Signatures of Λ-Σ mixing in the magnetic moments of hypernuclei

Carl B. Dover
Department of Physics, Brookhaven National Laboratory, Upton, New York 11973

Herman Feshbach
Center for Theoretical Physics, Laboratory for Nuclear Science and Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139

Avraham Gal
Racah Institute of Physics, The Hebrew University, Jerusalem 91904, Israel
(Received 19 October 1994)

We investigate the influence on hypernuclear magnetic moments of Λ-Σ mixing due to the strong $\Lambda N \leftrightarrow \Sigma N$ interaction. The case of a Λ coupled to an $I = 1$ nuclear core (example ${}^{15}_{\Lambda}$C) is found to be a particularly sensitive probe of this mixing. Qualitative estimates indicate a significant deviation of the magnetic moment from the Schmidt values. The examples of ${}^{3}_{\Lambda}$H, ${}^{4}_{\Lambda}$H(1^+) and ${}^{4}_{\Lambda}$He(1^+) have also been studied, as well as the influence of Λ-Σ mixing on the lifetime associated with the $1^+ \to 0^+$ electromagnetic transition in the $A = 4$ system. We also mention the π^+/π^- ratio in hypernuclear weak decay as another probe of the degree of Λ-Σ mixing.

PACS number(s): 21.80.+a, 21.10.Ky, 14.20.Jn, 13.75.Ev

I. INTRODUCTION

The magnetic moments μ of hypernuclei have been discussed as providing an excellent test of relativistic mean-field theory [1–5], a model of the nucleus based on the Dirac equation with strong scalar and vector potentials [6]. In this model, the reduced nucleon effective mass m_N^*, arising because of the large attractive scalar field, leads to a single-particle convection current enhanced by a factor m_N/m_N^* with respect to the nonrelativistic value. This enhancement is essentially cancelled by a core contribution to the total current ("backflow") [7], which restores the moment to its nonrelativistic Schmidt value ($\mu = \mu_\Lambda$ for a Λ in an $s_{1/2}$ state around a closed-shell nuclear core). In general, in hypernuclei, one might expect this cancellation to be less complete, since the Λ-nucleus and nucleon-nucleus potentials, responsible for the core response, differ considerably. Indeed, early calculations [1,2] appear to indicate measurable deviations from the Schmidt values. However, these calculations omitted the very important tensor coupling of the vector field (ω) to the Λ [8,9]. When this coupling, which is negligible for the nucleon, was included [3,4], the hypernuclear moments were restored to values very close to the Schmidt limit. Finally, we mention the work of Motoba *et al.* [10] and Tanaka [11]. The latter estimates the effect of conventional configuration mixing on hypernuclear magnetic moments, and finds small deviations from the Schmidt values in most cases. This summarizes the current theoretical situation. The experimental prospects have been addressed by Yamazaki [12] and Fukuda *et al.* [13].

What other mechanisms could produce deviations of hypernuclear moments from the Schmidt values? In this paper, we consider for the first time the influence of $\Lambda \leftrightarrow \Sigma$ mixing on magnetic moments. This arises because of a strong interaction $\Lambda N \to \Sigma N$ transition potential, which induces Σ components in the hypernuclear wave function. We treat these admixtures in perturbation theory and make some rough estimates.

The possibility of measurable deviations from the Schmidt limit is due to the large differences in hyperon moments. We have [14]

$$\begin{aligned} \mu_\Lambda &= -0.613 \pm 0.004\mu_N \ , \\ \mu_{\Sigma\Lambda} &= 1.61 \pm 0.08\mu_N \ , \\ \mu_{\Sigma^+} &= 2.42 \pm 0.05\mu_N \ , \\ \mu_{\Sigma^-} &= -1.160 \pm 0.025\mu_N \ , \end{aligned} \tag{1}$$

where μ_N is the nuclear magneton. Note that only the magnitude of $\mu_{\Sigma\Lambda}$, the Σ-Λ transition magnetic moment, is obtained from the measurement of the electromagnetic decay rate $\Sigma^0 \to \gamma\Lambda$. The positive sign of $\mu_{\Sigma\Lambda}$ results from the standard SU(3) phase convention. In addition, the value of μ_{Σ^0}, which is not experimentally accessible, can be estimated in the quark model as

$$\mu_{\Sigma^0} = \tfrac{2}{3}(\mu_p + \mu_n) - \tfrac{1}{3}\mu_\Lambda = 0.79\mu_N \ .$$

The quark model also gives

$$\mu_{\Sigma\Lambda} = \frac{\sqrt{3}}{5}(\mu_p - \mu_n) = 1.63\mu_N \ ,$$

in excellent agreement with the value quoted in Eq. (1). We note further that $\mu_{\Sigma\Lambda}$ is most important in determining the correction to the Schmidt value, since it is always multiplied by the *amplitude* for a Σ^0 admixture, whereas μ_{Σ^+}, μ_{Σ^-}, and μ_{Σ^0} are multiplied by Σ probabilities.

II. THE SIMPLEST HYPERNUCLEUS: ${}^{3}_{\Lambda}$H

The ${}^{3}_{\Lambda}$H system has spin $S = \frac{1}{2}$ and isospin $I = 0$ and is weakly bound (binding energy 0.13 ± 0.05 MeV). To first

0556-2813/95/51(2)/541(6)/$06.00

order, it can be thought of as a Λ bound to a deuteronlike core, with spin-wave function

$$\psi({}^3_\Lambda\text{H},\, I=0,\, S=\tfrac{1}{2},\, S_z=\tfrac{1}{2}) = \sqrt{\tfrac{1}{3}}\Lambda_\uparrow[pn]_0 - \sqrt{\tfrac{2}{3}}\Lambda_\downarrow[pn]_1 \ . \quad (2)$$

Via the $\Lambda N \to \Sigma N$ transition potential, this configuration mixes with the Σ state given by

$$\psi({}^3_\Sigma\text{H},\, I=0,\, S=\tfrac{1}{2},\, S_z=\tfrac{1}{2}) = \frac{1}{\sqrt{3}}\left(\Sigma^-_\uparrow\{pp\} - \Sigma^0_\uparrow\{pn\} + \Sigma^+_\uparrow\{nn\}\right) \ , \quad (3)$$

where we have defined [15,16]

$$[pn]_0 = {}^3S_1(I=0)\ pn \ \text{ pair with } S_z=0 \ ,$$

$$[pn]_1 = {}^3S_1(I=0)\ pn \ \text{ pair with } S_z=1 \ , \quad (4)$$

$$\{pp\},\ \{pn\},\ \text{and } \{nn\} = {}^1S_0(I=1)\ NN \ \text{ pairs } (S_z=0) \ .$$

These two basis states can be mixed to produce eigenstates of the quadratic Casimir operator F^2 of SU(3) defined by

$$F^2 = \sum_{\alpha=1}^{8} F_\alpha^2 \ , \quad (5)$$

where the F_α are SU(3) generators. For baryon-baryon potentials of the SU(3) invariant form

$$V = a + bF^2 \quad (6)$$

the eigenfunctions of the $A=3$ hypernucleus in the strong coupling or SU(3) limit are those that diagonalize F^2 (eigenvalue f^2), namely [15,16]

$$\psi(f^2=3) = \tfrac{1}{2}(\psi_\Lambda + \sqrt{3}\psi_\Sigma) \ ,$$
$$\psi(f^2=9) = \tfrac{1}{2}(\sqrt{3}\psi_\Lambda - \psi_\Sigma) \ . \quad (7)$$

It was shown in Ref. [16] that $b<0$ for pseudoscalar meson exchange as well as for magnetic couplings of vector mesons. Thus $\psi(f^2=9)$ would lie lowest in energy in the strong-coupling limit. From Eq. (7), we see that this state has a Λ probability of 3/4.

Now we investigate the change of magnetic moment as we pass from weak- to strong-coupling limits, evaluating the matrix elements of the magnetic moment operator

$$\hat{\mu} = \sum_i g_i \mu_i S_{iz} \ , \quad (8)$$

where $g_i = 2$ for Λ or Σ and $g_i = 1$ for the deuteronlike pair $[pn]_1$, for which we take $\mu_i = \mu_d = 0.857\mu_N$. For $\{NN\}$ pairs, we have $\mu_i = 0$. In the weak-coupling basis, we have

$$\mu({}^3_\Lambda\text{H}) = -\tfrac{1}{3}\mu_\Lambda + \tfrac{2}{3}\mu_d \cong 0.78\mu_N \ ,$$
$$\mu({}^3_\Sigma\text{H}) = \tfrac{1}{3}(\mu_{\Sigma^-} + \mu_{\Sigma^0} + \mu_{\Sigma^+}) \simeq 0.68\mu_N \ . \quad (9)$$

Note that ${}^3_\Sigma$H is expected to be unbound.

In the SU(3) limit [15,16], we have

$$\mu(f^2=9) = 0.75\mu_N \ ,$$
$$\mu(f^2=3) = 0.70\mu_N \ . \quad (10)$$

Thus we see that the magnetic moment of the $A=3$ hypernucleus is insensitive to the strength of the Σ-Λ coupling, since $\mu({}^3_\Lambda\text{H})$ and $\mu({}^3_\Sigma\text{H})$ are almost equal to each other. The transition moment $\mu_{\Sigma\Lambda}$ does not enter here due to the different isospin of the NN core states in ${}^3_\Lambda$H and ${}^3_\Sigma$H.

III. Λ-Σ MIXING IN $A = 4$ HYPERNUCLEI: LIFETIME AND MAGNETIC MOMENT OF 1^+ STATES

The ${}^4_\Lambda$H and ${}^4_\Lambda$He systems are each observed to have two bound states with $J^\pi = 0^+$ and 1^+, with the 0^+ lying lowest. The 1^+ state decays into the 0^+ by emitting an $M1$ γ ray of about 1.1 MeV [17]. The 1^+ states will have magnetic moments and lifetimes influenced by Λ-Σ mixing. We assess here the expected magnitude of this mixing effect.

To measure the magnetic moment, we need to prepare a *polarized* ${}^4_\Lambda\text{He}(1^+)$ or ${}^4_\Lambda\text{H}(1^+)$ nucleus. A possible way to do this is via the reactions

$${}^4\text{He}(K^-,\pi^-){}^4_\Lambda\text{He}(1^+) \ , \quad (11a)$$
$${}^4\text{He}(K^-,\pi^0){}^4_\Lambda\text{H}(1^+) \quad (11b)$$

at nonforward angles θ_π, where one can exploit the spin-flip amplitudes in the basic process $K^-N \to \pi\Lambda$. At 1.4 GeV/c, there are overlapping resonances in the $\bar{K}N$ system, and the spin-flip amplitude is sizable at $\theta_\pi \simeq 20°$ [18].

We first construct the Λ and Σ weak-coupling eigenstates ψ_{JM} for $A=4, I=1/2, J=0,1$ systems. For ${}^4_\Lambda$He and ${}^4_\Sigma$He, we have

$$\psi_{00}({}^4_\Lambda\text{He}) = \frac{1}{\sqrt{2}}(\Lambda_\uparrow \otimes {}^3\text{He}_\downarrow - \Lambda_\downarrow \otimes {}^3\text{He}_\uparrow) \ , \quad (12a)$$

$$\psi_{00}({}^4_\Sigma\text{He}) = \frac{1}{\sqrt{3}}(\Sigma^+_\uparrow \otimes t_\downarrow - \Sigma^+_\downarrow \otimes t_\uparrow) - \frac{1}{\sqrt{6}}(\Sigma^0_\uparrow \otimes {}^3\text{He}_\downarrow - \Sigma^0_\downarrow \otimes {}^3\text{He}_\uparrow) \ , \quad (12b)$$

$$\psi_{10}({}^4_\Lambda\text{He}) = \frac{1}{\sqrt{2}}(\Lambda_\uparrow \otimes {}^3\text{He}_\downarrow + \Lambda_\downarrow \otimes {}^3\text{He}_\uparrow) \ , \quad (12c)$$

$$\psi_{10}({}^4_\Sigma\text{He}) = \frac{1}{\sqrt{3}}(\Sigma^+_\uparrow \otimes t_\downarrow + \Sigma^+_\downarrow \otimes t_\uparrow) - \frac{1}{\sqrt{6}}(\Sigma^0_\uparrow \otimes {}^3\text{He}_\downarrow + \Sigma^0_\downarrow \otimes {}^3\text{He}_\uparrow) \ , \quad (12d)$$

$$\psi_{11}({}^4_\Lambda\mathrm{He}) = \Lambda_\uparrow \otimes {}^3\mathrm{He}_\uparrow \ , \qquad (12e)$$

$$\psi_{11}({}^4_\Sigma\mathrm{He}) = \sqrt{2/3}\Sigma^+_\uparrow \otimes t_\uparrow - \sqrt{1/3}\Sigma^0_\uparrow \otimes {}^3\mathrm{He}_\uparrow \ . \qquad (12f)$$

A similar construction holds for ${}^4_\Lambda$H and ${}^4_\Sigma$H. The moments of the uncoupled states are then

$$\mu({}^4_\Lambda\mathrm{He};1^+) = \langle\psi_{11}({}^4_\Lambda\mathrm{He})|\hat{\mu}|\psi_{11}({}^4_\Lambda\mathrm{He})\rangle \qquad (13)$$

and so forth. Of course, the magnetic moments of the 0^+ states vanish. We find

$$\mu({}^4_\Lambda\mathrm{He};1^+) = \mu_\Lambda + \mu_{^3\mathrm{He}} \ , \qquad (14a)$$

$$\mu({}^4_\Sigma\mathrm{He};1^+) = \tfrac{1}{3}(\mu_{\Sigma^0} + \mu_{^3\mathrm{He}}) + \tfrac{2}{3}(\mu_{\Sigma^+} + \mu_t) \ , \qquad (14b)$$

$$\mu({}^4_\Lambda\mathrm{H};1^+) = \mu_\Lambda + \mu_t \ , \qquad (14c)$$

$$\mu({}^4_\Sigma\mathrm{H};1^+) = \tfrac{1}{3}(\mu_{\Sigma^0} + \mu_t) + \tfrac{2}{3}(\mu_{\Sigma^-} + \mu_{^3\mathrm{He}}) \ . \qquad (14d)$$

Using the measured values, and assuming $\mu_{\Sigma^0} = 0.79\mu_N$ [cf. the discussion following Eq. (1)], we arrive at the predictions for uncoupled states shown in Table I. The experimental values $\mu(^3\mathrm{He}) = -2.13\mu_N$ and $\mu(^3\mathrm{H}) = 2.98\mu_N$ were used.

Now consider Λ-Σ mixing in a schematic model:

$$\psi_{11}({}^4_Y\mathrm{He}) = \alpha\psi_{11}({}^4_\Lambda\mathrm{He}) + \beta\psi_{11}({}^4_\Sigma\mathrm{He}) \ , \qquad (15a)$$

$$\psi_{11}({}^4_Y\mathrm{H}) = \alpha\psi_{11}({}^4_\Lambda\mathrm{H}) + \beta\psi_{11}({}^4_\Sigma\mathrm{H}) \ . \qquad (15b)$$

Then we arrive at the modified moments

$$\mu({}^4_Y\mathrm{He}) = \alpha^2\mu({}^4_\Lambda\mathrm{He}) + \beta^2\mu({}^4_\Sigma\mathrm{He}) - \frac{2\alpha\beta}{\sqrt{3}}\mu_{\Sigma\Lambda} \ , \qquad (16a)$$

$$\mu({}^4_Y\mathrm{H}) = \alpha^2\mu({}^4_\Lambda\mathrm{H}) + \beta^2\mu({}^4_\Sigma\mathrm{H}) + \frac{2\alpha\beta}{\sqrt{3}}\mu_{\Sigma\Lambda} \ . \qquad (16b)$$

The most significant correction to μ is due to the transition moment $\mu_{\Sigma\Lambda}$, since this term is *linear* in the Σ admixture β. This correction, however, cancels out in the isoscalar combination given by the sum of (16a) and (16b). For small β, we have

$$\Delta\mu({}^4_Y\mathrm{He}) = -1.86\beta \ , \qquad (17)$$

with the opposite sign for $\Delta\mu({}^4_Y\mathrm{H})$. At this point, a quantitative procedure would be to solve the coupled channel problem, assuming a $\Sigma \to \Lambda$ transition potential $V_{\Sigma N}$, perhaps taken from a meson exchange model [19]. Such calculations have been carried out for the $A = 3$ system by Afnan and Gibson [20] and for $A = 4, 5$ by Carlson [21]. To obtain a qualitative idea of the magnitude of $\Delta\mu$, we employ the perturbative estimate

$$\beta \sim \frac{\langle V_{\Sigma N\to\Lambda N}\rangle}{m_\Lambda - m_\Sigma} \ . \qquad (18)$$

TABLE I. Predicted magnetic moments of 1^+ states in $A = 4$ hypernuclei (units of μ_N).

System	$\mu(1^+)$
${}^4_\Lambda$He	−2.74
${}^4_\Sigma$He	3.15
${}^4_\Lambda$H	2.37
${}^4_\Sigma$H	−0.94

The transition potential $V_{\Sigma N\to\Lambda N}(r)$ depends on the meson exchange model employed. For example, in [20,22], we have $V_{\Sigma N\to\Lambda N}(r) < 0$, so $\beta > 0$. Taking the magnitude of the average matrix element $\langle V_{\Sigma N\to\Lambda N}\rangle$ to lie in the range 2–5 MeV, similar to diagonal ΛN or $\Lambda\Lambda$ matrix elements [23], we crudely estimate $\beta \simeq 0.03$–0.07, leading to $\Delta\mu({}^4_Y\mathrm{He}) \simeq -0.05\mu_N$ to $-0.13\mu_N$, a 2–5 % correction to the magnetic moment of ${}^4_\Lambda$He. This estimate of β is roughly consistent with the coupled channel calculations of Carlson [21], who obtains a Σ probability of about 0.8% in ${}^4_\Lambda\mathrm{H}(1^+)$. If the speculations of Refs. [15,16] prove to be correct, then β would be considerably larger.

We note that Eqs. (15a) and (15b) do not take account of any radial dependence of the wave function. In particular, Σ-Λ coupling will be strongest in the center of the nucleus, where the nucleon density is highest. In this region, one may even approach an SU(3) eigenstate [see Eq. (7) for the $A = 3$ case]. In the nuclear periphery, one will certainly approach the weak-coupling limit, and the Λ component will dominate.

We now consider the electromagnetic transition between the 1^+ and 0^+ states in ${}^4_\Lambda$He and ${}^4_\Lambda$H. We start from the standard formula for the rate of $M1$ transitions

$$\text{Rate } (M1) = 4.2\times10^{12}E^3(2J_i+1)^{-1}|\langle f||\hat{\mu}||i\rangle|^2 \ ,$$
$$\hat{\mu} = \hat{\mu}_\Lambda + \hat{\mu}_{\mathrm{core}} = g_\Lambda \mathbf{j}_\Lambda + g_{\mathrm{core}}\mathbf{J}_{\mathrm{core}} \qquad (19)$$

where the rate is in units of sec^{-1}, E is in MeV, and J_i, J_f are the initial and final nuclear spins within the hypernuclear doublet of levels with $\mathbf{J} = \mathbf{j}_\Lambda + \mathbf{J}_{\mathrm{core}}$ for $j_\Lambda = \frac{1}{2}$. The general expression in terms of g_Λ and g_{core} is given in [24], which for the $1^+ \to 0^+$ transitions in question becomes

$$\text{Rate } (M1) = 4.2\times10^{12}E^3(\mu_\Lambda - \mu_{\mathrm{core}})^2 \ , \qquad (20)$$

where

$$\mu_\Lambda - \mu_{\mathrm{core}} = \begin{cases} -3.59\mu_N & ({}^4_\Lambda\mathrm{H}) \\ +1.51\mu_N & ({}^4_\Lambda\mathrm{He}). \end{cases} \qquad (21)$$

Using $E = 1.1$ MeV [17], we then obtain the corresponding lifetime τ as

$$\tau = \frac{1}{\text{Rate } (M1)} = \begin{cases} 1.4\times10^{-14}\text{ sec} & ({}^4_\Lambda\mathrm{H}) \\ 0.8\times10^{-13}\text{ sec} & ({}^4_\Lambda\mathrm{He}). \end{cases} \qquad (22)$$

Such short lifetimes would be very difficult to measure experimentally [25].

We now consider the effect of Σ-Λ mixing on the $M1$ lifetimes. For ${}^4_Y$He, we consider mixed wave functions for the 1^+ and 0^+ states,

$$\psi({}^4_Y\mathrm{He};1^+) \approx \psi_{10}({}^4_\Lambda\mathrm{He}) + \beta_1\psi_{10}({}^4_\Sigma\mathrm{He}) \ ,$$
$$\psi({}^4_Y\mathrm{He};0^+) \approx \psi_{00}({}^4_\Lambda\mathrm{He}) + \beta_0\psi_{00}({}^4_\Sigma\mathrm{He}) \ , \qquad (23)$$

and work to first order in $\beta_{0,1}$. The relevant transition-matrix element involving $\mu_{\Sigma\Lambda}$ is given by

$$\langle\psi_{10}({}^4_\Sigma\text{He})|\hat{\mu}|\psi_{00}({}^4_\Lambda\text{He})\rangle = \langle\psi_{10}({}^4_\Lambda\text{He})|\hat{\mu}|\psi_{00}({}^4_\Sigma\text{He})\rangle = -\mu_{\Sigma\Lambda}/\sqrt{3}\ . \quad (24)$$

The modified lifetime of the mixed state ${}^4_Y\text{He}$ is then expressed in terms of the ratio

$$\frac{\tau({}^4_Y\text{He})}{\tau({}^4_\Lambda\text{He})} \approx \left[1-(\beta_0+\beta_1)\mu_{\Sigma\Lambda}/\sqrt{3}(\mu_\Lambda-\mu_{{}^3\text{He}})\right]^{-2} \approx [1-0.61(\beta_0+\beta_1)]^{-2}\ . \quad (25)$$

Thus, rather small Σ admixtures lead to noticeable deviations from $\tau({}^4_\Lambda\text{He})$, since the mixing amplitudes $\beta_{0,1}$ enter *linearly*. A similar consideration for ${}^4_Y\text{H}$ yields

$$\frac{\tau({}^4_Y\text{H})}{\tau({}^4_\Lambda\text{H})} \approx \left[1+(\beta_0+\beta_1)\mu_{\Sigma\Lambda}/\sqrt{3}(\mu_\Lambda-\mu_t)\right]^{-2} \approx [1-0.26(\beta_0+\beta_1)]^{-2}\ . \quad (26)$$

Here, the effect is less marked than in ${}^4_Y\text{He}$.

IV. Σ-Λ MIXING IN THE *p* SHELL

We close with some simple considerations on Σ-Λ mixing in the p shell, where more dramatic effects may be anticipated in some selected cases. First we note that if the Λ is coupled to an $I=0$ core, the Σ admixtures will be very small, since, by conservation of isospin, they only arise through $I=1$ core states. An example is the ground state of ${}^{13}_Y\text{C}$, which would have the composition

$$|{}^{13}_Y\text{C}(J=\tfrac{1}{2};\,I=0)\rangle \approx \alpha|\Lambda_{s_{1/2}}\otimes{}^{12}\text{C(g.s.)}\rangle + \beta|\Lambda_{s_{1/2}}\otimes{}^{12}\text{C}^*(J^\pi=0^+,1^+;I=0)\rangle + \gamma|\Sigma_{s_{1/2}}\otimes(A=12)^*(J^\pi=0^+,1^+;I=1)\rangle + \cdots\ , \quad (27)$$

where

$$|\Sigma_{s_{1/2}}\otimes(A=12)^*\rangle = (\Sigma^-_{s_{1/2}}\otimes{}^{12}\text{N}^* - \Sigma^0_{s_{1/2}}\otimes{}^{12}\text{C}^* + \Sigma^+_{s_{1/2}}\otimes{}^{12}\text{B}^*)/\sqrt{3}\ .$$

The matrix element involving the overlap of the ${}^{12}\text{C}$ ground state (g.s.) with the core excited states will be suppressed by recoupling coefficients and reduced radial overlaps, so γ will be small. Thus we expect $\mu({}^{13}_\Lambda\text{C}) \approx \mu_\Lambda$.

A more favorable case involves the coupling of a Λ to a $J=0, I=1$ nuclear core. Examples are ${}^{15}_\Lambda\text{C}$ and ${}^{19}_\Lambda\text{O}$, which could be fabricated via the reactions

$${}^{15}\text{N}(K^-,\pi^0){}^{15}_\Lambda\text{C},\quad {}^{15}\text{N}(\gamma,K^+){}^{15}_\Lambda\text{C},\quad {}^{19}\text{F}(K^-,\pi^0){}^{19}_\Lambda\text{O}. \quad (28)$$

We now write a schematic wave function of the form

$$|{}^{15}_Y\text{C}(J=\tfrac{1}{2};\,I=1)\rangle \approx \alpha|\Lambda_{s_{1/2}}\otimes{}^{14}\text{C(g.s.)}\rangle + \beta\frac{1}{\sqrt{2}}\left[\Sigma^0_{s_{1/2}}\otimes{}^{14}\text{C(g.s.)} - \Sigma^-_{s_{1/2}}\otimes{}^{14}\text{N}^*(J^\pi=0^+,\,I=1)\right] + \gamma\Sigma^-_{s_{1/2}}\otimes{}^{14}\text{N(g.s.)} + \cdots\ , \quad (29)$$

where ${}^{14}\text{N}^*(J^\pi=0^+,\,I=1)$, the isobaric analog of the ${}^{14}\text{C}$ ground state, is the first excited state of ${}^{14}\text{N}$, at 2.31 MeV above the $J^\pi=1^+$, $I=0$ ground state of ${}^{14}\text{N}$. To first order in β and γ, the term involving $\mu_{\Sigma\Lambda}$ again dominates, and we find

$$\mu({}^{15}_Y\text{C(g.s.)}) \approx \mu_\Lambda + \sqrt{2}\beta\mu_{\Sigma\Lambda} \approx \mu_\Lambda(1-3.71\beta)\ . \quad (30)$$

In this case, we obtain a considerable amplification factor multiplying β, so the effect of Σ coupling is enhanced. For β in the range +0.03 to +0.07, as estimated from Eq. (18), we find a suppression of $\mu({}^{15}_Y\text{C(g.s.)})$ in the range 11–26%. This is much more dramatic than the rather modest changes we exhibited for the $A=3$ and 4 systems. In the limit of SU(3) eigenstates for ${}^{15}_Y\text{C}$ [15,16], the results would be even more striking. Even in the weak-coupling limit, the effects of Σ-Λ coupling on the magnetic moment of ${}^{15}_Y\text{C}$ are likely to be much larger than the effects considered in [1–5] in the context of relativistic mean-field theory.

V. CONCLUSIONS

We have studied the problem of Σ-Λ mixing in hypernuclei, arising from the strong conversion process $\Sigma N \to \Lambda N$. In particular, we investigated the effect of this mixing on hypernuclear magnetic moments μ. Studies in the context of relativistic mean-field theory [1–5], which so far ignore Σ-Λ coupling, have indicated very small deviations of moments from the Schmidt limits, so we might hope to use such departures as signatures of the magnitude of Σ-Λ coupling. Our explorations, which are only qualitative, suggest some favorable cases in the nuclear p shell ($^{15}_{\Lambda}$C, for instance). The s shell ($^{3}_{\Lambda}$H,$^{4}_{\Lambda}$H,$^{4}_{\Lambda}$He) is less promising. In $^{3}_{\Lambda}$H, for instance, μ remains close to the Schmidt value even when Λ and Σ are coherently mixed to form SU(3) eigenstates. A full hypernuclear shell model calculation is required to evaluate these suggestions quantitatively. However, it is clear that Σ-Λ mixing becomes most effective when there is no change in the nuclear core state. This maximizes the radial overlaps and recoupling coefficients. For even A cores in the p shell, this requires that the core have $I = 1$.

Similar considerations follow for heavier core nuclei with neutron excess $N - Z > 0$. The configuration $\Lambda_{s_{1/2}} \otimes {}^{A}Z$(g.s.), with isospin $0 + \mathbf{T}_0 = \mathbf{T}_0$, where $T_0 = (N-Z)/2$, gets admixed most effectively with the $\Lambda \to \Sigma$ excited configuration $\Sigma_{s_{1/2}} \otimes {}^{A}Z$ of isospin $\mathbf{1} + \mathbf{T}_0 = \mathbf{T}_0$, where ${}^{A}Z$ stands for ${}^{A}Z$(g.s.) when $\Sigma = \Sigma^0$, and for the analog of ${}^{A}Z$(g.s.) when $\Sigma = \Sigma^-$. These types of admixtures were considered by Auerbach [26] who studied their effect on hypernuclear widths. His estimates give admixture amplitudes of order $|\beta| = 0.05$, within the range considered here by us.

Finally, we mention an old puzzle, where Σ-Λ coupling may play a significant role, namely, the ratio of π^+ to π^- emission in the weak decay of light hypernuclei. The measured ratio is

$$R = \frac{\Gamma(^{4}_{\Lambda}\mathrm{He} \to \pi^+ + (pnnn))}{\Gamma(^{4}_{\Lambda}\mathrm{He} \to \pi^- + (pppn))} = \begin{cases} 0.043 \pm 0.017 & \text{(Ref. [27])} \\ 0.054 - 0.069 & \text{(Ref. [28])}. \end{cases} \tag{31}$$

A π^+ decay may be the result of a $\Lambda \to n\pi^0$ decay, followed by $\pi^0 p \to \pi^+ n$ charge exchange, or a Σ^+ admixture in the $^{4}_{\Lambda}$He g.s. wave function, as in Eq. (23), followed by $\Sigma^+ \to n\pi^+$ weak decay. Early estimates [22,29,30] gave $R \le 0.01$, a significant shortfall. It would be interesting to perform a Σ-Λ coupled channel calculation, employing modern meson exchange potentials [19], in order to evaluate R. Note that R is proportional to the *square* of the Σ^+ admixture in the $^{4}_{\Lambda}$He wave function. Magnetic moments, on the other hand, can contain a term *linear* in the Σ admixture, multiplied by the transition moment $\mu_{\Sigma\Lambda}$. This can give a much larger effect, as in the case of the $^{15}_{\Lambda}$C moment, for instance.

There have been no measurements of hypernuclear magnetic moments, although some are planned at KEK in Japan [13]. Our considerations suggest that such measurements are well motivated, since they would shed light on the Σ admixtures in hypernuclear wave functions.

This manuscript was authored under Contract No. DE-AC02-76CH00016 with the U.S. Department of Energy. The research of A.G. was supported by the Israel Science Foundation.

[1] J. Cohen and R. J. Furnstahl, Phys. Rev. C **35**, 2231 (1987).
[2] J. Mares and J. Žofka, Phys. Lett. B **249**, 181 (1990).
[3] A. O. Gattone, M. Chiapparini, and E. D. Izquierdo, Phys. Rev. C **44**, 548 (1991).
[4] J. Cohen and J. V. Noble, Phys. Rev. C **46**, 801 (1992).
[5] J. Cohen, Phys. Rev. C **48**, 1346 (1993).
[6] B. D. Serot and J. D. Walecka, in *Advances in Nuclear Physics*, edited by J. W. Negele and E. Vogt (Plenum, New York, 1986), Vol. 16, pp. 1–327.
[7] J. A. McNeil *et al.*, Phys. Rev. C **34**, 746 (1986); W. Bentz *et al.*, Nucl. Phys. **A436**, 593 (1985); R. D. Furnstahl and B. D. Serot, *ibid.* **A468**, 539 (1987).
[8] C. B. Dover and A. Gal, *Progress in Particle and Nuclear Physics*, edited by D. Wilkinson, (Pergamon, Oxford, 1984), Vol. 12, pp. 171–239, see Eq. (2.34) in particular.
[9] B. K. Jennings, Phys. Lett. B **246**, 325 (1990).
[10] T. Motoba, H. Bandō, K. Ikeda, and T. Yamada, Prog. Theor. Phys. Suppl. No. 81 (Kyoto, 1985), Chap. III, pp. 42–103, see Table VIII.
[11] Y. Tanaka, Phys. Lett. B **227**, 195 (1989); Nuovo Cimento A **104**, 371 (1991).
[12] T. Yamazaki, Phys. Lett. **160B**, 227 (1985); Nucl. Phys. **A446**, 467c (1985).
[13] T. Fukuda, H. Ejiri, T. Shibata, H. Bandō, and T. Motoba, in *Proceedings of the 1986 INS International Symposium on Hypernuclear Physics*, edited by H. Bandō, O. Hashimoto, and K. Ogawa (Institute for Nuclear Study, Tokyo, 1986), pp. 170–176. In this paper, a proposed experiment to measure the magnetic moment of $^{55}_{\Lambda}$Fe is described, using the time-integral perturbed angular correlation method, in which the time-integrated precession of μ over the lifetimes of the state is measured. Predictions of the spin polarization of hypernuclei produced in (π^+, K^+) and (K^-, π^-) reactions are given by H. Ejiri *et al.*, Phys. Rev. C **36**, 1435 (1987) and H. Ejiri, T. Kishimoto, and H. Noumi, Phys. Lett. B **225**, 35 (1989).
[14] Review of particle properties, L. Montanet *et al.*, Phys. Rev. D **45** (1992).
[15] C. B. Dover and H. Feshbach, Phys. Rev. Lett. **59**, 2539 (1987).
[16] C. B. Dover and H. Feshbach, Ann. Phys. **198**, 321 (1990); **217**, 51 (1992); Note that the potential V also contains, in general, a term cG^3, where G^3 is the cubic Casimir operator. The phenomenological effects of this term are significant, but it is omitted here for simplicity of notation.
[17] A. Bamberger *et al.*, Nucl. Phys. **B60**, 1 (1973); M. Bedjidian *et al.*, Phys. Lett. **62B**, 467 (1976); **83B**, 252 (1979).
[18] T. Kishimoto, H. Ejiri, and H. Bandō, Phys. Lett. B **232**, 24 (1989).
[19] M. M. Nagels, T. A. Rijken, and J. J. deSwart, Phys. Rev. D **15**, 2547 (1977); **20**, 1633 (1979); P. M. M. Maessen, T. A. Rijken, and J. J. deSwart, Phys. Rev. C **40**, 2226 (1989).
[20] I. R. Afnan and B. F. Gibson, Phys. Rev. C **40**, R7 (1989).
[21] J. Carlson, in *Proceedings of the LAMPF Workshop on* (π, K) *Physics*, Los Alamos, New Mexico, 1990, edited by B. F. Gibson, W. R. Gibbs, and M. B. Johnson, AIP

Conf. Proc. No. 224 (AIP, New York, 1991), pp. 198–210.
[22] F. Von Hippel, Phys. Rev. **136**, B455 (1964). The transition potential $V_{\Sigma N \to \Lambda N}(r)$ is shown in Fig. 5(a) and is negative in sign.
[23] C. B. Dover, D. J. Millener, A. Gal, and D. H. Davis, Phys. Rev. C **44**, 1905 (1991).
[24] R. H. Dalitz and A. Gal, Ann. Phys. **116**, 167 (1978), see Eq. (2.26) in particular.
[25] E. Recknagel, in *Nuclear Spectroscopy and Reactions*, edited by J. Cerny (Academic, New York, 1974), Pt. C, p. 93.
[26] N. Auerbach, Phys. Rev. C **35**, 1798 (1987).
[27] G. Keyes, J. Sacton, J. H. Wickens, and M. M. Block, Nuovo Cimento **31A**, 401 (1976).
[28] C. Mayeur *et al.*, Nuovo Cimento **44**, 698 (1966); G. Bohm *et al.*, Nucl. Phys. **B9**, 1 (1969).
[29] R. H. Dalitz and F. Von Hippel, Nuovo Cimento **34**, 799 (1964).
[30] R. H. Dalitz and G. Rajasekharan, Phys. Lett. **1**, 58 (1962).