Importance of nucleon recoil in weak decay of hypernuclei

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Requiring the translational invariance in the nonrelativistic description of the weak decay of hypernuclei, we investigate its effects on an asymmetry in the angular distribution of the pion emitted from the polarized-${}^3_\Lambda$H decay.

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I. INTRODUCTION

Recently, studies of nuclei carrying strangeness quantum numbers have become very active due to remarkable improvements in experimental facilities [1–3]. Decay processes of hypernuclei are usually described by the interaction derived from an analysis of the free lambda-particle decay in the nonrelativistic limit

$$H = Gm_\pi^2 \left[A + B\frac{\boldsymbol{\sigma}\cdot\mathbf{q}}{2M_N} \right] \phi_\pi^{(-)*}(\mathbf{q};\mathbf{r}_\pi) \ , \tag{1}$$

where $Gm_\pi^2 = 0.2211 \times 10^{-6}$, $A = 1.06$, and $B = -7.10$ which were determined by an analysis of the free Λ decay and the emitted proton angular distribution [4]. $\mathbf{q}$ is the momentum of the emitted pion and M_N is the nucleon mass. The first term is the parity-violating interaction while the second term is the parity-conserving interaction.

In the relativistic theory the interaction should be Lorentz invariant, but in the nonrelativistic theory, the interaction should have a translationally invariant form. Since the lambda particle has a momentum in the coordinates fixed at the center of mass of the lambda hypernucleus, the interaction (1) should be modified to satisfy the translational invariance. Nevertheless, all previous calculations [5–9] without exception ignored the translational invariance. Its contribution may be negligibly small for certain physical quantities. However, this is not always true. In this paper, we will show an example revealing the importance of the translational invariance.

II. TRANSLATIONALLY INVARIANT INTERACTION

Starting from the general form of the interaction Lagrangian describing weak decay of a lambda particle,

$$\mathcal{L}_{\pi N\Lambda} = Gm_\pi^2 \Psi_N (A + B\gamma_5)\boldsymbol{\tau}\cdot\boldsymbol{\phi}_\pi \Psi_\Lambda + \text{H.c.}, \tag{2}$$

we can perform the nonrelativistic reduction to obtain the translationally invariant interaction as

$$H_\Lambda = Gm_\pi^2 \left[A + B\left(\frac{\boldsymbol{\sigma}\cdot\mathbf{p}_\Lambda}{2M_\Lambda} - \frac{\boldsymbol{\sigma}\cdot\mathbf{p}_N}{2M_N} \right) \right] \phi_\pi^{(-)*}(\mathbf{q};\mathbf{r}_\pi) \ . \tag{3}$$

The translational invariance of this form of interaction is obvious because the second term is actually proportional to a difference of velocities, $\mathbf{v}_\Lambda - \mathbf{v}_N$. By the momentum conservation $\mathbf{p}_\Lambda = \mathbf{p}_N + \mathbf{q}$, it can be rewritten as

$$H_\Lambda = Gm_\pi^2 \left[A + B\left(\frac{M_N}{M_\Lambda}\right)\frac{\boldsymbol{\sigma}\cdot\mathbf{q}}{2M_N} - B\left(1 - \frac{M_N}{M_\Lambda}\right)\frac{\boldsymbol{\sigma}\cdot\mathbf{p}_N}{2M_N} \right] \phi_\pi^{(-)*}(\mathbf{q};\mathbf{r}_\pi) \ . \tag{4}$$

The term proportional to $\mathbf{p}_N$ was completely ignored in previous works. Of course, the interaction (1) for a free lambda-particle decay can be reduced from the present interaction form (4) by the relation $\mathbf{p}_\Lambda = \mathbf{p}_N + \mathbf{q} = 0$. Taking the Jacobi coordinates for final states, ${}^3\text{He} + \pi^-$ and ${}^3\text{H} + \pi^0$, we have $\mathbf{q} = \mathbf{k}_{\mu\pi}$ and

$$\mathbf{p}_N = \mathbf{p}_r - [M_N/(M_p + M_n + M_N)]\mathbf{k}_{\mu\pi} \simeq \mathbf{p}_r - \tfrac{1}{3}\mathbf{k}_{\mu\pi} \ .$$

Then, the translationally invariant interaction is found in the form

$$H_\Lambda = \left\{ s_\pi - i\frac{p_\pi}{q_0}\boldsymbol{\sigma}\cdot\nabla_{\mu\pi} + i\frac{p'_\pi}{q_0}\boldsymbol{\sigma}\cdot\nabla_r^N \right\} \phi_\pi^{(-)*}(\mathbf{k}_{\mu\pi};\mathbf{r}_{\mu\pi}) \ , \tag{5}$$

where $q_0 = 100.4$ MeV/c and

$$s_{\pi^-} = -\sqrt{2}s_{\pi^0} = \sqrt{2}Gm_\pi^2 A = 3.31 \times 10^{-7} \ , \tag{6}$$

$$p_{\pi^-} = -\sqrt{2}p_{\pi^0} = \sqrt{2}Gm_\pi^2 \frac{q_0}{2M_p}\left(B' + \frac{B''}{3}\right) = -1.19 \times 10^{-7} \ , \tag{7}$$

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$$p'_{\pi^-} = \sqrt{2}Gm_\pi^2 \frac{q_0}{2M_p} B'' = -0.190 \times 10^{-7} \quad (8)$$

with $B' = (M_N/M_\Lambda)B$, $B'' = B - B'$, $B' = -5.97$, and $B'' = -1.13$. Here, m_π and M_N denote m_{π^-} and M_p for π^- emission while m_{π^0} and M_n for π^0 emission.

A. Lifetime of ${}^3_\Lambda$H hypernucleus

In order to examine the role of the nucleon recoil term, i.e., basically the last term in Eq. (4) or (5), let us first analyze the decay width for ${}^3_\Lambda\text{H} \to {}^3\text{He}+\pi^-$. For the wave function of ${}^3_\Lambda$H, we use the S-state wave function as

$$\psi_\Lambda(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3) = N_\Lambda V_\Lambda^m(1,2,3)\mathcal{Y}_0^0 f_\Lambda(r_{12},r_{23},r_{31}) \ , \quad (9)$$

where the lambda particle is located at $\mathbf{r}_3$. The spatial wave function is taken from the Downs and Dalitz six-parameter function [10] as

$$\begin{aligned} f_\Lambda(r_{12},r_{23},r_{31}) = & [\exp(-a_1 r_{23}) + x \exp(-a_2 r_{23})] \\ & \times[\exp(-a_1 r_{31}) + x \exp(-a_2 r_{31})] \\ & \times[\exp(-b_1 r_{12}) + y \exp(-b_2 r_{12})] \ . \quad (10) \end{aligned}$$

The totally mixed symmetric spin-isospin function V_Λ^m (1,2,3) is antisymmetrized for the two nucleons located at $\mathbf{r}_1$ and $\mathbf{r}_2$, and $\mathcal{Y}_0^0$ is the angular part of the wave function [11]. The final state is described as

$$\psi_A(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3) = N_A V_A^a(1,2,3)\mathcal{Y}_0^0 f_A(r_{12},r_{23},r_{31}) \quad (11)$$

where the spatial wave function is described by a Gaussian-type wave function as

$$f_A(r_{12},r_{23},r_{31}) = \exp\left(-\frac{\tilde{\alpha}^2}{2}(r_{12}^2 + r_{23}^2 + r_{31}^2)\right) \ . \quad (12)$$

The spin-isospin function $V_A^a(1,2,3)$ is totally antisymmetrized. The normalization factor was calculated as, $N_\Lambda^2 = 0.003\,561$ with $a_1 = 0.126$, $a_2 = 1.26$, $b_1 = 0.382$, $b_2 = 1.13$, $x = 2.13$, and $y = 2.23$ which are obtained for the binding energy $B_\Lambda = 0.13$ MeV by interpolating the parameters given for $B_\Lambda = 0.0$, 0.25, and 1.00 MeV with the intrisic range of 0.8411 fm in the Λ-nucleon potential [10]. And $N_A^2 = 0.042\,43$ is obtained with $\tilde{\alpha} = 0.384$ fm^{-1} [12]. When these wave functions, (9) and (11), are applied, the differential operators, $\boldsymbol{\nabla}_{\mu\pi}$ and $\boldsymbol{\nabla}_r^p$, in Eq. (5) should be replaced by $\frac{3}{2}\boldsymbol{\nabla}_{31}$ and $\nabla_{31}^p - \nabla_{23}^p$, respectively, and $\frac{2}{3}\mathbf{r}_{31}$ takes place for $\mathbf{r}_{\mu\pi}$. Then, the decay width can be calculated as

$$\begin{aligned} \Gamma_{\pi^-} = & \frac{1}{2J_i+1}\frac{1}{8\pi^2}\frac{k_{\mu\pi}}{1+(\omega_\pi/M_A)} \\ & \times \sum_{M_i M_f} \int |\langle A|\text{H}_\Lambda|{}^3_\Lambda\text{H}\rangle|^2 d\Omega_{\mu\pi} \ , \quad (13) \end{aligned}$$

where

$$\langle A|\text{H}_\Lambda|{}^3_\Lambda\text{H}\rangle = \langle s_{\pi^-}\rangle + \langle p_{\pi^-}\rangle + \langle p'_{\pi^-}\rangle \ , \quad (14)$$

$$\begin{aligned} \langle s_{\pi^-}\rangle = & s_{\pi^-}\int_D \psi_A^\dagger(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3)\phi_{\pi^-}^{(+)}(-\mathbf{k}_{\mu\pi};\tfrac{2}{3}\mathbf{r}_{31}) \\ & \times\psi_\Lambda(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3)d\tau \ , \quad (15) \end{aligned}$$

$$\begin{aligned} \langle p_{\pi^-}\rangle = & -i\frac{p_{\pi^-}}{q_0}\int_D \psi_A^\dagger(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3)\boldsymbol{\sigma} \\ & \times[\tfrac{3}{2}\nabla_{31}\phi_{\pi^-}^{(+)}(-\mathbf{k}_{\mu\pi};\tfrac{2}{3}\mathbf{r}_{31})]\psi_\Lambda(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3)d\tau \ , \quad (16) \end{aligned}$$

and

$$\begin{aligned} \langle p'_{\pi^-}\rangle = & 2i\frac{p'_{\pi^-}}{q_0}\int_D [-\boldsymbol{\sigma}\cdot\nabla_{31}^p\psi_A^\dagger(\mathbf{r}_1,\mathbf{r}_2\mathbf{r}_3)] \\ & \times\phi_{\pi^-}^{(+)}(-\mathbf{k}_{\mu\pi};\tfrac{2}{3}\mathbf{r}_{31})\psi_\Lambda(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3)d\tau \ , \quad (17) \end{aligned}$$

where M_A is the mass of ^{3}He, $d\tau = d\mathbf{r}_{12}d\mathbf{r}_{23}d\mathbf{r}_{31}$, and D is the domain $r_{12}+r_{23} \geq r_{31}$, $r_{23}+r_{31} \geq r_{12}$, $r_{31}+r_{31} \geq r_{23}$ in the triangular coordinates. The pion wave function,

$$\phi_{\pi^-}^{(-)*}(\mathbf{k}_{\mu\pi};\tfrac{2}{3}\mathbf{r}_{31}) = \phi_{\pi^-}^{(+)}(-\mathbf{k}_{\mu\pi};\tfrac{2}{3}\mathbf{r}_{31})$$

can be obtained by solving the Klein-Gordon equation with an optical potential. We use here the Landau optical potential [13] between the emitted pions and the residual nuclei, ^{3}He and ^{3}H. The pion wave function obtained with a Coulomb cutoff $R_{\text{cut}} = 6$ fm is complex, i.e., possessing both real and imaginary parts, and has been examined by analysis of ${}^3\text{He}(\pi^-,\pi^-){}^3\text{He}$ at $T_\pi^{\text{lab}} = 45.1$ MeV [14] as shown in Fig. 1. The numerical values of decay rates can be calculated as $\Gamma({}^3_\Lambda\text{H} \to {}^3\text{He} + \pi^-) = 0.5089$ μeV and for ${}^3_\Lambda\text{H}\to{}^3\text{H} + \pi^0$ channel, $\Gamma({}^3_\Lambda\text{H}\to{}^3\text{H} + \pi^0) = 0.2954$ μeV with the similar method

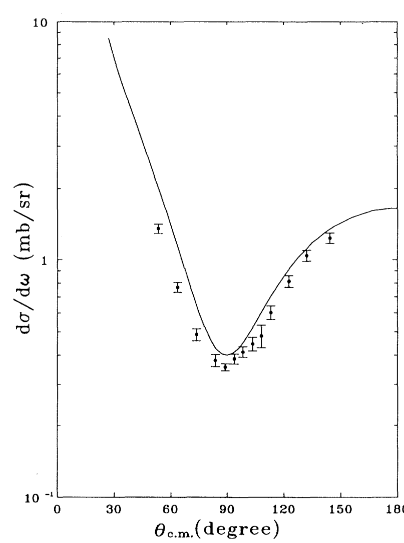

FIG. 1. The differential cross section for ${}^3\text{H}(\pi^-,\pi^-){}^3\text{H}$ at $T_\pi^{\text{lab}} = 45.1$ MeV. The solid curve was calculated by the Landau program [13] with the Thomas potential plus exact Coulomb effect. The cutoff $R_{\text{cut}} = 6$ fm was used. The experimental data were taken from [14].

above. Since the branching ratio of ${}^3_\Lambda\mathrm{H}\to{}^3\mathrm{He}+\pi^-$ channel, i.e.,

$$R_{\pi^-} = \Gamma({}^3_\Lambda\mathrm{H}\to{}^3\mathrm{He}+\pi^-)/\Gamma(\text{all }\pi^-)$$

is known as 0.3 [15], we can find the lifetime to be 0.245 ns assuming

$$R_{\pi^0} = \Gamma({}^3_\Lambda\mathrm{H}\to{}^3\mathrm{H}+\pi^0)/\Gamma(\text{all }\pi^0) = 0.3\ .$$

The measured value is $0.246^{+0.062}_{-0.041}$ ns [15] and $0.240^{+0.170}_{-0.100}$ ns [16]. Our result is in good agreement with the experimental data. In this analysis, we found that contributions from p_{π^-} and p'_{π^-} terms in Eq. (5) were only 1.2%. Since the decay rate is yielded almost by the s_{π^-} term along, the importance of the translation invariance cannot be visible in this process. However, it does not mean unimportance of the nucleon recoil effect.

Therefore, in order to examine the importance of the translational invariance, one has to investigate some characteristic processes where the p channel is comparatively enhanced. One of such processes is the asymmetry of the angular distribution of the negative pion emitted in the two-body decay of the polarized hypernucleus.

B. Asymmetry of pion angular distribution

Let us investigate the decay process, ${}^3_\Lambda\mathrm{H}\to{}^3\mathrm{He}+\pi^-$. After some mathematical procedures, one can easily find the angular distribution of the emitted pion in the form

$$\frac{d\sigma(\theta_\pi)}{d\Omega_{\mu\pi}} = \frac{1}{4\pi}[1+\alpha P({}^3_\Lambda\mathrm{H})\cos\theta_\pi]\ , \tag{18}$$

where $P({}^3_\Lambda\mathrm{H})$ is the polarization of ${}^3_\Lambda\mathrm{H}$, θ_π is the angle of π^- emission relative to this ${}^3_\Lambda\mathrm{H}$ polarization, and the decay asymmetry parameter α is defined as

$$\alpha = \frac{2\,\mathrm{Re}[\langle s_{\pi^-}\rangle^*(\langle p_{\pi^-}\rangle+\langle p'_{\pi^-}\rangle)]}{|\langle s_{\pi^-}\rangle|^2+|\langle p_{\pi^-}\rangle+\langle p'_{\pi^-}\rangle|^2}\ . \tag{19}$$

The numerical value of α can be obtained as

$$\alpha = 0.215\ . \tag{20}$$

If the portion recoil term is ignored, for Eq. (1) we find

$$\alpha_0 = 0.264\ . \tag{21}$$

Therefore, the contribution from the proton recoil term is

$$\frac{\alpha-\alpha_0}{\alpha_0} = -0.186\ , \tag{22}$$

i.e., 18.6%. This amount of modification may be observed by a slight improvement of precision in present asymmetry measurement. The experimental values of α_Λ for a free lambda particle were 0.45 ± 0.05 [17], 0.62 ± 0.07 [4], and 0.642 ± 0.013 [18]. Our present calculation shows that the value of asymmetry parameter reduces drastically in the nucleus.

III. SUMMARY

Our present investigation for ${}^3_\Lambda\mathrm{H}\to{}^3\mathrm{He}+\pi^-$ decay clarifies importance of the translational invariance, particularly for asymmetry in the angular distribution of the emitted pion. It has been found that the value of asymmetry parameter in ${}^3_\Lambda\mathrm{H}\to{}^3\mathrm{He}+\pi^-$ decay could be reduced by 18.1% by the translationally invariant term. This fact does not depend very much on the wave function of ${}^3\mathrm{He}$. The result obtained with the Irving-Gunn wave function with $\tilde{\alpha} = 0.7704\ \mathrm{fm}^{-1}$ [19] is 18.4%. And the lifetime is 0.256 ns.

As is mentioned in the Introduction, the contribution of the translational invariance can only be observed in the cases of hypernuclei. The translationally invariant term plays an important role not only in the asymmetry of pion angular distribution but also in the processes where the daughter proton is excited to the state different from the initial state of lambda. Therefore, an accurate measurement of the π^- angular distribution for ${}^3_\Lambda\mathrm{H}\to{}^3\mathrm{He}+\pi^-$ is highly requested to confirm our prediction.

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