Non-Markovian approach to the damping of giant monopole resonances in nuclei

S. Ayik,[1] M. Belkacem,[2] and A. Bonasera[2]
[1] *Tennessee Technological University, Cookeville, Tennessee 38505*
[2] *INFN–Laboratorio Nazionale del Sud, Viale Andrea Doria (ang. Via S. Sofia), 95123 Catania, Italy*
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The collisional relaxation rates of the giant monopole vibrations in nuclei are calculated using a semiclassical transport equation with a memory dependent collision term. Calculations show that depending on the mass the collisional damping accounts for about 10–20% of the experimental damping widths.

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Semiclassical transport models of the Boltzmann-Uhling-Uhlenbeck (BUU) type are often employed for studying nuclear collective vibrations [1]. Although these models give a good description of the average resonance energies, they are not realistic for describing the collisional relaxation rates. This is due to the fact that in the BUU-type models, the binary collisions are treated in a Markovian approximation by assuming that the duration time τ_d of binary collisions is much shorter than the time scale τ_u of the collective vibrations and the mean free time τ_λ between collisions, $\tau_d \ll \tau_u, \tau_\lambda$. As a result, the phase space of the $2p$-$2h$ states for damping is severely restricted by an incorrect energy conservation factor. In fact, as a result of the Markovian approximation, the collisional width of collective vibrations vanishes at zero temperature in the linearized limit [2]. In particular, the Markovian collision term gives rise to vanishing collisional width for the monopole mode at any temperature. Therefore, for a proper description of the collisional relaxation rates, it is necessary to improve the transport models by incorporating the memory effects associated with collective vibrations into the collision term [3–5].

A derivation of the collision term by including the memory effects in the semiclassical framework has been carried out recently, and the model has been applied to estimate the collisional relaxation rates of the giant quadrupole and the giant dipole vibrations [6] (also, see [7,8] on the memory effects in the Boltzmann-Langevin model). According to this model, the small deviations of the phase-space density around equilibrium,

$$\delta f(\mathbf{r},\mathbf{p};t) = f(\mathbf{r},\mathbf{p};t) - f_0(\epsilon) \equiv \chi(\mathbf{r},\mathbf{p};t)\frac{\partial}{\partial\epsilon}f_0 \ , \qquad (1)$$

are determined by a linearized transport equation,

$$\frac{\partial}{\partial t}\delta f - \{\delta h, f_0\} - \{h_0, \delta f\}$$
$$= \int d\mathbf{p}_2 d\mathbf{p}_3 d\mathbf{p}_4 \delta W[(1-f^0)(1-f_2^0)f_3^0 f_4^0 - f_3^0 f_2^0(1-f_3^0)(1-f_4^0)] \ , \qquad (2)$$

where $f_0(\epsilon) = \{\exp[(\epsilon-\epsilon_F)/T]+1\}^{-1}$ and the transition rate is given by

$$\delta W = W\int \frac{d\omega}{2\pi} e^{-i\omega t}\Delta\chi(\omega)\frac{Z(\omega)}{2} \qquad (3)$$

with $Z(\omega) = [\delta(\Delta\epsilon-\omega) - \delta(\Delta\epsilon+\omega)]/\omega$, $\Delta\epsilon = \epsilon_3+\epsilon_4-\epsilon_1-\epsilon_2$, and $\Delta\chi(\omega) = \chi_3(\omega)+\chi_4(\omega)-\chi_1(\omega)-\chi_2(\omega)$. Here W is the transition rate of the BUU collision term and $\chi_j(\omega) \equiv \chi(\mathbf{r},\mathbf{p};\omega)$ denotes the time Fourier transform of the distortion function $\chi(\mathbf{r},\mathbf{p};t)$. In the collision term of Eq. (2), collective phonons with energy $\hbar\omega$ can be emitted and absorbed during binary collisions [9], which strongly modifies the collisional relaxation rates as compared to the Markovian limit. In the Markovian limit, which is appropriate for slowly varying disturbances in time, the factor $Z(\omega)$ is approximated by its zero frequency limit $Z(\omega \to 0)$. In this case, the collision term of Eq. (2) goes over to the linearized form of the standard BUU collision term. This can be seen by noting that $Z \to 2\partial\delta(\epsilon_3+\epsilon_4-\epsilon_1-\epsilon_2)/\partial\epsilon_2$ and by carrying out one partial integration with respect to ϵ_2 in Eq. (2).

We define the relaxation rate associated with an isoscalar harmonic vibration with a mean frequency Ω as

$$\frac{1}{\tau} = -\frac{\int d\mathbf{r}\, d\mathbf{p}\, \chi\delta K}{\int d\mathbf{r}\, d\mathbf{p}\, \chi\delta f} = \frac{\int d\mathbf{r}\, d\mathbf{p}_1 d\mathbf{p}_2 d\mathbf{p}_3 d\mathbf{p}_4 W Z(\Omega)(\Delta\chi)^2 f_1^0 f_2^0(1-f_3^0)(1-f_4^0)}{4\int d\mathbf{r}\, d\mathbf{p}\, \chi^2(\partial/\partial\epsilon)f_0} \ , \qquad (4)$$

where $\chi \equiv \chi(\mathbf{r},\mathbf{p};\Omega)$. It is possible to derive a similar expression for the relaxation rate of the isovector vibrations by considering the proton and the neutron degrees of freedom explicitly. In Ref. [6], these expressions have been used to estimate the relaxation rates of the giant quadrupole and the giant dipole vibrations by parametrizing the distortion of the momentum distribution in terms of the Legendre functions as $\chi = p^2 P_2(\theta)$ and $\chi = pP_1(\theta)$, respectively.

Using Eq. (4), we can also calculate the relaxation rates of the giant monopole vibrations [10]. In the scaling model description of the monopole mode with a velocity

field $\phi = r^2/2$, the Fermi surface remains spherical but oscillates radially with the monopole frequency. Therefore the relaxation rate, and hence the collisional width $\Gamma = \hbar/\tau$, can be estimated by taking $\chi = p^2$ for the distortion function in Eq. (4) to give[1]

$$\Gamma = \hbar\left(\frac{\hbar\Omega}{2}\right)^2 \times \frac{\int d\mathbf{r}\, d\mathbf{p}_1 d\mathbf{p}_2 d\mathbf{p}_3 d\mathbf{p}_4 W Z f_1^0 f_2^0 (1-f_3^0)(1-f_4^0)}{\int d\mathbf{r}\, d\mathbf{p}\, \epsilon^2 (\partial/\partial\epsilon) f_0} . \tag{5}$$

In the Markovian limit, i.e., $\Omega \to 0$, the collisional width vanishes at any temperature. When the collective energy $\hbar\Omega$ and the temperature T are small compared to the Fermi energy, $\hbar\Omega, T \ll \epsilon_F$, the two-body transitions in Eq. (5) are concentrated in the vicinity of the Fermi surface. In this case, utilizing an approximate method developed in condensed matter physics, the momentum integrals in Eq. (5) can be evaluated analytically [11]. For an isotropic cross section and uniform nuclear density ρ, we find

$$\Gamma \approx \hbar v_F \sigma \rho \left(\frac{\hbar\Omega}{4\epsilon_F}\right)^2 \left[\left(\frac{\hbar\Omega}{\epsilon_F}\right)^2 + \left(\frac{2\pi T}{\epsilon_F}\right)^2\right] . \tag{6}$$

In Fig. 1, the dotted line shows the results of the analytical approximation to the collisional width at zero temperature as a function of the atomic mass number of nuclei. In the formula (6) we take the nuclear matter parameters as $\rho = 0.16$ fm^{-3}, $v_F = 0.28c$, $\epsilon_F = 37$ MeV, and the total cross section $\sigma = 40$ mb. For the mass dependence of the resonance energies for the medium weight and heavy nuclei ($A \geq 70$) we use the formula

$$\hbar\Omega = 31.2A^{-1/3} + 20.6A^{-1/6} \text{ MeV} \tag{7}$$

and for $A < 70$ we take $\hbar\Omega = 17.5$ MeV.

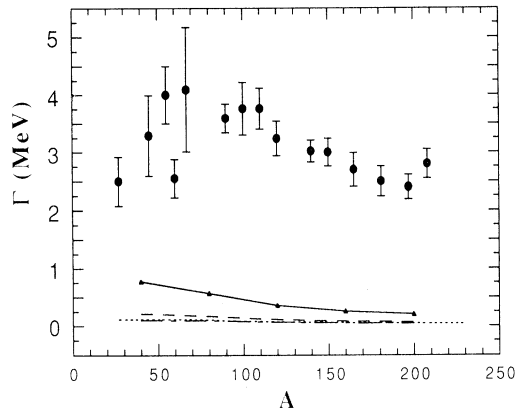

FIG. 1. The collisional widths of giant monopole resonances at zero temperature are plotted as a function of the mass number. The dotted line shows the analytical approximation (6). The dashed-dotted line, the dashed line, and the solid line show the Monte Carlo calculations of (5) with the nuclear mater parameters and constant cross section, with the surface effect and constant cross section, and with the surface effect and energy-angle dependent cross section, respectively. The experimental widths are shown by solid dots with error bars.

In order to check the accuracy of the analytical approximation, we carry out an evaluation of the momentum integrals in Eq. (5) by the Monte Carlo method with the same nuclear matter parameters. The results of the Monte Carlo calculations are shown in Fig. 1 by the dashed-dotted line. This result indicates that the analytical formula (6) provides a good approximation for Eq. (5) in the nuclear matter limit, but it gives rise to a very small collisional width. We can also incorporate the surface effects into the calculations in a local density approximation by parametrizing the nuclear density distribution as

$$\rho(r) = \rho_0\{\exp[(r-R)t] + 1\}^{-1} , \tag{8}$$

where $R = 1.12A^{1/3}$ fm denotes the sharp nuclear radius, ρ_0 is fixed by the normalization, and $t = 0.545$ fm is the thickness parameter assumed to be the same for all nuclei. The integrands in Eq. (5) depend on r through the Fermi energy

$$\epsilon_F(r) = \frac{\hbar^2}{2m}\left[\frac{3\pi^2}{2}\rho(r)\right]^{2/3} . \tag{9}$$

We repeat the Monte Carlo evaluation of Eq. (5) by including the surface effect but with the same isotropic cross section $\sigma = 40$ mb as used above. These results are indicated by a dashed line in Fig. 1. It is seen that the surface effect increases the collisional widths, in particular of the lighter nuclei. The reason for this is that the average Fermi energy becomes smaller due to the surface effect, which increases the ratio $\hbar\Omega/\epsilon_F$ as seen from the approximate formula (6).

In addition to the geometric effect, the nuclear surface has a dynamical influence on the collisional widths, as well. In the vicinity of the nuclear surface the effective cross section becomes larger as a result of the reduced Fermi motion. In order to include this effect, we employ an energy-angular dependent cross section as parametrized in Ref. [1] and evaluate Eq. (5) for the collisional width with the Monte Carlo method. The result of this calculation is shown by the solid line in Fig. 1 as a function of the atomic mass number. The calculated widths including surface effects with energy-angular dependent cross section are about a factor of 2–4 larger than those obtained with constant cross section, but account only for a small fraction of the experimental damping widths of the giant monopole resonances, which are indicated by solid dots in Fig. 1. We note that Monte Carlo calculations of Eq. (5) performed with various inputs have about 10% statistical error. The collisional widths

[1]In Ref. [6], the distortion function in the monopole mode was incorrectly taken as $\chi = \text{const}$ which gives zero collisional width.

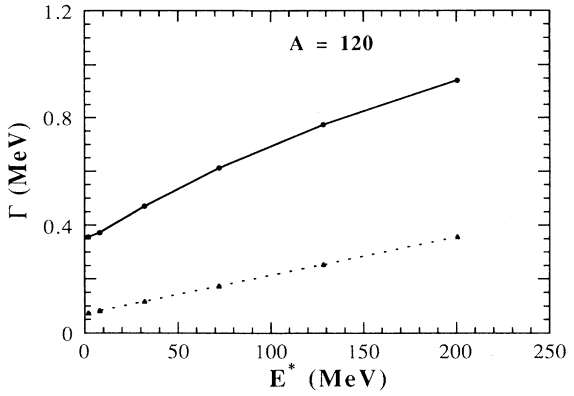

FIG. 2. The collisional width for $A = 120$ is plotted as a function of temperature. The dotted line and the full line show the analytical approximation and the Monte Carlo calculation with the surface effect and energy-angle dependent cross section, respectively.

increase with the temperature or the excitation energy of the system. As an example, in Fig. 2, the collisional width is plotted as a function of the excitation energy E^* which is related to the temperature through $E^* = aT^2$ with $a = \pi^2 A/4\epsilon_F$ for the mass number $A = 120$. In this figure, the dotted line shows the result of the analytical approximation (6) and the full line shows the Monte Carlo calculations of Eq. (5) including the surface effect with the energy-angular dependent cross section. Note that the Monte Carlo calculations do not show a T^2 dependence like the one in Eq. (6). This is due to the energy dependent elementary cross section; with increasing temperature the particles that collide have higher and higher energies while the cross section stabilizes to 40 mb as in the analytical estimate.

In summary, employing a memory incorporated transport model we calculate the collisional relaxation rates of the giant monopole vibrations in nuclei. The collisional widths are small and depending on the mass constitute about 10–20 % of the total damping widths.

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