)L_2S_2T_2 : LST\rangle. \quad (23)$$

Extracting from the lhs by means of the FP expansion the subsystem of N_2 nucleons with numbers $n+1, ..., N_2+n$ we can integrate over these variables since they are not acted upon by the operator

$$\hat{R} \equiv N_{n\Lambda}(\mathbf{U}^+\cdot\mathbf{U}^+)^{(n-\Lambda)/2}Y_{\Lambda M}(\mathbf{U}^+), \quad (24)$$

standing in Eq. (23).

As a result,

$$K_b = \Omega_b \left(\begin{array}{c} N_2+n \\ n \end{array}\right)^{\frac{1}{2}} \sum_{S_1T_1S_2T_2} \langle[\tilde{n}]S_1T_1, [\tilde{f}_{p_2}]S_2T_2 \mid [\tilde{f}_2]ST\rangle \times\langle p^{N_2+n}[f_2](\lambda_2'\mu_2')\mathcal{L}ST \mid p^n[n](n0)\Lambda S_1T_1; p^{N_2}[f_{p_2}](\lambda_2\mu_2)L_2S_2T_2\rangle\langle p^n[n](n0)\Lambda S_1T_1 \mid \hat{R} \mid s^n[n](00)0S_1T_1\rangle, \quad (25)$$

where $(\lambda_2'\mu_2')$ has only one possible value, because the signatures of the Young schemes $[f_2]$ and $[f_{p_2}]$ are uniquely connected with the signatures of the quantum numbers $(\lambda_2'\mu_2')$ and $(\lambda_2\mu_2)$, respectively. The last factor in Eq. (25) can be reduced to the simplest cluster coefficient,

$$\langle p^n[n](n0)\Lambda S_1T_1 \mid \hat{R} \mid s^n[n](00)0S_1T_1\rangle = \langle p^n[n](n0)\Lambda \mid n\Lambda, n0[n](00)0\rangle \left(\frac{n}{b}\right)^{\frac{n}{2}}, \quad (26)$$

$$K_n \equiv \langle p^n[n]\Lambda \mid n\Lambda, n0[n]0\rangle = \left(\frac{n!}{n^n}\right)^{\frac{1}{2}}, \quad (27)$$

which can be easily obtained if the $(\mathbf{U}^+)$ operator is expressed in terms of single-nucleon oscillator creation operators. Substituting expressions (26) and (27) into (25), we can factorize the FPC $\langle p^{N_2+n} \mid p^n, p^{N_2}\rangle$ into orbital and spin isospin parts, with the resulting elimination of spin-isospin quantum numbers due to orthonormality of the SU(4) Clebsch-Gordan coefficients. So the scalar part $\| K_b \|$ of the cluster coefficient K_b can be written as

$$\| K_b \| \equiv \langle p^{N_2+n}[f_{p_2}](\lambda_2'\mu_2') \mid (n0), bN_2[f_2](\lambda_2\mu_2)\rangle = \Omega_b \left(\begin{array}{c} N_2+n \\ n \end{array}\right)^{\frac{1}{2}} \sqrt{n_{f_2}/n_{f_{p_2}}}K_n \quad (28)$$

[we see that Ω_b is just the reduced cluster coefficient (28) for the particular case of n=0]. Finally, what remains to do is to calculate the value of Ω_b following formula (16) [the sign of Ω_b is not essential, since only the first term of the expansion (17) takes part in our constructions]. Thus, we return to the projection operator $\hat{P}_{000}$, which can be written as [13]

$$\hat{P}_{000} = \exp : -(\mathbf{U}^+\mathbf{U}) := \sum_k (-1)^k : (\mathbf{U}^+\mathbf{U})^k : /k!. \quad (29)$$

Here, : : is the symbol of the ordered product of operators where all $\mathbf{U}^+$ operators stand to the left of $\mathbf{U}$ operators. Further, we exploit the expansion

$$: (\mathbf{U}^+\cdot\mathbf{U})^r := r!(-1)^r \sum_l N_{r,l}^2 : (\mathbf{U}^+\cdot\mathbf{U}^+)^{(r-l)/2} \left[Y_l(\mathbf{U}^+)\cdot Y_l(\mathbf{U})\right]_{00} (\mathbf{U}\cdot\mathbf{U})^{(r-l)/2}, \quad (30)$$

where the coefficients $N_{r,l}$ [see also Eqs. (3) and (20)] arise in analogy with those in Ref. [14] where a similar expansion of the scalar products $(\mathbf{r}_1\cdot\mathbf{r}_2)^k$ was used. Introducing the intermediate states and taking into account the Hermitian properties of the operators $\mathbf{U}^+$, $\mathbf{U}$ we obtain

$$\Omega_b^2 = \sum (-1)^l N_{r,l}^2 \langle s^\nu p^{b-\nu}[f](\lambda\mu)LM \mid (\mathbf{U}^+\cdot\mathbf{U}^+)^{(r-l)/2}Y_{lm}(\mathbf{U}^+) \mid s^{\nu+r}p^{b-\nu-r}[f](\lambda'\mu')L'M'\rangle^2, \quad (31)$$

TABLE I. SU(3)-reduced cluster coefficients $\| K_b \|$ for d, t, and α clusters with $[f]$=[2], [3], and [4], respectively.

b	2		3			4			
N_b	0	2	0	2	3	0	2	3	4
$\| K_b \|$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{3}$	$\frac{1}{\sqrt{3}}$	$\frac{2}{3}$	$\frac{\sqrt{3}}{4\sqrt{2}}$	$\frac{\sqrt{3}}{4}$	$\frac{1}{2}$	$\frac{\sqrt{15}}{4\sqrt{2}}$

with the summation being carried out on r, l, L', $(\lambda'\mu')$, M', and m. Based on the experience of the derivation of formula (28), it is easy to arrive at the final result

$$\Omega_b^2 = \sum_{r[f_p]} (-1)^r \binom{\nu + r}{r} \binom{A - \nu}{r} \left(\frac{r}{A}\right)^r \times K_r^2 \frac{n_{f_p'}}{n_{f_p}} u^2([\tilde{\nu}][\tilde{r}][\tilde{f}][\tilde{f}_p'] : [\tilde{\nu} + r][\tilde{f}_p]). \quad (32)$$

Here, the Racah coefficients of the SU(4) group are discussed in Ref. [15], $\nu + r \leq 4$, $[f_p]$ is the Young scheme of configuration $p^{b-\nu}$, the signature of (λ, μ) is defined as $\lambda = f_{p_1} - f_{p_2}$, $\mu = f_{p_2} - f_{p_3}$, $b - \nu = f_{p_1} + f_{p_2} + f_{p_3}$, and the connection between the quantum numbers $[f_p']$ and (λ', μ') of the configuration $p^{b-\nu-r}$ is similar.

In our specific situation of d, $t(h)$, and α clusters when scattering amplitudes are diagonal on $[f]$ the above Racah coefficients are equal to unity. So we need only the quantities $\| K_b \|$ and the SU(3) Clebsch-Gordan coefficients which are displayed in Tables I and II, respectively. Multiparticle fractional parentage coefficients of the usual shell model can be found in Ref. [16].

Our algebraic treatment has the remarkable common features with Ref. [17] where only ground-state virtual clusters were considered but the residual nucleus A-b was permitted to have an arbitrary hole excitation.

IV. A FEW REACTIONS ON p-SHELL NUCLEI

The first example is the $^{16}O(p, pt)^{13}N$ reaction to the three lowest levels $1/2^-$, $3/2^-$, and $5/2^-$ of the residual nucleus ^{13}N. Figure 1 demonstrates the expected Θ_q anisotropy at the kinematical conditions p^2=0.6 $(\mathrm{GeV}/c)^2$, q=90 MeV/c, and φ_q=15°. We see that the anisotropy is rather substantial; the ratio of maximum to minimum is 2–5, but the above ratio is still a few times less than for the $^{12}C(p, p\alpha)^{12}C(0^+; 2^+; 4^+)$ reaction [3]. This feature is typical for clusters lighter than the α particle. Figure 2 shows the calculated momentum distri-

TABLE II. Clebsch-Gordan coefficients of the SU(3) group $\langle(\lambda\mu)L \mid (\lambda'\mu')L', (\lambda''\mu'')L''\rangle$.

$(\lambda'\mu')^a$	(20)						(30)	
$(\lambda\mu)L$ $L'L''$	SP	DP	SS	DS	SD	DD	PP	FP
$(30)P$	$\sqrt{\frac{5}{9}}$	$\sqrt{\frac{4}{9}}$						
$(30)F$	-	1						
$(40)S$			$\sqrt{\frac{5}{9}}$	-	-	$\sqrt{\frac{4}{9}}$	1	-
$(40)D$			-	$\sqrt{\frac{7}{18}}$	$\sqrt{\frac{7}{18}}$	$\sqrt{\frac{4}{18}}$	$\sqrt{\frac{7}{10}}$	$\sqrt{\frac{3}{10}}$
$(40)G$			-	-	-	1	-	1

[a] If (λ, μ)=$(\lambda 0)$ and $(\lambda'\mu)$=$(\lambda', 0)$, then $(\lambda''\mu'')$=$(\lambda - \lambda', 0)$.

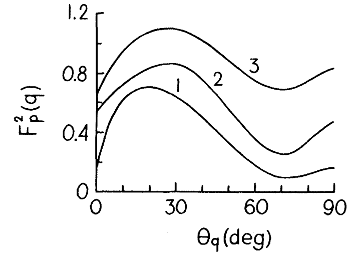

FIG. 1. Θ_q anisotropies of the final nucleus momentum distribution for the $^{16}O(p, pt)^{13}N$ reaction, $p^2 = 0.6$ $(\mathrm{GeV}/c)^2$, q=90 MeV/c, and $\varphi_q = 15°$. Curves 1, 2, and 3 correspond to the lowest levels of the ^{13}N nucleus with $j^\pi = \frac{1}{2}^-, \frac{3}{2}^-$, and $\frac{5}{2}^-$, respectively.

butions of the recoil nucleus ^{13}N in the corresponding states $1/2^-$, $3/2^-$, or $5/2^-$ after averaging on the orientation angles Θ_q and φ_q. The pronounced minima of the form factors, which are typical characteristics of our theory with no deexcitation effects, become apparent when the above effects are taken into account, and the cross section increases from 2 to 10 times. So, even though the form factors are being averaged over the orientations of the recoil momentum $\mathbf{q}$, they still show the essential influence of deexcitation effects.

Various approximations to our theory are compared in Figs. 3–5 showing the results for the deuteron, triton, and α-particle knockout from ^{14}N, ^{12}C, and ^{16}O nuclei, respectively, with the transition to the ground states of the residual nuclei. Typical eikonal parameters of cluster distorted waves are given in Table III. For potentials of clusters bound in the initial nuclei, see Ref. [18]. It is instructive to see from these figures that deexcitation effects produce a few times increase of cross sections. However, if the values of Θ_q and φ_q are fixed, some times the shapes of the form factors can be similar for theories with and without deexcitation effects, respectively. So the comparison of results for a few different $\mathbf{q}$ orientations is of importance here, which means the investigation of Θ_q and φ_q anisotropies (see Table III).

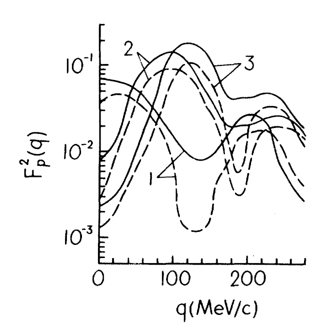

FIG. 2. Momentum distributions of the final nucleus for the $^{16}O(p, pt)^{13}N$ reaction averaged over orientations of the recoil momentum $-\mathbf{q}$, $p^2 = 0.75$ $(\mathrm{GeV}/c)^2$. Curves 1, 2, and 3 correspond to the lowest levels of the ^{13}N nucleus with $j^\pi = \frac{1}{2}^-, \frac{3}{2}^-$, and $\frac{5}{2}^-$, respectively (solid line, with deexcitation; dashed lines, with the ground-state virtual cluster only).

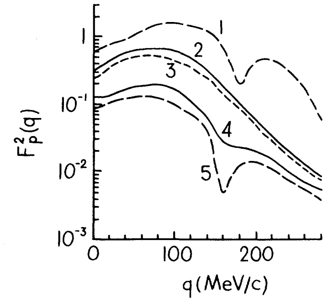

FIG. 3. Momentum distribution of the final nucleus for the $^{14}N(p,pd)^{12}C(0_1^+)$ reaction, $p^2 = 0.5$ (GeV/c)^2, $\Theta_q = 22°$, $\varphi_q = 74°$. Curves 1, 2, 3, 4, and 5 correspond to plane waves with deexcitation, distorted waves with deexcitation, distorted waves with deexcitation but with oscillator asymptotics of the bound cluster wave function, distorted waves with no deexcitations, and plane waves with no deexcitations, respectively.

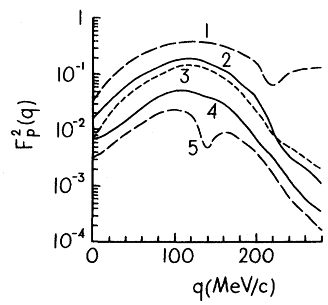

FIG. 4. Same as in Fig. 3 for the $^{12}C(p,pt)^9B_{g.s.}(\frac{3}{2}^-)$ reaction, $p^2 = 1.2$ (GeV/c)^2, $\Theta_q = 57°$, $\varphi_q = 88°$.

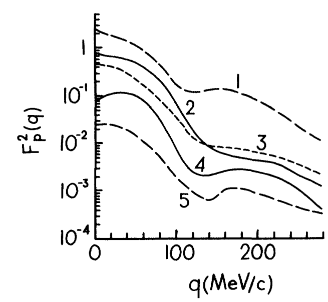

FIG. 5. Same as in Fig. 3 for the $^{16}O(p,p\alpha)^{12}C(0_1^+)$ reaction, $p^2 = 1.2$ (GeV/c)^2, $\Theta_q = 42°$, $\varphi_q = 62°$.

TABLE III. The eikonal parameters of distorted waves $\beta(\gamma)$.

$E_{b,A\text{-}b}$(MeV)	15	30	45	60
$^{12}C(p,pt)^9B$	0.55 (0.057)	0.60 (0.072)	0.64 (0.088)	0.67 (0.101)
$^{14}N(p,pd)^{12}C$	0.32 (0.036)	0.41 (0.053)	0.47 (0.068)	0.50 (0.085)
$^{16}O(p,p\alpha)^{12}C$	0.67 (0.090)	0.74 (0.103)	0.79 (0.114)	0.82 (0.120)

The asymptotic character at the bound cluster wave function is of influence too; it corresponds either to the Woods-Saxon potential well or to the oversimplified case of the oscillator potential. Finally, the form factor of the $^{16}O(p,p\alpha)^{12}C(0^+)$ reaction is, in general, similar to that of the $^{12}C(p,p\alpha)^8Be(0^+)$ reaction — see Ref. [3]. We must remember in all the above discussions that the theory is valid in the region of small recoil momenta $q < 300$ MeV/c. At higher q values three-body rescattering effects [7,8] are expected to be visible. However, their magnitude in general is a few times less than that of cluster deexcitation effects in the region of small q values.

V. CONCLUSION

The principal results of the present paper are the following.

(1) Eikonal approximation formulas for a cluster quasielastic knockout theory are given. This numerically economical version of a generalized DWIA approach was confirmed by the method of a partial-wave expansion of the distorted waves.

(2) A method of calculation of excited cluster spectroscopical amplitudes is presented with the emphasis on practical details.

(3) Numerical DWIA results for the $^{16}O(p,pt)^{13}N(1/2^-, 3/2^-, 5/2^-)$ reaction show the essential Θ_q anisotropy, which is, however, a few times less than the anisotropies expected for the $(p,p\alpha)$ reaction on a ^{12}C nucleus [3]. This feature is typical for knocked-out clusters lighter than the α particle. In general, Θ_q and φ_q anisotropies originating in cluster deexcitation effects are much more strong than those connected with three-body rescattering effects, etc. [7,8].

(4) The contribution of cluster deexcitation amplitudes to the momentum distribution of the recoil nucleus A-b is quite visible even if this distribution is averaged over the orientations of the recoil momentum q.

(5) The investigation of the q dependence of the form factors at a fixed **q** orientation can be especially instructive when the results for a few different orientations can be compared to each other.

So, in general, the experimental investigation of quasielastic knockout of clusters from p-shell nuclei by 1 GeV protons is a very promising area of research where many original features are expected.

The actual direction of further theoretical investigations should take into account the corrections to Glauber-Sitenko multiple scattering theory (especially, spin- and isospin-dependent effects) to be able to cover the energy range E_p=400–800 MeV. We strongly suggest experimenters extend to this energy range the productive current experimental research of cluster quasielastic knockout at bombarding energies of 100–200 MeV [1] having in mind here the low-energy modification of new effects discussed in Ref. [3] and in our present paper.

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