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Off-shell ρ-ω mixing through quark loops with a nonperturbative meson vertex and quark mass functions

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The momentum dependence of the off-shell ρ-ω mixing amplitude is calculated through a two-quark loop diagram, using nonperturbative meson-quark vertex functions for the ρ and ω mesons, as well as nonperturbative quark propagators. Both these quantities are generated self-consistently through an interlinked Bethe-Salpeter equation (BSE) cum Schwinger-Dyson equation (SDE) approach with a three-dimensional 3D support for the BSE kernel with two basic constants that are prechecked against a wide cross section of both meson and baryon spectra within a common structural framework for their respective 3D BSE's. With this precalibration, the on-shell strength works out at $-2.434\,\delta(m_q^2)$ in units of the change in "constituent mass squared," which is consistent with the e^+e^- to $\pi^+\pi^-$ data for a u-d mass difference of 4 MeV, while the relative off-shell strength (0.99 ± 0.01) lies midway between quark-loop and QCD–sum rule (SR) results. We also calculate the photon-mediated ρ-ω propagator whose off-shell structure has an additional pole at $q^2 = 0$. The implications of these results *vis-à-vis* related investigations are discussed.

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I. INTRODUCTION

During the past few years the problem of charge symmetry breaking (CSB), which has been one of the longstanding questions in nuclear physics [1–3], has attracted a good deal of attention in theoretical quarters [4–11], stimulated by new experiments [12,13]. Thus it has been claimed [4,5] that an understanding of the Nolen-Schiffer anomaly [1] is possible in terms of the so-called class III and class IV forms of the CSB potential [3]. In this respect the dominant contributor on which there is a broad consensus [2,4–7,10] seems to be the short-range ρ^0-ω mixing term in the one-boson-exchange (OBE) potential, while other terms, e.g., the long-range [one-pion-exchange (OPE)] CSB effect arising from the n-p mass difference, give much smaller contributions [4]. A more sensitive test of the intrinsically small CSB terms comes from the results of the TRIUMF [12] and IUCF-Saclay [13] experiments on polarized n-p scattering, which are free from Coulomb effects. The crucial experimental parameter in this regard is the (small) difference $\Delta A(\theta) = A_n(\theta) - A_p(\theta)$ between the neutron and proton analyzing powers measured at an angle θ_0 corresponding to the vanishing of the average analyzing power. This quantity in the Born approximation is proportional to the CSB potential $V_{\rm CSB}$ whose contribution from the ρ^0-ω mixing effect may be schematically expressed in a fairly standard notation as in Ref. [7]:

$$V_{\rm CSB}^{\rho\text{-}\omega} = \langle NN|H_{\rm int}|NN\omega\rangle G_0 \langle\omega|H_{\rm CSB}|\rho^0\rangle \times G_0\langle\rho^0 NN|H_{\rm int}|NN\rangle + (\rho^0 \Leftrightarrow \omega). \quad (1)$$

Here G_0 is the appropriate V-meson propagator and $\langle\omega|H_{\rm CSB}|\rho^0\rangle$ gets its dominant theoretical contribution from the strong CSB effect of the u-d mass difference δm_q with $H_{\rm CSB} \sim \rho_\mu\omega_\mu\delta(m_q^2)$, and partly from the e.m. chain $\rho \Rightarrow \gamma \Rightarrow \omega$ via vector-meson dominance (VMD) and/or quark loop. Alternatively, the matrix element can be estimated from the experimental $e^+e^- \Rightarrow \pi^+\pi^-$ amplitude at the ω pole [5], and its consistency, if any, with the quark loop picture would contribute a test of the latter. [The other pieces in Eq. (1) which refer to the strong interaction (CS conserving) background are not of immediate interest for this discussion.] A fit to the $\Delta A(\theta)$ from the TRIUMF experiment at 477 MeV [12] has been claimed in Ref. [6]. However, the effect is energy dependent, as suggested by the IUCF-Saclay experiment at 183 MeV [13]. Further, the significance of any agreement with theory is tempered by the possibility of competing CSB mechanisms (e.g., n-p mass difference in OPE exchange versus ρ-ω mixing effect [11,13]), with considerable freedom in their respective parametrizations, unless such competing mechanisms stem from some *common* theoretical framework capable of demarking their relative strengths (thus necessarily giving the question a *quantitative* orientation).

A more serious issue concerns the behavior of the ρ-ω mixing amplitude, which is the dominant contender for the CSB effect, when it is extrapolated from its "on-shell," timelike value (measured in $e^+e^- \Rightarrow \pi^+\pi^-$ at the ω pole) to its *off-shell* structure [7] which is relevant to the corresponding N-N potential (1) in the spacelike region of q^2. Indeed, it has been claimed [10] that the momentum dependence of this mixing amplitude, whether computed in terms of $q\bar{q}$ loops [7] or of $N\bar{N}$ loops [10], can be so strong that the ρ-ω contribution to the CSB potential is greatly suppressed even in the $r \sim 0.9$ fm region [10,11] where the occurrence of a node in the N-N potential should make this (intrinsically small) ampli-

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tude relatively more visible. Unfortunately, theoretical estimates [6–11] of its off-shell effect have varied rather widely so that the issue is no longer a qualitative one. On the other hand, the "nuclear" stakes on the ρ-ω mixing effect, which seems to play such a crucial role in the CSB scenario, are high enough to warrant a more systematic quantitative evaluation of the off-shell amplitude which should leave little scope for parametric uncertainties. To that end it is useful first to express the off-shellness of ρ-ω amplitude $\theta(q^2)$, defined linearly as a function of q^2 [11], in terms of a dimensionless parameter λ as

$$\theta(q^2) = \theta(M^2)[1 - \lambda(1 + (q^2/M^2))] \tag{2}$$

in the notation of Ref. [11] (but adapted to the Euclidean metric), where M is the average ρ-ω mass. The estimates of λ have been found to vary over a wide range, from relatively smaller values (0.6–0.7) predicted by quark loop mechanisms [7,8] to much larger values (1.7) obtained by the QCD sum rule method [11,14] with the nucleon loop method [10] yielding an intermediate value (~ 1.0).

The object of this investigation is to offer an independent estimate of the off-shell measure λ as well as of the related on-shell quantity $\theta(M^2)$ which is hopefully free from the large scale parametric uncertainties inherent in any phenomenological approach to these important quantities. This is sought to be achieved within the framework of the quark loop method, with the u-d mass difference δm_q as one key parameter. (We do *not* address any other questions on the CSB aspects of the N-N interaction.) The dynamical method used for this purpose is an interlinked Bethe-Salpeter equation (BSE) cum Schwinger-Dyson equation (SDE) framework characterized by two basic constants ω_0, C_0 [15] which are precalibrated against hadron spectra ($q\bar{q}, qqq$) as well as several other observable hadronic parameters. However, since such a framework is hardly novel and has been employed by many workers in the past [16] and present [17,18] we have first attempted (in Sec. II) to give a comparative assessment of our approach *vis-à-vis* the contemporary literature on the subject, without going afresh into the detailed motivations which have been recently described elsewhere [15], the basic concern being to avoid the introduction of any fresh input parameters beyond the two already introduced [15], plus the u-d mass difference δm_q. In Sec. III we recapitulate only the essential aspects of our three-dimensional (3D) BSE-cum-SDE formalism [15] in which the active ingredients are the hadron quark vertex function $\Gamma_h(\hat{q})$ and the dynamical quark mass function $m(\hat{p})$, while referring to the several recent publications [19–22] for their detailed derivations. Section IV outlines the derivations of the $\rho \Rightarrow \omega$ amplitude at the quark level, with its proportionality to the $u - d$ mass difference δm_q being facilitated by following a simple device of differentiation with respect to m_q [7]. This is a quark-level alternative to its hadron-level counterpart which emphasizes the proportionality to $(m_\omega^2 - m_\rho^2)$. The detailed steps which are otherwise routine are skipped for brevity, without fear of any possible misunderstanding that our calculation necessarily refers to a ρ-ω quark loop which is proportional to δm_q, and not, e.g., to a ρ-ρ quark loop which is not so proportional [because of an obvious difference in phase which comes about from the isospin structures ($u\bar{u} \mp d\bar{d}$) of ρ^0 and ω, respectively]. See also the Appendix. The contribution of the ρ-γ-ω chain to the ρ-ω amplitude is also included for completeness. The last section gives a brief discussion of the results *vis-à-vis* contemporary investigations [7–11] through a direct comparison of their effect on the ρ-ω mixing potential, Eq. (1), at the "node" $r \sim 0.9$ fm [10,11] of the N-N potential.

II. QUARK LOOP METHOD IN 4D VS 3D BSE

In the QCD–sum rule (SR) method, the inputs are $\langle q\bar{q}\rangle, \delta\langle q\bar{q}\rangle$, and the current u, d masses which are more or less under control. However, the uncertainties arise from the matching of the two sides of the "duality" equation by the standard methods [14] to obtain the necessary stability in the sum rule structure, which is the key to the determination of the hadronic parameter λ [11].

In the quark loop method [7,9], on the other hand, the dynamics resides in the hadron-quark vertex function and constituent quark mass [7], or, better, the dynamical mass function, and these two quantities in turn are interlinked through the Schwinger-Dyson and Bethe-Salpeter equations, a formalism which, though well known since the 1970s [16], has been greatly revived in the 1990s [17,18]. These approaches also require in practice a generous degree of parametrization [17,18] of the basic entities, since any exact solution of these coupled equations is still a distant dream. Therefore any attempt to adapt this methodology to the present problem [9] must also share the corresponding parametric uncertainties [18], without prior checks from other sectors of hadron physics, notably spectroscopy as well as some sensitive coupling constants and form factors. A broad perspective on these approaches [17,18] has been discussed more fully elsewhere [15], in the context of an alternative program with a similar philosophy [19–22] whose continued emphasis on the spectroscopy sector stems from its sensitivity to the "gluonlike propagator" in the infrared region (a paraphrase for the q-$\bar{q}$ potential in the more ordinary language).

We shall not go into the relative merits of the QCD-SR [14] and the BSE-cum-SDE methods, which have been discussed elsewhere [15]. However, in the context of the latter it will be useful to make a distinction between two broad types, the "spectroscopy-oriented" type [19,20] which depends on a basically 3D approach, and the more "orthodox" 4D type [17,18]. For purposes of this paper we shall term these two BSE-cum-SDE methods as "3D" and "4D" BSE's, respectively, for short. Both make use of the two-quark loop integration to calculate the ρ-ω amplitude but this difference in terminology emphasizes the difference in the parametrizations of the infrared region of the gluonic propagator, which are 3D and 4D, respectively. While the 4D form is *prima facie* more natural, the theoretical reasons for the 3D form are no less persuasive and the interested reader will find the necessary details on its theoretical motivations [23] from various angles in [15,22]. Here we shall merely cite the chief experimental reason, viz., the O(3)-like spectra in the Particle Data Group tables [24] continually for four decades which provide the bedrock of foundation for any theoretical effort at a microscopic description of quark structure, and our 3D BS program [19–22] has been specifically designed to meet this requirement. On the other hand, a literal consequence of the 4D form of parametrization of the infrared part (confining) of the gluon propagator [18] would be to predict O(4)-like spectra which contradicts experiment [15]. The reason why such O(4)-like results are not entirely visible in some of these spectral predictions [18] is merely because of their consideration of mainly the ground state masses $L = 0$, since the fuller implications of the 4D forms would not start showing up until the predictions include the L_- excited states [15].

Since some quark loop results of the 4D BSE [18] are already available [9], along with those of the QCD-SR analysis [11], it should be of considerable interest even without prejudice to the question of O(3)- versus O(4)-like spectra (important as it may be in its own right), to record for comparison the corresponding results of the 3D BSE [15] in view of their parameter-free nature. This may be useful in the context of the current controversy [7,9–11] on the off-shell strength of the ρ-ω mixing amplitude which is an important parameter for charge symmetry breaking in the N-N force. We recall in this connection that the 3D BSE formalism is specifically calibrated to both q-$\bar{q}$ [20] and qqq [21] spectra, both in excellent accord with data [24], as well as to a representative list of hadron couplings [25,15,19]. All this has been obtained with just two basic constants C_0 and ω_0 common to both types, since a third input, the quark mass (constituent), gets *dynamically generated* through the (chiral symmetry breaking) solution of the SDE [15,16], so that in this (spectroscopy-oriented) BSE-cum-SDE approach there is practically no scope for any free parametrization beyond the ones noted above, a condition which is probably important for the determination of the rather sensitive parameters $\theta(M^2)$ and λ under study.

III. 3D BSE-CUM-SDE FORMALISM

The purpose of this paper is to present the results of this calculation in the most economical fashion, omitting all but the essential details. To that end we shall use [11] for the definitions and notations of the crucial parameters involved, and calibrate our language to that of [7,9] as far as possible for the definition of the loop integrals except for the implicit understanding of a Euclidean metric notation underlying our formulation, and the use of P_μ for their q_μ notation since the latter has, in all our formulations [15,19,22], stood for the internal four-momentum of the quarks within the hadron, while P_μ is the four-momentum of the (composite) hadron. As to the 3D BSE formalism itself, we shall make free use of [15,19] but recapitulate some essential results so as to keep the paper within reasonably self-contained limits.

The quantities we shall explicitly calculate in our formalism are the following. (i) The function $\theta(p^2)$ with an explicit proportionality to $\delta(m_q^2)$ where m_q^2 is the constituent (dynamical) quark mass squared, which is obtained directly from an analytical formula (given below) for $\Pi(P^2)$ [7], by a simple process of differentiation with respect to m_q^2. (See Appendix for derivation.) (ii) The parameter λ [11] which can be explicitly identified from the linear dependence of this quantity on the inverse meson propagator $(P^2 + M^2)$. (iii) The analogous ρ-ω potential mediated by an intermediate photon, so that its full off-shell structure is a chain of two linear off-shell quantities $g_{\rho\text{-}\gamma}$ and $g_{\omega\text{-}\gamma}$ to be compared with the θ function, Eq. (2) for ρ-ω mixing due to the "strong" effect of u, d difference which involves this linear factor in the off-shell quantity $(P^2 + M^2)$ only once. [Despite the comparative weakness of the e.m. effect, its off-shell scenario is, on this account, somewhat different from that of the (strong) u-d effect: this point is discussed further at the end.]

As to the actual numerical values the only unknown quantity in our formalism is the u-d mass difference which we shall keep as a free multiplicative factor for $\theta(M^2)$, to facilitate discussion at the end on this point. The main results are

$$\theta(M^2) = -1289(\text{MeV}) \times \delta(m_q), \quad \lambda = 0.99 \pm 0.01, \tag{3}$$

where for $\delta(m_q)$ it is usual [9] to take $m_d - m_u$.

Our formalism is based on the covariant instaneity ansatz (CIA) [19] which gives the Bethe-Salpeter kernel $K(q, q')$ for a quark-antiquark system a 3D support expressed through its dependence on the component of q_μ transverse to P_μ for which we use a caret notation, i.e.,

$$\hat{q}_\mu = q_\mu - (qP)P_\mu/P^2, \tag{4}$$

so that $K(q, q') = K(\hat{q}, \hat{q}')$ for a 3D support [19]. As a result of this ansatz, there is an exact interconnection between the 3D and 4D forms of the BSE [19] and the hadron-quark vertex function $\Gamma_H(q, P)$ becomes a function $\Gamma_H(\hat{q})$ of a single argument $\hat{q}_\mu$. It is usually convenient to take out the Dirac matrix from this structure, viz., γ_5 for a pion, $i\gamma$ for a ρ meson, etc.; the multiplying scalar factor $\Gamma_h(\hat{q})$ which carries the dynamical information has the following universal structure [19]:

$$\Gamma_h(\hat{q}) = N_H D(\hat{q})\phi(\hat{q})/(2\pi)^{5/2}. \tag{5}$$

Here $D(\hat{q})$ is a 3D denominator function and $\phi(\hat{q})$ the corresponding wave function, which together satisfy a Lorentz-covariant Schrödinger-like equation of the form $D\phi = \int K\phi$, representing the 3D reduction of the 4D BSE as a result of the above ansatz. The quantity N_H is the standard 4D BS normalizer which goes with the vertex function (5). The D function for equal mass kinematics has the simple form

$$D(q) = 4\omega(\omega^2 - M^2/4), \quad \omega^2 = m_q^2 + \hat{q}^2, \tag{6}$$

while the ϕ function is model dependent. In particular, a Gaussian form

$$\phi(\hat{q}) = \exp[-\hat{q}^2/(2\beta^2)] \tag{7}$$

emerges (for harmonic confinement) as a solution of the 3D BSE, with β^2 obtained analytically from the input structure of the BS kernel [15,20] and checked against spectroscopy [24]. Its value for the ρ-ω case is 0.0692 [20]. In Eq. (6), M is the hadron mass and m_q the constituent (dynamical) quark mass. Its momentum dependence was obtained in [15] by simply relating the quark mass function to the pion vertex function which must reduce to each other in the chiral limit of vanishing pion mass ($M_\pi = 0$), by virtue of the Ward-Takahashi identity for the axial-vector vertex function [16]. Therefore by specializing Eqs. (5)–(7) to the pion case in the limit $M_\pi = 0$, one immediately obtains the formula [15]

$$m(\hat{p}) = m_q^{-2}(m_q^2 + \hat{p}^2)^{3/2}\exp(-\hat{p}^2/2\beta^2), \tag{8}$$

where the quantity β^2 (=0.031) for the pion case [20] is still governed by the same BS dynamics [15,20], but now (because of the Goldstone nature of the pion in the chiral limit) the normalization has had to be fixed anew by identifying the "constituent" mass m_q with this function at its zero-momentum limit [$m(0) = m_q$]. In terms of $m(\hat{p})$ the nonperturbative quark propagator $S_F(p)$ is now given by

$$[S_F(p)]^{-1} = i[m(\hat{p}) + i\gamma \cdot p], \tag{9}$$

where the Landau gauge is understood [$A(p^2) = 1$ [17,15]] and $m(\hat{p})$ is given by (8). This nonperturbative mass function was employed in [15] for evaluating the quark condensate $\langle q\bar{q}\rangle$ as an explicit quadrature:

$$\langle q\bar{q}\rangle = \frac{6}{\pi^2}\int d^3\hat{p}\frac{m(\hat{p})}{\sqrt{\hat{p}^2 + m(\hat{p})^2}},$$

giving a value in the QCD-SR range [28]; the meaning of $\hat{p}$ was also clarified therein.

An important property of the structure (5) for the quantity $D(\hat{q})$ in the hadron-quark vertex function is that it prevents, through a general cancellation mechanism [19,22], the occurrence of overlapping pole effects due to integration over the timelike component of the loop momentum in any quark loop integral, and thus automatically preempts the possibility of any "free" propagation of quarks that might otherwise occur. It thus may be regarded as a simple 3D alternative to the construction of quark propagators as entire functions through more elaborate models [18,9], but with the added benefit of a parameter-free description (cf. [9]). This structure will play a key role in simplifying the loop integral for the meson self-energy operator from its 4D scalar form, Eq. (11), to the 3D form, Eq. (13), as given below.

IV. THE ρ-ω AMPLITUDE

After collecting these essential ingredients of the 3D BSE formalism, we now turn to the central quantity, viz., the two-loop contribution to the meson self-energy operator $\Pi_{\mu\nu}(P^2)$ [7,9,11] which is expressible as

$$\rho_\mu\Pi_{\mu\nu}\omega_\nu = i(2\pi)^4\int d^4q[\Gamma_h(\hat{q})]^2 \times\mathrm{Tr}[i\gamma\cdot\rho S_F(\tfrac{1}{2}P + q)i\gamma\cdot\omega S_F(\tfrac{1}{2}P - q)], \tag{10}$$

where $\Gamma_h(\hat{q})$ is the scalar part of the vertex function defined by Eq. (5), and the ρ, ω symbols on both sides of (10) stand for their respective polarization vectors. At this stage the scalar vertex function is common to ρ and ω, since their mass difference due to the u-d effect will be automatically taken care of via standard differentiation with respect to m_q^2, cf. [7] (see below). Simplifying the trace in Eq. (10) and checking on current conservation (which is routinely satisfied) we can write $\Pi_{\mu\nu}(P^2)$ as $(\delta_{\mu\nu} - P_\mu P_\nu P^{-2})\Pi(P^2)$ where, following any one of [19,22],

$$\Pi(P^2) = 2i(2\pi)^{-1}N_V^2 \times\int d^4qD^2(\hat{q})\phi^2(\hat{q})\frac{\Delta_1 + \Delta_2 - P^2 - \frac{4}{3}\hat{q}^2}{\Delta_1\Delta_2}, \tag{11}$$

where [17,20,25]

$$\Delta_{1,2} = m_q^2 + (P/2 \pm q)^2 \quad (P^2 = -M^2). \tag{12}$$

The integration over the longitudinal (timelike) component of q_μ, viz., $M\,d\sigma$ (σ equals qP/P^2), is carried out again as in [19,22] wherein the structure (6) of the D function ensures an exact cancellation of the effects of overlapping singularities arising from the σ-pole residues. The resultant 3D integration over $d^3\hat{q}$ is expressible as

$$\Pi(P^2) = -2N_V^2\int d^3\hat{q}\,\phi^2(\hat{q})[D^2(\hat{q})/\omega - D(\hat{q}) \times(P^2 + 4q^2/3)]. \tag{13}$$

Equation (13) brings out explicitly, without further ado, the *linear* structure of the mass operator in the off-shell variable P^2. The BS normalizer N_V^2 in Eq. (13) is itself an integral of the same kind as $\Pi(P^2)$, and is formally defined for any V meson through the equation [25,19]

$$2iP_\mu N_V^{-2} = (2\pi)^{-1}\int d^4q[\Gamma_h(\hat{q})]^2\mathrm{Tr}\{i\gamma\cdot VS_F(\tfrac{1}{2}P + q)i\gamma_\mu \times S_F(\tfrac{1}{2}P + q)i\gamma\cdot VS_F(\tfrac{1}{2}P - q)\}, \tag{14}$$

whose integral over the timelike component of q_μ can be carried out exactly as above to give a formula analogous to Eq. (13):

$$N_V^{-2} = 2\int d^3\hat{q}\,\phi^2(\hat{q})4\omega[\omega^2 - q^2/3] = 0.0502\ \mathrm{GeV}^{-6}. \tag{15}$$

The quantities ω and D in both Eqs. (13) and (15) are defined as in Eq. (6), which in turn carries the explicit information on the m_q^2 dependence of both these quantities. This fact facilitates a simple differentiation with respect to m_q^2 under the integral signs in Eqs. (13) and (15) in order to evaluate $\delta\Pi(P^2)$ which precisely represents, with no further normalization, the desired quantity $\theta(P^2)$ defined in Eq. (2), while the values of its two crucial parameters as predicted by this model are already listed in Eq. (3). In obtaining the latter we have used the equality of $\delta(m_q^2)$ with $2m_q\delta(m_q)$, and employed the "spectroscopic" value 265 MeV for m_q, the constituent mass [20,21]. For the evaluation of the integrals (13) and (15) we have not explicitly considered the momentum variation (8) of the dynamical mass, but left it at its "constituent" value m_q corresponding to zero momentum. This has been done mainly for simplicity and transparency in carrying out the differentiation process. Although not strictly valid, the scope of error on this account is likely to be small for two reasons: (i) the main burden of momentum variations in the two integrals (13) and (15) is carried by the meson-quark vertex function whose effect has been fully incorporated via Eqs. (5)–(7); (ii) the mass function, Eq. (8), maintains a sort of plateau (250–300 MeV) in the region of integration which provides the bulk contributions to the integrals. Our estimate of error, based on some trial runs with the momentum-dependent mass function, is about 10%. On the other hand, the explicit analytic structure in m_q^2 of the integrals (13) and (15) greatly minimizes the possibility of further numerical errors that would be inherent in the differentiation process in the absence of a (nonperturbative) analytical form which is usually more difficult to ensure than, e.g., in a point vertex structure [7] without additional parametric assumptions on the way, e.g., [9].

Before comparing our results with others we wish to record for completeness the predictions of this model on the photon-mediated chain of ρ-γ-ω mixing amplitude which we denote by $\theta_\gamma(P^2)$ in the same relative normalization as Eq. (2). Here we need no longer distinguish between m_u and m_d and take a simple proportionality of the ρ-γ and ω-γ amplitudes to a common dynamical quantity $g_V(P^2)$ defined by

$$g_V(P^2)V_\mu = -i\int d^4q\,\Gamma_h(\hat{q})\mathrm{Tr}[i\gamma\cdot V S_F(q+P/2)i\gamma_\mu \times S_F(-q+P/2)]/\sqrt{2}, \tag{16}$$

the multiplicity factors being e and $e/3$, respectively, and V_μ standing collectively for ρ or ω. The other symbols are as defined in Eq. (10) and earlier. The evaluation of $g_V(P^2)$ is on lines similar to Eq. (10), but actually simpler and leads to the explicit formula

$$g_V(P^2) = 4\sqrt{\tfrac{3}{2}}\beta^3 N_V[2m_q^2 + 4\beta^2 - (P^2+M^2)/2]. \tag{17}$$

Writing it in a form analogous to Eq. (2), we have

$$g_V(P^2) = f_V(M^2)[1-\mu(1+P^2/M^2)], \tag{18}$$

where the on-shell value $f_V(M^2)$ and the off-shell coefficient μ are

$$f_V(M^2) = g_V(M^2) = 0.1608\ \mathrm{GeV}^2,$$
$$\mu = M^2/4(m_q^2+2\beta^2) = 0.7197. \tag{19}$$

The final result for the complete photon-mediated ρ-ω amplitude is

$$\theta_\gamma(P^2) = \frac{e^2}{3}g_V(P^2)\frac{1}{-P^2}g_V(P^2), \tag{20}$$

where we have explicitly shown the photon propagator in the middle to bring out the "extended" nature of the off-shell extrapolation due to the photon-mediated mixing compared to that due to the u-d effect, despite the smallness of (20) compared to (2). Unlike (2) there is no uncertainty in (20) within this model, though the on-shell value ($P^2=-M^2$) is a bit too high (see the discussion below):

$$\theta_\gamma(M^2) = +1316\ \mathrm{MeV}^2. \tag{21}$$

The off-shell effect, on the other hand, is best expressed through the corresponding N-N potentials [11] which are given by

$$V(\rho\text{-}\omega) = -[\theta(M^2)/2M][1-(2\lambda/Mr)]\exp(-Mr), \tag{22}$$

$$V(\rho\text{-}\gamma\text{-}\omega) = [\theta_\gamma(M^2)^2/M^4]\{(1-\mu)^2/r + [(2\mu-1)/r] \times\exp(-Mr) - (M/2)\exp(-Mr)\}, \tag{23}$$

respectively, where a common factor $g(\rho N)g(\omega N)$ [11] has been suppressed from the last equations. Equation (22) has no counterpart of the $1/r$ term in (23).

V. RESULTS AND DISCUSSION

To put the results of this investigation in perspective with those in [7–11] we should first note that in this spectroscopy-rooted approach there is little scope for any significant variation of the input parameters (ω_0, C_0, and m_q) whose respective values (158 MeV, 0.27, and 265 MeV) can be traced all the way back to the BS kernel itself [20], without effecting a simultaneous change in the (already good) fits [20] to the observed meson spectra [24], and in the more recent (equally good) fits [21] to the baryon spectra [24] with these very parameters. It is with this constraint that the numbers obtained above may be viewed *vis-à-vis* those in [7–11], especially in respect of the off-shell parameter λ, Eq. (2), which can be compared with almost all of them. However, the on-shell value $\theta(M^2)$, Eq. (3), is quite specific in this model, and could at best be compared with the predictions of, say, chiral Lagrangian models [8], except that the available prediction [8] refers to π^0-η mixing and cannot be used for a direct comparison.

The only uncertainty in our on-shell value, Eq. (3), arises from a corresponding uncertainty in the value of $\delta(m_q)$ for which a natural substitute, following Politzer [26], would be $(m_d - m_u)$. The latter quantity has been discussed in great detail in [11] to which we refer the interested reader, but for a definitive estimate it should be reasonable to take a value of, say, 4 MeV [9], which is well within the limits of the [11] analysis. With this value we get $\theta(M^2)$ equal to $-5156\ \mathrm{MeV}^2$, which should be compared with the value ($-4520 + 600$) obtained from $e^+e^- \to \pi^+\pi^-$ data [5] after taking account of the $\rho\to\gamma\to\omega$ chain which gives a smaller contribution of opposite sign, viz., $+1316$, Eq. (19). Its inclusion gives the net value -3840 which is still within the experimental range [5] (taking account of the uncertainties of the u-d mass difference).

The somewhat larger value of $\theta_\gamma(M^2)$ compared to the VMD value [27] 610 MeV^2 quoted in [11] may in turn be related to the quantity $g_V(M^2)$, Eq. (19), which gives 0.1608 GeV^2. This number, when divided by $M_\rho = 0.775$ GeV, precisely translates, in the OCD-SR [28] notation, to the result $f_\rho = 215$ MeV, to be compared to the quoted value of 200 MeV [28] needed for agreement with the $\rho \to e^+e^-$ width. This is the extent of our overestimate of $\theta(M^2)$ compared to the VMD value [27,11], but nevertheless tolerable enough to warrant a discussion (below) of the off-shell aspects of ρ-γ-ω mixing along with those of the main (u-d) term.

The off-shell prediction is dominated by the parameter λ, Eq. (2), at the value 0.99, Eq. (3), and its photonic counterpart μ defined in Eq. (19) at the value 0.720. Our value of λ is rather below the QCD-SR range (1.43–1.85) [11], implying a "softer" off-shell effect in this quark loop model than the "harder" effect in the QCD-SR approach, as already noted in [11] for QCD-SR versus quark loop methods: A smaller value of λ would tend to postpone the onset of attenuation of the ρ-ω mixing potential due to the off-shell effects to somewhat shorter distances, as measured by the "critical distance" [11] $r = 2\lambda_M$, which is also seen from Eq. (22). In a similar way, the off-shell effect of the photon-mediated ρ-ω mixing, as measured by the parameter $\mu = 0.720$, Eq. (19), produces the potential Eq. (23), but its $(1/r)$ term has no counterpart in Eq. (22). Taking note of the opposite signs of the two effects, the following scenario emerges. The two short-range terms of (22) get duly reduced by the two corresponding terms of (23) by about 20–25 %. However, the long-range $(1/r)$ term of (23), which has no counterpart in (22), reinforces the $\exp(-Mr)$ term of the latter, again by about 20–25% near the critical distance [9,11], but continues with increasing strength down to shorter distances and therefore further postpones the attenuation by another (small) amount. For brevity we omit further discussion [11].

Finally, we wish to comment on the magnitude of our λ value, 0.99±0.01, *vis-à-vis* other determinations [7–11]. We have checked on the possible variation in this quantity due to the (neglected) effect of the momentum dependence of the dynamical mass Eq. (8), and found the effect to be $\leq 10\%$. There is little scope for further variation in this otherwise "rigid" description, unless a totally different set of input parameters (C_0, ω_0, m_q) from the ones [20,21] employed here produces an equally good fit [20,21] to the observed spectra [24], which is rather unlikely. Nevertheless, this value seems to lie about midway between other quark loop calculations [7–9] and QCD-SR results [11], though somewhat nearer to the former than to the latter; rather surprisingly, it is quite close to the nucleon loop value [10] of about unity [11]. It is also in fair agreement with the corresponding results [8] for π^0-η mixing obtained from chiral Lagrangian models [29], though a similar result for the analogous case of ρ-ω mixing by the same method [29] is not yet available. Of course a *nonlinear* dependence of $\theta(P^2)$ on P^2, such as attempted in [9], may well change this (linear) scenario, but this requires more effort.

To summarize, we have outlined an explicit calculation of the ρ-ω mixing amplitude, both on and off shell, in the form expressed by Eqs. (2) and (3), using a 3D BSE-cum-SDE approach which is attuned to hadron spectroscopy of both varieties simultaneously [20,21]. The on-shell value agrees with experiment [5], while the off-shell parameter λ is rather close to unity, signifying a change of sign for $\theta(q^2)$ in just the transition region between spacelike and timelike momenta.

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APPENDIX

The self-energy $\Pi(P^2)$, Eq. (13), for ρ or ω, can be straightforwardly adapted to give the ρ^0-ω transition self-energy denoted by $\delta\Pi(P^2)$ which is directly identifiable with the desired quantity $\theta(P^2)$, Eq. (12), off the mass shell. The quantity $\delta\Pi(P^2)$ which, as the notation implies, is obtained by differentiation with respect to m_q^2 under the sign of integration in Eq. (13), arises as follows. Since the isospin functions of ρ^0 and ω are $|u\bar{u} \mp d\bar{d}\rangle/\sqrt{2}$, respectively, and since they are each normalized to unity, the self-energy formula via the respective quark loops is formally given by Eq. (13) for either meson as a 2×2 matrix, to take care of the mass difference between the u and d quarks which affects the quantity m_q^2 appearing inside the integral. However, the ρ^0-ω transition mass, which works out as an overlap of $\langle u\bar{u} - d\bar{d}|$ with $|u\bar{u} + d\bar{d}\rangle$, will be equal to the *difference* between the corresponding expressions $\Pi(P^2)$ with m_q^2 corresponding to the u and d quarks, respectively, and will "vanish" unless this difference is properly taken into account. This last is most simply achieved through differentiation with respect to m_q^2, cf. Ref. [7].

In the present case, the functional dependence on m_q^2 is carried primarily by the quantities N_V given by Eq. (15) and $D(\hat{q}), \omega$ given by Eq. (6). This is a full-fledged nonperturbative counterpart of the corresponding perturbative calculation in Ref. [7]. Additional m_q^2 dependence comes about in principle from the structure of the inverse range parameters β^2 because of their *dynamical* dependence on the input parameters (ω_0, C_0) of the BS kernel [15,20,25] so as to relate to the spectroscopy [20,21]. This is unlike the free parametrization of such quantities (in Gaussian form) usually employed in other phenomenological descriptions of the $q\bar{q}$ wave functions [30] in which no such dynamic significance was sought for them. However, in this paper we shall not press this detailed aspect of the dynamics any further at this stage, and rest content with taking the average value of β^2 at 0.069 GeV^2 [20] for ρ, ω, so that the m_q^2 dependence comes about only from the explicit structures (15) and (6) for N_V and (D, ω), respectively.

To compute $\delta\Pi(P^2)$, it is convenient to rewrite Eq. (13) as

$$\Pi(P^2) = I(m_q, P^2)/J(m_q^2), \tag{A1}$$

where $J = N_V^{-2}$, and I stands for the integral on the right-hand side (RHS). The linear dependence of I on

P^2, and hence also of $\delta\Pi(P^2)$ after differentiation, comes about from routine simplifications leading from Eq. (10) to Eq. (13), using methods already explained in Refs. [19,22,25]. Note that $J(m_q^2)$ does not depend on P^2 since the normalization is calculated on shell ($P^2 = -M^2$). Then

$$\delta\Pi(P^2) = \delta m_q^2[J\partial I/\partial m_q^2 - I\partial J/\partial m_q^2]/J^2, \qquad \text{(A2)}$$

which shows the explicit proportionality to δm_q^2 as well as the linear dependence of the RHS on P^2. Since $\delta\Pi(P^2)$ is exactly equal to $\theta(P^2)$, its identification with Eq. (2) leads to the numerical results given in Eq. (3) of the text, on the contribution of the strong CSB effect arising from the u-d mass difference.

The off-shell P^2 dependence of the γ-mediated ρ^0-ω mixing effect is already given in Eqs. (16)–(21) of the text. In particular, Eq. (20) shows that the P^2 dependence of this contribution is quite different from linear. The resultant effect is discussed in Sec. V of the text.

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