Reply to "Comments on 'New interpretation of the lowest $K = 0$ collective excitation of deformed nuclei as a phonon excitation of the γ band'"

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This is a reply to the preceding Comments.

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One Comment claims that we are premature in our conclusions [1] and the other [2] that he did it first. We have objections to both, which we will present below, but, of course, we welcome a discussion of this fundamental and interesting problem concerning the nature of the lowest $K = 0$ [$K = 0_2^+$] excitations in deformed nuclei. We argue [3] that, in many deformed nuclei these excitations have large amplitudes for phonon excitations built on the γ band, that is, two-phonon excitations, rather than β bands. We will first present the key arguments, supplementing information in Ref. [3], and then relate these directly to the claims of Burke and Sood and Kumar.

Clearly, a key criterion, certainly a necessary condition, to describe the $K = 0_2^+$ excitations as phonons built on the γ band, is that they preferentially decay to the γ band. (In this, we do agree with Kumar.) Therefore a requirement for our interpretation is that the branching ratio R' be large, where R' is defined as

$$R' \equiv \frac{B(E2: 0_2^+ \rightarrow 2_\gamma^+)}{B(E2: 0_2^+ \rightarrow 2_1^+)} . \tag{1}$$

If R' is very large, we would interpret this as evidence for a significant two-phonon amplitude in the $K = 0_2^+$ excitation. [Note that this definition, with the prime and the $0_2^+ \rightarrow 2_\gamma^+$ transition in the numerator, is the inverse of that (called R) used in Ref. [3]: This is done in order to exhibit clearly and transparently the dominance of decay to the γ band.] The data supporting our contention that $R' \gg 1$ are extensive. We tabulate all existing data for well-deformed nuclei in Table I. Only a few of these data involve $0_2^+ \rightarrow 2_\gamma^+$ transitions directly. These transitions are low in energy and often not observed. However, there are far more data from higher-spin states. We have extracted results for all available cases for $\Delta I = 2$ transitions (thus avoiding uncertainties with possible $M1$ components) and converted the branching ratios to R' values by appropriate Clebsch-Gordon coefficients, thus, in effect, comparing branching ratios for the intrinsic matrix elements.

The experimental results in Table I demonstrate an *overwhelming* preponderance of decay to the γ band. Aside from ^{166}Er (and, to a lesser extent, ^{164}Er where the error is large), the average value of R' in the table far exceeds 100; that is, they show a *preference of decay of the K=0_2^+ band to the γ band over the ground band by two orders of magnitude.* We argue that such a dominance is so strong that it suggests a substantial two-phonon character in the $K = 0_2^+$ excitations of these nuclei.

To put the R' values in Table I in perspective, we compare with the recent calculations of Soloviev [5]. He obtains R' values near or slightly above unity — ranging up to a maximum of 5.6 in ^{168}Er—in several rare earth nuclei. The experimental values for ^{168}Er are 56(9) and 110(40), for two $K = 0_2^+$ band branching ratios, and are even higher in most of the other nuclei. The $K = 0_2^+$ wave functions in Soloviev's calculations are admixtures of several components. The squared amplitudes for two-phonon γ vibrational components range up to 6%. Reference [5] thus predicts both a (small) preference for decay to the γ band and small two-phonon amplitudes. This is certainly an encouraging improvement in the model, but, at the same time, these calculations, in fact, clearly underestimate the overwhelming dominance for decay to the γ band in ^{168}Er and most of the other cases in Table I, usually by two orders of magnitude of more. We argue that the experimental dominance is so large (an order of magnitude or more larger than in Soloviev's cal-

TABLE I. Experimental $B(E2)$ ratios for decay of the $K = 0_2^+$ band in deformed rare earth nuclei.

Nucleus	$I_{K=0}$	I_γ	I_g	R' [Eq. (1)]$\equiv \frac{B(E2:0_2^+\rightarrow 2_\gamma^+)}{B(E2:0_2^+\rightarrow 2_g^+)}$ [a]
^{158}Gd	4	2	2	$3.6(8)\times 10^2$
^{160}Gd	4	2	6	$6.1(41) \times 10^2$
^{160}Gd	4	2	2	$1.6(8) \times 10^3$
^{160}Dy	4	2	6	$3.0(15) \times 10^2$
^{162}Dy	2	4	4	$1.2(2) \times 10^2$
^{162}Dy	2	4	0	$1.6(11) \times 10^3$
^{164}Er	0	2	2	1.9(10)
^{166}Er	0	2	2	0.9(1)
^{168}Er	4	2	2	$5.6(9) \times 10^1$
^{168}Er	6	4	4	$1.1(4) \times 10^2$

[a]The table gives either the ratio R' [Eq. (1)] directly or the equivalent value of R' obtained from data for other spin states of the $K = 0_2^+$, γ, and g bands after multiplying by the appropriate Clebsch-Gordon coefficients. Results are obtained from the latest Nuclear Data Sheet compilations, references cited therein, and Ref. [4]. Multiple values for the same nucleus are obtained from different combinations of $K = 0_2^+$, γ, and ground-band spin states. Their variations, especially near the edge of the deformed region, reflect band-mixing effects.

culations [5]) that it suggests a much larger two-phonon component in the $K = 0_2^+$ excitation and, indeed, perhaps the dominance of this component. We also stress, of course, that a quantitative determination of the wave function structure is elusive.

The idea of substantial collective two-phonon character is further supported by noting that the absolute $K = 0_2^+ \rightarrow \gamma\ B(E2)$ values are actually comparable to or greater than $\gamma \rightarrow$ ground values. While some of the enhancement of $K = 0_2^+ \rightarrow \gamma$ transitions is probably a band-mixing effect (see Ref. [6]), the collective character of these transitions seems clear. Moreover, in regard to the comparison to the interacting-boson approximation (IBA), Burke and Sood considered only the $0_2^+ \rightarrow 2_\gamma^+$ transitions, following the lead of Ref. [3], and found that the experimental R' values for ^{166}Er and a pair of transitional Os nuclei were one to two orders of magnitude smaller than predicted. However, when the additional results, for other nuclei, obtained from higher spins, are included as in Table I, it is seen that most of the values agree with IBA predictions. It will be noted from Fig. 2 (lowest curve) of Ref. [3] that the IBA value of $R'(\mathrm{IBA}) \sim 100$ is, if anything, even less than the experimental dominance.

We further stress that the fluctuations in R' in Table I do *not* argue against a two-phonon interpretation. While it is true that collective properties should vary smoothly, we recall that R' involves *two* transitions $K = 0_2^+ \rightarrow \gamma$ *and* $K = 0_2^+ \rightarrow g$. If the $K = 0_2^+$ excitation is a collective mode built on the γ vibration, the $K = 0_2^+ \rightarrow$ ground-band transitions would be forbidden, and it is hardly surprising that they would fluctuate. In fact, the $B(E2{:}0_2^+ \rightarrow 2_1^+)$ values are known to fluctuate by more than an order of magnitude in the rare earth region, and therefore, independently of the behavior of $B(E2{:}0_2^+ \rightarrow 2_\gamma^+)$ values, it is natural that R' itself should fluctuate.

The criterion of smoothly varying character for a collective excitation is, nevertheless, important to address. This smoothness, however, should be relative to the *base* on which a given excitation is built. In the case of the $K = 0_2^+$ mode, we should then expect a smooth dependence of its relation to the γ *band.* This is, in fact, the case empirically. As shown in Ref. [3] (see Fig. 1), the energy ratio $E(0_2^+)/[E(2_\gamma^+) - E(2_g^+)]$ has a regular, sawtoothed, pattern across the rare earth nuclei. While a smooth phenomenology could also result [7] for an excitation of β vibrational character, it is certainly compatible with and expected for a phonon excitation built on the γ band as well. Other correlations of $K = 0_2^+$ and γ-band properties have been discussed in Ref. [8].

The same energy ratios lead to a further argument for such a structure. Most of the ratios $E(0_2^+)/[E(2_\gamma^+) - E(2_g^+)]$ range empirically from 0.8 to 1.8. While it might be thought that this is a low for a phonon mode built on the γ band, it is, in fact, in good agreement with the IBA predictions in which the $K = 0_2^+$ mode does have such a two-phonon structure. In the IBA, calculated values are 1.0 for SU(3) and, for realistic deformed nuclei, lie between 1.2 and 1.8. Recall that the model could have, in principle, predicted a wide range of values for this ratio (and, for nondeformed nuclei, gives values of 2–3) and that a β vibration or quasiparticle excitation could also have virtually *any* energy relative to the γ band. We therefore feel that the comparison between empirical values (the bulk between 0.8 and 1.8) and the IBA predictions [1.0 for SU(3), 1.2–1.8 for typical calculations in deformed nuclei] is actually surprisingly good.

We now relate these points to the Comment by Burke and Sood. They make the valid point that one should not judge the nature of an excitation from a single piece of data and stress that it is difficult to extract a quantitative value for the two-phonon amplitude in the 0_2^+ excitation. Further, they are undoubtedly correct in stating that the empirical $K = 0_2^+$ bands contain admixtures of components, and only one of them is a phonon excitation built on the γ band. We differ with them in the relative importance of these components.

Burke and Sood present four arguments in support of their case: that there are only a few data on the branching ratio for the $E2$ decay of the 0_2^+ state to the γ and ground bands, that they do not show a clear preference for decay to the γ band, and that the branching ratio data disagree with the IBA; that the rapid changes in $K = 0_2^+$ properties from nucleus to nucleus argue against a collective mode; that the low energy of the $K = 0_2^+$ band is incompatible with a phonon structure built on the γ band and again disagrees with the IBA; and that Soloviev and colleagues calculate that the dominant structure of the $K = 0_2^+$ band is that of a β vibration.

Our responses to these are now made very simply by reference to the above discussion. As noted, Burke and Sood followed the lead of Ref. [3] in defining the $E2$ branching ratio from the 0_2^+ state and cited three R' values. One of their cases, ^{166}Er, is indeed anomalous and does not fit our picture. We will comment on this below. Though the other two, ^{188}Os and ^{190}Os, support our case since they show a preference for decay to the γ band by factors of 5–10, we did not include them in Table I because they are transitional nuclei. In any case, when the broader set of data in Table I are considered, it is clear that the $K = 0_2^+$ bands show a dominance of decay of the 0_2^+ band to the γ band by about two orders of magnitude (or more) over decay to the ground band in the clear majority of deformed nuclei where the data exist. Moreover, $K = 0_2^+ \rightarrow \gamma\ E2$ matrix elements are comparable to $\gamma \rightarrow$ ground matrix elements. Also, the empirical R' values in Table I (except ^{164,166}Er) are compatible with the IBA. We note that, owing to their low energy, γ-ray transitions from the $K = 0_2^+$ band to the γ band are necessarily weak in intensity. In most cases, their observation requires sensitive spectrometers such as the GAMS facility in Grenoble. The lack of data on $K = 0_2^+ \rightarrow \gamma$ transitions in other nuclei than in Table I does not therefore imply that such transitions are unobserved because they are weaker, but only that the requisite GAMS experiments have not been done.

Second, we have argued that the rapid changes in the $K = 0_2^+$ mode that Burke and Sood refer to (fluctuations in R' in Table I) are largely due to the forbidden nature of the transitions from the two-phonon $K = 0_2^+$ band to the ground band. These indeed fluctuate widely (as we observed in Ref. [3] explicitly), and they *should.* Other properties of the $K = 0_2^+$ band, however, such as its energy, behave quite smoothly *relative to the* γ *band* on which we argue it is built. While this is not, *per se*, evidence for such phonon structure, it is consistent with it. Third, the low energy of the $K = 0_2^+$ mode is not an argument against a two-phonon structure. In the IBA it has that structure and yet is low lying. Moreover although Burke and Sood disagree, we feel that its empirical energy ratio to the γ band (0.8–1.8) is actually in surprisingly *good* agreement with the IBA (1.2–1.8). In summary, we feel that the evidence discussed above suggests a pattern of behavior consistent with a substantial component of two-phonon character in the $K = 0_2^+$ excitation as a phonon built on the γ vibration. Other data, such as single-nucleon transfer reaction cross sections, point to specific quasiparticle amplitudes as well. Our principal difference with Burke and Sood is in the relative magnitude of these structural components.

Finally, as noted above, the model of Soloviev referred to by Burke and Sood has recently been undergoing substantial improvements, especially in regard to two-phonon excitations. For example, in ^{168}Er, the two-phonon component of the $K = 4$ band has increased from $\sim 1\%$ in earlier calculations to 30% in recent work [9]. To date, this model, with two-phonon strength of 2%–6% in the $K = 0_2^+$ excitation, achieves R' values up to ~ 6, but cannot reproduce the *extent* of the dominance of decay to the γ band seen in Table I. It will be most interesting to follow future theoretical developments.

It is worth also commenting on the anomalous character of ^{166}Er (and perhaps ^{164}Er) in Table I. (This is the case aptly highlighted by Burke and Sood.) Here the ratio R' in Eq. (1) is indeed approximately unity and raises the question of whether the $K = 0_2^+$ band has very different character than in most other rare earth nuclei. It may be that the two-phonon 0^+ excitation lies higher in ^{166}Er. Further theoretical and experimental study is called for, and we are grateful to Burke and Sood for noting this case.

We come now to the Comment [2] of Kumar. He makes two points: that he interpreted the lowest excited $K = 0^+$ bands as $\gamma\gamma$ excitations in his conference paper [10] in 1983 and that his calculations with the dynamic deformation model (DDM) agree with the data. Regarding the first, Kumar gave in his 1983 paper [10] a theoretical criterion for defining a $\gamma\gamma$ excitation—related to Eq. (1)—when he wrote, "The excited $K = 0$ band *should* be called a $\gamma\gamma$ band and not a β band *if* it decays mainly to the γ band," (our italics). This is a *definition* which Kumar made (and repeated nearly verbatim four paragraphs later) in the context of his discussion of ^{168}Er. Kumar further states [2] that his 1983 reference "gives several figures comparing [the DDM with] experimental spectra, where such bands are clearly labeled $\gamma\gamma$ bands." This is true, but there are *no* comparisons with experimental branching ratios from *any* bands labeled $\gamma\gamma$. Moreover, he showed *no branching ratio data* that the $K = 0_2^+$ bands decay predominantly to the γ band in *any* other nuclei and certainly made no contention that it is so in the majority of deformed rare earth nuclei.

It is, in fact, just in this aspect of suggesting that most empirical $K = 0_2^+$ bands have substantial amplitudes as phonon excitations built on the γ band (albeit not necessarily $\gamma\gamma$ excitations *per se*) that our contribution centers.

Kumar adds [2] that "There is also ample comparison and agreement to conclude that such bands are described in the DDM as well as the γ bands." In replying, we note that Kumar's model has great merit as an overall description of low-lying collective bands (see, for example, Figs. 5–10 in his Ref. [10]). It seems to have problems, though, in the detailed description of individual nuclei. This is hardly surprising given his goal to encompass the entire Nuclear Chart and that he adjusts no free parameters for individual nuclei: It is an achievement that the model succeeds as well as it does.

Nevertheless, for ^{168}Er, one notes that the DDM calculations [10] predict a highly perturbed energy sequence for the $K = 0_2^+$ band, namely, 2^+, 0^+, 4^+, 6^+, whereas the data show a normal rotational sequence 0^+, 2^+, 4^+, 6^+. More importantly, his Table 3 shows that the DDM calculations predict a near-complete K mixing of the $K = 0_2^+$ and γ bands, in complete disagreement with the data. Specifically, his Table 3 gives the calculated K components of what he labels the "$K = 0_2^+$" and "γ" bands in ^{168}Er. The squared amplitudes of $K = 0$ in the γ band for $I = 4$, 6, and 8 are 34%, 54%, and 58%. The $K = 0_2^+$ band is the orthogonal admixture. However, a Mikhailov plot analysis [6] gives the experimental (spin-independent part of the) mixing matrix element of the $K = 0_2^+$ and γ bands as 0.28(5) keV. With an energy difference of ~ 400 keV, incorporating the proper spin function, this corresponds (for $I = 6$, for example) to a total K-mixing amplitude of 0.036 or a squared amplitude of 0.15% Thus, experimentally, the $K = 0_2^+$ and γ bands have nearly *pure* K. Thus his calculations cannot be taken as a basis for assigning a $\gamma\gamma$ structure.

Finally, regarding his question about what the phonon structure of the $K = 0_2^+$ excitation would be if it is not a $\gamma\gamma$ mode, our point was to merely exercise due caution. The data tell us that the $K = 0_2^+$ excitation is a phonon built on the γ band: The data do not directly reveal the microscopic structure of that excitation.

Despite our disagreement with the Comments by Burke and Sood and Kumar, they highlight the interesting point that, after nearly half a century of study, even the nature of the most basic low-lying collective modes of deformed nuclei is still actively debated. These Comments reflect the intense interest in these excitations, and this on-going discussion points to the need for renewed microscopic theoretical studies.

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