# Spin-dependent structure functions of nuclei in the meson-nucleon theory

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A theoretical approach to the investigation of spin-dependent structure functions in deep inelastic scattering of polarized leptons off polarized nuclei, based on the effective meson-nucleon theory and operator product expansion method, is proposed and applied to deuteron and <sup>3</sup>He. The explicit forms of the moments of the deuteron and <sup>3</sup>He spin-dependent structure functions are found and numerical estimates of the influence of nuclear structure effects are presented.

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# I. INTRODUCTION

The large body of experiments on deep inelastic leptonnucleon scattering performed in the 1970s and 1980s, which resulted in the amazing agreement of the data with the quark-parton model and the confirmation of the phenomenon of logarithmic violation of Bjorken scaling, provided the basis for the foundation of quantum chromodynamics (QCD) as the theory of strong interactions. However, the latest experimental results in this field seem to display deviations from the predictions of the quarkparton model ("free QCD") and perturbative QCD [1,2]. In this context, it should be stressed that basic theorems (sum rules) of QCD require the knowledge of the spin structure function (SSF)  $g_1^n(x)$  of the neutron.

This has been recently extracted by deep inelastic scattering of polarized leptons off polarized nuclear targets, e.g., deuteron [Spin Muon Collaboration (SMC) [3]] and <sup>3</sup>He (E142 experiment [4]) and these data, combined with earlier data of the European Muon Collaboration (EMC) on the proton [1], are being currently used to check the predictions of QCD [5,6].

The Bjorken sum rule has been computed from E142 data and, using the result  $\int_0^1 g_1^p(x) dx = 0.126 \pm 0.010 \pm 0.015$  obtained by the EMC Collaboration [1], the value

 $\int_0^1 (g_1^p - g_1^n) \, dx = 0.148 \pm 0.022 \text{ has been found [4], to be compared with the theoretical prediction of <math>0.187 \pm 0.004$ . At the same time, the SMC estimated for the first time the first moment of SSF of the "isoscalar" nucleon:  $M_n \equiv \int_0^1 g_1^N \, dx = 0.023 \pm 0.02 \pm 0.015$ , and combining again these data with the EMC proton data, the Bjorken sum rule was found to be  $\int_0^1 (g_1^p - g_1^n) \, dx = 0.20 \pm 0.05 \pm 0.04$ .

New experiments are planned at DESY [7], CERN [3], and SLAC [4], also aimed at an improved measurement of this fundamental prediction of QCD.

It is worth stressing here that all information on the neutron SSF have been and will be obtained by analyzing deep inelastic scattering (DIS) off polarized *nuclear* targets, in particular <sup>2</sup>H and <sup>3</sup>He. Therefore such information can in principle depend upon nuclear structure effects, which however are considered to provide only a minor correction.

As a matter of fact, the possibility to measure the near free isoscalar nucleon structure functions by using polarized <sup>2</sup>H target is motivated by the fact that typical nuclear effects in the deuteron are small and are predominantly determined by well-known spin-orbital structure of the deuteron wave function (cf. Refs. [8.9]). At the same time, the interest in polarized  $^{3}$ He targets stems from the observation [10] that since the dominant component of the <sup>3</sup>He wave function is a fully symmetric S wave with the two protons in a spin singlet state, a polarized <sup>3</sup>He can be associated to a large extent with a polarized neutron. It should however be pointed out that the precision required by a significant check of the Bjorken sum rule would demand a quantitative estimate of all possible nuclear effects. For example, in the case of <sup>3</sup>He, the small S' and D wave components of the realistic three body wave function generate a proton contribution to the <sup>3</sup>He polarization and asymmetry, which has to be subtracted in order to obtain information on the neutron properties. Recently, the proton contribution to the process  ${}^{3}\vec{\mathrm{He}}(\vec{e},e')X$  has been quantitatively

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evaluated by full convolution approach, where not only Fermi motion [11] but binding as well [12] were taken into account by generalizing the usual convolution approach (see, e.g., Ref. [13]), by introducing the concept of spin dependent spectral function [14,15], and nuclear effects have been indeed found to be small. Relativistic light cone calculations have been performed in Ref. [16] using the spin dependent spectral function of Ref. [15]; the obtained results practically do not differ from the nonrelativistic ones obtained in Ref. [14].

The underlying reason for the applicability of the convolution model is the existence of two typical momentum scales in deep inelastic processes, which leads to the factorization of the amplitude of the reaction into two pieces, depending, respectively, on the "large" external momentum, i.e., the structure function of the nucleon, and the "low" typical momenta of nucleons in the nucleus, i.e., the momentum distribution of the nucleons [17]. A rigorous method to analyze such a factorization is based upon the Wilson operator product expansion (OPE). A theoretical approach to investigate deep inelastic scattering on nuclei by using the OPE method within the effective meson-nucleon theory with one-boson-exchange (OBE) interaction has been suggested in Refs. [18,19]; these calculations have been performed within well-defined approximations, such as the leading twist approximation in the OPE and the lowest order approximation on the meson-nucleon coupling constant. Such an approach allows one to derive a convolution formula which includes binding effects and, at the same time, preserves the energy-conservation sum rule.

In this paper the model is extended to deep inelastic scattering of polarized leptons off polarized nuclear targets. To this end, the set of operators, providing the basis for OPE, is extended by considering the axial operators in terms of nucleon fields interacting with meson fields. Using the nonrelativistic reduction, an explicit form of the operators relevant to describe polarized deep inelastic scattering are found. Particular attention is paid to the investigation of the properties of the physical nucleon, e.g., the physical mass, the meson cloud, the renormalization constants, and the SSF's which appear in the calculation. The explicit expressions for moments of SSF of the lightest nuclei, the deuteron, and <sup>3</sup>He, are found and the inverse Mellin transform reconstructs the corresponding nuclear SSF in the form of convolution of the nucleon SSF and effective distributions of the nucleons in a nucleus. The obtained formulas are generalized to the case of heavy nuclei and found to be similar to those used in the conventional convolution approach [12]. As an example of applications of the method, the SSF of the polarized deuteron and <sup>3</sup>He are numerically estimated and compared with recent experimental data.

The paper is organized as follows. In Sec. II the basic formalism is presented. The antisymmetric part of the Compton scattering amplitude is defined in terms of the axial twist-two operators in the OPE method and their explicit form in the nonrelativistic limit is computed. In Sec. III the moments of the spin-dependent structure function  $g_1^N(x)$  for the physical nucleon are evaluated in terms of the corresponding moments of bare nucleons and meson cloud contribution. The moments and the SSF  $g_1^D(x)$  and  $g_1^{^{3}\text{He}}(x)$  are calculated in Sec. IV, where an extension to complex nuclei is proposed and a formal comparison with the conventional convolution approach is illustrated. Preliminary results for A = 2 have already been presented in Ref. [20]. The results of numerical calculations are presented in Sec. V and the conclusions in Sec. VI, respectively.

# **II. BASIC FORMALISM**

# A. Kinematics and notation

The spin-dependent structure function can be determined experimentally by measuring the asymmetry in the reaction with polarized particles

$$\vec{l} + \vec{A} \longrightarrow l' + X.$$

In the one-photon-exchange approximation the relevant part of the cross section can be written in terms of the antisymmetric parts of the leptonic  $L_{\mu\nu}$  and hadronic  $W_{\mu\nu}$  tensors:

$$\frac{d^2\sigma}{dE'd\Omega} = \frac{\alpha^2}{Q^4} \frac{E'}{M_A E} L_{[\mu\nu]} W^{[\mu\nu]}, \qquad (2.1)$$

where  $[\mu\nu]$  are the antisymmetric indices, E and E' are the initial and final energies of the lepton,  $Q^2 \equiv -q^2$  is the square of four-momentum transfer,  $\alpha$  stands for the electromagnetic fine structure constant, and  $M_A$  is the mass of the target.

The leptonic tensor is obtained from quantum electrodynamics and the antisymmetric part of the hadronic tensor  $W_{\mu\nu}$  is expressed in terms of two independent spin structure functions  $G_{1,2}$ :

$$W_{[\mu\nu]}(p_A, q, S) = i\epsilon_{\mu\nu\lambda\sigma} q^{\lambda} \left( G_1(q_0, Q^2) S^{\sigma} + [(p_A \cdot q) S^{\sigma} - (S \cdot q) p_A^{\sigma}] \frac{G_2(q_0, Q^2)}{M_A^2} \right),$$
(2.2)

where  $p_A$  denotes the 4-momentum of the target,  $q_0$ stands for the time component of the 4-vector q (in the rest frame of the target we use the notation  $q_0 \equiv \nu$ ), and S is the polarization four-vector, normalized as  $S \cdot S = -1$ and satisfying the relation  $S \cdot p_A = 0$ . In what follows we will consider deep inelastic scattering off polarized targets with spin one-half (the nucleon and <sup>3</sup>He) and one (the deuteron), in which case the vector S may be computed in quantum field theory as the mean value of the canonical spin operator defined by Noether's theorem [21].<sup>1</sup> We

<sup>&</sup>lt;sup>1</sup>For example, its components are  $S_{\lambda} = (0, 0, 0, \langle \sigma_z \rangle)$  for the nucleon and  $S_{\lambda} = -(i/M_D)\epsilon_{\lambda\mu\nu\rho}\mathcal{E}^{\mu}\mathcal{E}^{\star\nu}p_D^{\rho}$ , for the deuteron with total momentum  $p_D$  and polarization 4-vector  $\mathcal{E}^{\mu}$ .

choose the quantization axis in the opposite direction to the photon momentum,  $q^{\mu} = (\nu, 0, 0, -|\mathbf{q}|)$ .

In order to measure the SSF  $G_1$  and  $G_2$  appearing in Eq. (2.2), one has to consider the difference between the cross sections corresponding to parallel and antiparallel electron and target spins, respectively, for one has

$$\frac{d^2\sigma}{dE'd\Omega} (\uparrow\uparrow) - \frac{d^2\sigma}{dE'd\Omega} (\uparrow\downarrow)$$
$$= \frac{4\alpha^2 E'}{EQ^2} \left[ (E + E'\cos\theta) G_1 - Q^2 G_2/M_A \right], \quad (2.3)$$

where  $\theta$  is the electron scattering angle. In the Bjorken limit  $(Q^2 \to \infty, \nu \to \infty, x \equiv Q^2/2p_A q$  is fixed), which will be considered from now on, the functions  $G_{1,2}$  are predicted to depend only upon x, yielding the "true" dimensionless spin-dependent structure functions  $g_1(x) = \nu G_1(\nu, Q^2)$  and  $g_2(x) = \nu^2 G_2(\nu, Q^2)/M_A$ .

Defining the asymmetry  $A_{\parallel}(x)$  as the ratio

$$A_{\parallel}(x) = \frac{d^2\sigma/dE'd\Omega\left(\uparrow\uparrow\right) - d^2\sigma/dE'd\Omega\left(\uparrow\downarrow\right)}{d^2\sigma/dE'd\Omega\left(\uparrow\uparrow\right) + d^2\sigma/dE'd\Omega\left(\uparrow\downarrow\right)} \quad , \quad (2.4)$$

one obtains

$$A_{\parallel}(x) = \frac{2xg_1(x)}{F_2(x)},$$
(2.5)

where  $F_2(x)$  is the usual spin independent structure function. The asymmetry (2.5) and, consequently, the SSF are the main objects of experimental and theoretical investigations.

Since any structure function, either spin independent or spin dependent, can be represented as a linear combinations of helicity amplitudes, it is instructive to perform further analysis in the helicity basis. For a thorough analysis of helicity amplitudes in case of targets with an arbitrary spin we refer the interested reader to [22]. We define the amplitudes  $A_{\lambda \ M,\lambda' \ M'}$  as

$$A_{\lambda \mathcal{M},\lambda' \mathcal{M}'} = \varepsilon_{\lambda}^{\mu \star} W_{[\mu\nu]}(S) \varepsilon_{\lambda'}^{\nu}, \qquad (2.6)$$

where  $\lambda, \lambda'$  and  $\mathcal{M}, \mathcal{M}'$  are the spin projections of the photon and target, respectively, along the z axis,  $\varepsilon_{\lambda}^{\mu}$ is the polarization vector of a helicity  $\lambda$  photon,  $\varepsilon_{\pm}^{\mu} = \mp (0, 1, \pm i, 0)/\sqrt{2}$ ,  $\varepsilon_{0}^{\mu} = (-|\mathbf{q}|, 0, 0, \nu)/\sqrt{Q^{2}}$ , and the other notations are self-explanatory. A simple calculation shows that the structure function  $g_{1}$  in the Bjorken limit is defined by the amplitudes (2.6)

$$A_{\pm\pm,\pm} = \mp g_1 \pm \frac{Q^2}{\nu^2} g_2 \tag{2.7}$$

and it reads

$$g_1 = -\frac{1}{2} \left( A_{++,++} - A_{+-,+-} \right). \tag{2.8}$$

Actually, the amplitudes (2.6) may contain the symmetric part too, which involves the structure functions  $F_1(x, Q^2)$  and, for spin one targets,  $b_1(x, Q^2)$  [22]. In our case this part is irrelevant, since it does not contribute to the asymmetry (2.5) and to the SSF.

The SSF, as well as the hadronic tensor  $W_{[\mu\nu]}$ , can be directly calculated, via the optical theorem, from the imaginary part of the amplitude for forward Compton scattering of virtual photons off hadronic targets

$$W_{[\mu\nu]} = rac{1}{2\pi} {
m Im} T_{[\mu\nu]}.$$
 (2.9)

In what follows the Compton amplitude  $T_{[\mu\nu]}$  for nuclear targets and its relation with SSF through Eqs. (2.6) and (2.8) will be obtained.

# B. The Compton scattering amplitude for nuclear targets

To establish a direct connection between SSF and the amplitude  $T_{[\mu\nu]}$  we decompose the latter in the same form as (2.2), viz.,

$$T_{[\mu\nu]} = \epsilon_{\mu\nu\lambda\sigma} q^{\lambda} \frac{i}{\nu} \left\{ \alpha_1(x,Q^2) S^{\sigma} + \left[ (p_A \cdot q) S^{\sigma} - (S \cdot q) p_A^{\sigma} \right] \right\} \frac{\alpha_2(x,Q^2)}{\nu M_A} \quad , \quad (2.10)$$

where the Compton spin-dependent structure functions  $\alpha_{1,2}(x)$  are related to the Compton helicity amplitudes  $h_{\lambda\mathcal{M},\lambda'\mathcal{M}'}$  and the deep inelastic SSF by the following relations:

$$h_{\lambda\mathcal{M},\lambda'\mathcal{M}'} = \varepsilon_{\lambda}^{\mu\star} T_{[\mu\nu]}(S) \varepsilon_{\lambda'}^{\nu}, \qquad (2.11)$$

$$\alpha_1(x) = -\frac{1}{2}(h_{++,++} - h_{+-,+-}), \qquad (2.12)$$

$$g_1(x) = \frac{1}{2\pi} \text{Im } \alpha_1(x).$$
 (2.13)

Using the dispersion relation for the function  $\alpha_1(x)$  and Eq. (2.13) we get the expression for  $\alpha_1(x)$  in terms of the moments of the SSF  $g_1$ 

$$\alpha_1(x) = 4 \sum_{n=0,2,\cdots}^{\infty} \left(\frac{1}{x}\right)^{n+1} \int_0^1 dy \, y^n \, g_1(y).$$
 (2.14)

The integral on the right-hand side (RHS) in Eq. (2.14) is the n+1 moment  $M_{n+1}(Q^2)$  of the SSF. We will use Eq. (2.14) to obtain the structure functions from the explicit expressions of the nuclear Compton amplitude  $T_{[\mu\nu]}$ .

The computation of the amplitude  $T_{[\mu\nu]}$  requires the treatment of two relatively independent questions: (i) the analysis of the properties of the time ordered product of electromagnetic current operators at high momentum transfer  $(Q^2 \to \infty)$ , which characterizes the short distance physics and (ii) the determination of the vectors of the nuclear ground state  $|p_A\rangle$ , which essentially characterizes the large distance physics. This is seen explicitly from the expression of the amplitude (2.10) which is of the form

$$T_{[\mu\nu]} = i \int d^4\xi \exp(iq\xi) \langle p_A \mathcal{M} | T \left[ J_\mu(\xi) J_\nu(0) \right] | p_A \mathcal{M} \rangle.$$
(2.15)

The behavior of the T product of the electromagnetic currents may be established in a general form directly from Eq. (2.15). In the Bjorken limit  $(Q^2 \to \infty)$  the main contribution in the integral (2.15) comes from small space-time intervals,  $\xi^2 \to 0$ . In this limit the arguments of electromagnetic currents coincide and T product contains singularities. A consistent method to analyze these singularities is based on the Wilson operator product expansion [23]. According to the OPE on the light cone [24], the product of two arbitrary operators Aand B factorizes into two pieces when their arguments are separated by a small space-time interval; the first piece contains singularities (the *c*-number coefficient functions, or Wilson's coefficients) and the second one appears as a set of regular local operators, which are well defined in field theory. Then the local operators are expanded in a series

$$A(\xi)B(0) \sim \sum_{n} C_{n}(\xi^{2})\xi_{\mu_{1}}\cdots\xi_{\mu_{n}}O_{n}^{\mu_{1}\cdots\mu_{n}}(0). \quad (2.16)$$

The operators  $O_n^{\mu_1\cdots\mu_n}$  are defined to be symmetric and traceless in all Lorentz indices  $\mu_1\cdots\mu_n$ . The quantity n is the Lorentz spin. Another quantity, the twist, defined as  $\tau = d_n - n$  ( $d_n$  is the canonical dimension of the operator  $O_n$ ) plays an important role in the theory of deep inelastic scattering processes.

Namely, only the lowest values of  $\tau$  contribute to the matrix elements of the Compton amplitude [17]. Therefore in the leading order of the twist ( $\tau = 2$ ), the RHS of Eq. (2.15) can be rewritten as

$$T_{[\mu\nu]} = i\epsilon_{\mu\nu\lambda\sigma}q^{\lambda}\sum_{t;n=0,2,\cdots}^{\infty} C_{n,t}(Q^2) \left(\frac{2}{Q^2}\right)^{n+1} q_{\mu_1}\cdots q_{\mu_n} \langle p_A \mathcal{M} | \hat{O}_t^{\{\sigma\mu_1\cdots\mu_n\}}(0) | p_A \mathcal{M} \rangle,$$
(2.17)

where t tags the fundamental fields of the theory under consideration and  $\hat{O}_t^{\{\sigma\mu_1\cdots\mu_n\}}(0)$  are the relevant twist-two operators constructed from these fields. The transformation properties of the amplitude  $T_{[\mu\nu]}$  restrict the Lorentz spin, (n+1), in Eq. (2.17) to take only odd values.

It is worth emphasizing that in (2.17), due to the factorization in the OPE, the coefficient functions  $C_n$  are related to short distance, "subhadronic," physics (depending on the large momentum q), whereas the matrix elements of the operators  $O^{\{\sigma\mu_1\cdots\mu_n\}}$  characterize the large distance physics (depending on typical nuclear momenta).

In order to separate the part contributing to  $g_1(x)$ , it is convenient to rearrange the symmetric Lorentz indices  $\{\sigma\mu_1\cdots\mu_n\}$  so as to obtain two operators, one with no definite symmetry  $\hat{O}^{\sigma\{\mu_1\cdots\mu_n\}}$ , and the other with mixed symmetry [25], viz.,

$$\hat{O}^{\{\sigma\mu_{1}\cdots\mu_{n}\}} = \left(\hat{O}^{\sigma\{\mu_{1}\cdots\mu_{n}\}} + \frac{1}{n+1} \left[\sum_{i=1}^{n} \hat{O}^{\mu_{i}\{\mu_{1}\cdots\sigma\mu_{i+1}\cdots\mu_{n}\}} - n \cdot \hat{O}^{\sigma\{\mu_{1}\cdots\mu_{n}\}}\right]\right).$$
(2.18)

Then  $g_1$  gets contributions from  $\hat{O}^{\sigma\{\mu_1 \dots \mu_n\}}$  only. The mixed symmetry operators enclosed in the square brackets of Eq. (2.18), define the twist-2 part of  $g_2(x)$ , the so-called Wandzura-Wilczek contribution [25,26]. In this paper we consider the amplitude  $T_{[\mu\nu]}$  defined by Eqs. (2.10) and (2.17) with only the first operator from Eq. (2.18), that is, the part concerning the structure function  $g_1(x)$  only. The structure function  $g_2(x)$  is expected to be very small, (in the parton model it is exactly zero), and will not be considered here (for the investigation of nuclear effects in  $g_2(x)$  see, for instance, Ref. [27]).

The form (2.17) for the amplitude (2.15) near the light cone is valid in the framework of any renormalizable field theory [23,24]. Thus, the problem of its analysis is formulated now as a consistent calculation of both the coefficient functions  $C_{n,t}$  and the matrix elements of the twist-two operators  $O^{\sigma\{\mu_1\cdots\mu_n\}}$  sandwiched between nuclear ground state vectors  $|p_A\rangle$ . So far there does not exist a realistic field theory by which one could compute simultaneously both pieces, since if one of them is calculated in a more or less self-consistent way, then the other one relies on phenomenological approaches.

For example, within perturbative QCD, because of asymptotic freedom it is possible to analyze the properties of the Wilson's coefficients  $C_{n,t}(\alpha_s, Q^2)$  by computing directly, at least in principle, the corresponding Feynman graphs up to the desired order in  $\alpha_s$  [28].

Making use of the renormalization group equations, the  $Q^2$  dependence of these coefficients may be established as well. However, in practical calculations of the structure functions and comparison with experimental data one needs the matrix elements of the operators  $\hat{O}_t(0)$ , which are related to the nonperturbative region of QCD and, hence, are parametrized from the experimental data. Since the basis of operators in OPE, Eq. (2.17), and the Wilson coefficients are target independent, all information about the target is contained in the unknown matrix elements. Therefore in the treatment of the "QCD motivated" models of nuclear effects in deep inelastic scattering, one is forced to introduce parameters, which neither can be computed theoretically from QCD, nor can be fixed from independent experiments [29].

The problems related to the short and large distances are formally solved, in the case of deep inelastic scattering, within nonasymptotically free theories with spinor fields (nucleons) interacting with the massive bosons (mesons) via pseudoscalar and vector couplings [30,31]. These field theoretical models with a renormalizable interaction allow a perturbative investigation of the Wilson's coefficients and of the corresponding matrix elements, by summing the leading logarithmic corrections. These examples can be considered as an idealized meson-nucleon theory, since the realistic models necessarily involve phenomenological corrections, such as vertex meson-nucleon form factors, effective coupling constants and meson masses [32,33]. In realistic meson-nucleon models, the exact results of Refs. [30,31] serve as a hint for a formal approach and further phenomenological adjustments. It is obvious that the target independent Wilson's coefficients are not calculable within such a theory and they ought to be parametrized from experiments, for instance, from the experiments on deep inelastic scattering off free nucleons.

We apply the effective meson-nucleon theory and the OPE method to deep inelastic scattering off nuclei (cf., Refs. [34,18,19,27]). This theory allows one to describe fairly well the NN interaction at relatively small energies, the nuclear bound states  $|p_A\rangle$ , the binding energy and other properties of light nuclei [35]. Then, the axial twist-two operator  $O^{\sigma\{\mu_1\cdots\mu_n\}}$  for the system of interacting nucleon and meson fields may be written down in the form

$$\hat{O}_{n}^{\sigma\{\mu_{1}\cdots\mu_{n}\}}(0) = \left(\frac{i}{2}\right)^{n} \times \{:\bar{N}(0)\gamma^{\sigma}\gamma_{5} \stackrel{\leftrightarrow}{\partial^{\mu_{1}}} \cdots \stackrel{\leftrightarrow}{\partial^{\mu_{n}}} N(0):\},$$
(2.19)

where N stands for the nucleon spinor fields. Note the implicit presence of the meson degrees of freedom, via interaction, in the definition of the operator  $\hat{O}_n^{\sigma\{\mu_1\cdots\mu_n\}}(0)$ . In order to obtain the explicit expressions of the operator (2.19) and calculate its matrix elements, we need the Hamiltonian of the system. In order to achieve self-

consistency, this Hamiltonian has to provide simultaneously the equation of motion for the interacting fields and the target ground state:

$$\dot{N} = i \left[ H, N \right], \tag{2.20}$$

$$H|p_A\rangle = M_A|p_A\rangle. \tag{2.21}$$

Equation (2.21) for the nuclear ground state has been solved within the nonrelativistic limit using effective Hamiltonians containing  $\pi$ ,  $\sigma$ ,  $\omega$ ,  $\rho$ ,  $\eta$ , and  $\delta$  mesons [one-boson-exchange (OBE) approximation] [32,35]. Consequently, the operators (2.19) are also to be calculated within the nonrelativistic limit. The strategy of our calculation is therefore as follows: (i) we choose the appropriate covariant Lagrangian, giving the classical equation of motion for interacting meson and nucleon fields; (ii) these equations are reduced nonrelativistically and the effective nonrelativistic Hamiltonian is obtained; (iii) using the same procedure of nonrelativistic reduction, we compute the explicit form of the operators (2.19).

The procedure of nonrelativistic reduction of classical equations of motion for the interacting meson and nucleon fields has been established in a number of papers and could be found in details, for instance, in Refs. [36,37].

Below we perform explicitly the calculations with pseudoscalar-isovector coupling, the pion-nucleon interaction. The result is generalized to the case of other kinds of couplings. Introducing the isospin formalism we redefine the Wilson's coefficients in Eq. (2.17) as a diagonal  $(2 \times 2)$  matrix in the isospin space with the proton and neutron coefficients on the main diagonal,  $(\hat{C}_n)_{\alpha\beta} = C_{n,\alpha}\delta_{\alpha\beta}, \ \alpha, \ \beta = 1, 2$ , and use them into the definition of the operators (2.19).

The contribution of the operators  $\hat{O}^{\sigma\{\mu_1\cdots\mu_n\}}$  (2.19) to the Compton helicity amplitude then becomes

$$h_{+\mathcal{M},+\mathcal{M}} = -\sum_{n=0,2,\cdots}^{\infty} \left(\frac{2\nu}{Q^2}\right)^{n+1} \langle O_n^+ \rangle_A,$$
  
$$\langle O_n^+ \rangle_A = \left(\frac{i}{2}\right)^n \langle p_A \mathcal{M} | : \bar{N}(0) \hat{C}_n(Q^2) \gamma^+ \gamma_5 \overleftrightarrow{\partial}_-^n N(0) : | p_A \mathcal{M} \rangle, \qquad (2.22)$$

where  $\gamma_+ = \gamma_0 + \gamma_z$  and  $\partial_- = \partial_0 - \partial_z$ .

We perform the nonrelativistic transition of the fields N(0), following the method described in Refs. [36,37], and, using the equation of motion (2.21), we compute their nth  $\partial_{-}$  derivatives. The resulting operators, being composed from interacting nucleon fields, explicitly involve the meson degrees of freedom. Skipping some rather cumbersome details of calculations we write below the explicit nonrelativistic form of the operators  $O_n^+$  as a sum of operators up to second order in the coupling constant  $g_{\pi}$ :

$$O_n^{\lambda} = O_{N,n}^{\lambda} + O_{N\pi,n}^{\lambda} + O_{NN,n}^{\lambda}, \qquad (2.23)$$

$$O_{N,n}^{\lambda} = m^n \int \frac{d\mathbf{p}_1 d\mathbf{p}_2}{(2\pi)^6} N_n^{(1)}(\mathbf{p}_1, \mathbf{p}_2) \left[ \Sigma^{\lambda}(\mathbf{p}_1, \mathbf{p}_2) \right]_{s_1 s_2} a^{\dagger}(\mathbf{p}_1, s_1) \hat{C}_n a(\mathbf{p}_2, s_2),$$
(2.24)

$$O_{N\pi,n}^{\lambda} = m^{n} \frac{ig_{\pi}}{2m} \int \frac{d\mathbf{p}_{1} d\mathbf{p}_{2}}{(2\pi)^{9}} \frac{d\mathbf{k}}{\sqrt{2\omega(\mathbf{k})}} \left( N_{n}^{(-)}(\mathbf{k}) b_{j}^{\dagger}(\mathbf{k}) - N_{n}^{(+)}(\mathbf{k}) b_{j}(\mathbf{k}) \right) \\ \times \left[ \Sigma^{\lambda}(\mathbf{p}_{1}, \mathbf{p}_{2}) \boldsymbol{\sigma} \cdot \mathbf{k} \right]_{s_{1} s_{2}} a^{\dagger}(\mathbf{p}_{1}, s_{1}) \hat{C}_{n} \tau^{j} a(\mathbf{p}_{2}, s_{2}) + \text{H.c.}, \qquad (2.25)$$

$$O_{NN,n}^{\lambda} = -m^{n} \frac{g_{\pi}^{2}}{4m^{2}} \int \frac{d\mathbf{p}_{1} d\mathbf{p}_{2} d\mathbf{p}}{(2\pi)^{12}} \frac{d\mathbf{k}}{2\omega(\mathbf{k})} \left[ \Sigma^{\lambda}(\mathbf{p}_{1},\mathbf{p}_{2}) \boldsymbol{\sigma} \cdot \mathbf{k} \right]_{s_{1} s_{2}} \left( \tilde{N}_{n}^{(-)}(\mathbf{k}) + \tilde{N}_{n}^{(+)}(-\mathbf{k}) \right) \\ \times [\boldsymbol{\sigma} \cdot \mathbf{k}]_{s_{3} s_{4}} : a^{\dagger}(\mathbf{p}_{1},s_{1}) \hat{C}_{n} \tau^{j} a(\mathbf{p}_{2},s_{2}) a^{\dagger}(\mathbf{p},s_{3}) \tau_{j} a(\mathbf{p}+\mathbf{k},s_{4}) : +\text{H.c.},$$
(2.26)

where *m* is the mass parameter of the Lagrangian, the mass of bare nucleons,  $\tau_j$  are the isospin matrices,  $a^{\dagger}(\mathbf{p}, s)$  $(a(\mathbf{p}, s))$  is the creation (annihilation) operator of a bare nucleon, proton or neutron, with spin *s* and momentum  $\mathbf{p}, b_j^{\dagger}(\mathbf{k})$   $(b_j(\mathbf{k}))$  creates (annihilates) the *j*th pion with momentum  $\mathbf{k}; \omega = \sqrt{m_{\pi}^2 + \mathbf{k}^2}$   $(m_{\pi}$  is the pion mass),  $[\Sigma^{\lambda}]_{s_1 s_2} = \chi_{s_1}^+ \Sigma^{\lambda} \chi_{s_2}; \chi_s$  is the Pauli spinor, and the operator  $\Sigma$  is the nonrelativistic analogue of the four-vector spin operator for the spin- $\frac{1}{2}$  particles

$$\Sigma^{\lambda}(\mathbf{p}_{1},\mathbf{p}_{2}) = \begin{cases} \frac{1}{2m}\boldsymbol{\sigma} \cdot (\mathbf{p}_{1}+\mathbf{p}_{2}), & \lambda = 0\\ \sigma^{\lambda} \left(1 - \frac{\mathbf{p}_{1}^{2} + \mathbf{p}_{2}^{2}}{8m^{2}}\right) \\ + \frac{1}{4m^{2}} \left(\boldsymbol{\sigma} \cdot \mathbf{p}_{1} p_{2}^{\lambda} + \boldsymbol{\sigma} \cdot \mathbf{p}_{2} p_{1}^{\lambda} - \mathbf{p}_{1} \cdot \mathbf{p}_{2} \sigma^{\lambda} - i[\mathbf{p}_{1} \times \mathbf{p}_{2}]^{\lambda}\right), \quad \lambda = x, y, z. \end{cases}$$
(2.27)

The functions  $N_n^i$  in Eqs. (2.24)-(2.26) depend only upon momenta and are of the form

$$N_n^{(1)}(\mathbf{p}_1, \mathbf{p}_2) = \left(1 + \frac{\mathbf{p}_1^2 + \mathbf{p}_2^2}{4m^2} + \frac{p_{1z} + p_{2z}}{2m}\right)^n,$$
(2.28)

$$N_n^{(\pm)}(\mathbf{k}) = \pm \frac{1}{\omega_+(\mathbf{k})} \left( \left[ 1 \pm \frac{\omega_+(\mathbf{k})}{2m} \right]^n - 1 \right),$$
(2.29)

$$\tilde{N}_{n}^{(\pm)}(\mathbf{k}) = \frac{1}{\omega(\mathbf{k})} N_{n}^{(\pm)}(\mathbf{k}) \mp \frac{1}{\omega(\mathbf{k})} \frac{\omega_{+}(\mathbf{k})}{k_{z}^{2}} \left( \left[ 1 \pm \frac{k_{z}}{2m} \right]^{n} - 1 \right), \qquad (2.30)$$

where  $\omega_+ \equiv \omega + k_z$ . The operators (2.24)-(2.26) are the basic result of nonrelativistic OPE method within the OBE approximation. Their matrix elements will determine the polarized deep inelastic scattering of leptons on those nuclear targets, e.g., the two and three nucleon systems, which are well described within the effective meson-nucleon theory with OBE potential.

Note that our method is based upon perturbation theory within the effective meson-nucleon theory. Since the effective meson-nucleon coupling constants are large, it can be argued whether a perturbation expansion can be applied at all. We simply follow here the customary use of perturbation theory in the effective meson-nucleon theory [32,33], justified by the success of OBE model. The price one has to pay in such an approach is the use of some phenomenological ingredients. For example, since the nuclear forces are strongly repulsive at small distances, the physics at such distances is mocked up by a short-range repulsive core, which is handled partly by introducing meson-nucleon vertex form factors. Thus, in the calculation of the corresponding matrix elements the uncertainties of the nuclear force at very short distances are insignificant. Analogously the short distance contributions to the nucleon and nuclear structure functions are hidden into the Wilson's coefficients, which are the new "effective constants" in the effective meson-nucleon theory [18,19,27,34].

From the QCD point of view this situation corresponds to the picture when hadrons, viewed as a core of valence quarks surrounded by a sea quark-antiquark pairs and gluons, are approximated by a core of bare nucleons with a correlated color neutral quark-antiquark pairs, the meson cloud. In OPE such a picture means that much of detailed dynamics of the quarks is embedded in the coefficient functions  $C_n(Q^2)$ , the influence of the meson cloud is rather included into the matrix elements of the operators  $O^{\sigma\{\mu_1\cdots\mu_n\}}$ . In case of the effective meson-nucleon theory a preliminary investigation of the twist behavior of eq. (2.17) is hindered by the fact that all the renormalization effects are included into the effective constants of the theory (coupling constants, meson masses, vertex form factors, etc.), so that the renormalization group equations are here, in a sense, inapplicable. The use of the twist-two as a leading term in Eq. (2.17) is to be regarded as an assumption and it should be verified a posteriori, by looking at the quality of the final results and, possibly, by investigating the higher twist corrections.

# III. MOMENTS OF THE NUCLEON STRUCTURE FUNCTIONS

As mentioned above, the nonrelativistic expressions for the operators (2.24)-(2.26) and the Compton amplitude (2.22) involve, in particular, the bare (unknown) parameters, i.e., the mass parameter from the Lagrangian of the theory and the Wilson's coefficients  $C_n(Q^2)$  which are not calculable within our approach. Whereas the mass parameter may be fixed by introducing into the Lagrangian a corresponding counterterm, the coefficients  $C_n(Q^2)$  are to be related with the SSF of the physical nucleons. Therefore we first investigate the ground state and the SSF of the physical nucleons.

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#### A. Nucleon ground state

For the investigation of the nuclear ground states in the effective meson-nucleon theory it is convenient to use the Tamm-Dancoff method.

For a physical nucleon with an isospin index  $\alpha$ , momentum **q**, and a given z projection of the spin s, the Tamm-Dancoff decomposition is given by

$$|N\rangle_{\mathbf{q},\,\alpha,s} = \sqrt{1 - Z_N}\varphi_0|\bar{N}\rangle_{\mathbf{q},\,\alpha,s} + \varphi_1|\bar{N}\pi\rangle_{\mathbf{q},\,\alpha,s} + \cdots, \qquad (3.1)$$

where  $Z_N$  is the normalization constant defined by the condition  $\langle N|N\rangle = 1$  and  $|\bar{N}\rangle$  and  $|\bar{N}\pi\rangle$  represent the basis vectors of the states with one bare nucleon, and with one bare nucleon and one meson, respectively. The coefficients  $\varphi_i$  in the expansion (3.1) are the operators in the momentum space and define the corresponding wave functions

$$\varphi_0 |\bar{N}\rangle_{\mathbf{q},\,\alpha,s} = a^{\dagger}_{\alpha}(\mathbf{q},s)|0\rangle,\tag{3.2}$$

$$\varphi_1|\bar{N}\pi\rangle_{\mathbf{q},\,\alpha,s} = \int \frac{d\mathbf{k}d\mathbf{p}}{(2\pi)^6} \varphi_{\mathbf{1}s,\,\alpha}^{\alpha',\,s',\,j}(\mathbf{p},\mathbf{k},\mathbf{q}) a_{\alpha'}^{\dagger}(\mathbf{p},s') b_j^{\dagger}(\mathbf{k})|0\rangle. \tag{3.3}$$

The explicit expressions for the wave functions  $\varphi_1$  is found from the condition that the  $|N\rangle$  is the state of the physical nucleon, which means that it obeys Eq. (2.21) with the physical nucleon mass,<sup>2</sup> viz.,

$$\varphi_{1M,\alpha}^{\alpha',s',j}(\mathbf{p},\mathbf{k},\mathbf{q}) = -(2\pi)^3 \delta^{(3)}\left(\mathbf{p}+\mathbf{k}-\mathbf{q}\right) \frac{ig_{\pi}}{2m_N} \frac{[\boldsymbol{\sigma}\cdot\mathbf{k}]_{s'M}}{\omega(\mathbf{k})\sqrt{2\omega(\mathbf{k})}} \left[\tau^j\right]_{\alpha'\alpha}.$$
(3.4)

In particular, as it can be seen from (3.4), in the nonrelativistic approximation there is no interaction between the nucleon and the pion with the angular orbital momentum  $l \neq 1$ . Finally one can find the mass counterterm  $\delta m$  and the renormalization constant  $Z_N$  that are of the form

$$\delta m = -\frac{g_{\pi}^2}{4m^2} \int \frac{d\mathbf{k}}{(2\pi)^3} \frac{3\mathbf{k}^2}{2\omega^2(\mathbf{k})}, \qquad Z_N = \frac{g_{\pi}^2}{4m^2} \int \frac{d\mathbf{k}}{(2\pi)^3} \frac{3\mathbf{k}^2}{2\omega^3(\mathbf{k})}.$$
(3.5)

#### B. The matrix elements

Now we are in the position to compute the matrix elements of the operators (2.24) and (2.25) with nucleon ground state vectors (3.1). The operator (2.26) is of two-body origin, hence it does not contribute to the nucleon matrix elements.

Schematically the matrix elements for a nucleon at rest are as follows:

M.E. 
$$\sim_{\alpha,s_z} \langle \bar{N} | \varphi_0 \sum_n O^+_{N,n} \varphi_0 | \bar{N} \rangle_{\alpha,s_z} (1 - Z_N)$$
 (IA + renorm.) (3.6)

$$+_{\alpha,s_{z}}\langle \bar{N},\pi|\varphi_{1}\sum_{n}O_{N,n}^{+}\varphi_{1}|\bar{N},\pi\rangle_{\alpha,s_{z}}$$
(recoil) (3.7)

$$+_{\alpha,s_{z}}\langle \bar{N},\pi|\varphi_{1}\sum_{n}O_{N\pi,n}^{+}\varphi_{0}|\bar{N}\rangle_{\alpha,s_{z}} + \text{H.c.} \qquad (\text{interaction}).$$
(3.8)

The four different matrix elements given by Eqs. (3.6)–(3.8) are known, respectively, as the impulse approximation (scattering off bare constituents), the renormalization and recoil contributions, and the term of pure interaction origin (self-energy-like correction). The contribution of these matrix elements with given isospin to the helicity amplitude (2.22), sandwiched between the states  $\alpha$ , may be explicitly written in the form

<sup>&</sup>lt;sup>2</sup>In Eq. (3.1) we keep only the first two terms. The next term,  $\varphi_2 | \bar{N} \pi \pi \rangle$ , in spite of being proportional to  $g_{\pi}^2$ , does not contribute to the nucleon SSF, as it can be seen from Eqs. (2.24)-(2.30) and (3.1)-(3.3).

$$-\frac{1}{2}h_{++,++}^{\alpha\alpha} = \sum_{n=0,2,\cdots}^{\infty} \left(\frac{1}{x}\right)^{n+1} \left[ \left(\hat{C}_n\right)_{\alpha\alpha} \left\{ \left(1 - \frac{\delta m}{m_N}\right)^n - Z_N \right\} \right]$$
(3.9)

$$+\left(\tau^{j}\,\hat{C}_{n}\,\tau_{j}\right)_{\alpha\alpha}\frac{g_{\pi}^{2}}{4m_{N}^{2}}\left\{\int\frac{d\mathbf{k}}{(2\pi)^{3}}\frac{2k_{z}^{2}-\mathbf{k}^{2}}{\omega^{2}(\mathbf{k})}\left(\frac{1}{2\omega(\mathbf{k})}-N_{n}^{(-)}(\mathbf{k})\right)\right\}$$
(3.10)

$$-\left(\hat{C}_{n}\right)_{\alpha\alpha}\frac{g_{\pi}^{2}}{4m_{N}^{2}}\int\frac{d\mathbf{k}}{\left(2\pi\right)^{3}}N_{n}^{(+)}(\mathbf{k})\frac{3\,\mathbf{k}^{2}}{\omega^{2}(\mathbf{k})}\right],\tag{3.11}$$

where  $\alpha = 1, 2$  corresponds to the matrix elements on proton and neutron, respectively.

The physical meaning of the obtained result becomes clear if Eqs. (3.9)–(3.11) are depicted in terms of Feynman diagrams. Figure 1 represents the helicity amplitude for the proton. It can be seen that the proton amplitude is determined not only by the proton Wilson coefficients  $C_{n,1}$ , but by the neutron coefficients  $C_{n,2}$  as well as multiplied by a factor of 2 which comes from the relation between the pion-nucleon coupling constants:  $g_{\pi^{\pm}N} = \sqrt{2} g_{\pi^0 N}$ . From Eqs. (2.13) and (2.14) it is easy to show that in the present approach the Wilson's coefficients in the OPE are proportional to the moments of SSF of the bare nucleons [proton (p) on neutron (n)]

$$C_{n,1(2)} = 2\,\bar{\mathsf{M}}_{n+1}(\bar{g}_1^{p(n)}),\tag{3.12}$$

with

$$\mathsf{M}_{n}(f) \equiv \int_{0}^{1} dx \, x^{n-1} f(x). \tag{3.13}$$

Relations (3.9)-(3.13) determine the moments of SSF of the physical nucleons in the effective meson-nucleon theory. Accordingly, the moments should be parametrized in order to describe the scattering off free physical nucleons, and they should be replaced by this parametrization whenever they appear in nuclear matrix elements. The first (n = 0) moments of the nucleon SSF play an important role in deep inelastic scattering of polarized particles, since they define some integral relations among the SSF, known as sum rules. As n = 0 the interaction term in Eqs. (3.10) and (3.11) vanishes [see also Eqs. (2.27)-(2.29)] and one obtains

$$\begin{split} \mathsf{M}_{1}^{p(n)}(g_{1}) &= \bar{\mathsf{M}}_{1}^{p(n)} \left(1 - Z_{N}\right) \\ &+ \left(\bar{\mathsf{M}}_{1}^{p(n)} + 2\bar{\mathsf{M}}_{1}^{n(p)}\right) \\ &\times \frac{g_{\pi}^{2}}{4m_{N}^{2}} \int \frac{d\mathbf{k}}{(2\pi)^{3}} \frac{2k_{z}^{2} - \mathbf{k}^{2}}{\omega^{2}(\mathbf{k})}. \end{split}$$
(3.14)



The second term on the RHS of Eq. (3.14) is the contribution of recoil diagrams, which is partially canceled by the renormalization term [38]. The remaining part is the additional counterterm to renormalize the composite axial operator (2.19). For n = 0 the operator (2.19) is the spin operator for the spinor fields and its matrix element, (3.14), is the mean value of the spin projection over polarized states of the physical nucleon. Equation (3.14) demonstrates that the spin projection of the physical nucleon differs from the mean value of the spin projection of the bare nucleons. This is expected, since the interaction between the core and the meson cloud, with orbital momentum l = 1, slightly redistributes the spin among the constituents. The total angular momentum of a meson-nucleon system, is a sum of the orbital momentum l = 1 and the spin s = 1/2, so that both parallel and antiparallel polarizations of the core contribute to the nucleon polarization. From the point of view of the present approach, the orbital moment of the meson cloud affects the spin distribution of bare nucleons inside a polarized physical nucleon. The effects of the meson cloud on unpolarized deep inelastic scattering on nucleon have been investigated in Refs. [39,40] and are known as the Sullivan processes.

Another interesting consequence of Eq. (3.14) may be derived by analyzing the difference between the first moments of SSF of the proton and neutron, the Bjorken sum rule. In this case one finds that the Bjorken sum rule on free nucleons may be computed by considering the bare constituents and the meson cloud corrections:

$$\int [g_1^p(x,Q^2) - g_1^n(x,Q^2)] dx$$
  

$$\equiv \mathsf{M}_1^p(Q^2) - \mathsf{M}_1^n(Q^2)$$
  

$$= [\bar{\mathsf{M}}_1^p(Q^2) - \bar{\mathsf{M}}_1^n(Q^2)]$$
  

$$\times \left(1 - \frac{g_\pi^2}{3m_N^2} \int \frac{d\mathbf{k}}{(2\pi)^3} \frac{\mathbf{k}^2}{\omega^3(\mathbf{k})}\right), \qquad (3.15)$$

where the RHS of Eq. (3.15) is the BSR for bare nucleons

FIG. 1. The dressing diagrams for the helicity amplitude corresponding to the forward Compton scattering off the proton. Black dots denote the scattering off a bare nucleon and the dashed line the pion propagator. The first diagram is the contribution of the impulse approximation and renormalization terms, the second and third diagrams represent the recoil effects and the last two diagrams are the self-energy-like interaction terms.

corrected by the interaction with the meson cloud. Explicit numerical estimates of the role of the meson cloud in polarized deep inelastic scattering processes on nucleons is ambiguous and requires a proper investigation. In fact one needs a consistent procedure for the regularization of integrals appearing in Eq. (3.15) and for the inclusion of the vertex form factor into the meson-nucleon vertex. Also, some assumptions about the properties of the bare nucleon core are necessary. The role of the orbital momentum of the virtual pions in determining the spin of the nucleon has to be investigated as well. These problems are beyond the goal of the present paper and will be considered elsewhere.

# **IV. NUCLEAR MATRIX ELEMENTS**

In the previous section we explained in details the basic formalism and apply it to the physical nucleons. The

basic idea of the calculations of nuclear matrix elements remains the same: once the nonrelativistic expressions for the axial operators have been established in terms of nucleon and meson fields, the vectors of nuclear ground states are to be defined in the same manner, i.e., making use of the same nonrelativistic Hamiltonian which has been used to derive Eqs. (2.24)-(2.30) and (3.1)-(3.3). In what follows we keep the terms up to the second order in the meson-nucleon coupling constant g, which corresponds to the usual approximations in nuclear physics in deriving the potential and Schrödinger equation,  $q^2$  approximation. In this sense the present approach pretends to be self-consistent. The condition of a consistency is that the Hamiltonian, with OBE interaction, should indeed describe the real nucleus. We choose the realistic OBE potentials, such as the Bonn [35] or Reid [41] ones, which give a good description of light nuclei.

Below the explicit expressions for the moments of the  $^{2}$ H and  $^{3}$ He SSF's are derived.

#### A. The deuteron

The vectors of the deuteron ground state in the Tamm-Dancoff approximation are given by a relation similar to Eq. (3.1); viz.,

$$\varphi_0 \left| \bar{N} \bar{N} \right\rangle = \varphi_0^{s_1 s_2} \left( \mathbf{p_1}, \mathbf{p_2} \right) \left( 2\pi \right)^3 \delta(\mathbf{p_1} + \mathbf{p_2}) \, a_\alpha^\dagger(\mathbf{p_1}, s_1) \frac{\epsilon^{\alpha\beta}}{\sqrt{2}} a_\beta^\dagger(\mathbf{p_2}, s_2) \left| 0 \right\rangle, \tag{4.1}$$

$$\varphi_1 \left| \bar{N} \bar{N} \pi \right\rangle = \varphi_1^{s_1 s_2 \alpha \beta j} \left( \mathbf{p_1}, \mathbf{p_2}, \mathbf{k} \right) (2\pi)^3 \delta(\mathbf{p_1} + \mathbf{p_2} + \mathbf{k}) a_\alpha^{\dagger}(\mathbf{p_1}, s_1) a_\beta^{\dagger}(\mathbf{p_2}, s_2) b_j^{\dagger}(\mathbf{k}) \left| 0 \right\rangle, \tag{4.2}$$

where  $\epsilon^{\alpha\beta}$  is the Levi-Civita tensor which describes the isospin function of the deuteron and the nucleon spins  $s_1, s_2$ and orbital momentum in  $\varphi_0^{s_1s_2}(\mathbf{p_1}, \mathbf{p_2})$  are summed to the total angular momentum J = 1. The quantum numbers  $\alpha, \beta$ , and j are combined so as to give the total deuteron isospin T = 0. The physical meaning of the coefficient  $\varphi_0$ can be understood by projecting the bare deuteron state onto the state with two nucleons located at points  $\mathbf{r_1}, \mathbf{r_2}$  in coordinate space

$$\langle \mathbf{r}_{1}, \, \mathbf{r}_{2} | \varphi_{0} \bar{N}_{1} \bar{N}_{2} \rangle = \int \frac{d\mathbf{k}_{1} d\mathbf{k}_{2}}{(2\pi)^{6}} e^{i\mathbf{k}_{1} \cdot \mathbf{r}_{1} + i\mathbf{k}_{2} \cdot \mathbf{r}_{2}} (2\pi)^{3} \delta(\mathbf{p}_{1} + \mathbf{p}_{2}) \, \varphi_{0}^{s_{1}s_{2}} \left(\mathbf{p}_{1}, \mathbf{p}_{2}\right) \chi_{\tilde{s}_{1}} \chi_{\tilde{s}_{2}} \eta_{\tilde{\alpha}} \eta_{\tilde{\beta}} \frac{\epsilon^{\alpha\beta}}{\sqrt{2}} \\ \times \langle \, 0 | a_{\tilde{\alpha}}(\mathbf{k}_{1}, \tilde{s}_{1}) a_{\tilde{\beta}}(\mathbf{k}_{2}, \tilde{s}_{2}) a_{\alpha}^{\dagger}(\mathbf{p}_{1}, s_{1}) a_{\beta}^{\dagger}(\mathbf{p}_{2}, s_{2}) | 0 \, \rangle.$$

$$(4.3)$$

It is clear from Eq. (4.3) that  $\varphi_0(\mathbf{p}, \mathbf{p})$  with spin,  $\chi_s$ , and isospin,  $\eta_\alpha$ , functions is the conventional deuteron wave function in the momentum space, obeying the Schrödinger equation with OBE potential [36]. In what follows a more conventional notation for  $\varphi_0$  will be adopted, viz.,  $\varphi_0(\mathbf{p}, \mathbf{p}) \equiv \Psi_{\mathcal{M}}^D(\mathbf{p})$ . For the function  $\varphi_1$  we have

$$\varphi_{1}^{s_{1}s_{2}\alpha\beta,j}(\mathbf{p}_{1},\mathbf{p}_{2},\mathbf{k}) = -(2\pi)^{3}\delta^{(3)}\left(\mathbf{p}_{1}+\mathbf{p}_{2}+\mathbf{k}\right)\frac{ig_{\pi}}{2m_{N}}\frac{1}{\omega(\mathbf{k})\sqrt{2\omega(\mathbf{k})}}$$
$$\times \left\{\varphi_{0}^{s_{1}s}\left(\mathbf{p}_{1},\mathbf{p}_{1}\right)\frac{\epsilon^{\alpha\alpha'}}{2}\left[\boldsymbol{\sigma}\cdot\mathbf{k}\right]_{s\,s_{2}}\left[\tau^{j}\right]_{\alpha'\beta}-(1\leftrightarrow2)\right\}.$$

$$(4.4)$$

The renormalization constant for the deuteron state contains the nucleon contribution (3.5) and the exchange part,  $Z_D = 2Z_N + \tilde{Z}_D$ , where

$$\tilde{Z}_{D} = -\int \frac{d\mathbf{p}d\mathbf{k}}{(2\pi)^{6}} \Psi_{\mathcal{M}}^{+D}(\mathbf{p}) \frac{V_{\pi}(\mathbf{k})}{\omega(\mathbf{k})} \Psi_{\mathcal{M}}^{D}(\mathbf{p} + \mathbf{k}).$$
(4.5)

We proceed now with an analysis of the moments of the deuteron SSF. To begin with, let us define the isoscalar nucleon SSF,  $g_1^N(x) \equiv [g_1^p(x) + g_1^n(x)]/2$ . Then the *n*th moment of the deuteron solely depends on  $M_n(g_1^N)$ . Using Eqs. (3.9)-(3.11) we get

$$\begin{split} \mathsf{M}_{n+1}(g_1^N) &= \bar{\mathsf{M}}_{n+1}(g_1^{\bar{N}}) \left\{ 1 - \frac{g_{\pi}^2}{4m_N^2} \int \frac{d\mathbf{k}}{(2\pi)^3} \frac{3}{\omega^2(\mathbf{k})} \left( N_n^{(+)}(\mathbf{k}) \, \mathbf{k}^2 + N_n^{(-)}(\mathbf{k}) \left( 2k_z^2 - \mathbf{k}^2 \right) \right) \right\} \\ &+ \bar{\mathsf{M}}_{n+1}(g_1^{\bar{N}}) \left\{ -Z_N + \frac{g_{\pi}^2}{4m_N^2} \int \frac{d\mathbf{k}}{(2\pi)^3} \frac{3}{2\omega^3(\mathbf{k})} \left( 2k_z^2 - \mathbf{k}^2 \right) + n \frac{\delta m}{m_N} \right\}. \end{split}$$
(4.6)

The moments of the deuteron SSF's are determined by matrix elements similar to those in Eqs. (3.6)-(3.8). In the nuclear case all the operators (2.24)-(2.26) contribute to the corresponding matrix elements. While evaluating the matrix elements, one obtains contributions corresponding to Eqs. (3.6)-(3.8) computed with the deuteron wave functions (4.1) and (4.2); representing the scattering off a bare nucleon and self-energy-like corrections (see also Fig. 2). These diagrams provide the contribution to the Fermi motion of "dressed" nucleons. Besides, there are terms of a pure exchange origin which reflect the fact that nucleons in the deuteron are "off-mass-shell," and terms with renormalization and recoil contributions:

$$\frac{1}{2} \left(\frac{M_D}{m_N}\right)^n \mathsf{M}_{n+1}^D(g_1^D) = \frac{1}{2} \sum_{\mathcal{M}=\pm 1} \mathcal{M} \bar{\mathsf{M}}_{n+1}^N \left[ \left(1 - \frac{3}{2} P_D\right) \left( -\frac{\tilde{Z}_D}{2} + \frac{\mathsf{M}_{n+1}(g_1^N) - \bar{\mathsf{M}}_{n+1}(g_1^{\bar{N}})}{\bar{\mathsf{M}}_{n+1}(g_1^{\bar{N}})} \right) \right. \\ \left. + \int \frac{d\mathbf{p}}{(2\pi)^3} N_n^{(1)}(\mathbf{p}, \mathbf{p}) \Psi_{\mathcal{M}}^{*D}(\mathbf{p}) \Sigma_{(12)}^+(\mathbf{p}) \Psi_{\mathcal{M}}^D(\mathbf{p}) - \int \frac{d\mathbf{p} d\mathbf{k}}{(2\pi)^6} \Psi_{\mathcal{M}}^{*D}(\mathbf{p}) S_z V_{\pi}(\mathbf{k}) \left( \frac{1}{\omega(\mathbf{k})} - N_n^{(3)}(\mathbf{k}) \right) \Psi_{\mathcal{M}}^D(\mathbf{p} + \mathbf{k}) \right],$$
(4.7)

where **S** is the total spin of the nucleons,  $\mathbf{S} = \frac{1}{2}(\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2)$  (with its projection on to the z axis  $S_z$ ),  $V_{\pi}(\mathbf{k})$  is the one pion exchange potential,  $\langle S_z \rangle_D = (1 - 3/2P_D)$ ,  $P_D$  is the D-wave probability in the deuteron,  $\Sigma^+_{(12)}(\mathbf{p}) = \frac{1}{2}[\Sigma^+_1(\mathbf{p}, \mathbf{p}) + \Sigma^+_2(\mathbf{p}, \mathbf{p})]$  [see Eqs. (2.24)-(2.27)] and  $N_n^{(3)}(\mathbf{k})$  is defined by

$$N_n^{(3)}(\mathbf{k}) = \frac{1}{k_z} \left[ \left( 1 + \frac{k_z}{2m_N} \right)^n - \left( 1 - \frac{k_z}{2m_N} \right)^n \right].$$
(4.8)

The expressions in Eq. (4.7) still contain the bare moments,  $\bar{M}_n(g_1^{\bar{N}})$ . However they may be expressed through the physical moments by making use of

$$\left(1 - \frac{3}{2}P_D\right) \left(\mathsf{M}_{n+1}(g_1^N) - \bar{\mathsf{M}}_{n+1}(g_1^{\bar{N}})\right) = \langle S_z \rangle_D \left(\mathsf{M}_{n+1}(g_1^N) - \bar{\mathsf{M}}_{n+1}(g_1^{\bar{N}})\right) \\ \approx \langle \Sigma_{(12)}^+ \cdot N_n^{(1)}(\mathbf{p}) \rangle_D \left(\mathsf{M}_{n+1}(g_1^N) - \bar{\mathsf{M}}_{n+1}(g_1^{\bar{N}})\right).$$

$$(4.9)$$

In obtaining expression (4.9), we have used that fact that  $M_n(g_1^N) - \bar{M}_n(g_1^{\bar{N}}) \sim g_{\pi}^2$ ; then the moments of the deuteron, up to  $g_{\pi}^2$  terms, contain only well-defined quantities, viz.,



FIG. 2. The helicity amplitude for the forward Compton scattering off the deuteron. The notations are the same as in Fig. 1. The first three diagrams represent the impulse approximation with physical nucleons plus renormalization effects, and the remaining two diagrams are the deuteron recoil and interaction terms, respectively.

$$\frac{1}{2} \left( \frac{M_D}{m_N} \right)^n \mathsf{M}_{n+1}(g_1^D) = \mathsf{M}_{n+1}(g_1^N) \, \Delta Z \\
+ \mathsf{M}_{n+1}(g_1^N) \int \frac{d\mathbf{p}}{(2\pi)^3} f_D^{\mathrm{IA}}(\mathbf{p}) \left( 1 + \frac{p_z}{m_N} + \frac{\mathbf{p}^2}{2m_N^2} \right)^n \\
+ \mathsf{M}_{n+1}(g_1^N) \int \frac{d\mathbf{p}d\mathbf{k}}{(2\pi)^6} f_D^{\mathrm{int}}(\mathbf{p}, \mathbf{k}) \frac{1}{k_z} \left[ \left( 1 + \frac{k_z}{2m_N} \right)^n - \left( 1 - \frac{k_z}{2m_N} \right)^n \right],$$
(4.10)

where

$$f_D^{\mathrm{IA}}(\mathbf{p}) = \frac{1}{2} \sum_{\mathcal{M}=\pm 1} \mathcal{M} \Psi_{\mathcal{M}}^{\star D}(\mathbf{p}) \left( \frac{\mathbf{S} \cdot \mathbf{p}}{m_N} + S_z + \frac{\mathbf{S} \cdot \mathbf{p}}{2m_N^2} \, p_z \right) \Psi_{\mathcal{M}}^D(\mathbf{p}), \tag{4.11}$$

$$f_D^{\text{int}}(\mathbf{p}, \mathbf{k}) = \frac{1}{4} \sum_{\mathcal{M}=\pm 1} \mathcal{M} \Psi_{\mathcal{M}}^{*D}(\mathbf{p}) \{ S_z, V_{\pi}(\mathbf{k}) \} \Psi_{\mathcal{M}}^D(\mathbf{p} + \mathbf{k}),$$
(4.12)

where  $\{S_z, V_\pi\}$  stands for the anticommutator of  $S_z$  and  $V_\pi$  and  $\Delta Z$ , which is the difference between the renormalization and recoil contributions, has the same origin as in the case of nucleons and reads as

$$2\Delta Z = \frac{1}{2} \sum_{\mathcal{M}=\pm 1} \mathcal{M} \int \frac{d\mathbf{p} d\mathbf{k}}{(2\pi)^6} \Psi_{\mathcal{M}}^{*D}(\mathbf{p}) \left\{ \frac{\langle S_z \rangle - S_z}{2\omega(\mathbf{k})}, V_{\pi}(\mathbf{k}) \right\} \Psi_{\mathcal{M}}^D(\mathbf{p} + \mathbf{k}).$$
(4.13)

Figure 2 illustrates the dressing of the moments of the deuteron. The sum of the first three diagrams gives the impulse approximation, that is, the scattering off a physical, already dressed nucleon. Comparing these diagrams with those in Fig. 1, it is seen that in the impulse approximation the helicity amplitude on the deuteron is determined by the scattering amplitudes off free physical nucleons, which ought to be fixed from other experiments. In this context, the present approach allows one to avoid the problem as to what amplitudes should be associated with "off-mass-shell" nucleon in impulse approximation (see, for instance, Ref. [42]). In our approach the binding effects are taken into account by terms which are of a pure exchange origin (Fig. 2, last diagram) which contain implicitly, via the potential  $V_{\pi}(\mathbf{k})$ , the contribution of the meson degrees of freedom. The fourth diagram is the recoil contribution terms.

Applying the inverse Mellin transform to (4.10) and omitting the  $\Delta Z$  part, the deuteron structure function  $g_1^D$  can be obtained in the convolution form

$$\frac{1}{2}g_{1}^{D}(x) = \int_{x}^{M_{D}/m} \frac{dy}{y} g_{1}^{N}\left(\frac{x}{y}\right) \left[f_{D}^{IA}(y) + f_{D}^{int}(y)\right], \qquad (4.14)$$

where the distribution functions  $f_D^{IA}(y)$  and  $f_D^{int}(y)$  are given by

$$f_D^{IA}(y) = \int \frac{d\mathbf{p}}{(2\pi)^3} f_D^{IA}(\mathbf{p}) \,\delta\left(y - 1 - \frac{p_z}{m_N} - \frac{\mathbf{p}^2}{2m_N^2}\right),\tag{4.15}$$

$$\mathbf{f}_{D}^{\mathrm{int}}(y) = \int \frac{d\mathbf{p}d\mathbf{k}}{(2\pi)^{6}} f_{D}^{\mathrm{int}}(\mathbf{p}, \mathbf{k}) \frac{1}{k_{z}} \left[ \delta \left( 1 - y + \frac{k_{z}}{2m_{N}} \right) - \delta \left( 1 - y - \frac{k_{z}}{2m_{N}} \right) \right] \theta(y), \tag{4.16}$$

where  $f^{IA}$  corresponds to the impulse approximation, or Fermi motion correction, with "on-mass-shell" nucleons, and  $f^{int}$  accounts for the binding of the nucleon inside the deuteron.

Equations (4.10) and (4.14)–(4.16) are the basic result for the determination of the moments and the SSF's of the deuteron within the OPE-OBE approach. It will be shown later on that Eqs. (4.14)–(4.16) lead, if proper assumptions are made, to the phenomenological convolution model approach used in Ref. [12]. It is worth recalling here the problem as to whether the so-called flux factor has to be considered in the convolution formula for polarized deep inelastic scattering [13,43]. In our approach the nonrelativistic flux factor ~  $(1 + p_z/m_N)$  comes automatically, as it can be seen from Eq. (4.11).

Formulas (4.10)-(4.16) have been obtained for the pseudoscalar isovector coupling. In this case the deuteron wave function  $\Psi^D(\mathbf{p})$  appears to be the solution of the Schrödinger equation with the one pion exchange NN potential  $V_{\pi}(\mathbf{k})$ . Obviously, this wave function and the one pion exchange potential are not yet sufficient to describe the properties of the deuteron. To this end it is necessary to take into account other mesons contributing to the OBE potential, viz., the  $\sigma$ ,  $\omega$ ,  $\rho$ ,  $\eta$ , and  $\delta$  mesons [32]. Including these mesons in our approach leads to contributions similar to (4.10) and (4.16), except that the wave function  $\Psi^D(\mathbf{p})$  is replaced by the solution of the Schrödinger equation with the full OBE potential. The convolution formula can be written in a more compact form by expanding the  $\delta$ functions in (4.16) and retaining terms up to  $k_z^2/m_N^2$ ; one gets

$$\frac{1}{2}g_{1}^{D}(x) = g_{1}^{IA}(x) - \frac{d}{dx} \left[ xg_{1}^{N}(x) \right] \frac{\langle S_{z}, V_{OBE}(\mathbf{k}) \rangle_{D}}{m_{N}},$$
(4.17)

where

$$g_1^{\mathrm{IA}}(x) = \int_x^{M_D/M} \frac{dy}{y} g_1^N\left(\frac{x}{y}\right) \mathsf{f}^{\mathrm{IA}}(y) \quad , \tag{4.18}$$

and  $\langle S_z V_{OBE} \rangle_D$  is the spin-weighted mean value of the potential of the nucleon in the polarized deuteron.

The second term in Eq. (4.17) is the correction to the impulse approximation due to the binding of nucleons. It can be seen that this contribution is small (~  $\langle V_{OBE} \rangle / m_N$ ) and depends on the behavior of the nucleon structure function  $g_1^N(x)$  and its first derivative.

#### B. The <sup>3</sup>He

In this section the Compton helicity amplitude  $h_{+\mathcal{M},+\mathcal{M}}$  pertaining to the scattering of the virtual helicity (+) photon off the polarized (with projection  $\mathcal{M} = \pm \frac{1}{2}$ ) <sup>3</sup>He nucleus will be presented. Using the Tamm-Dancoff decomposition for the state vector  $|{}^{3}He\rangle_{\mathbf{Q}=0,\mathcal{M}}$  and the operators  $O_{t}^{\lambda}(n)$ , the following form for the helicity amplitude can be obtained:

$$h_{+\mathcal{M},+\mathcal{M}} = -\frac{2M_{^{8}\text{He}}}{m_{N}} \sum_{n=0,2,\dots}^{\infty} \left(\frac{1}{x}\right)^{n+1} \int dV_{^{3}\text{He}} \Psi_{\mathcal{M}}^{^{3}\text{He}*}(\mathbf{p}_{1},\mathbf{p}_{2},\mathbf{p}_{3}) \left\{ \left[ \int \frac{d\mathbf{k}}{(2\pi)^{3}} \frac{g_{\pi}^{2}}{4m_{N}^{2}} \frac{1}{\omega^{2}} \right] \times \sum_{i=1}^{3} \left\{ 3\mathbf{k}^{2}\sigma_{z}^{(i)}\hat{C}_{n}^{(i)}\left(\frac{1}{2\omega} - N_{n}^{(+)}(\mathbf{k})\right) + \left(2k_{z}\left(\sigma^{(i)}\cdot\mathbf{k}\right) - \mathbf{k}^{2}\sigma_{z}^{(i)}\right) \tau^{(i)}\hat{C}_{n}^{(i)}\tau^{(i)}\left(\frac{1}{2\omega} - N_{n}^{(-)}(\mathbf{k})\right) \right\}$$
(4.19)

$$+ (1 - Z_{^{3}\text{He}}) \sum_{i=1}^{3} N_{n}^{(1)}(\mathbf{p}_{i}) \Sigma_{i}^{+}(\mathbf{p}_{i}) \hat{C}_{n}^{(i)} \Bigg] \Psi_{\mathcal{M}}^{^{3}\text{He}}(\mathbf{p}_{1}, \mathbf{p}_{2}, \mathbf{p}_{3})$$

$$(4.20)$$

$$+ \int \frac{d\mathbf{k}}{(2\pi)^3} \sum_{i < j} \left[ \{ V_{\pi i j}, S_{z i j} \} \left( \frac{1}{\omega} + N_n^{(3)}(\mathbf{k}) \right) + \frac{V_{\pi i j}}{\omega} \sigma_z^{(k)} \hat{C}_n^{(k)} \right] \Psi_{\mathcal{M}}^{He}(\mathbf{p}_i + \mathbf{k}, \mathbf{p}_k, \mathbf{p}_j - \mathbf{k}) \right\},$$
(4.21)

where  $S_{zij} = \frac{1}{2} \left( \sigma_z^{(i)} \hat{C}_n^{(i)} + \sigma_z^{(j)} \hat{C}_n^{(j)} \right), dV_{^3\text{He}} \equiv \prod_{i=1}^3 \frac{d\mathbf{p}_i}{(2\pi)^6} \delta(\mathbf{p_1} + \mathbf{p_2} + \mathbf{p_3}), \text{ and in Eq. (4.21) } i, j \text{ list two nucleon pairs}$ 

exchanging a meson while the remaining kth nucleon interacts with the incoming lepton,  $k \neq i, j$ . In Eq. (4.19) the reader may easily recognize the corresponding "dressing" part for bare nucleons, Eqs. (3.10) and (3.11), which being included into Eq. (4.20), gives the impulse approximation with the *physical* nucleons. The last term in Eq. (4.21) and the term depending upon the renormalization constant  $Z_{^{3}\text{He}}$  in Eq. (4.20) correspond to the recoil and renormalization contributions, similar to those obtained in the deuteron case. The moments of the <sup>3</sup>He SSF's can easily be obtained by recalling Eqs. (2.14) and (3.13): the inverse Mellin transform gives the SSF in the convolution form

$$g_{1}^{^{3}\text{He}}(x) = \int_{x}^{M_{^{3}\text{He}}/m_{N}} \frac{dy}{y} \left[ g_{1}^{p}\left(\frac{x}{y}\right) f_{^{3}\text{He}}^{p}(y) + g_{1}^{n}\left(\frac{x}{y}\right) f_{^{3}\text{He}}^{n}(y) \right],$$
(4.22)

where the effective distribution functions  $f_{^{3}He}^{p,n}(y)$  of nucleons in <sup>3</sup>He contain contribution from the Fermi motion of "on-mass-shell" nucleons (impulse approximation) and from the nuclear binding

$$f_{^{s}He}^{p,n}(y) = f_{IA}^{p,n}(y) + f_{int}^{p,n}(y),$$
(4.23)

where

$$f_{IA}^{p,n}(y) = \int dV_{{}^{3}\text{He}} n_{\parallel}^{p,n}(\mathbf{p}_{1},\mathbf{p}_{2},\mathbf{p}_{3}) \,\delta\left(y - 1 - \frac{p_{z_{1}}}{m_{N}} - \frac{\mathbf{p}_{1}^{2}}{2m_{N}^{2}}\right),\tag{4.24}$$

$$\mathbf{f}_{\text{int}}^{\boldsymbol{p},\boldsymbol{n}}(\boldsymbol{y}) = \int \frac{dV_{s_{\text{He}}}d\mathbf{k}}{\left(2\pi\right)^{3}} n_{\text{int}}^{\boldsymbol{p},\boldsymbol{n}}(\mathbf{p}_{1},\mathbf{p}_{2},\mathbf{p}_{3},\mathbf{k}) \frac{\theta(\boldsymbol{y})}{k_{z}} \left[\delta\left(1-y+\frac{k_{z}}{2m_{N}}\right)-\delta\left(1-y-\frac{k_{z}}{2m_{N}}\right)\right],\tag{4.25}$$

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with the spin-dependent momentum distributions  $n_{||}^{p,n}$  and  $n_{\rm int}^{p,n}$  being defined by

$$n_{\parallel}^{p,n}(\mathbf{p}_{1},\mathbf{p}_{2},\mathbf{p}_{3}) = \sum_{\mathcal{M}=\pm 1/2,i} \mathcal{M} \, \Psi_{\mathcal{M}}^{^{3}\mathrm{He}*} \Sigma_{i}^{+}(\mathbf{p}_{i}) \left\{ [1 \pm \tau_{3}(i)]/2 \right\} \Psi_{\mathcal{M}}^{^{3}\mathrm{He}}, \tag{4.26}$$

$$n_{\rm int}^{p,n}(\mathbf{p}_1,\mathbf{p}_2,\mathbf{p}_3,\mathbf{k}) = \sum_{\mathcal{M}=\pm 1/2, i\neq j} \mathcal{M} \Psi_{\mathcal{M}}^{^{3}{\rm He}*}(\mathbf{p}_1,\mathbf{p}_2,\mathbf{p}_3) V_{\pi,ij} S_{zij} \{ [1 \pm \tau_3(j)]/2 \} \Psi_{\mathcal{M}}^{^{3}{\rm He}}(\mathbf{p}_i + \mathbf{k},\mathbf{p}_k,\mathbf{p}_j - \mathbf{k}).$$
(4.27)

Equation (4.22) can be cast in a more manageable form by expanding the  $\delta$  functions as in the case of the deuteron

$$g_{1}^{^{3}\mathrm{He}}(x) = \int_{x}^{M_{^{3}\mathrm{He}}/m_{N}} \frac{dy}{y} \left[ g_{1}^{p}\left(\frac{x}{y}\right) \mathsf{f}_{\mathrm{IA}}^{p}(y) + g_{1}^{n}\left(\frac{x}{y}\right) \mathsf{f}_{\mathrm{IA}}^{n}(y) \right] \\ - \frac{d}{dx} \left[ x g_{1}^{p}(x) \right] \frac{2 \langle \sigma_{z}^{p} V_{\mathrm{OBE}}^{p} \rangle_{^{3}\mathrm{He}}}{m_{N}} - \frac{d}{dx} \left( x g_{1}^{n}(x) \right) \frac{\langle \sigma_{z}^{n} V_{\mathrm{OBE}}^{n} \rangle_{^{3}\mathrm{He}}}{m_{N}},$$

$$(4.28)$$

where  $\langle \sigma_z^i V_{\text{OBE}}^i \rangle_{^{3}\text{He}}$  is the spin-weighted mean value of the potential of the nucleon (i = p, n) in the polarized <sup>3</sup>He.

#### C. Generalization to heavy nuclei

Let us generalize our approach to heavy nuclei. To this end, let us first discuss the unpolarized case. The comparison will be carried up to a given order in  $(p/m_N)^n$ , namely, to n = 2.

Guided by the results for A = 2, one can write for a generic isoscalar nucleus A [18]<sup>3</sup>

$$F_{2}^{A}(x) = F_{2}^{IA}(x) - x \frac{dF_{2}^{N}(x)}{dx} \frac{\langle V \rangle_{A}}{m_{N}} , \qquad (4.29)$$

where

$$F_2^{\mathrm{IA}}(x) = \int F_2^N\left(\frac{x}{y}\right) f_A^{\mathrm{IA}}(y) dy, \qquad (4.30)$$

 $\mathbf{and}$ 

$$f_A^{\mathrm{IA}}(y) = \int \left(1 + \frac{p_z}{m_N}\right) n(p) \delta\left(y - \left[1 + \frac{\mathbf{p}^2}{2m_N} + \frac{p_z}{m_N}\right]\right) d\mathbf{p}.$$
(4.31)

In Eq. (4.29)  $\langle V \rangle_A$  is the mean potential energy of the nucleon interacting with the lepton; it is linked to the nuclear mean potential energy per nucleon  $\langle V \rangle \equiv \langle \Psi_A | \sum_{i < j} v_{ij} | \Psi_A \rangle / A$  by the relation  $\langle V \rangle_A = 2 \langle V \rangle$ . The well-known relationships between the total energy per nucleon  $\epsilon_A = E_A / A$ , the mean nucleon kinetic energy  $\langle T \rangle = \langle \Psi_A | \sum_i t_i | \Psi_A \rangle / A$ , and the mean removal energy per nucleon  $\langle E \rangle$  are given by  $\epsilon_A = \langle T \rangle + \langle V \rangle$  and  $\langle E \rangle = 2 |\epsilon_A| + (A - 2) \langle T \rangle / (A - 1)$ .

Now we turn to the deuteron Eqs. (4.11), (4.12), and (4.14)–(4.16). In the definition of the distribution functions in impulse approximation, Eq. (4.11), it is easy to identify the nonrelativistic analogue of the nucleon spin vector  $S_z + (\mathbf{S} \cdot \mathbf{p})/2m_N^2$ , which may be obtained by applying the Lorentz boost operator to the spin vector of a nucleon at rest. In the nonrelativistic limit the spin vector is the same in any reference frame. Consequently, in the extreme nonrelativistic limit it should be replaced by  $S_z$  only. Furthermore, in the matrix element of  $(\mathbf{S} \cdot \mathbf{p})/m_N$  in Eq. (4.11) only the z components of the scalar product  $(\mathbf{S} \cdot \mathbf{p})$  give a contribution as it can be checked by direct computation. So that in this case the distribution function  $f^{IA}(\mathbf{p})$  becomes

$$f^{\mathrm{IA}}(\mathbf{p}) = \frac{1}{2} \sum_{\mathcal{M}=\pm 1} \mathcal{M} \Psi_{\mathcal{M}}^{*D}(\mathbf{p}) \left( 1 + \frac{p_z}{m_N} \right) S_z \Psi_{\mathcal{M}}^D(\mathbf{p}).$$
(4.32)

Expanding the  $\delta$  function in (4.16) around the "on-mass-shell" y,  $\delta(y-1-\mathbf{p}^2/2m_N^2-p_z/m_N) \approx \delta(1-y) - \delta'(1-y)(\mathbf{p}^2/2m_N^2+p_z/m_N)$  and substituting in Eq. (4.16) the difference of two  $\delta$  functions by its first derivative we get

$$f_{\parallel}^{N/D}(y) \equiv f^{\mathrm{IA}}(y) + f^{\mathrm{int}}(y) \approx \frac{1}{2} \sum_{\mathcal{M}=\pm 1} \mathcal{M} \int \frac{d\mathbf{p}}{(2\pi)^3} \Psi_{\mathcal{M}}^{*D}(\mathbf{p}) \left(1 + \frac{p_z}{m_N}\right) S_z \Psi_{\mathcal{M}}^D(\mathbf{p}) \\ \times \left[\delta(1-y) - \delta'(1-y)(\mathbf{p}^2/2m_N^2 + p_z/m_N + \varepsilon_D/m_N - \langle T \rangle/m_N)\right].$$
(4.33)

<sup>&</sup>lt;sup>3</sup>Note that in Eq. (16) of Ref. [18] the "plus" sign in Eq. (16) should be replaced by a "minus" sign.

In deriving Eq. (4.33) we used the Schrödinger equation to express  $\langle S_z, V \rangle$  through the deuteron binding energy  $\varepsilon_D$  and the kinetic energy  $\langle T \rangle$  of nucleons. It can be easily shown that Eq. (4.33) can be cast in the form

$$f_{\parallel}^{N/D}(y) = \int \frac{d\mathbf{p}}{(2\pi)^3} dE \,\mathcal{P}_{\parallel}^D(\mathbf{p}, E) \left(1 + \frac{p_z}{m_N}\right) \delta\left(y - \left[\frac{m_N - E - \mathbf{p}^2/2m_N + p_z}{m_N}\right]\right),\tag{4.34}$$

where we have introduced the deuteron spin dependent spectral function

$$\mathcal{P}^{D}_{\parallel}(\mathbf{p}, E) \equiv \Psi^{*D}_{\mathcal{M}=1}(\mathbf{p}) \, S_{z} \, \Psi^{D}_{\mathcal{M}=1}(\mathbf{p}) \, \delta(E - |\varepsilon_{D}|) \tag{4.35}$$

giving the probability to find in the deuteron a nucleon with momentum  $\mathbf{p}$ , removal energy  $E = |\varepsilon_D|$ , and spin projection  $S_z$ . In order to generalize Eq. (4.34) to a heavy nucleus, we notice that the quantity  $m_N - E - \mathbf{p}^2/2m_N$ appearing in the  $\delta$  function of Eq. (4.34) is nothing but the time component of the four momentum of an off-shell nucleon in the deuteron, viz.,  $p_0 = M_D - \sqrt{\mathbf{p}^2 + m_N^2} \simeq m_N - |\varepsilon_D| - \mathbf{p}^2/2m_N$ .

nucleon in the deuteron, viz.,  $p_0 = M_D - \sqrt{\mathbf{p}^2 + m_N^2} \simeq m_N - |\varepsilon_D| - \mathbf{p}^2/2m_N$ . Formula (4.34) can therefore be generalized to the case of any nuclear mass number A > 2 by substituting the "deuteron spin-dependent spectral function"  $\mathcal{P}^D_{\mathcal{M}}(\mathbf{p}, E)$  with the corresponding nuclear spin dependent spectral function [14,44]. Then Eq. (4.34) exactly coincides with the phenomenological convolution approach used in [14,12] (see next section).

# D. Comparison with the conventional convolution approach

Using the concept of spin-dependent spectral func-[14], the SSF of <sup>3</sup>He has been recently calcution lated [12] within the so-called convolution approach in which the lepton is assumed to interact with off-shell nucleons with four momentum  $p \equiv (p_0, \mathbf{p})$ , with  $p_0 =$  $M_A - \sqrt{(E - m_N + M_A)^2 + \mathbf{p}^2}$ , where  $M_A$  is the mass of the target nucleus and  $E = M_{A-1} + m_N - M_A + E_{A-1}^*$ is the nucleon removal energy with  $M_{A-1}$  being the mass of the spectator A - 1 system and  $E_{A-1}^*$  its intrinsic excitation energy. Since in our calculations we have used the results of Ref. [14], it is worth comparing the two approaches from a formal point of view. To this end, we start with the unpolarized case. Then we can use the analogous of (4.17) for an isoscalar nucleus in the unpolarized case and take its generalization to any value of A, as we did in the previous section obtaining Eqs. (4.29)- (4.31). The convolution formula for off-shell nucleons reads as follows [13]:

$$F_2^A(x) = \int F_2^A\left(\frac{x}{y}\right) f_A(y) dy, \qquad (4.36)$$

where

$$f_A(y) = \int \left(\frac{p^+}{m_N}\right) P(|\mathbf{p}|, E) \delta\left(y - \frac{p^+}{m_N}\right) d\mathbf{p} dE \quad , \tag{4.37}$$

with  $p^+ = p_0 + p_z$  and  $P(|\mathbf{p}|, E)$  being the unpolarized nucleon spectral function. It can be seen that whereas in Eq. (4.17) the binding effect is explicitly displayed through the mean potential energy  $\langle V \rangle$ , in the conventional convolution approach the binding effects are hidden in the definition of the light cone momentum distribution  $f_A(y)$ . Let us, however, write down the latter in the order  $p^2/m_N^2$  as in Eq. (4.17). To this end, we expand the  $\delta$  function in Eq. (4.37) around the point  $y - \left[1 + rac{p^2}{2m_N^2} + rac{p_z}{m_N}
ight]$  and keep only terms of the order  $\mathbf{p}^2/m_N^2$ ; we get

$$f_A(y) \simeq \int \left(1 + \frac{p_z}{m_N}\right) n(|\mathbf{p}|) \\ \times \delta \left(y - \left[1 + \frac{p^2}{2m_N^2} + \frac{p_z}{m_N}\right]\right) d\mathbf{p} \\ + \frac{\langle E \rangle + \langle T \rangle A/(A-1)}{m_N} \delta'(y-1), \qquad (4.38)$$

where  $n(|\mathbf{p}|) = \int P(|\mathbf{p}|, E) dE$  and  $\langle E \rangle$ =  $\int EP(|\mathbf{p}|, E) d\mathbf{p} dE$ . Substituting (4.38) into (4.36) and using the relationship  $\langle E \rangle + \langle T \rangle = 2 \langle V \rangle$  we recover Eq. (4.29). Thus we have demonstrated that the convolution formulas arising from the OPE and from the conventional treatment of unpolarized DIS coincide up to the order  $p^2/m_N^2$ ; likewise we have shown that at this order the binding effect in the latter approach arises from the average potential energy of the nucleon hit by the incoming lepton. Let us turn now to the polarized case and let us analyze the <sup>3</sup>He case. The conventional approach yields [12]

$$g_1^{^{3}\mathrm{He}}(x) = \sum_{i=n,p} \int_x^A \frac{dy}{y} g_1^i\left(\frac{x}{y}\right) G^i(y) \quad , \tag{4.39}$$

where the light cone momentum distribution  $G^{i}(y)$  is given by the following expression:

$$G^{i}(y) = \int dE \int d\mathbf{p} P_{\parallel}^{i}(\mathbf{p}, E) \delta\left(y - \frac{p_{0} + p_{z}}{m_{N}}\right) , \quad (4.40)$$

where  $E = M_D + m_N - M_{{}^{\mathfrak{s}}\operatorname{He}} + E_{D^{\star}}$  is the nucleon removal energy  $(E_{D^{\star}}$  being the energy of the spectator np pair in the continuum),  $p_0 = M_{{}^{\mathfrak{s}}\operatorname{He}} - [(E - m_N + M_{{}^{\mathfrak{s}}\operatorname{He}})^2 + |\mathbf{p}|^2]^{\frac{1}{2}}$ is the energy of a bound "off-mass-shell" nucleon, and  $P_{\parallel}^i(\mathbf{p}, E)$  is the spin dependent spectral function (cf. Eqs. (9) and (16) of Ref. [12]). The integral of the spin dependent spectral function represents the spin dependent momentum distribution

$$n^{i}_{\parallel}(\mathbf{p}) = \int dE P^{i}_{\parallel}(\mathbf{p}, E) \quad . \tag{4.41}$$

By expanding the  $\delta$  function in (4.41) around  $y - \left[1 + \frac{p^2}{2m_N} + \frac{p_z}{m_N}\right]$  and considering the recoil of the two body system nonrelativistically ( $E_R = p^2/4m_N$ ), and taking into account the differences between the proton and neutron spectral functions (see [13]), it can be shown that (4.22) and (4.28) are recovered.

# **V. RESULTS OF CALCULATIONS**

#### A. The deuteron

For explicit numerical calculation of the nuclear SSF,  $g_1^D(x)$  and  $g_1^{^{3}\text{He}}(x)$ , one needs, as it can be seen from Eqs. (4.14) and (4.28), a suitable parametrization of the isoscalar nucleon SSF  $g_1^N(x)$ . Within the present approach this function contains all the information about the Wilson's coefficients and the influence of the meson cloud on bare nucleon; moreover, according to the main assumptions of the effective meson-nucleon theory, it includes all the dynamic at distances shorter than the core of OBE potential. The parametrization of  $g_1^N$  should be fixed from the experiment; in principle, there exist nowadays experimental data of  $g_1$  on both the proton [1] and the neutron [4] structure functions, but they are not yet fully complete, especially at very small values of x. In this interval some assumptions about the behavior of the nucleon structure function are unavoidable. Moreover, the choice of the isoscalar structure function  $g_1^N(x)$  determines whether the Bjorken and Ellis-Jaffe sum rules will be fulfilled or not. In this sense our results depend on the parametrization of the nucleon SSF, however this dependence is found to be insignificant (see below). We have chosen here two different parametrizations of the nucleon SSF, describing quite well the EMC data on proton and satisfying the Bjorken sum rule. We use the parametrization from Ref. [45] as a basic input in all our calculations, and in order to analyze the dependence of the results from the chosen parametrization we use the parametrization from Ref. [46].

In our numerical calculations for the deuteron we use the wave function obtained from the Bonn interaction, yielding  $P_D \approx 0.0428$  [35]. Figure 3 displays the numerical estimate of the ratio  $R_D = g_1^D/g_1^N$ , which illustrates the effects of nuclear structure on  $g_1^D$ . The results presented in this figure, deserve the following comments: (i) within the impulse approximation [curve 1; Eq. (4.14) without  $\mathbf{f}^{\text{int}}$ ], the ratio in the interval 0.2 < x < 0.7is governed by the destructive contributions of the *D*wave admixture which generates a polarization of the deuteron along the *z* direction even if the nucleons have their spins aligned in the direction opposite to the polarization. Thus it can be concluded that the effect of the *D* wave is the most relevant nuclear contribution



FIG. 3. The ratio of the deuteron and the isoscalar nucleon spin-dependent structure functions (SSF). Curve 1: the impulse approximation; curve 2: impulse approximation plus interaction term  $\langle \{S_z, V_\pi\} \rangle$ . Both curves have been computed using the parametrization of the nucleon SSF from Ref. [45]; curve 3: the same as in curve 2, with the parametrization of the nucleon SSF from Ref. [46]; curve 4: the same as in curve 2, plus the contribution from the interaction term  $\langle \{S_z, V_{OBE}\} \rangle$  representing the total contribution af all mesons considered in the one-boson-exchange (OBE) interaction. The arrow indicates the value  $(1 - \frac{3}{2}P_D)$  (see text).

within the impulse approximation; (ii) as for the interaction term, according to (4.17), the binding corrections to the deuteron SSF is governed by  $\langle S_z, V_{OBE} \rangle_D$ . Let us first of all estimate the contribution due to pions; a direct numerical computation gives

$$\langle S_z, V_\pi \rangle_D \sim -5 \text{ MeV.}$$
 (5.1)

If model ambiguities given, e.g., by different choices for the form factors and wave functions are considered, one gets  $\langle S_z, V_{\pi} \rangle_D \sim -(3-5)$  MeV.] It can be seen from Fig. 3 that the effects from binding due to  $\pi$  mesons is similar to the one occurring in the unpolarized case, but significantly smaller. It can also be seen that the results are not sensitive to the parametrization of the nucleon SSF (cf. curves 2 and 3). We turn now to the estimate of the contributions due to other mesons, although it could be anticipated that the pions give the dominant contribution to the binding effects, since the pion contribution in the deuteron is the most significant one [37,47]. In order to estimate the most general boson contribution to the deuteron SSF one should, in principle, calculate the matrix elements  $\langle S_z, V_{OBE} \rangle_D$  for all kinds of bosons considered by the Bonn potential model. To this end one can use the Schrödinger equation to get

$$\langle S_z, V_{\text{OBE}} \rangle = \left(1 - \frac{3}{2}P_D\right) \varepsilon_D - \left(\langle T \rangle_0 - \frac{1}{2}\langle T \rangle_2\right), \quad (5.2)$$

where  $\langle T \rangle_{0,2}$  are the mean values of the nucleon kinetic energy in the S and D waves (for the deuteron wave function of the Bonn potential we have  $\langle T \rangle_0 \approx 10.2$  MeV and  $\langle T \rangle_2 \approx 4.4$  MeV). Taking into account Fermi motion and the full interaction effects by Eq. (5.2) results in a EMC-like effect, as in the unpolarized case (curve 4 in Fig. 3). By comparing curves 1 (only Fermi motion) and 4 (Fermi motion plus binding), it can be concluded that the main nuclear structure effect in the deuteron SSF

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FIG. 4. (a) The weighted deuteron spin-dependent structure function  $xg_1^D(x)$ ; (b) the first moment  $M_1(g_1^D) = \int_{x_{\min}}^{m_D/m_N} g_1^D(x) dx$  versus the lower limit of integration  $x_{\min}$ . The dotted (solid) line corresponds to calculations with the parametrization of the nucleon SSF from Ref. [45] (Ref. [46]). Experimental data from Ref. [3].

comes from the presence of the D wave, with the binding effect remaining a rather small correction. Note that all curves in Fig. 3 are not shown at values of x smaller then  $x \approx 0.2$ . The reason is that realistic parametrizations of the nucleon SSF have nodes at small x, which lead to "poles" in the ratio  $g_1^D/g_1^N$  simulating nuclear effects. At values of x smaller then these poles, all curves in Fig. 3 tend to the limit  $(1 - \frac{3}{2}P_D)$  as  $x \to 0$  (this is a general result for all kinds of parametrization no more singular at the origin than 1/x).

Figures 4(a) and 4(b) display the calculations of the absolute values of the deuteron structure function and the comparison with the SMC experimental data: a good agreement between our results and the experimental data can be observed. The numerical estimate of the first moment of  $g_1^D(x)$  within our approach,  $\int dx g_1^D(x) \approx 0.03$ , is also in an agreement with the experimental result  $\int dx g_1^{D(SMC)}(x) = 0.023 \pm 0.02 \pm 0.015$  [3].

### B. <sup>3</sup>He

As is well known [10] the interest in polarized <sup>3</sup>He targets stems from the fact that such a system can be considered to a large extent as an effective polarized neutron target. As a matter of fact, the two protons in <sup>3</sup>He

are mostly in the singlet  ${}^{1}S_{0}$  configuration so that the polarization of the  ${}^{3}$ He is mainly determined by the polarization of the neutron [11,10]. For such a reason DIS experiments off polarized  ${}^{3}$ He are aimed at obtaining information on the neutron SSF's. However, the nonvanishing proton contribution to the total polarization and nuclear structure effects could in principle hinder the direct extraction of the neutron SSF. We have calculated the  ${}^{3}$ He spin structure function  $g_{1}^{{}^{3}}$ He [Eq. (4.22)] using the same approximation as in the case of  ${}^{2}$ H, i.e., by using Eq. (4.28).

The interaction term, obtained using the Schrödinger equation and a three-body wave function containing S, S' and D waves, reads as

$$\langle \sigma_z^p V_{\text{OBE}}^p \rangle_{{}^{\mathfrak{g}}\text{He}} = 2 \left( \frac{2}{3} P_S' - \frac{2}{3} P_D \right) \overline{\varepsilon}_{{}^{\mathfrak{g}}\text{He}} - 2 \left( \frac{2}{3} \langle T \rangle_{S'} - \frac{2}{3} \langle T \rangle_D \right) = 2 p^p \overline{\varepsilon}_{{}^{\mathfrak{g}}\text{He}} - 2 \langle T \rangle_{\parallel}^p,$$
 (5.3)

$$\langle \sigma_z^n V_{\text{OBE}}^n \rangle_{{}^{3}\text{He}} = 2\left(P_S + \frac{1}{3}P_{S'} - P_D\right)\overline{\varepsilon}_{{}^{3}\text{He}} -2\left(\langle T \rangle_S + \frac{1}{3}\langle T \rangle_{S'} - \langle T \rangle_D\right) = 2p^n \overline{\varepsilon}_{{}^{3}\text{He}} - 2\langle T \rangle_{\parallel}^n,$$
(5.4)

where  $P_{\parallel}^{i} = \int n_{\parallel}^{i}(\mathbf{p}) d\mathbf{p}/(2\pi)^{3}$ ,  $\langle T \rangle_{\parallel}^{i} = \int n_{\parallel}^{i}(\mathbf{p}) \mathbf{p}^{2}/2m_{N} d\mathbf{p}/(2\pi)^{3}$ , and  $\overline{\varepsilon}_{^{3}\mathrm{He}}$  is the mean value of the binding energy per nucleon in <sup>3</sup>He.

The spin-dependent momentum distributions, the effective nuclear polarizations, and the mean values of the kinetic energies have been taken from Ref. [12], where these quantities have been calculated from the spindependent spectral function obtained from the Reid soft core interaction. The results are

$$\langle \sigma_z^p V_{\text{OBE}}^p \rangle_{^{\mathfrak{s}}\text{He}} \approx 2.4 \text{ MeV},$$
 (5.5)

$$\langle \sigma_z^n V_{\text{OBE}}^n \rangle_{^{3}\text{He}} \approx -17.8 \text{ MeV}.$$
 (5.6)

In Fig. 5 the ratio  $R_{^{3}\text{He}} = g_{1}^{^{3}\text{He}}/g_{1}^{n}$  calculated using the proton and neutron spin structure functions from Refs. [45,46] is presented, whereas in Fig. 6 the SSF  $g_1^{^{3}\text{He}}$ is compared with the free neutron SSF. It can be seen that, as in the deuteron case, the contribution of binding effects is rather small. It should be pointed out that the results presented in Figs. 5 and 6 can hardly be distinguished from the ones obtained within the conventional convolution approach of Ref. [12] [Eq. (4.39)]. According to the conclusions of Sec. IV C this means that relativistic effects (terms of order larger than  $\frac{p^2}{m_N^2}$ ) are small, as also demonstrated in Ref. [16]. Our results fully confirm the finding of [12], namely, that nuclear structure effects in DIS of polarized electrons off polarized <sup>3</sup>He are those due to the effective proton and neutron polarizations generated by the S' and D waves of <sup>3</sup>He wave functions, so that the relation

0.2

 $\frac{g_1^{3}(x)}{g_1^{n}(x)} \xrightarrow{1.20} 2$ 

FIG. 5. The ratio of the <sup>3</sup>He and free neutron SSF calculated within the impulse approximation (dotted) and within the impulse approximation plus the interaction term  $\langle \{S_z, V_{OBE}\}\rangle$  (solid). Curves 1 (2): parametrization of the nucleon SSF from Ref. [45] (Ref. [46]).

$$g_1^{^{3}\mathrm{He}}(x) \approx 2p_p g_1^p(x) + p_n g_1^n(x),$$
 (5.7)

 $p_p = -0.030$  and  $p_n = 0.88$  being the effective nucleon polarization, represents a reliable approximation of Eq. (4.39) at  $x \leq 0.6$ . The smallness of the difference between the free neutron structure function  $g_1^n(x)$  and  $g_1^{^3\text{He}}(x)$ is due to the smallness of the nuclear structure effects and is largely independent of the form of the chosen parametrization for the nucleon SSF. For instance, using the parameters from Ref. [45], the results presented in Figs. 5 and 6 change by 20%, i.e., by a quantity well below the experimental errors, hence, Eq. (5.7) may be considered as a good approximation for the extraction of the neutron SSF.

#### VI. CONCLUDING REMARKS

The necessity of plausible and precise data on the neutron spin-dependent structure function is obvious. Since the information about the internal neutron structure is predominantly obtained from nuclear data, usually from the polarized deuteron and <sup>3</sup>He, an appropriate nuclear model for subtracting of the effects of nuclear structure is requested.

In this paper a theoretical approach for the analysis of polarized deep inelastic scattering off light nuclei was proposed, which allows a self-consistent consideration of the role of the Fermi motion and the meson degrees of freedom. Since our model relays on the operator product expansion method within OBE approximation, the *n*th moment of nuclear SSF have been found as a product of two *n*-dependent functions. The contribution of the impulse approximation and of the nuclear corrections to the moments have been separated in an explicit form. As a consequence, the inverse Mellin transform yields back the nuclear SSF in a convolution form with *two* distribution functions: one of them describes the electromagnetic in-



FIG. 6. The SSF for <sup>3</sup>He calculated using the parametrization of the neutron SSF from Ref. [45]. Dashed line: the impulse approximation; solid line: the contribution of the impulse approximation plus binding effects. Dotted line: the free neutron SSF.

teraction of the lepton with an on-shell nucleon, whereas the other one describes the strong interaction of the hit nucleon with the other nucleons of the nucleus.

A generalization of the model to the case of heavy nuclei has been proposed and the comparison with the convolution approach based upon lepton scattering off bound off-shell nucleons [13] was performed with the result that up to order  $p^2/m_N^2$  the two approaches coincide. The numerical calculations of the spin-dependent structure functions for polarized deuteron and <sup>3</sup>He show that the nuclear corrections are relatively small and essentially depend on the spin-orbital structure of the corresponding nucleus, in agreement with the finding of Ref. [14], based upon the conventional convolution approach. For the deuteron the main effect of the nuclear structure is due to the destructive role of the orbital motion of nucleons with L = 2 and is of the order of magnitude  $\sim (1 - \frac{3}{2}P_D)$ . The comparison with the SMC experimental results [3] shows a reasonable agreement of our calculations with the data. In case of polarized <sup>3</sup>He the main nuclear structure corrections come from the S' and D wave admixtures to the ground state wave function. Since they lead to a partial depolarization of the neutron inside polarized <sup>3</sup>He and to an effective proton polarization,  $g_1^{\frac{1}{3}}$  slightly differs from the free neutron spin-dependent structure function  $g_1^n$ . The binding effects in both cases are found to be small. These results agree with those obtained within the convolution approach [14,12,9].

To sum up, we can conclude that the convolution approaches so far proposed describe fairly well the peculiarities of polarized DIS off polarized nuclei, so that nuclear corrections can be estimated in a reliable way. In closing, we would like to point out that our method based upon the OPE within the effective meson nucleon theory, allows one to microscopically understand the origin of the binding effect which is present in the convolution approach. Moreover, since the expressions for moments have been obtained explicitly, this allows one to estimate the influence of nuclear effects on the Bjorken sum rule via direct calculations of the first moments.

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