Nature of the lowest excited $K^\pi = 0^+$ states of even-even deformed nuclei

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A recent paper by Casten and von Brentano [Phys. Rev. C **50**, R2180 (1994)] has proposed on the basis of interacting-boson approximation calculations that the lowest excited $K^\pi = 0^+$ states of deformed nuclei should be given a new interpretation as phonon excitations based on the gamma vibrations. The main argument for this reinterpretation is the observation of large $E2$ strengths coupling the $K^\pi = 0^+$ and gamma bands in many nuclei. However, these strengths could arise from rather minor double-γ-phonon admixtures in the $K^\pi = 0^+$ bands, and the dominant components of these complex states could have a quite different character.

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Casten and von Brentano [1] have pointed out the difficulties encountered historically in attempts to describe the structures of excited $K^\pi = 0^+$ states in deformed even-even nuclei, the lowest of which have often been loosely referred to as beta vibrations. They propose a new interpretation for these states, as phonon excitations based on the well-known $K^\pi = 2^+$ gamma vibrations, in which case they would be some type of double-phonon state. The strongest argument leading to this proposal is that there are large $E2$ strengths connecting the $K^\pi = 0^+$ bands to the gamma bands in many nuclides, much stronger than those connecting the $K^\pi = 0^+$ bands to the ground state bands.

The purpose of this Comment is to point out that although such $E2$ strengths suggest the presence of a double-phonon component in the $K^\pi = 0^+$ state, a quantitative measure for the magnitude of this component has not been extracted. It will be argued below that the observed $E2$ strengths may well arise from a rather minor double-γ-phonon component in the $K^\pi = 0^+$ states, and that the dominant component could have a quite different character.

Casten and von Brentano report *sd* interacting-boson approximation (IBA) calculations which predict much larger $E2$ strengths from the lowest $K^\pi = 0^+$ states to the γ band than to the ground state band. The ratio R defined in their Eq. (9) is the relative $E2$ transition strength from the $K^\pi = 0^+$ bandhead to the $I^\pi = 2^+$ members of the ground state and gamma bands, respectively. This ratio is one of the properties which is reported to agree very well with experiment, although no experimental data are shown. A search through the appropriate Nuclear Data Sheets for the region from ^{150}Nd to ^{190}Os turned up only three nuclides for which the information needed for an experimental determination of this ratio was available. These were ^{166}Er, ^{188}Os, and ^{190}Os, for which the empirical values for R are 1.34, 0.22, and 0.099, respectively (after taking the E_γ^5 dependence of the $E2$ transitions into account).[1] Figure 2 of Casten and von Brentano shows that the values of R for all these cases are predicted to be less than 0.02, indicating that the limited number of empirical ratios available are one or two orders of magnitude larger than the *sd* IBA predictions. Further examination shows that the situation must be more complicated. The IBA predicts that properties such as these ratios should vary slowly and smoothly as the boson number changes, whereas experimentally there are large differences in the behavior for the neighboring cases of ^{166}Er and ^{168}Er. The value of R=1.34 above for ^{166}Er shows that the decay from the $K^\pi = 0^+$ band to the ground band is stronger than that to the gamma band, whereas in ^{168}Er the transitions to the γ band dominate the decay [7]. This dramatic change from one even-even nuclide to the next is more typical of a behavior which depends upon the microscopic structure of the nucleus. In fact, calculations with the quasiparticle phonon nuclear model (QPNM) for the structure of $K^\pi = 0^+$ states in this region [8] reproduce the above differences in nature for the $E2$ branchings between ^{166}Er and ^{168}Er. These calculations show that the low-

[1]Some inconsistencies exist in the literature, concerning the branching ratio for the decay of the 1460 keV 0^+ level of ^{166}Er, which is used to obtain the ratio R=1.34. The intensity ratio of 2.09:100 for the 674 and 1380 keV gamma rays adopted in the Nuclear Data Sheets [2] is consistent with the measurements of Reich and Cline [3], but Allab *et al.* [4] report a ratio of about 2:100 in a level scheme and about 20:100 in a table. A ratio of 1.6:9.7 (which is comparable to the value of 20:100) was reported by Bondarenko *et al.* [5] in measurements from the $(n, n'\gamma)$ reaction. However, an examination of the gamma-ray spectrum for the decay of 27 h ^{166}Ho presented by Heath [6] suggests that the ratio is much closer to the value 2:100 than to 20:100.

est $K^\pi = 0^+$ states have a complex structure, but that the dominant component ($\geq 60\%$) in each case is a beta vibration, with no double-phonon component having an admixture as large as 10%. (Here, and elsewhere in this report, values for admixtures and components given as percentages indicate the squares of wave function amplitudes.) In ^{166}Er the lowest $K^\pi = 0^+$ state is calculated to have a very small double-γ-phonon admixture, which results in a small $B(E2)$ value to the gamma band, as observed. In ^{168}Er the lowest $K^\pi = 0^+$ state is calculated to have a somewhat larger, although still rather small (4%), double-γ-phonon component. It is important to note that the calculations show that this minor component is large enough to explain the observed dominance of $E2$ decays to the gamma band over those to the ground state band [8]. It is realized that the small values quoted above for double-γ-phonon components may have significant uncertainties, as possible future adjustments in parameters of the QPNM could change the percentage admixtures for *small* components by a factor of several. However, past experience has shown that such adjustments do not often affect the qualitative structure, in that the dominant components usually remain the dominant ones. The important point is that the $B(E2)$ results do not require the existence of a large double-phonon component in the $K^\pi = 0^+$ wave function, as small admixtures of the double γ phonon (at the level of a few percent) appear to be adequate to explain the observed results. Thus, the $B(E2)$ data must be used with caution because a very small part of the wave function may be responsible for the dominant mode of decay.

It appears there may be a parallel between the situation for these $K^\pi = 0^+$ bands and that for the low-lying $K^\pi = 4^+$ bands of even-even deformed rare earth nuclei, for which the double-phonon description was recently disputed [9]. These $K^\pi = 4^+$ bands were assigned as double γ phonons on the basis of the large $B(E2)$ values coupling them with the γ bands [10,11]. However, other types of experimental data, such as two-quasiparticle admixtures from single-nucleon-transfer reactions and other experiments, conflicted strongly with this interpretation [9]. A much more successful explanation of all the experimental data, including $E4$ strengths, $B(E2)$ values between bands, etc., is obtained by considering the $K^\pi = 4^+$ bands as predominantly hexadecapole, or g-boson, structures. Here also, the presence of minor (typically a few percent, and in the largest cases 20%–30%) double-γ-phonon admixtures were seen to explain the observed $E2$ strengths of several Weisskopf units coupling these bands to the γ bands. Calculations with the *sd* IBA were capable of reproducing the level energies and $B(E2)$ values, with the $K^\pi = 4^+$ bands assumed to have a double-γ-phonon character. However, calculations with the *sdg* IBA revealed that these $K^\pi = 4^+$ bands have a predominantly hexadecapole nature [12–14]. This indicates that the double-γ-phonon interpretation for these states, invoked to explain the large $E2$ strengths to the γ bands, arose from the truncation to the *sd* IBA space, which may not be justified for these cases.

Another feature of the proposed new interpretation [1] concerns the excitation energies of the $K^\pi = 0^+$ and $K^\pi = 2^+$ bands. A survey of even-even deformed rare earths [15] lists 50 nuclides for which the lowest excited $K^\pi = 0^+$ and $K^\pi = 2^+$ states have both been assigned. There are 22 cases for which the $K^\pi = 0^+$ state has the lower energy. It seems unreasonable to expect that if, in general, the $K^\pi = 0^+$ states were excitations based upon the $K^\pi = 2^+$ ones, they would have a lower energy in almost half of the instances. It is much easier to accept the explanation that the lowest $K^\pi = 0^+$ bands have small admixtures of higher-lying $K^\pi = 0^+$ double-phonon (including the double-γ-phonon) states.

An indication that the lowest $K^\pi = 0^+$ states may be described only approximately by the *sd* IBA is seen by considering the ratios of energies given by Casten and von Brentano in their Fig. 1(a). It has been stated [16] that "... the IBA unavoidably predicts ..." this ratio to be within the range of 1.2–1.8. This condition is satisfied for only 20 of the 50 cases in the survey mentioned above [15]. More than half of the cases actually have a ratio less than 1.2.

We agree wholeheartedly with the authors that this problem is an interesting and difficult one, and encourage further studies. In addition to the properties suggested by Casten and von Brentano for more detailed examination, we would like to stress that theoretical works should consider all the experimental data available, rather than concentrating mainly on energies and $B(E2)$ values. A considerable amount of data in the form of $E0$ strengths, two-quasiparticle admixtures, and two-neutron transfer strengths is available and should be explained by any successful model. In many nuclei (p,t) and (t,p) reactions have shown large amounts of $L = 0$ strength to excited $K^\pi = 0^+$ states, and the amount can vary significantly from one nuclide to the next. Calculations with a microscopic model were able to reproduce these variations [17], providing further indication that the microscopic structures of these $K^\pi = 0^+$ states are important.

In summary, the $K^\pi = 0^+$ states are probably quite complex in structure, and one should make use of all the experimental data available to study as many aspects of their character as possible. It seems premature to reinterpret them as proposed by Casten and von Brentano because it has not been demonstrated that the two-phonon component is the dominant one, and it would be misleading to label them according to one of their minor components.

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