Description of low-lying vibrational and two-quasiparticle states in ^{166}Er

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The energies and wave functions of nonrotational states below 2.3 MeV in ^{166}Er are calculated within the quasiparticle-phonon nuclear model. It is shown that the wave functions have a dominating one-phonon component. The contribution of the two-phonon configurations to the wave function of $K^\pi \neq 0^+$ and 4^+ states below 2.3 MeV is less than 6%. The existence of the double-γ vibrational state with $K^\pi = 4^+$ at 2.0–2.2 MeV in ^{166}Er is predicted. The calculated excitation energies and $B(E2)$ and $B(E3)$ values for transitions from the ground state are found to be in agreement with experimental data. Large spectroscopic factors of one-nucleon transfer reactions are explained by the large two-quasiparticle components of the one-phonon terms of the wave functions.

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I. INTRODUCTION

The lowest excitation in doubly even well-deformed nuclei has been extensively studied experimentally during the recent decades. At higher energies, our knowledge is still fragmentary. A rich amount of experimental data especially regarding γ-ray transitions between excited states has been obtained in ^{168}Er. The nucleus ^{166}Er has received a great deal of attention because its levels are populated in decay of three radioactive parents of quite different spins: ^{166}Ho (0^-), ^{166m}Ho (7^-), and ^{166}Tm (2^+). The spins and parities of many levels up to excitation energies of 2.3 MeV have been obtained from these studies [1–5]. Vibrational levels have been studied through the $(n, n'\gamma)$ reaction [6] and the Coulomb excitation [7]. The levels of ^{166}Er have been studied in [8] by using single-nucleon transfer reactions such as ^{167}Er(d,t), ^{167}Er(^{3}He,α), ^{165}Ho(^{3}He,d) and ^{165}Ho(α,t). These single-nucleon transfer experiments were performed in order to determine the two-quasiparticle components of the wave functions of excited states. The excited 0^+ states have been investigated in [9] by using the (p,t) and (t,p) reactions. Several calculations for ^{166}Er have been carried out and published in [10, 11] and other papers. Much less detailed calculations for ^{166}Er compared with ^{156}Gd, ^{178}Hf, and especially ^{168}Er have been made.

The quasiparticle-phonon nuclear model (QPNM) [12–18] is used for a microscopic description of the low-spin, small-amplitude vibrational states in spherical nuclei not far from closed shells and in well-deformed nuclei. Ground state correlations in these nuclei are very small. The energies and wave functions of the vibrational and two-quasiparticle states in ^{156}Gd [19], ^{158}Gd [19], ^{168}Er [15, 20], and other nuclei have been calculated in the QPNM. The γ-ray transitions between excited states in ^{168}Er, ^{156}Gd, and ^{158}Gd have been calculated in [19, 20].

In this article, the energies, wave functions, and $E1$, $E2$, and $M1$ transition rates between excited states in ^{166}Er are calculated within the QPNM. The results of present calculations are compared with the corresponding experimental data. The energies, wave functions, and γ-ray transition rates obtained in this investigation are presented in Sec. II. The specific properties of the excited states in ^{166}Er are discussed and conclusions are presented in Sec. III.

II. NONROTATIONAL STATES AND γ-RAY TRANSITIONS BETWEEN EXCITED STATES IN ^{166}Er

The energies and wave functions of nonrotational states are calculated within the QPNM in the same way as in Ref. [20] with the wave functions consisting of one- and two-phonon terms. The random phase approximation (RPA) equation for the $K^\pi = 0^+$ states is given in [14, 18], for the $K^\pi = 0^-$ and 1^- states in [21], and for other K^π states in [13, 15, 18].

The calculations are performed with the Woods-Saxon potential for $A = 165$ with quadrupole $\beta_2 = 0.28$, hexadecapole $\beta_4 = -0.01$, and γ equal to zero equilibrium deformations. The isoscalar constants $\kappa_0^{\lambda\mu}$ of the particle-hole (ph) multipole interactions are fixed to reproduce the experimental energies of the first $K^\pi_{\nu=1}$ nonrotational states. The calculations are performed with the isovector constant $\kappa_1^{\lambda\mu} = -1.5\kappa_0^{\lambda\mu}$ for the ph interactions and the constant $G^{\lambda\mu} = \kappa_0^{\lambda\mu}$ for the particle-particle (pp) interactions. We have used the constants of the multipole interaction, phonon basis, and effective charges for ^{166}Er which are close to ^{168}Er [20].

We calculated the excitation energies and structure of nonrotational states in ^{166}Er and $B(E\lambda)\uparrow$ values for $E2$, $E3$, $E4$, and $E5$ transitions from the ground $0^+_{\text{g.s.}}$ to excited $I^\pi K_\nu$ states. The experimental data [1, 4, 5, 7–9] as well as the results of our calculations in ^{166}Er are given in Table I. The calculated structure is given as a contribution of the one-phonon $\lambda\mu i$ and two-phonon $\{\lambda_1\mu_1 i_1, \lambda_2\mu_2 i_2\}$ components to the normalization of the

TABLE I. Nonrotational states in ^{166}Er.

K^π_ν	Experiment			Calculation in the QPMN			
	E_ν (MeV)	$B(E\lambda)\uparrow$ (s.p.u.)	Structure	E_ν [a] (MeV)	$B(E\lambda)\uparrow$ [b] (s.p.u.)	Structure[c]	%
2^+_1	0.786	5.2	(d,t)[d]	0.8	5.0	221:98	
						221: $nn523\downarrow -521\downarrow$	24
						$pp411\uparrow +411\downarrow$	23
						$nn521\uparrow +521\downarrow$	16
						$pp413\downarrow -411\downarrow$	6
						$nn633\uparrow -651\uparrow$	5
2^-_1	1.458	5.3	(d,t): $nn633\uparrow -521\uparrow \approx 58\%$	1.4	5.3	321:97 {201,321}: 2	
			$pp523\uparrow -411\uparrow \approx 4\%$			321: $nn633\uparrow -521\uparrow$	40
						$pp523\uparrow -411\uparrow$	10
						$nn642\uparrow -521\downarrow$	10
0^+_1	1.460	0.66		1.4	0.5	201:94 {321,321}:4	
			$\tilde{S}(t,p)$[e] $= 0.15$	$\tilde{S}(t,p)$[e] $= 0.16$		201: $pp411\downarrow -411\downarrow$	48
			$\tilde{S}(p,t)$[e] ≤ 0.0025	$\tilde{S}(p,t) = 0.004$		$nn512\uparrow -512\uparrow$	11
						$nn521\downarrow -521\downarrow$	10
						$pp523\uparrow -523\uparrow$	9
4^-_1	1.572		$(^3\mathrm{He},d)$[d]: $pp523\uparrow +411\downarrow$ is large	1.5	1.0	541:97 {221,321}: 2	
			(d,t): $nn633\uparrow +521\downarrow \approx 4\%$			541: $pp523\uparrow +411\downarrow$	88
						$nn633\uparrow +521\downarrow$	8
0^-_1	1.662	2.8		1.8	2.8	301:99	
						301: $nn642\uparrow -523\downarrow$	26
						$nn642\uparrow -512\uparrow$	8
						$pp411\downarrow -541\downarrow$	2
2^+_2	(1.703)			1.9	0.3	222:97 {321,541}:2	
						222: $nn523\downarrow -521\downarrow$	60
						$pp411\uparrow +411\downarrow$	37
0^+_2	1.713			1.8	0.4	202:91; 203:2	
			$\tilde{S}(t,p) = 0.14$	$\tilde{S}(t,p)$=0.09		{221,221}:5	
						202: $nn521\downarrow -521\downarrow$	28
			$\tilde{S}(p,t)$=0.14	$\tilde{S}(p,t) = 0.04$		$nn512\uparrow -512\uparrow$	14
						$pp523\uparrow -523\uparrow$	14
						$pp411\downarrow -411\downarrow$	12
1^+_1	1.812			1.8	1.7	211:98	
2^+1_1	1.910		(d,t): $nn633\uparrow -642\uparrow$ is large			211: $nn633\uparrow -642\uparrow$	79
						$pp523\uparrow -514\uparrow$	9
1^-_1	1.830		from ^{166}Ho $K^\pi = 0^-$ logft=5.2 $nn633\uparrow -523\downarrow$ is large	1.8	3.0	311:97 312:1	
						311: $nn633\uparrow -523\downarrow$	56
						$nn633\uparrow -512\uparrow$	9
						$pp523\uparrow -402\uparrow$	6
6^-_1	1.910		(d,t): $nn633\uparrow +523\downarrow$	1.9	0.1	761:100	
			$(^3\mathrm{He},\alpha)$: is large			761: $nn633\uparrow +523\downarrow$	97
3^-_1	1.916		$(^3\mathrm{He},d)$: $pp523\uparrow -411\downarrow$ is large	1.9	0.4	331:97	
						331: $pp523\uparrow -411\downarrow$	86
						$nn633\uparrow -521\downarrow$	6
3^+_1	(1.938)			1.96	0.6	431:79 432:20	
						431: $nn523\downarrow +521\downarrow$	80
						$nn512\uparrow +521\downarrow$	11
0^+_3	1.935			2.0	0.006	203:86 204:4	
			$\tilde{S}(p,t) = 0.08$	$\tilde{S}(t,p)$=0.014		202:3; 205:2 {221,221}:4	
				$\tilde{S}(p,t)$=0.0012		203: $nn633\uparrow -633\uparrow$	40
						$nn521\downarrow -521\downarrow$	37
4^+_1	1.979		(α,t): $pp523\uparrow +541\downarrow$ is large	1.96	1.1	441:76 {221,221}:21	
			(d,t): $nn633\uparrow +660\uparrow$ is noticeable			441: $nn523\downarrow +521\uparrow$	37
						$pp523\uparrow +541\downarrow$	32
						$nn633\uparrow +660\uparrow$	6
5^+_1				2.0	1.0	651:99	
						651: $nn523\downarrow +512\uparrow$	71
						$pp404\downarrow +411\uparrow$	18
7^-_1	1.990		$(^3\mathrm{He},d)$: $pp523\uparrow +404\downarrow$	2.5	–	771:100	
			(α,t): is large			771: $pp523\uparrow +404\downarrow$	100
2^-_2	(2.022)			2.0	0.1	322:99	
						322: $nn642\uparrow -521\downarrow$	78
						$pp523\uparrow -411\uparrow$	12
						$nn633\uparrow -521\uparrow$	7
4^-_2	2.002		(d,t): $nn633\uparrow +521\downarrow$ is large	2.0	0.03	542:99	
						542: $nn633\uparrow +521\downarrow$	90
						$pp523\uparrow +411\downarrow$	9
3^+_2				2.0	1.3	432:78 431:21	
						432: $nn512\uparrow +521\downarrow$	47
						$nn523\downarrow +521\downarrow$	19
						$nn633\uparrow -660\uparrow$	10
						$pp404\downarrow -411\downarrow$	8

TABLE I. (*Continued*).

K^π_ν	Experiment E_ν (MeV)	$B(E\lambda)\uparrow$ (s.p.u.)	Structure	Calculation in the QPMN E_ν [a] (MeV)	$B(E\lambda)\uparrow$ [b] (s.p.u.)		Structure[c]	%
4^+_2				2.05	0.01		{221,221}:73	
							442: 3 441:23	
2^+_3				2.08	0.01		223:97	
							{321,541}:2	
						223:	$nn521\uparrow+521\downarrow$	55
							$pp411\uparrow+411\downarrow$	30
							$nn523\downarrow-521\downarrow$	8
							$nn633\uparrow-651\uparrow$	6
3^-_2	(2.080)		(d,t): $nn633\uparrow-521\downarrow$ is large	2.1	0.1		332:99	
						332:	$nn633\uparrow-521\downarrow$	86
							$pp523\uparrow-411\downarrow$	9
6^+_1				2.1	0.02		661:100	
						661:	$nn633\uparrow+642\uparrow$	99.5
1^-_2				2.1	2.3		312:97	
						312:	$nn633\uparrow-523\downarrow$	44
							$nn633\uparrow-512\uparrow$	17
							$pp523\uparrow-402\uparrow$	8
3^+_3	(2.132)		$(^3\mathrm{He},d)$: $pp523\uparrow-541\downarrow$ (α,t): is large $\log ft$=5.6 from ^{166}Tm	2.24	0.001		433:95	
							{201,433}:4	
						433:	$pp523\uparrow-541\downarrow$	95
2^+_4	2.160		for $I^\pi K_\nu = 3^+2_4$ (d,t): $nn633\uparrow-651\uparrow$ is large	2.2	0.002		224:97	
							{221,441}:1	
						224:	$nn633\uparrow-651\uparrow$	82
							$nn521\uparrow+521\downarrow$	16
0^+_4	2.196			2.1	0.01		204:89 203:3	
							{221,221}:2	
							201:1 202:1	
			$\tilde{S}(t,p)\leq 0.03$	$\tilde{S}(t,p)$=0.001		204:	$nn523\downarrow-523\downarrow$	28
			$\tilde{S}(p,t)$=0.08	$\tilde{S}(p,t)=0.04$			$pp404\downarrow-404\downarrow$	17
							$pp523\uparrow-523\uparrow$	10
4^+_3				2.2	0.01		442:88	
							{221,221}: 5	
						442:	$pp523\uparrow+541\downarrow$	60
							$nn523\downarrow+521\uparrow$	34
3^-_3				2.2	0.3		333:98	
							{221,311}:1	
						333:	$nn642\uparrow+521\downarrow$	87
0^-_2	(2.2)			2.2	0.8		302:99	
						302:	$nn642\uparrow-512\uparrow$	23
							$nn642\uparrow-523\downarrow$	21
2^-_3	(2.055)		$(^3\mathrm{He},d)$: $pp523\uparrow-411\uparrow$ (α,t) is large	2.2	0.3		323:91	
							{201,321}:3	
							{202,321}:2	
						323:	$pp523\uparrow-411\uparrow$	50
							$nn633\uparrow-521\uparrow$	45
0^+_5				2.2	0.02		205:95	
							204:2 203:2	
						205:	$nn633\uparrow-633\uparrow$	40
							$nn512\uparrow-512\uparrow$	7
							$nn523\downarrow-523\downarrow$	6
							$pp404\downarrow-404\downarrow$	4
1^+_2	(2.378)			2.2	0.01		212:99	
						212:	$nn521\uparrow-521\downarrow$	98
5^-_1	2.244			2.2	0.1		551:99	
			(d,t): $nn633\uparrow+521\uparrow$ is large			551:	$nn633\uparrow+521\uparrow$	89
							$nn642\uparrow+523\downarrow$	5
							$pp514\uparrow+411\downarrow$	4
1^+_3				2.3	0.6		213:93 214:2	
							{321,331}:2	
						213:	$pp411\uparrow-411\downarrow$	69
							$pp514\uparrow-523\uparrow$	16
							$nn633\uparrow-642\uparrow$	7
5^-_2				2.3	0.05		552:99	
						552:	$nn523\downarrow+642\uparrow$	89
							$nn633\uparrow+521\uparrow$	5
							$pp514\uparrow+411\downarrow$	4
1^+_4	(2.464)			2.4	1.0		214:93 213:2	
							{321,331}:3	
						214:	$pp514\uparrow-523\uparrow$	42
							$pp411\uparrow-411\downarrow$	30
							$nn633\uparrow-642\uparrow$	12
6^-_2				2.4	0.02		762:100	
						762:	$nn633\uparrow+512\uparrow$	96
3^+_4	(2.293)			2.5	0.5		434:96	
			(d,t): $nn633\uparrow-660\uparrow$ is large				{221,211}:1	
						434:	$nn633\uparrow-660\uparrow$	41

TABLE I. (*Continued*).

K^{π}_{ν}	Experiment E_{ν} (MeV)	$B(E\lambda)\uparrow$ (s.p.u.)	Structure	Calculation in the QPMN E_{ν} [a] (MeV)	$B(E\lambda)\uparrow$ [b] (s.p.u.)	Structure [c]	%
						$nn512\uparrow+521\downarrow$	35
						$pp404\downarrow-411\downarrow$	15
4^+_4	(2.318)			2.6	0.3	443:92	
	(2.633)					{201,441}:2	
					443:	$nn633\uparrow+660\uparrow$	53
						$nn523\downarrow+521\uparrow$	20
						$nn642\uparrow+651\uparrow$	11
6^-_3	(2.608)			2.6		763:100	
			(^{3}He,d): $pp523\uparrow+402\uparrow$			763: $pp523\uparrow+402\uparrow$	94
			(α,t): is large				
0^-_3	(≈ 2.7)			2.6	1.1	303:96	
						{221,321}:2	
					303:	$nn642\uparrow-512\uparrow$	18
						$nn633\uparrow-514\downarrow$	16
						$pp523\uparrow-404\downarrow$	4
0^-_4				2.8	0.2	304:95	
					304:	$nn633+-514\downarrow$	25
						$pp523\uparrow-404\downarrow$	22
0^-_5	(≈ 2.8)			2.9	1.0	305:89	
						{221,321}:8	
					305:	$pp523\uparrow-404\downarrow$	16
						$nn633\uparrow-514\downarrow$	7
						$nn651\uparrow-521\uparrow$	7
8^+_1	(3.075)			2.8	–	881:100	
			(^{3}He,d): $pp523\uparrow+514\uparrow$			$pp523\uparrow+514\uparrow$	100
			(α,t): is large				
9^-_1	(2.494)		(d,t): $nn633\uparrow+505\uparrow$	3.4	–	991:100	
			(^{3}He,α): is large			$nn633\uparrow+505\uparrow$	100

[a] Due to accuracy of the calculation the energies are given, as a rule, with one decimal digit.

[b] The $B(E\lambda)\uparrow$ are equal to $B(E\lambda; 0^+0_{\text{g.s.}} \rightarrow I=\lambda K^{\pi}_{\nu})$ and are given in single-particle units

$$B(E\lambda)\uparrow_{\text{s.p.}} = \frac{2\lambda+1}{4\pi}\left(\frac{3}{\lambda+3}\right)^2 (0.12A^{1/3})^{2\lambda}(10\text{ fm})^{2\lambda}.$$

[c] The structure is given by the one- and two-phonon contribution to the normalization of the wave function and the phonon is denoted by $\lambda\mu i$, where i is the root number of the RPA equation; the two-quasiparticle configurations are labeled through the Nilson asymptotic quantum numbers.

[d] The (d,t) and (^{3}He,d) denote that the large intensities for the (d,t) reaction were observed at the angle of 45° and those for the (^{3}He,d) reaction at the angle of 50° and the (^{3}He,d) and (d,t) denote that the large intensities were observed at the angle of 60° for the excitation of the corresponding bands (Tables I and II in Ref. [8]).

[e] The (t,p) and (p,t) reaction strengths are given relative to the ground-to-ground strengths $\tilde{S}_{\nu}(t,p) = S_{\nu}(t,p)/S_{\text{g.s.}}(t,p)$ and $\tilde{S}_{\nu}(p,t) = S_{\nu}(p,t)/S_{\text{g.s.}}(p,t)$.

wave function. The Pauli principle is taken into account in the contribution of two-phonon components. Then, we list the asymptotic quantum numbers $Nn_z\Lambda\uparrow$ at $K = \Lambda + 1/2$ and $Nn_z\Lambda\downarrow$ at $K = \Lambda - 1/2$ for the largest two-quasineutron nn and two-quasiproton pp components of the wave functions of the one-phonon state $\lambda\mu i$. All the calculated nonrotational states with energies below 2.3 MeV and several other states are given in Table I.

The experimental and calculated $E1$ transitions to the ground state and the calculated γ-ray transitions between excited states in ^{166}Er are given in Table II. The calculations are performed for reduced $E1$, $E2$, and $M1$ transition probabilities for which there are experimental data for the intensities of the γ transitions or $B(E1)$ values. In this table, experimental energy levels and $I^{\pi}K_{\nu}$ values of the initial and final states are given as well. The experimental $B(E1)$ values were taken from Refs. [22,23] and $B(E\lambda)\downarrow$ and $B(M1)\downarrow$ values are given in $e^2\,\text{fm}^{2\lambda}$ and $\mu_N\,\text{fm}^{2\lambda-2}$ units, respectively. The experimental information involves the intensities of the γ transition from the initial to final states $n_f = 1, 2, \ldots$. This information is used to obtain the ratio of intensities. The experimental and calculated branching ratios of transitions from an initial to final $n_f = 2, 3, \ldots$ states relative to the final $n_f = 1$ state are given in Table II.

There are numerous experimental data on 0^+ excited states in deformed nuclei. The description of several first 0^+ states in deformed nuclei in the QPNM with ph interactions cannot be considered to be satisfactory. The role of the pp interaction in the description of excited 0^+ states is essential since with the change of G^{20} the energies of several low-lying poles of the RPA secular equations also change. The $B(E2)$ value for the excitation of the $I^{\pi}K_{\nu} = 2^+0_1$ state and the energies of the 0^+_2, 0^+_3, and 0^+_4 states decrease at $G^{20} = \kappa^{20}_0$ in comparison with $G^{20} = 0$, and the structure of the 0^+ states changes as well. The RPA wave functions of the 0^+ states are a superposition of a great number of two-quasiparticle configurations.

Let us consider the excited 0^+ states in ^{166}Er. The experimental [9] and calculated (t,p) and (p,t) reaction strengths are given in Table I relative to the ground-to-ground strengths $\tilde{S}_{\nu}(t,p) = S_{\nu}(t,p)/S_{\text{g.s.}}(t,p)$ and $\tilde{S}_{\nu}(p,t) = S_{\nu}(p,t)/S_{\text{g.s.}}(p,t)$. The calculated energies

and (t,p) and (p,t) reaction strengths do not contradict experimental data [9]. It is difficult to interpret the first $K^\pi_\nu = 0^+_1$ state as a β-vibrational state due to a relatively small $B(E2)$ value for the transition to the ground state band. The dominance of the $E2$ reduced transition probabilities from the 0^+_1 state to the $K^\pi_\nu = 2^+_1$ state over that to the ground state has been observed in ^{168}Er and ^{162}Dy. According to our calculation, the ratio

$$\frac{B(E2;0^+0_\nu \to 2^+2_1)}{B(E2;0^+0_\nu \to 2^+0_{\text{g.s.}})}$$

equals 0.01, 0.9, and 36 for $\nu = 1$, 2, and 3, respectively. The contribution of the two-phonon configuration $\{221,221\}$ to the normalization of the wave function of the 0^+_2 and 0^+_3 states equals 0.05 and 0.04. The dominance of the decay from the 0^+_3 to the 2^+_1 state over that to the ground state band is due to a very small $B(E2;0^+0_3 \to 2^+0_{\text{g.s.}})$ value.

We describe the energy and $B(E2)\uparrow$ value of the γ-vibrational 2^+_1 state. According to our calculations, the first $K^\pi_\nu = 4^+_1$ state in ^{166}Er is predominantly hexadecapole and, in agreement with the experimental data [9], should be strongly excited in the (α,t) reaction. The $B(E2;4^+4_1 \to 2^+2_1)$ is equal to 115 $e^2\,\text{fm}^4$. The first $K^\pi = 4^+$ $\{221,221\}$ two-phonon pole has the energy 2.05 MeV. The higher $K^\pi = 4^+$ poles are the following: $\{321,321\}$ 3.37 MeV, $\{301,541\}$ 3.39 MeV, $\{201,441\}$ 3.6 MeV, and so on. The RPA energies of the hexadecapole 4^+ states are 2.02 MeV, 2.34 MeV, 2.60 MeV, and so on. The situation with a two-phonon $\{221,221\}$ $K^\pi = 4^+$ state in ^{166}Er is very specific due to a small density of the $K^\pi = 4^+$ state near the $\{221,221\}$ pole and a very small value for the function, responsible for a coupling between one- and two-phonon configurations. Therefore, the two-phonon configuration $4^+\{221,221\}$ in ^{166}Er is weakly fragmented. According to our calculation at $\kappa_0^{22} = 0.02185\ \text{fm}^2\,\text{MeV}^{-1}$ and $\kappa_0^{44} = 0.030\ \text{fm}^2\,\text{MeV}^{-1}$, the second $K^\pi_\nu = 4^+_2$ state is a double-γ-vibrational state

TABLE II. $E1$ transition to ground state and $E1$, $M1$, and $E2$ transitions between excited states in ^{166}Er.

Initial state		$E\lambda$ or $M1$	n_f	Final state		$B(E\lambda)\downarrow(e^2\,\text{fm}^{2\lambda})$ or $B(M1)\downarrow(\mu_N^2)$		Branching ratios $\frac{W^\lambda(i\to n_f)}{W^{\lambda'}(i\to n_f=1)}$	
$I^\pi K_\nu$	E_ν (MeV)			$I^\pi K_\nu$	E_ν (MeV)	expt.	calc.	expt. [1]	calc.
0^+0_1	1.460	$E2$		2^+2_1	0.786		1.1		
1^-0_1	1.662	$E1$		$0^+0_{\text{g.s.}}$	0	$(8.9\pm0.5)\times10^{-3}$	30×10^{-3}		
0^+0_2	1.713	$E2$		2^+2_1	0.786		95		
1^-1_1	1.830	$E1$		$0^+0_{\text{g.s.}}$	0	$\approx 1\times10^{-3}$	7×10^{-3}		
3^-3_1	1.918	$E1$	1	2^+2_1	0.786		7×10^{-6}	1	1
		$M1$	2	2^-2_1	1.458		0.03	10	3
0^+0_3	1.935	$E2$		2^+2_1	0.786		60		
4^+4_1	1.978	$E2$		2^+2_1	0.786		115		
4^+4_2	2.05[a]	$E2$		2^+2_1	0.786		500		
3^+3_2	2.133	$M1$	1	2^+2_1	0.786		0.001	1	1
		$E2$					0.75		0.07
		$E1$	2	2^-2_1	1.458		10^{-4}	2.3	1.0
		$E1$	3	4^-4_1	1.572		3×10^{-6}	0.06	0.02
		$M1$	4	2^+2_1	1.703		4×10^{-4}	0.07	0.01
		$E1$	5	3^-3_1	1.916		2×10^{-5}	4.75	0.005
		$E1$	6	4^-4_2	2.002		3×10^{-6}	0.47	10^{-4}
3^+2_4	2.160	$E2$	1	$2^+0_{\text{g.s.}}$	0.081		0.08	1	1
		$M1$	2	2^+2_1	0.786		9×10^{-4}	0.89	1
		$E2$					0.05		0.1
		$E2$	3	2^+0_1	1.528		0.11	0.011	0.003
1^-0_2	$\approx$2.2	$E1$		$0^+0_{\text{g.s.}}$	0	$\approx 3\times10^{-3}$	7×10^{-3}		
4^+4_3	2.2[a]	$E2$		2^+2_1	0.786		70		
3^-3_2	2.216	$E1$	1	2^+2_1	0.786		10^{-4}	1	1
		$M1$	2	2^-2_1	1.458		0.01	15	0.15
		$M1$	3	4^-4_1	1.572		5×10^{-5}	0.14	3×10^{-4}
		$E2$	4	1^-1_1	1.830		0.12	0.09	10^{-5}
		$M1$	5	3^-3_1	1.916		6×10^{-3}	1.15	0.004
		$M1$	6	4^-4_2	2.002		3×10^{-4}	0.4	10^{-4}
3^-3_3	2.243	$E1$	1	2^+2_1	0.786		10^{-5}	1	1
		$M1$	2	3^-2_1	1.514		2×10^{-4}	1.28	0.02
4^+4_4	2.6[a]	$E2$		2^+2_1	0.786		0.42		
1^-0_3	$\approx$2.7	$E1$		$0^+0_{\text{g.s.}}$	0	1×10^{-3}	12×10^{-3}		
1^-0_4	$\approx$2.8	$E1$		$0^+0_{\text{g.s.}}$	0	4×10^{-3}	8×10^{-3}		

[a]The calculated energies.

and the $K^\pi_\nu = 4^+_3$ and 4^+_4 states are predominantly hexadecapole. These results are presented in Tables I and II. According to our calculation at $\kappa^{22}_0 = 0.0215$ fm^2 MeV^{-1}, the first $K^\pi_\nu = 4^+_1$ state has energy of 1.97 MeV. The contribution of the {441} and {221,221} components to the normalization of the wave function is equal to 94% and 3% and $B(E2; 4^+4_1 \to 2^+2_1) = 8.7\ e^2$ fm^4. The second $K^\pi_\nu = 4^+_2$ state is a double-γ-vibrational state with the {221,221} component equal to 82%. The $4^+\{221,221\}$ and 442 strengths are mixed between two levels with the energies 2.30 and 2.35 MeV if $\kappa^{22}_0 = 0.0207$ fm^2 MeV^{-1}. In the last case, the energy of the $K^\pi_\nu = 2^+_1$ state is 0.2 MeV higher than the experimental one. If we use $\kappa^{22}_0 = 0.0225$ fm^2 MeV^{-1}, then the first $K^\pi_\nu = 4^+_1$ state is a double-γ-vibrational one, but the energy of the $K^\pi_\nu = 2^+_1$ state is 0.15 MeV smaller than the experimental one.

According to experimental data [1, 3, 4, 8], there are several $I^\pi = 2^+$, 3^+, and 4^+ states in ^{166}Er. The interpretation of these states, except the $K^\pi_\nu = 2^+_1$ state, is not clear. The calculation in terms of the Nilsson model with the pairing and Coriolis interactions has been performed in [8]. Of course, our wave function of the $K^\pi = 1^+$, 2^+, 3^+, and 4^+ states can be used for further Coriolis coupling calculations. About a dozen large cross sections in the (d,t) reaction at forward angles indicative of the $l = 0$ neutron transfer have been observed in [8]. The only configurations in this mass region which would be expected to have large $l = 0$ (d,t) cross section are those formed by transferring a 400↑ neutron. To explain similar multiple strong $l = 0$ neutron transfers, the $\Delta N = \pm 2$ mixing between the single-particle states $n400\uparrow$ and $n660\uparrow$ and the states $n402\downarrow$ and $n651\uparrow$ has been calculated in [8]. All the assignments made on the basis of the comparison of calculated and experimental cross sections are considered to be tentative.

The mixing between the neutron single-particle states 400↑ and 660↑ and 402↓ and 651↓ has been investigated in [24] in terms of the Woods-Saxon wave functions, with pairing and quasiparticle-phonon interactions. It has been shown that a strong mixing of these pairs takes place in the Sm, Gd, and Dy isotopes at equilibrium deformation β_2 =0.30–0.33 and $\beta_4 = 0.04$. A quasiparticle-phonon interaction leads to an increase in an interval for β_2 where the $\Delta N = \pm 2$ mixing occurs as compared to the single-particle model. There is no noticeable $\Delta N = \pm 2$ mixing at $\beta_2 = 0.28$ and $\beta_4 = -0.01$ used in our calculation. One has to change the parameters of the Woods-Saxon potential to get the $\Delta N = \pm 2$ mixing at equilibrium deformation of the ^{166}Er nucleus.

The energies and wave functions of the $K^\pi = 0^-$ and 1^- states are calculated in the RPA by taking the isoscalar and isovector ph and *pp* and isovector ph dipole interactions into account. The $B(E1)$ values calculated in [19, 21] for the excitation of the $I^\pi K_\nu = 1^-0_1$ states in ^{156,158}Gd, ^{164}Dy, and ^{168}Er are 3–5 times as large as experimental ones. According to [21], the total $E1$ strength for excitation of the $K^\pi = 0^-$ states in ^{164}Dy and ^{168}Er is 3–4 times as large as that for the excitation of the $K^\pi = 1^-$ states. The strong correlation between the $B(E1)$ and $B(E3)$ values takes place for transitions from the ground state $0^+_{\rm g.s.}$ to the same band. According to our calculation, the $B(E1; 1^-0_\nu \to 0^+0_{\rm g.s.})$ values for $\nu = 1$, 2, 3, and 5 are 3–12 times as large as experimental ones [22, 23]. The first $K^\pi_\nu = 2^-_1$ state with energy 1.458 MeV is the second nonrotational state in ^{166}Er. There is a gap between the 2^+_1 and 2^-_1 states. We describe correctly the energy, $B(E3)$ value, and the two-quasiparticle structure of the 2^-_1 state. According to our calculation, the energies of the 2^-_2 and 2^-_3 states equal 2.0 and 2.2 MeV.

The first $K^\pi_\nu = 4^-_1$ state with energy 1.572 MeV is well described in the QPNM. There are several $I^\pi = 3^-$ and 4^- states. Interpretation of these states is not clear. According to our calculation of the branching ratios of the $E1$, $E2$, and $M1$ transitions between excited states and the comparison with the relevant experimental data [1], the $I^\pi = 3^-$ state with energy 1.918 MeV cannot be interpreted as the $I^\pi K_\nu = 3^-1_1$ state. It is possible to consider this state as the $I^\pi K_\nu = 3^-3_1$ one. The $I^\pi = 3^-$ 2.216 MeV and $I^\pi = 3^-$ 2.243 MeV states cannot be interpreted as $I^\pi K_\nu = 3^-3_2$ and 3^-3_3 states.

The calculated energies of the two-quasiparticle states with $K^\pi_\nu = 6^-_1$, 5^-_1, and 6^-_3 are in agreement with experimental ones, but the calculated energy of the $K^\pi = 7^-$ state is 0.5 MeV higher than the experimental ones. Predictions of the energies and structure of several nonrotational states below 2.3 MeV in ^{166}Er were made.

III. DISCUSSION AND CONCLUSION

Let us consider specific properties of the ^{166}Er nucleus. In the spectrum of nonrotational states there is a gap between the first nonrotational excited state with $K^\pi_\nu = 2^+_1$ and next $K^\pi_\nu = 2^-_1$ and other excited states. This gap equals 0.672 MeV. There is no such a gap in the spectrum of other double-even deformed nuclei. Due to this gap, the energies of the low-lying poles are the following: 2.14 MeV with $K^\pi = 4^+\{221,221\}$, 2.47 with $0^+\{221,221\}$, 2.60 with $4^-\{221,321\}$, 2.70 with $2^-\{221,541\}$, and 2.97 MeV with $2^+\{221,441\}$. The energy of other poles is larger than 3 MeV. Therefore, the contribution of the two-phonon configurations to normalization of the wave function of the $K^\pi \neq 0^+$ and 4^+ exited states with energies below 2 MeV and in the range 2.0–2.3 MeV is smaller than 3% and 6%, respectively. Due to this gap, the contributions of the two-phonon configurations in ^{166}Er are smaller than in other doubly even nuclei in the rare-earth region.

Let us consider the situation with the two-phonon states in doubly even well-deformed nuclei. A state is determined as a two-phonon state if the contribution of the two-phonon configuration to the wave function normalization exceeds 50%. The energy centroids of the two-phonon collective states in deformed nuclei have been calculated in Ref. [25]. It has been shown that due to a shift of the two-phonon poles, the density of levels in the energy region of the first two-phonon poles is large. Therefore, the two-phonon states should be strongly fragmented. Based on the QPNM calculations of the energy centroids of two-phonon states it has been concluded in Ref. [25] that collective two-phonon states cannot exist in

well-deformed nuclei. This prediction is fulfilled in many cases.

The investigation of low-lying states in doubly even well-deformed nuclei has shown that the one-phonon states below 2.5 MeV are weakly fragmented. Therefore, the lowest two-phonon states are not strongly fragmented in several specific cases. An effect of three phonon terms in the wave function has been taken into account in Refs. [19, 20] as an additional shift of two-phonon poles. Therefore, a contribution of the two-phonon configuration to the wave function of low-lying states was calculated instead of fragmentation of the two-phonon state.

The nuclei ^{168}Er, ^{166}Er, and ^{164}Dy are most favorable for observation of the $K^\pi = 4^+$ double-γ-vibrational states in the energy range 2.0–2.3 MeV. Much attention has been paid to ^{168}Er. Experimental investigations [26–28] have established a large double-γ-vibrational component in the first $K^\pi_\nu = 4^+_1$ state in ^{168}Er. According to the multiphonon method [29], *sdg* interactive boson model (IBM) [30], and self-consistent-collective-coordinate method [31], the first $K^\pi_\nu = 4^+_1$ state in ^{168}Er should be a two-phonon state. According to the QPNM calculation [20], the contribution of hexadecapole {441} one-phonon and double-γ-vibrational {221,221} components to the normalization of the wave function of the 4^+_1 state in ^{168}Er equals 60% and 30%, respectively. The calculated energies of the $K^\pi_\nu = 2^+_1$, 4^+_1, and 4^-_1 states and $B(E2; 2^+_1 \rightarrow 0^+_{\text{g.s.}})$ and $B(E1; 4^+_1 \rightarrow 4^-_1)$ values are in good agreement with experimental data analyses of Ref. [32].

According to the present calculation, the double-γ-vibrational strength with $K^\pi = 4^+$ in ^{166}Er is concentrated on one or two levels at energy 2.0–2.2 MeV. When the energy and $B(E2; 0^+0_{\text{g.s.}} \rightarrow 2^+2_1)$ value of the γ-vibrational state are correctly described, as is shown in Tables I and II, the second $K^\pi_\nu = 4^+_2$ state at the energy 2.05 MeV should be the double-γ-vibrational state. It is expected that a more rigorously calculated effect of the three-phonon terms on fragmentation of the two-phonon $4^+\{221,221\}$ configuration in the ^{166}Er is small.

The situation with the $K^\pi = 4^+$ double-γ-vibrational state in ^{164}Dy is not yet clear. According to the present calculation, the largest part of the $4^+\{221,221\}$ strength in ^{164}Dy is concentrated on one or two levels in the energy range 2.15–2.30 MeV. The results of calculations depend on the constants κ_0^{22} and κ_0^{44} and truncation of the space of the single-particle levels. By fitting the constants κ_0^{22} and κ_0^{44} and using a full single-particle space it is possible to obtain fragmentation of the $4^+\{221,221\}$ state in the energy range 2.1–3.0 MeV.

There are many nonrotational states in ^{166}Er. The γ-vibrational and first $K^\pi_\nu = 0^-_1$, 1^-_1, and 2^-_1 octupole states are collective states. There are many collective or weakly collective states. In most cases it is difficult to separate nonrotational states into collective and weakly collective states. Only the $K^\pi_\nu = 6^-_1$, 7^-_1, 9^-_1, and 8^-_1 states are pure two-quasiparticle ones. The experimental data in ^{166}Er demonstrate richness of the properties of nuclear excitations. Most of the experimental data are fairly well described in the QPNM. The predictions were made for energy and structure of the nonrotational states below 2.3 MeV.

Satisfactory description of the first quadrupole and octupole states in deformed nuclei has been obtained in the IBM and the QPNM. As has been shown in Ref. [33], there is an essential difference between the IBM and the QPNM in describing the states above the first quadrupole and octupole states in well-deformed nuclei. This difference is due to the fact that in the IBM the quadrupole or octupole strength is strongly fragmented but belongs to one unique d or f boson. The QPNM quadrupole or octupole strength is due to a large number of one-phonon states with a different quasiparticle structure. The two-quasiparticle components of the wave function of the excited state in ^{166}Er are demonstrated by one-nucleon transfer reactions and β-decay measurements.

On the basis of the calculation of energies and wave functions in ^{166}Er performed in the QPNM we can conclude the following.

(1) Fairly good description of energies and structure of the most nonrotational excited states has been obtained.

(2) Large intensities in the (d,t), $(^3\text{He},\alpha)$, $(^3\text{He},d)$, and (α,t) reactions are explained by the corresponding large two-quasiparticle configuration of the one-phonon terms of the wave functions.

(3) The wave function of excited states with an energy below 2.3 MeV is dominated by the one-phonon term. The contributions of the two-phonon components to the normalization of the wave functions in ^{166}Er are smaller than in ^{156}Gd, ^{158}Gd, ^{162}Dy, and ^{168}Er. The small anharmonicity of the nonrotational states with an energy below 2.3 MeV is a general property of the well-deformed doubly even nuclei in the rare-earth region.

(4) We predict the existence of the $K^\pi = 4^+$ two-phonon {221,221} state at energy 2.0–2.2 MeV. The existence of the $K^\pi = 4^+$ double-γ-vibrational state in ^{166}Er is due to a very small density of the $K^\pi = 4^+$ states near the {221,221}pole and a very small value for the function responsible for a coupling between one- and two-phonon terms in the wave function.

(5) There are no other two-phonon states, including $K^\pi = 0^+$ {221,221}, below 2.5 MeV.

(6) Information on the large components of the wave function of excited states can be obtained in experimental and theoretical investigation of γ-ray transition rates between excited states. This is a new information on the nuclear structure in addition to that from inelastic scattering, Coulomb excitation, one- and two-nucleon transfer reactions, and β decays.

We hope that the present calculation will be useful in experiments at a new generation of accelerators and detectors. Of great interest is experimental study of excited states of deformed nuclei at 2–3 MeV.

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