Geometrical description for the negative correlation between forward and transverse energies in ultrarelativistic heavy-ion collisions

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Recent experimental data on the negative correlation between forward energy and transverse energy, which have been reported for the interactions of 14.6A GeV ^{28}Si with Al, Cu, Au and Pb, 60A and 200A GeV ^{16}O and ^{32}S with S, Cu, Ag and Au, are investigated by a model based on the description of geometrical properties of nucleus-nucleus collision processes. A parameter, ellipticity e, which might describe the conversion of initial energy into that of radiated final-state hadrons is proposed.

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I. INTRODUCTION

As well known, the advent of ultrarelativistic heavy-ion collisions in the laboratories [1], up till now at the Brookhaven Alternating Gradient Synchrotron (BNL AGS) and the CERN Super Proton Synchrotron (SPS), and in future at the BNL Relativistic Heavy Ion Collider [2] and the CERN Large Hadron Collider [3], offers the best chance to create high temperature and/or high density environment and gives high hopes to explore nuclear matter in the entirely uncharted regions. At the extreme environment a phase transition from the normal hadronic state toward the quark-gluon plasma (QGP) has been predicted to occur by the statistical QCD [4].

To bear the brunt of an attack an initial concern should be the question whether the collision between two ultrarelativistic nuclei would actually transfer enough energy to make the system excited. This is based on the knowledge of how much energy bombarding nuclei could deposit, which is determined by the nuclear stopping power and measured usually by the so-called transverse energy resulting from degradation of the initial longitudinal energies of the participating nuclei during interpenetration of the projectile and target nuclei. The transverse energy can be expressed as

$$E_T = \sum_i E_i \sin\theta_i \,,$$

where E_i and θ_i are the observed energies and scattering angles, respectively.

The theoretical idea of nuclear stopping power (or nuclear transparency another way round) in collisions was first introduced by Bethe [5] in 1940 and the work of emerging the experimental data were especially pioneered by Busza and Goldhaber [6] 40 years later.

Recently a common approach for experimentally determining the nuclear stopping power have been adopted by several experiments [7–11]. The measurements of global event quantities such as the transverse energy E_T, the forward energy E_{ZD}, and their correlation have proved to be valuable tools for obtaining an understanding of the reaction mechanism. As pointed out by Tannenbaum [12], at first glance their correlation appears to be a trivial consequence of energy conservation: the energy not observed in the forward direction will be observed as transverse energy. Early measurements clearly demonstrated the negative correlation between E_T and E_{ZD} by the WA80 experiment for ^{16}O+Au interactions [11] and confirmed the intuitive notion that the "central collisions" exhibit small forward energies and large transverse energies. It is believed that the detailed shape of the correlation curves, as a function of nucleon numbers of projectile and target and incident energies, can provide stringent constraints on the reaction dynamics of heavy-ion collisions at ultrarelativistic energies.

In general, the motivation of such measurements for nucleus-nucleus collisions at high energies is based on two assumptions. First, the forward energy is a measure of the number of projectile spectator nucleons which might be related to the impact parameter b. Second, the efficiency for conversion of energy originally carried by the projectile participant nucleons into energy, which suffices to the formation of produced particles and is determined by the reaction dynamics, can be inferred from the transverse energy production. The correlation between forward energy and transverse energy would help us to understand the nuclear stopping power. There have existed several models, e.g., the wounded nucleon model (WNM) [13], the wounded projectile nucleon model (WPNM) [14], the FRITIOF model [15], etc., some of which are successful in the analysis of transverse energy spectra and one can get more detailed information from the excellent review by Sorensen [16].

In this paper we propose a geometrical description of heavy-ion collisions to explain the recent experimental data on the correlation between forward and transverse energies. The experimental data used in our analysis are those for the interactions of 60A and 200A GeV ^{16}O and ^{32}S with targets of S, Cu, Ag, and Au reported by the NA35 experiment [8], the interactions of 14.6A GeV ^{28}Si with Al and Au by the E802 experiment [9], and the interactions of 14.6A GeV ^{28}Si with Al, Cu, and Pb by the E814 experiment [10].

We will take the connection of energy distribution between longitudinal and transverse directions into account by using a rather simple picture of ellipsoidal decay. In Sec. II we recall the basic participant-spectator picture of geometrical model on nucleus-nucleus collisions. In Sec. III we describe the details of a theoretical assumption for transverse energy created by the excited system ellipsoidally decaying. In Sec. IV we make the comparison of our results with the experimental data. The conclusions appear in Sec. V.

II. PARTICIPANT-SPECTATOR PICTURE

The model we considered contains three distinct assumptions some of which are rather different from those usually contained in other fireball models.

(i) The numbers of participants from projectile and target nuclei can be estimated simply from nuclear geometry before collisions.

(ii) The reference frame is chosen to be the center-of-mass frame of participants (here denoting the c.m.p. frame), which is different for different impact parameters, and the available kinetic energy is calculated in that frame. The c.m.p. frame is different from the nucleon-nucleon center-of-mass frame and the geometrical c.m. frame which is defined as the projectile (small) plus the region of target (large) grilled by the projectile at zero impact parameter.

(iii) The excited energy of the producing-particle system in the c.m.p. frame is estimated by deducting the spectators of projectile and target.

Denote B and A as the numbers of nucleons inside two colliding nuclei, respectively (always let $B \leq A$), and for convenience the incident energy per nucleon $E_{\rm lab}$ and the mass of nucleon $m_N = 1$ are in units of GeV. In the participant-spectator picture of nucleus-nucleus collisions, we denote the number of nucleons inside their participants as $B_p(b)$ and $A_p(b)$, respectively, both of which are functions of the impact parameter b,

$$B_p(b) = f_B(b)B\,, \quad A_p(b) = f_A(b)A\,, \tag{1}$$

where the fractional functions $f_B(b)$ and $f_A(b)$ can be derived [17] in the following three regions: (i) region $0 < b < R_A - R_B$,

$$f_B(b) = 1\,,$$
$$f_A(b) = \left[1 - (1-\alpha^2)^{3/2}\right]\left[1 - (1+\alpha)^2(1-\xi)^2\right]^{1/2}\,; \tag{2}$$

(ii) region $R_A - R_B < b < R_A + R_B$,

$$f_B(b) = \xi^2\left(\frac{1+\alpha}{\alpha}\right)^2\left[\frac{3\alpha}{4(1+\alpha)^{3/2}} - \frac{\xi}{8}\left(\frac{3}{(1+\alpha)^{1/2}} - 1\right)\right],$$
$$f_A(b) = \xi^2\left(\frac{1+\alpha}{\alpha}\right)^2\left\{\frac{3\alpha^{7/2}}{4(1+\alpha)^{3/2}} - \frac{\xi}{8}\left[\frac{3\alpha^{5/2}}{(1+\alpha)^{1/2}} - [1-(1-\alpha^2)^{3/2}]\,[1-(1-\alpha)^2]^{1/2}\right]\right\}; \tag{3}$$

(iii) region $b > R_A + R_B$,

$$f_B(b) = f_A(b) = 0\,. \tag{4}$$

In above equations, $\xi = 1 - b/(R_B + R_A)$ and $\alpha = R_B/R_A$.

In terms of the above expressions by which the nucleon numbers of participants from projectile and target are given, we can estimate the excited energy of producing-particle system and the forward energy. The total excited energy in the c.m.p. frame for an event with impact parameter b can be obtained simply by

$$E^*(b) = \sqrt{B_p^2(b) + A_p^2(b) + 2B_p(b)A_p(b)E_{\rm lab}} - B_p(b) - A_p(b)\,. \tag{5}$$

The forward energy is estimated by only considering the contribution from the projectile spectator, that is,

$$E_{ZD}(b) = \begin{cases} [B - B_p(b)]E_{\rm lab}, & \text{if projectile} < \text{target}\,, \\ [A - A_p(b)]E_{\rm lab}, & \text{if projectile} > \text{target}. \end{cases} \tag{6}$$

III. ELLIPSOIDAL DECAY

In this model we assume tersely that the transverse energy is proportional to the total excited energy at every fixed b

$$E_\perp = \lambda_\perp E^*(b).$$

The ratio of the transverse part to the total excited energy, which measures the efficiency of the deposition of initial energy, is given by the following equation:

$$\lambda_\perp = \frac{\int_\Omega \varepsilon \sin\Theta \, dxdydz}{\int_\Omega \varepsilon \, dxdydz}\,, \tag{7}$$

where ε is the average energy density and $\sin\Theta = \sqrt{x^2+y^2}/\sqrt{x^2+y^2+z^2}$.

In the calculation of $\lambda_\perp$ in some models, the space is usually regarded to be isotropical and one obtains $\lambda_\perp = \pi/4$. This implies that all participants have lost historical memories after collisions. In fact, due to the transparency of the nucleus at high energies all participants will not lose historical vestiges and some of the produced hadrons will carry their parent's memories of motion. This picture will lead to the unequivalence in longitudinal and transverse directions. So we assume that the excited system decays ellipsoidally. In terms of the ellipsoidal coordinates

$$x = \mu\varrho \sin\vartheta \cos\varphi,$$
$$y = \mu\varrho \sin\vartheta \sin\varphi,$$
$$z = \nu\varrho \cos\vartheta,$$

where μ and ν are elliptical major and minor axes, respectively, and the integral range $\Omega : 0 < \varrho < 1, 0 < \vartheta < \pi, 0 < \varphi < 2\pi$. The ratio defined by Eq. (7) can be derived as

$$\lambda_\perp(e) = \frac{1}{2}\int_0^\pi \frac{\sin^2\vartheta}{\sqrt{\sin^2\vartheta + e^{-2}\cos^2\vartheta}} d\vartheta. \quad (8)$$

Here we have defined an ellipticity

$$e = \mu/\nu\,. \quad (9)$$

It should be mentioned that the different ellipticity parameter e seems roughly to be just for different parametrization of the nuclear stopping power when $e < 1$ and it is indeed related to the ratio between the transverse energy and the total excited energy by Eq. (8).

Let us consider a collision process. Although participants experience complex collisions some participants may still be able to move in the original directions with a small change of momentum. The produced hadrons will, therefore, carry their parent's kinematical information. This persistence make the longitudinal direction more populated than the transverse direction. In this case we have $e < 1$.

From the asymptotical properties of the complete elliptic integrals (see Appendix) we have

$$\lim_{e\to 0} \lambda_\perp(e) = 0 \quad (10)$$

corresponding to the case when participants interpenetrate almost freely with little transverse energies produced, and

$$\lim_{e\to 1} \lambda_\perp(e) = \frac{\pi}{4}\,. \quad (11)$$

It is obvious that the case of $e = 1$ accords with the result of the isotropical decay of the excited system, and this case was taken to calculate the maxium transverse energy by some models.

IV. COMPARISON WITH EXPERIMENTS AND SOME PREDICTIONS

To make comparison with experiments we analyze the available data reported by the NA35, E802, and E814

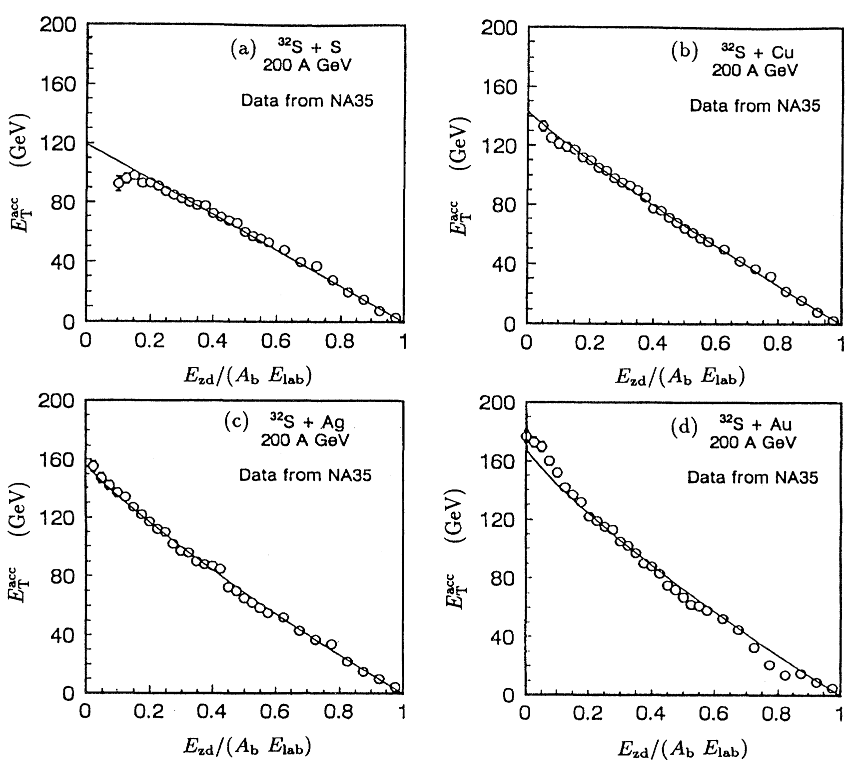

FIG. 1. Correlation between transverse energy E_T^{acc} and forward energy E_{ZD}. The horizontal axis is scaled by the product of projectile nucleon number A_b with their incident energy E_{lab}. The vertical axis is still adopted as the acceptance data measured by a given detector for comparison with experiment. Results of our model calaulation are given as solid curves. Data are taken from the NA35 experiment [8]. In (a), (b), (c), and (d) the mean E_T^{acc} are plotted as a function of E_{ZD} for sulfur-induced interactions at 200A GeV with S, Cu, Ag, and Au, respectively.

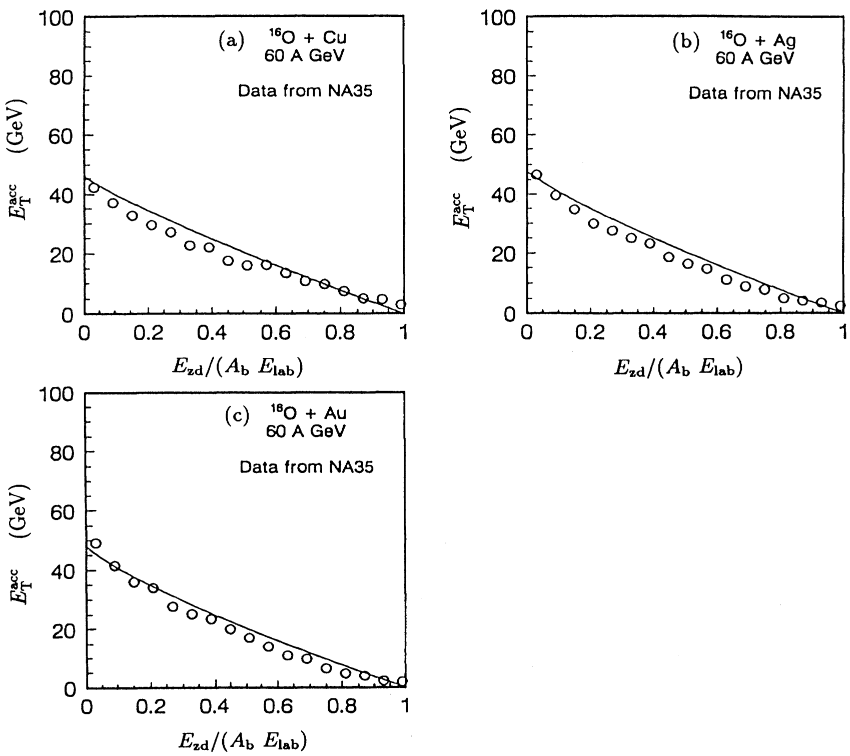

FIG. 2. Same as Fig. 1 with data taken from the NA35 experiment [8] for oxygen-induced interactions at 60A GeV with Cu, Ag, and Au in (a), (b), and (c), respectively.

Collaborations. In the following analyses we have paid attention to the case that the values of experimentally measuring data, $E_T^{\rm acc}$, taken from some given detectors of the above Collaborations do not correspond to the total transverse energy emitted. To extrapolate data for central collisions (b=0) to 4π acceptance, we employ acceptance correction factors to get total transverse energy $E_T^{4\pi}$. The values of $E_T^{4\pi}(b=0)$ for the NA35 data are taken from Table 8 in Ref. [8] and those for the E802 and E814 data from Refs. [9, 10].

The NA35 Collaboration have given correlation between $E_T^{\rm acc}$ and E_{ZD} from interactions induced by 200A GeV sulfur with S, Cu, Ag and Au targets and from those induced by 200A GeV oxygen with Au and by 60A GeV oxygen with Cu, Ag, and Au at the CERN SPS. Results from our model calculation comparing with the above data are shown in Fig. 1, Fig. 2, and Fig. 3, respectively. It shows good agreement. One can see from Fig. 1(a) that $E_T^{\rm acc}$ depends almost linearly on $E_{ZD}/A_bE_{\rm lab}$. (A_b is the nucleon number of the beam nucleus.) In our model, the center of mass of participants coincides with the center of mass of the nuclei for symmetric collision, i.e., the velocity of the fireball (along the collision axis) is the same for all impact parameters and an approximately linear dependence of $E_T^{\rm acc}$ on $E_{ZD}/A_bE_{\rm lab}$ can be obtained. For an asymmetric collision, on the other hand, the velocity of the fireball depends on the impact parameter, and one expects obvious deviations from the linear dependence as shown in other figures of Figs. 1 and 2. In Fig. 3 we compare data between different projectile nuclei with the same target at the same energy and those between different energies with the same colliding system.

Results from the E802 experiment at the BNL AGS is also illustrated in Fig. 4, with measurements from 14.6A GeV ^{28}Si interactions with Al and Au targets. The data in Fig. 4 do not simply follow the nuclear geometry

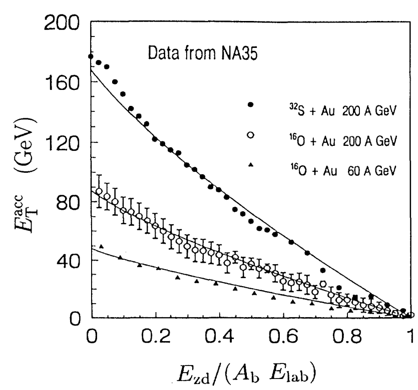

FIG. 3. Same as Fig. 1 with data taken from the NA35 experiment [8] for comparison between different projectile nuclei ^{32}S and ^{16}O with Au at 200A GeV, and that between different energies 200A and 60A GeV induced interactions of ^{16}O with Au.

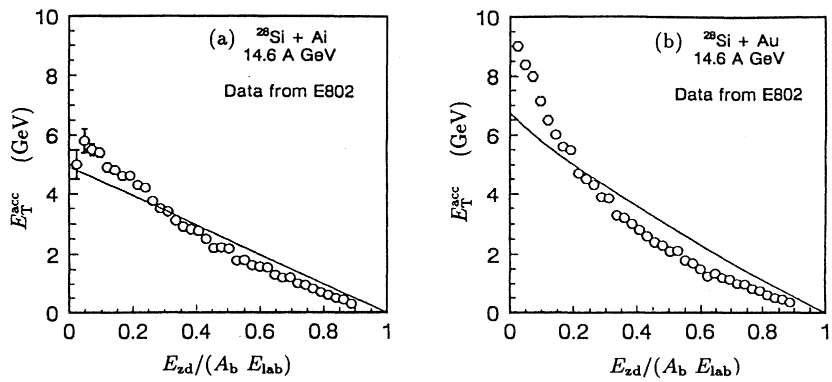

FIG. 4. Same as Fig. 1 with data taken from the E802 experiment [9] for silicon-induced interactions at 14.6A GeV with Al and Au in (a) and (b), respectively.

calculation by our model. The observed small E_T^{acc} parts seem to be the same for the Al and the Au targets, but a clear target dependence is there for "central collisions." In Fig. 5 we give the comparison of the calculation with the E814 data, which are 14.6A GeV ^{28}Si interactions with Al, Cu, and Pb targets; similar situation like the E802 data is shown.

The values of parameter e for fitting the three experiments (NA35, E802, E814) are plotted in Fig. 6. It is shown from the previous figures that our model can give better agreements at higher energies. The higher the incident energy is, the more powerful the geometrical description will be. From Fig. 6 it is shown that there exists an obvious dependence of the ellipticity parameter e on the nucleon numbers of projectile and target nuclei and a rather weak incident energy dependence. The values of ellipticity will be increased with increasing size of colliding nuclei, but with decreasing incident energy. The shape of negative correlation curves will deviate the line-like dependence if the difference between sizes of projectile and target nuclei is enlarged; see Fig. 7. The dependence of the ellipticity on the impact parameter has not been taken into account in this paper, we shall remark on it in the following section.

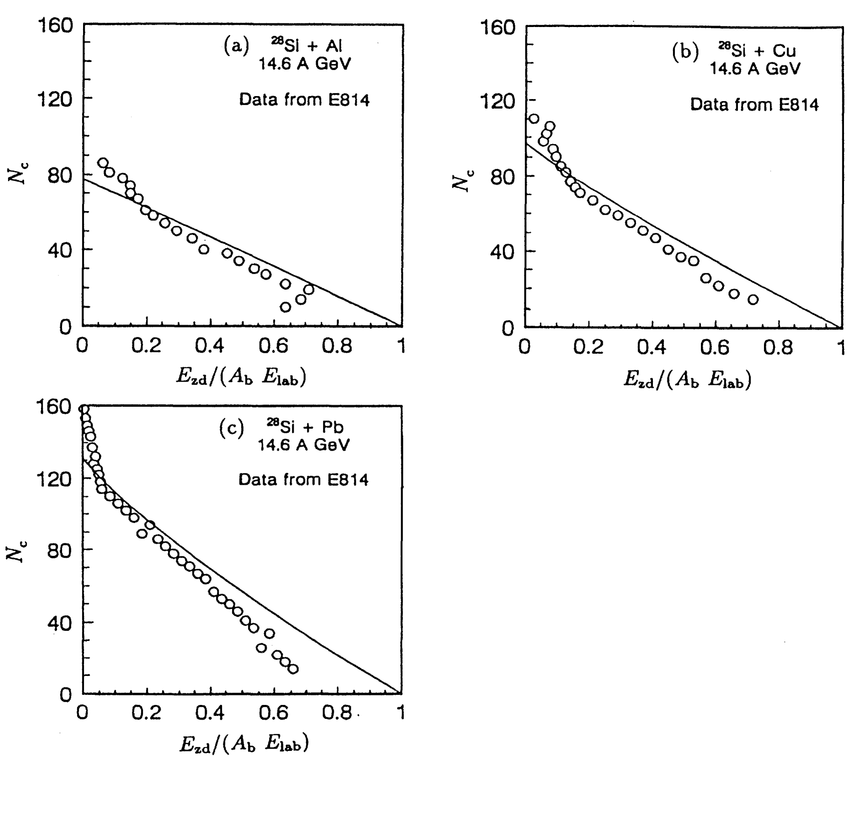

FIG. 5. Correlation between scaled forward energy E_{ZD}/A_bE_{lab} and the charged particle multiplicity N_c. Data are taken from the E814 experiment [10] for silicon induced interactions at 14.6A GeV with Al, Cu, and Pb in (a), (b), and (c), respectively.

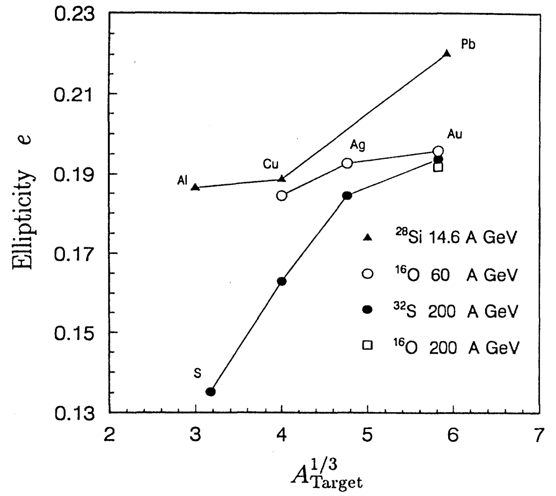

FIG. 6. The fitting parameter, ellipticity e is plotted as a function of the nucleon number of target nucleus, $A_{\rm T}^{1/3}$, for sulfur-induced interactions at 200A GeV with S, Cu, Ag, and Au, oxygen-induced interactions at 200A GeV with Au, oxygen-induced interactions at 60A GeV with Cu, Ag, and Au and for silicon-induced interactions at 14.6A GeV with Al, Cu and Pb.

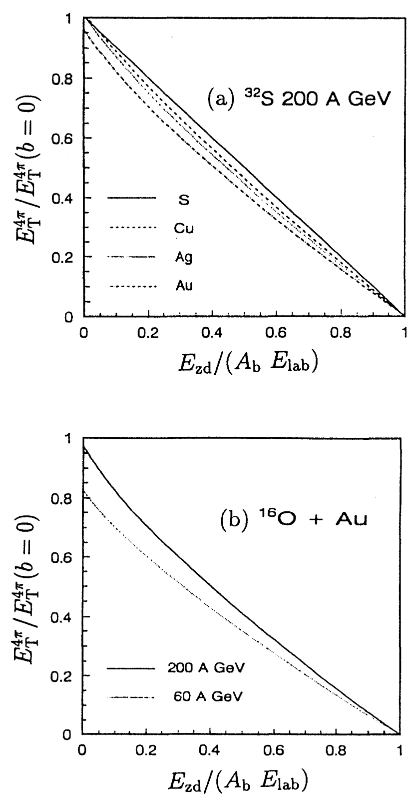

FIG. 7. (a) Four curves are correlations between the scaled transverse energy and the scaled forward energy for 200A GeV ^{32}S-induced interactions with S, Cu, Ag, and Au. (b) Two curves for ^{16}O+Au interactions at 200A and 60A GeV.

V. CONCLUSIONS

Here we consider the present experiments and give some conclusions about their analyses by our model. (i) The ellipticity e is higher at the BNL AGS than that at the CERN SPS, which as mentioned that the stopping power is larger at the AGS than at the SPS, since e is simply a measure of the nuclear stopping power. (ii) Nuclear stopping power should increase with the size of projectile and target nuclei. In data fitting of previous sections it is obvious that ellipticity e depends strongly on the size of target nucleus. At the moment we only have few data about dependence of projectile size. (iii) The negative correlation between transverse and longitudinal energies shows line-like shape only for symmetic or near-symmetric colliding systems. Nonlinear negative correlation appears evidently in asymmetric systems.

In this paper we have assumed that the ellipticity is approximately independent of the impact parameter in Sec. III for explicity and simplicity in establishing our ellipsoidal decay model at the first step. It has been presented that the inclusive data on $p_\perp$ spectra, stangeness ratios, etc., show that a peripheral collision of heavy nuclei resembles very much a central collision of light nuclei, with the same number of participants. In fact, if the ellipticity (i.e., the stopping power) is higher for heavy nuclei than for light nuclei, one expects naturally that it is also higher for a central collision than for a peripheral collision. It might precisely be the origin of the discrepancy between our model calculation and the data, especially in Figs. 4 and 5 (but also in Fig. 3), which show that the stopping power is larger for central collisions than for peripheral collisions. So it might be reasonable to think that the ellipticity does depend on the impact parameter, as well as it depends on the nucleon numbers of projectile and target, but it should be a rather small variation. Using the model one could study how the stopping power depends on the impact parameter, and, independently, how it depends on the nucleon numbers of colliding nuclei, and compare both results. This analysis will be done elsewhere.

The model of mainly geometrical description for heavy-ion collisions should be suited to higher incident energies. The utility of the correlation between forward and transverse energies as a valuable probe for the study of the dominant "soft" multiparticle production processes in ultrarelativistic nuclear physics has been elaborated.

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APPENDIX

Taking $\kappa_1^2 = 1 - e^2$ (when $e < 1$) we get the following expression from Eq. (8):

$$\lambda_\perp(e) = \frac{e}{\kappa_1^2}[K(\kappa_1) - E(\kappa_1)], \qquad \text{(A1)}$$

where $K(\kappa)$ and $E(\kappa)$ are the first and second kinds of complete elliptic integrals [18]

$$K(\kappa) = \int_0^1 (1-t^2)^{-1/2}(1-\kappa^2 t^2)^{-1/2} dt$$

and

$$E(\kappa) = \int_0^1 (1-t^2)^{-1/2}(1-\kappa^2 t^2)^{1/2} dt,$$

respectively. A curve of $\lambda_\perp$ versus e is plotted in Fig. 8.

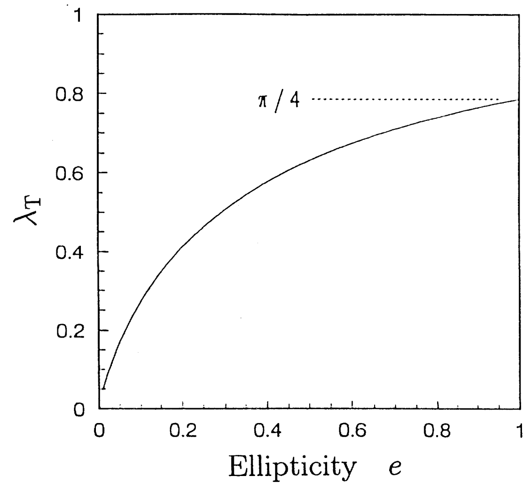

FIG. 8. The solid curve is the dependence of $\lambda_\perp$ versus e. The dashed line of $\pi/4$ corresponds to isotropic decay.

Taking $\kappa_2^2 = 1 - e^{-2}$ (when $e > 1$) we obtain

$$\lambda_\perp(e) = (1 - \kappa_2^{-2})K(\kappa_2) + \kappa_2^{-2}E(\kappa_2). \qquad \text{(A2)}$$

We can also have the asyptotical property

$$\lim_{e\to\infty} \lambda_\perp(e) = 1. \qquad \text{(A3)}$$

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