

## $\alpha$ - $d$ capture with formation of ${}^6\text{Li}$ and the isoscalar $E1$ multipole

G.G. Ryzhikh and R.A. Eramzhyan

*Institute for Nuclear Research of the Russian Academy of Sciences, Moscow 117312, Russia*

S. Shlomo

*Cyclotron Institute, Texas A&M University, College Station, Texas 77843*

(Received 21 September 1994)

We present results of a calculation of the low-energy fusion reaction  $\alpha + d \rightarrow {}^6\text{Li} + \gamma$ . In this calculation  ${}^6\text{Li}$  was treated as a three body  $\alpha$ - $2N$  system in the framework of the multicluster dynamic model with Pauli projection (MDMP). To construct the  ${}^6\text{Li}$  wave function with a high precision a large basis is used for the radial wave function of  $\alpha$ - $2N$  relative motion in the ground state of  ${}^6\text{Li}$ . We pay special attention to the asymptotic region of this radial function. We obtained good agreement with experimental data on both angular distribution and total cross section by taking into account the contribution of the isoscalar  $E1$  multipole.

PACS number(s): 21.45.+v, 21.60.Gx, 25.10.+s, 27.20.+n

### I. INTRODUCTION

Among many low-energy fusion reactions

$$A + B \rightarrow C + \gamma,$$

those reactions in which each participating nucleus has isospin  $T = 0$  are of particular interest. In such reactions the lowest multipoles ( $E1, M1$ ) are strongly suppressed and usually this transition proceeds via the multipoles like  $E2$  or higher. At very low energies the contribution of the suppressed multipoles strongly increases and this fact may be important, for example, for astrophysical processes.

In this paper we will consider the

$$\alpha + d \rightarrow {}^6\text{Li} + \gamma \quad (1)$$

reaction. It was already discussed in several papers [1-3]. From the experimental data it follows [4] that the dipole transition is important in this reaction at low energies. If indeed this is the case, the following question arises: What is the origin and the magnitude of this dipole transition? We point out that in previous works [1,2] the dipole transition in this reaction was associated with an isoscalar  $E1$  multipole. Since these early works, the theoretical description of the nuclear structure of the participants and the accuracy of the numerical calculation have been improved significantly. These improvements may lead to a better physical insight into the problem of the contribution of the dipole transition. That is why we are returning in this work to the discussion of this reaction. The improvements made in the present study are the following.

(i) A very large basis was used to construct the radial wave function of  $\alpha$ - $d$  motion in the ground state of  ${}^6\text{Li}$ .

(ii) Since the reaction cross section at very low energies is extremely sensitive to the asymptotic behavior of the  $\alpha$ - $d$  relative motion wave function in the ground state,

we make special effort to treat it accurately.

(iii) The population of the  $J^\pi T = 3^+0$  level of  ${}^6\text{Li}$  at  $E^* = 1.2$  MeV was included in the calculation.

The paper is organized as follows. In Sec. II we discuss the corrections to the isoscalar  $E1$  multipole operator which may contribute to the cross section, particularly at very low energies. In Sec. III we give details of the nuclear model used to construct the ground state and continuum state wave functions. Then in Sec. IV we present the results for the total capture cross section to the ground state and to the  $J^\pi T = 3^+0$  excited state, and in Sec. V the differential cross sections are given for four energies. In Sec. VI we present our summary and conclusions.

### II. MULTIPOLE TRANSITION OPERATORS IN THE LONG-WAVE APPROXIMATION

We will limit our consideration to energies not exceeding  $E = 10$  MeV. In this energy range one can employ the long wave approximation. In this approximation, the isoscalar  $E2$  multipole operator is given by

$$\hat{T}_{2m}(E2) = -\sqrt{\frac{\pi}{60}} e k_\gamma^2 \sum_{i=1}^6 r_i^2 Y_{2m}(\hat{r}_i). \quad (2)$$

Here  $r_i$  is the coordinate of nucleon  $i$  relative to the center of mass.

In the long-wave approximation the  $E1$  transition in the reaction (1) is forbidden. There are various corrections to the  $E1$  operator in this approximation which can generate  $E1$  transition.

(i) Spin-dependent correction

$$\sqrt{\frac{\pi}{6}} ie \left( \frac{k_\gamma^2}{M} \right) (\mu_p + \mu_n) \sum_{i=1}^6 r_i [Y_1(\hat{r}_i) \otimes \sigma_1(i)]_{1m}. \quad (3)$$

(ii) A retardation effect proportional to  $ek_\gamma^3 \sum_{i=1}^6 r_i^3 Y_{1m}(\hat{r}_i)$ .

(iii) Effects of isospin mixing of the nuclear states. The mixing of  $T = 1$  components in the  ${}^6\text{Li}$  ground state and in the  $\alpha$ - $d$  continuum state wave function is mainly due to Coulomb polarization of the dinucleon (deuteron) in the field of the  $\alpha$  particle. The polarization manifests itself as a deviation of the charge distribution, considered from the center of mass of the system, leading to a dipole transition (see, for example, Ref. [5]).

(iv) Correction due to the fact that the  $\alpha$  cluster and the deuteron cluster are bound systems [binding effect BE]. Due to this fact the center of mass of the  $\alpha$ - $d$  system is shifted relative to the six-nucleon center of mass. Thus the  $E1$  operator has the form

$$\vec{d}_\rho = e \vec{\rho} \frac{m_\alpha m_d}{m_\alpha + m_d} \left[ \frac{Z_\alpha}{m_\alpha} - \frac{Z_d}{m_d} \right], \quad (4)$$

where  $\rho$  is the radial coordinate of the  $\alpha$ - $d$  relative motion, and  $m_\alpha$  and  $m_d$  are the masses of the  $\alpha$  particle and the deuteron, with  $Z_\alpha = 2$  and  $Z_d = 1$ , respectively.

Of course, one cannot consider this expression as a rigorous one. Indeed in the derivation of Eq. (4) the distortion of clusters, relativistic corrections to their wave functions, and meson exchange currents that may be of importance for such a tiny effect were not taken into account. The consistent treatment of all these effects is extremely difficult. That is why (following the paper [1]) we multiply the expression (4) by a factor  $\kappa$ . The numerical value of  $\kappa$  will be determined by a fit to the experimental data.

In our calculations we took into account all these effects. It has been noted in Refs. [1–3] that the contribution of the terms (i)–(iii) are very small. Our results support this conclusion. Therefore we will concentrate below on the contribution of (iv), see Eq. (4).

### III. THE NUCLEAR MODEL

The wave function of the  ${}^6\text{Li}$  ground state was constructed in the multicluster dynamic model with the Pauli projection (MDMP) which treats this nucleus as a three-body  $\alpha$ - $2N$  system. A projection procedure was used to take into account in an approximate way the Pauli principle by using a deep  $\alpha$ - $N$  potential with forbidden states. The details of this approach are given in Ref. [6]. For the interaction between the outer nucleons the full Reid soft-core potential was used [7]. The  $\alpha$ - $N$  potential was constructed [8] by fitting the free  $\alpha$ - $N$  phase shifts up to 20 MeV. To reproduce the  $S$ ,  $P$ , and  $D$  phases with high accuracy the constructed potential appears to be dependent on parity.

The  ${}^6\text{Li}$  ground state wave function was constructed as a sum over the quantum numbers  $\omega = \lambda l S$ :

$$\Psi_{JM} = \sum_{\omega} \Phi_{\omega}(r, \rho) F_{\omega}(r, \rho). \quad (5)$$

Here  $\lambda$  and  $l$  are the partial angular momenta conjugated

to the Jacoby coordinates  $\vec{\rho}$  and  $\vec{r}$ , respectively. The vector  $\vec{r}$  connects the two outer nucleons and the vector  $\vec{\rho}$  connects the center of mass (c.m.) of the  $\alpha$  particle and the c.m. of two outer nucleons.  $L$  and  $S$  are the angular and spin momenta of the whole system, respectively.  $F_{\omega}$  is the spin-angular function:

$$F_{\omega} = \sum_{M_L, M_S} \langle LM_L SM_S | JM \rangle \mathcal{Y}_{\lambda l}^{LM}(\hat{r}, \hat{\rho}) \chi_{SM_S}(N_1, N_2). \quad (6)$$

The radial wave function was written as a sum of

$$\Phi_{\omega}(r, \rho) = \sum_i C_i r_i^{\lambda} \rho_i^l \exp(-\alpha_i r^2 - \beta_i \rho^2), \quad (7)$$

where the nonlinear parameters  $\alpha_i$  and  $\beta_i$  are chosen on the generalized Tchebyshev grid [6]. The radial wave function is obtained by solving the Schrödinger equation using the variational method on the nonorthogonal basis.

The Pauli principle in the MDMP is taken into account by excluding from the solutions the Pauli-forbidden components. This is done by using the pseudopotential [6]

$$\hat{V}_{\alpha N} = V_{\alpha N} + \eta(\Gamma_{\alpha N_1} + \Gamma_{\alpha N_2}). \quad (8)$$

Here  $\Gamma_{\alpha N_i}$  is the projector on the forbidden states in the  $\alpha$ - $N_i$  subsystems,  $\eta$  is a large constant ( $\eta \rightarrow \infty$ ).

It would be preferable to use an accurate three-body approach for the  $\alpha$ - $d$  continuum state function too by taking into account the distortion (polarization) of deuteron in the field of the  $\alpha$ -particle. But as we will show below the capture cross section at low energies depends mainly on the peripheral part of the wave functions. It means that the results of calculations mainly depend on the  $\alpha$ - $d$  phase shifts but not on the precise form of the  $\alpha$ - $d$  interaction. In Ref. [9] it was demonstrated that the low-energy  $\alpha$ - $d$  phase shifts calculated in the framework of our  $\alpha$ - $2N$  three-body approach are in good agreement with experimental data. Therefore, it seems to be justified to use here a more simple two-body scheme for the  $\alpha$ - $d$  continuum state wave function with the  $\alpha$ - $d$  potential fitted to the phase shifts. Thus the continuum wave function of the  $\alpha$ - $d$  system was obtained numerically by solving the two-body Schrödinger equation with the  $\alpha$ - $d$  potential given in Ref. [10]. This potential has a central and a spin-orbit part. It reproduces accurately the phase shifts for the  $l=0, 1$ , and  $2$  partial waves and the low-lying resonances in the  $\alpha$ - $d$  system with  $L=2$  and  $J=1, 2$ , and  $3$ .

Three sets of  ${}^6\text{Li}$  ground state wave functions were used in the calculation. The first set is a standard one, which we call FUNCTION-92 (F-92). It is given in Ref. [8]. In this set the number of Gaussians over the  $\alpha$ - $d$  coordinate used to construct the main  $S$  component of the ground state wave function is equal to 7 and the nonlinear parameters  $\beta_i$  [see Eq. (7)] were varied within the interval

$$2.2 \times 10^{-2} \geq \beta_i \geq 4.7.$$

With this wave function the binding energy is equal to 3.25 MeV, compared with the experimental value

of  $E = 3.7$  MeV. So in this version of the calculations,  ${}^6\text{Li}$  is underbound by approximately 0.5 MeV. At large  $\alpha$ - $d$  distances this three-body wave function is factorized with a high precision into the two-body form  $\Phi(r, \rho) = \phi_d(r)\phi_{\alpha d}(\rho)$ . To avoid the numerical problems with the description of the asymptotic region by a limited Gaussian basis, the function  $\phi_{\alpha d}(\rho)$  was matched to the standard Whittaker function at the distance of  $\rho = 10$  fm.

In the second set we changed slightly the  $\alpha$ - $N$  potential. In odd partial waves of  $\alpha$ - $N$  interaction we took the value of  $k = 0.4265$  fm instead of  $k = 0.43216$  fm, where  $k$  is a nonlinear parameter which determines the width of the potential (see Ref. [8] for details). Such a modification of the potential results in only 1% effect in the phase shifts, but it is enough to bring the calculated binding energy ( $E = 3.691$  MeV) close to the experimental value  $E_{\text{exp}} = 3.7$  MeV. In addition, the number of Gaussians used in this set to describe the  $\alpha$ - $d$  relative motion was almost doubled ( $N_g=15$ ) and the parameters  $\beta_i$  were varied within the very broad interval

$$3.2 \times 10^{-5} \geq \beta_i \geq 1.6 \times 10^3.$$

Such a broad basis allows us to describe very accurately the  ${}^6\text{Li}$  wave functions both at small and at large  $\alpha$ - $d$  distances simultaneously. In this way, an accurate asymptotic behavior of the ground state wave function with respect to the  $\alpha$ - $d$  coordinate was achieved. We denote this version of the wave function as the modified large basis F-92 (MLBF-92). Again at the distance of  $\rho=10$  fm this function was matched to the Whittaker one. Note that unlike the earlier calculations [1–3] the wave function of  ${}^6\text{Li}$  in both cases has larger numbers of partial angular momenta  $\lambda$  and  $l$ . Such additional components can be important especially for the angular distribution of the capture cross section.

It is known [11] that the asymptotic part of the  $\alpha$ - $d$  wave function is the most important in determining the capture processes at the low-energy region. Therefore, in the third version (W-92) of the calculations we used as the  ${}^6\text{Li}$  ground state wave function the pure asymptotic form

$$\psi_{6\text{Li}} = \phi_\alpha \phi_d \left( 2\sqrt{2\mu_{\alpha d}E} \right)^{1/2} C_{\alpha d} W_{\alpha d}(E, \rho).$$

Here  $\mu_{\alpha d}$  is the reduced mass of the  $\alpha$ - $d$  system,  $C_{\alpha d}$  is a dimensionless asymptotic normalization constant, and  $W_{\alpha d}(E, \rho)$  is the Whittaker wave function which corresponds to the  $\alpha$ - $d$  binding energy  $E$ . We use here the experimental values of  $C_{\alpha d} = 2.9$ , extracted recently from low-energy  $\alpha$ - $d$  phase shift analyses [11,12], and of  $E = E_{s\text{Li}} - E_d = 1.48$  MeV. It is important to note that both these values are in excellent agreement with those found for the second set of function: MLBF-92.

To calculate the matrix elements here the numerical integration was carried out starting from  $\rho = 6$  fm, where the strong interaction between the  $\alpha$  particle and the deuteron is already very small. That is, we neglect in matrix elements the contribution from the internal part of the wave functions. It turns out that this assumption works quite well in the low energy region. This

means that the calculated values of cross section for the deuteron capture processes at low energies which are of interest to astrophysics are practically model independent.

All necessary expressions to calculate the differential and integral cross sections of the reaction (1) are given in the Appendix.

## IV. THE TOTAL CAPTURE RATE

### A. Transition to the ground state

To demonstrate the sensitivity of the total capture cross section to the details of the wave functions we have plotted in Fig. 1(a) the results of the calculation for the three sets of wave functions. When calculating the cross section, we took into account the  $M1$ ,  $E1$ , and  $E2$  multipoles with retardation effects. In the electric multipoles the spin part of the corresponding operator was also taken into account. Qualitatively all three versions of the wave function used reproduce the energy dependence of the total cross section. However, at low energies we obtain better agreement with data for the wave function MLBF-92 [see the solid line in Fig. 1(a)]. This is mainly due to the fact that this wave function accurately reproduces the experimental value of the  $\alpha$ - $d$  binding energy [compare with the dotted line in Fig. 1(a)]. We note that already the Whittaker function alone (with  $C = 2.9$ ) when used as the wave function of the  $\alpha$ - $d$  motion in the ground state of  ${}^6\text{Li}$  gives results which are not far from the experimental data. The MLBF-92 version of the  ${}^6\text{Li}$  ground state wave function reproduces fairly well the experimental data [4] up to  $E_{\text{c.m.}} = 3$  MeV, including the recently obtained data at low energies [13].

It is easy to take into account in our  $\alpha$ - $2N$  approach the effect of dinucleon Coulomb polarization in the  ${}^6\text{Li}$  ground state by adding in the variational basis the components with isospin  $T = 1$ . Their admixture to the  ${}^6\text{Li}$  ground state is due to both the  $\alpha$ - $p$  Coulomb interaction and the proton-neutron mass difference. It was found that the weight of this component is about 0.004% only. This magnitude is in good agreement with the estimation of Ref. [14]. As a result the contribution of the corresponding isovector  $E1$  multipole is at least one order of magnitude smaller than that needed to reproduce the experimental data on  $\alpha$ - $d$  capture.

The effect of the Coulomb distortion of the incoming clusters should be significantly less important because the mean intercluster distance in this case is much larger than that for the ground state.

Thus one can conclude that the effects of Coulomb distortion are not large enough to reproduce the existing experimental data. Therefore one has to include the additional BE mechanism of generating the dipole term, Eq. (4).

In Fig. 1(b), the total capture cross section at very low energies ( $E \leq 100$  keV) is given. The solid and dashed lines correspond to the function MLBF-92 and W-92, respectively. To demonstrate the role of the BE isoscalar  $E1$  multipole, in this region of energy, we also plotted

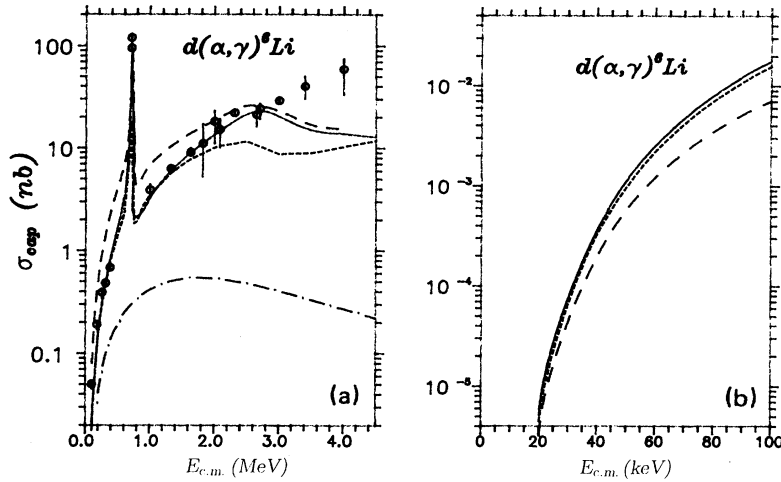


FIG. 1. (a),(b) Total deuteron capture cross section to the ground state of  ${}^6\text{Li}$ . The solid line is the result of the calculation with the modified large basis function-92 (MLBF-92). The long dashed line is the result with function-92. The short dashed line is the result with asymptotic representation of  ${}^6\text{Li}$  wave function. The dash-dotted line is the result of the  $E1$  part with  $\kappa = 0.7$  for MLBF-92.

(dash-dotted line) the cross section, which is only due to this multipole. The scaling factor  $\kappa$  in Eq. (4) was taken to be  $\kappa = 0.70$ . This value was obtained from the analysis of the experimental data on angular distribution, as will be discussed in the next section. We first note that for the total cross section, the  $E2$  contribution dominates (by a factor larger than 2) at energies above 300 keV. The nice agreement between the solid and dashed lines demonstrates that at these low energies only the asymptotic behavior of the relative  $\alpha$ - $d$  wave function is needed.

### B. Transition to the $J^\pi T = 3^+0$ level

The wave function of the  $J^\pi T = 3^+0$  level was constructed in the version of F-92 as a resonance state. The cross section is given in Fig. 2 together with that for the ground state. It is seen from Fig. 2 that the cross section to the  $J^\pi T = 3^+0$  state becomes dominant already at excitation energy of  $E_{c.m.} = 5$  MeV and above. Precise theoretical consideration of such processes requires the calculation of transition between two continuum state

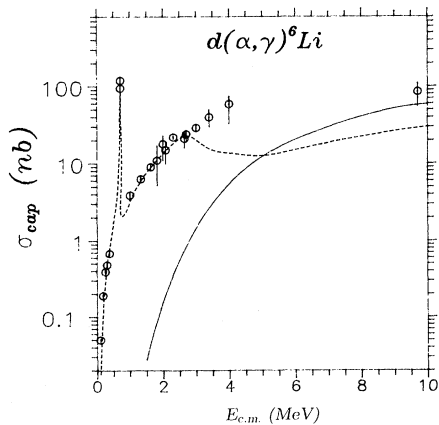


FIG. 2. Deuteron capture cross section to the  ${}^6\text{Li}$  ground state (short dashed line) and excited  $3^+0$  state (solid line).

functions and is very difficult. So our calculation can only be considered as an estimation of the effect.

### V. ANGULAR DISTRIBUTION

Experimental data on the angular distribution are available for three energies,  $E_{c.m.} = 1.33, 1.63,$  and  $2.08$

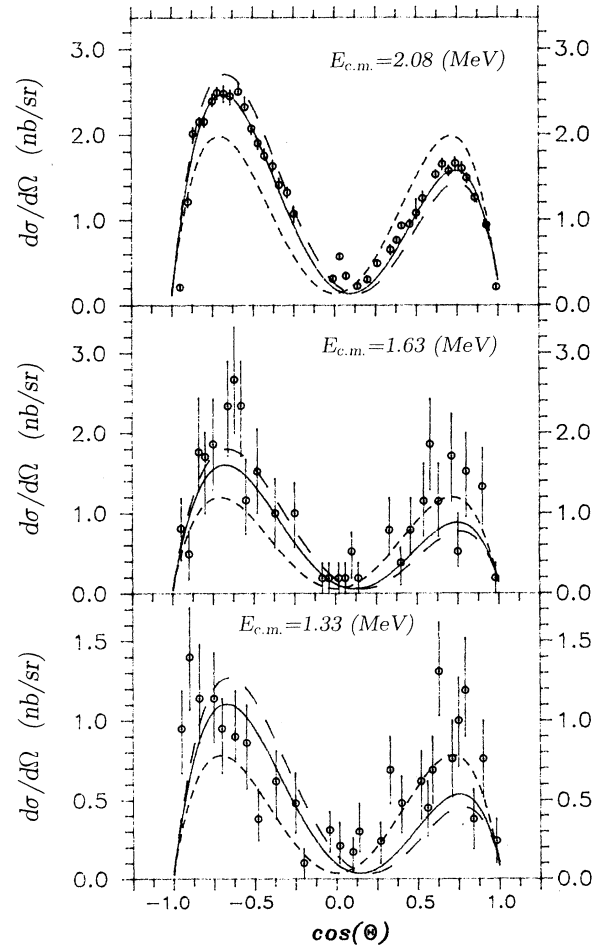


FIG. 3. Angular distributions calculated with the MLBF-92 function including the isoscalar  $E1$  multipole with  $\kappa = 1$  (long dashed),  $0.7$  (solid), and  $0.0$  (short-dashed) curve. The experimental data are from Ref. [5].

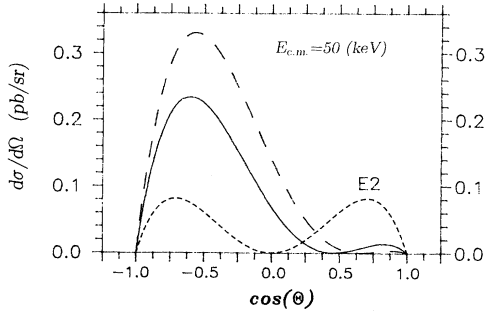


FIG. 4. Angular distribution of the deuteron capture at  $E_{c.m.} = 50$  keV. The notation is the same as in Fig. 3.

MeV [4]. However, only at  $E_{c.m.} = 2.08$  MeV are the experimental data of good quality. Unfortunately, the experimental data for angular distribution in Ref. [4] are given in arbitrary units. We normalized this differential cross section by using the data for the total cross section given in the same paper. Since these data are given only in the figures some uncertainties still persist. The data at this energy allow us to determine only a limit on the value of the isoscalar  $E1$  multipole. The normalized differential cross section is shown in Fig. 3. The dashed, solid, and dash-dotted lines were obtained using the MLBF-92 wave functions for the following values of  $\kappa = 1, 0.7,$  and  $0,$  respectively. It is seen from Fig. 3 [see, in particular, Fig. 3(a)] that the assumed form of the isoscalar  $E1$  multipole, Eq. (4), is sufficient to reproduce the data with a fitted value of  $\kappa = 0.7$ . The value  $\kappa = 0.7$  seems to give the best fit.

Note that the fitted value of the parameter  $\kappa = 0.7$  in our calculation is larger than that deduced in Ref. [15]. As a result [see Fig. 3(a)] the overall description of the experimental angular distributions with this value of  $\kappa$  is better than in Ref. [15]. This is due to the fact that we have used a more accurate wave function for the  ${}^6\text{Li}$  ground state in our calculation.

To demonstrate the important effect of the isoscalar  $E1$  multipole we show in Fig. 4 the calculated differential cross section at very low energy of 50 keV. The dashed, solid, and dash-dotted lines correspond to the values of  $\kappa = 1, 0.7,$  and  $0.$  The angular distribution of the pure  $E2$ -multipole ( $\kappa = 0$ ) differs markedly from those obtained by including the isoscalar  $E1$  multipole. Note here that neither the inclusion of the  $M1$  multipole nor the corrections (i)–(iii) could be practically seen in the figures since they are so small.

## VI. CONCLUSION

We have performed a very careful analysis of the reaction  $\alpha + d \rightarrow {}^6\text{Li} + \gamma$ . The main interest for studying this reaction is to find out whether cluster effects are responsible for generating the very specific corrections to the  $E1$  multipole, given in Eq. (4). These effects can be seen only in the angular distributions at low energies or in the total cross section at very low energies. To calculate this angular distribution with high precision we paid special

attention to the asymptotic behavior of the ground state wave function. It appears that this asymptotic region is responsible for the main contribution to the total cross section at low energies. Moreover, for energies of astrophysical interest the  $\alpha$ - $d$  capture cross section can be quite accurately described by using only an asymptotic form for the  ${}^6\text{Li}$  wave function. Therefore a more reliable theoretical prediction of reaction cross section can be established for low energies.

We have shown that the contributions of (i)–(iii) to the  $E1$  multipole are too small to explain the experimental data. That is why the contribution of the BE effect to the  $E1$  multipole, given by Eq. (4), is needed to reproduce correctly the available experimental data for the total cross section and differential cross section [see Figs. 1(a) and 3].

At the same time it seems to be interesting to derive the expression for the BE correction in a consistent manner taking into account the relativistic corrections and meson exchange currents.

We point out that a detailed knowledge of this multipole will allow us to increase the reliability of predictions for the cross section in the region of interest to astrophysics. It was shown that at energies  $E_{c.m.} \leq 100$  keV the  $E1$  multipole dominates in the cross section. Thus one could expect a higher abundance of  ${}^6\text{Li}$  as compared with the result of a calculation in which the  $E1$  multipole is not taken into account.

Measurements of angular distribution with higher precision at low energies could play the decisive role in testing the theoretical approaches for this reaction.

We have also shown that the magnitude of the capture cross section to the excited  $J^\pi T = 3^+ 0$  level of  ${}^6\text{Li}$  becomes very important already at  $E_{c.m.} \geq 4$  MeV.

## ACKNOWLEDGMENTS

Two of us (G.G.R. and R.A.E.) would like to express their thanks to the Cyclotron Institute at Texas A&M University for kind hospitality. S.S. would like to thank the European Center for Theoretical Studies in Nuclear Physics and Related Areas (ECT\*), Trento, Italy for the kind hospitality. This work was supported in part by the National Science Foundation Grants No. PHY-9107008 and No. PHY-9413872 and Russian Fund of Fundamental Researches No. 93-02-3376.

## APPENDIX

The differential cross section for the reaction (1) reads

$$\frac{d\sigma}{d\Omega} = (8\pi\mu k/q) \frac{1}{2J_d + 1} \sum_n a_n P_n(\cos\theta), \quad (\text{A1})$$

where  $\theta$  is the angle between the outgoing photon and the incoming deuteron momenta in the c.m. system. The angular distribution coefficients  $a_n$  are expressed through the nuclear matrix elements as follows:

$$a_n = \sum_{l_1 l_2 j_1 j_2 J_1 J_2} i^{l_1 - l_2 - n} \frac{(-1)^{J_f + j_1 + j_2}}{\hat{j}_1 \hat{j}_2} Z_\gamma(J_1 j_1 J_2 j_2; J_f n) \\ \times Z(l_1 j_1 l_2 j_2; 1n) \text{Re} \left\{ T_{J_2}^*(j_2 l_2) T_{J_1}(j_1 l_1) \right\}. \quad (\text{A2})$$

Here  $J_1 = J_\alpha = 0$ ,  $J_2 = J_d = 1$ ,  $J_f$  is the angular momentum of  ${}^6\text{Li}$ ,  $\hat{j} = \sqrt{2j+1}$ .  $T_J(lj)$  is a reduced matrix element of rank  $J$ , where  $l$  and  $j$  are the orbital and the total angular momenta of the  $\alpha$ -d continuum state wave function, respectively. The functions  $Z$  and  $Z_\gamma$  have the following forms [16]:

$$Z(l_1 j_1 l_2 j_2; S n) = i^{n - l_1 + l_2} (-1)^{l_1 + j_1 + l_2 + j_2 + n} \hat{l}_1 \hat{j}_1 \hat{l}_2 \hat{j}_2 \hat{n} \\ \times \left\{ \begin{matrix} l_1 & j_1 & S \\ j_2 & l_2 & n \end{matrix} \right\} \left( \begin{matrix} l_1 & l_2 & n \\ 0 & 0 & 0 \end{matrix} \right), \quad (\text{A3})$$

$$Z_\gamma(J_1 j_1 J_2 j_2; J_f n) = (-1)^{J_1 + j_1 + J_2 + j_2 + n} \hat{J}_1 \hat{j}_1 \hat{J}_2 \hat{j}_2 \hat{n} \\ \times \left\{ \begin{matrix} j_1 & J_1 & J_f \\ J_2 & j_2 & n \end{matrix} \right\} \left( \begin{matrix} J_1 & J_2 & n \\ 1 & -1 & 0 \end{matrix} \right). \quad (\text{A4})$$

The total cross section in the c.m. is the sum of the square of matrix elements of all multipolarity squared:

$$\sigma_{\text{dis}}^{\text{tot}} = (4\pi)^2 \frac{\mu k}{q} \frac{2}{2J_d + 1} \sum_{l_j J} |T_J(l_j)|^2. \quad (\text{A5})$$

- 
- [1] K. Langanke, Nucl. Phys. **A457**, 351 (1986).  
 [2] N.A. Burkova, K.A. Zhaksibekova, M.A. Zhusupov, and R.A. Eramzhyan, Phys. Lett. B **248**, 15 (1990).  
 [3] R. Crespo, A.M. Eiro, and J.A. Tostevin, Phys. Rev. C **42**, 1646 (1990).  
 [4] R.G. Robertson *et al.*, Phys. Rev. Lett. **47**, 1867 (1981).  
 [5] S. Jang, Phys. Rev. C **47**, 286 (1993).  
 [6] V.I. Kukulin, V.M. Krasnopolsky, V.T. Voronchev, and P.B. Sazonov, Nucl. Phys. **A417**, 128 (1984).  
 [7] R.V. Reid, Ann. Phys. (N.Y.) **50**, 393 (1968); B.D. Day, Phys. Rev. C **24**, 1203 (1981).  
 [8] V.I. Kukulin, V.N. Pomerantsev, Kh.D. Rasikov, V.T. Voronchev, and G.G. Ryzhikh, Nucl. Phys. **A586**, 151 (1995).  
 [9] G.G. Ryzhikh, R.A. Eramzhyan, and V.I. Kukulin, in contributed paper to the *14th International Union of Pure and Applied Physics Conference on Few-body Problems in Physics*, edited by F. Gross (North-Holland, Amsterdam, in press), p.165; and (unpublished).  
 [10] V.I. Kukulin and V.N. Pomerantsev, Yad. Fiz. **51**, 376 (1990).  
 [11] L.D. Blokhintsev and A.M. Mukhamedzhanov, Ref. [9], p. 39.  
 [12] L.D. Blokhintsev, V.I. Kukulin, A.A. Sakharuk, and D.A. Savin, Phys. Rev. C **48**, 2390 (1993).  
 [13] J. Kiener *et al.*, Phys. Rev. C **44**, 2195 (1991).  
 [14] N. Auerbach, Phys. Rep. **98**, 274 (1983).  
 [15] S. Typel, G. Blüge, and K. Langanke, Z. Phys. **A339**, 335 (1991).  
 [16] A.M. Baldin, V.I. Goldanski, and I.L. Rosenthal, *Kinematics of Nuclear Reactions* (Oxford University Press, London, 1961).