Effects of finite excitation energy of environment on fast quantum tunneling

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Quantum tunneling under the influence of an environment with nearly degenerate spectrum is considered. We first give a general derivation of the zero point motion formula in the sudden tunneling limit. The effects of a finite excitation energy of the environment are then considered by a perturbation method. Examples of linear oscillator coupling and rotational coupling show that the finite excitation energy can be represented by a dissipation factor and reduces the tunneling probability estimated in the limit of sudden tunneling. These examples clearly show that the applicability of the sudden tunneling approximation is governed by the details of the coupling as well as the relative time scales of the tunneling and the environmental degrees of freedom. We discuss some applications to heavy-ion fusion reactions.

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I. INTRODUCTION

Quantum tunneling in systems with many degrees of freedom has attracted much interest during the past decade in many fields of physics and chemistry [1,2]. This problem is often called macroscopic quantum tunneling. One of the major interests in this area is to assess the effects of the environment on the tunneling rate of a macroscopic variable.

In nuclear physics, heavy-ion fusion reactions are typical examples of this problem. When the bombarding energy is below the Coulomb barrier, fusion takes place by quantum tunneling of the relative motion between heavy ions. It is now well established that the fusion cross section at energies below the Coulomb barrier is enhanced by several orders of magnitude due to the coupling of the relative motion to nuclear intrinsic motions [3]. A standard way to address this problem is to numerically solve the relevant coupled-channels equations for the relative motion. A drawback of this approach is that the number of coupled-channels equations becomes very large if many channels are considered. It requires a long computer time to solve the equations, and sometimes it is not easy to understand physically the origin of the effects of channel coupling. For these reasons, simplified treatments in the adiabatic or in the sudden tunneling approximations are often used [4]. The adiabatic formula is applicable if the motion of the macroscopic degree of freedom is extremely slow compared with that of the internal degrees of freedom [5–7]. This corresponds to cases where the excitation energies of the internal motions are very high. In the opposite limit, where the tunneling motion is very fast or where the internal states have an almost degenerate spectrum, the so-called zero point motion formula is applicable and gives a clear understanding of the effects of channel coupling in terms of the barrier distribution [4,8–13].

In most of the realistic cases, however, it is important to take deviations from these limits into account in order to quantitatively estimate the tunneling rate. In previous papers [5,6], we developed formulas to handle deviations from the limit of adiabatic tunneling by introducing the concept of the mass renormalization [5] or the dynamical norm factor [6]. In this paper we consider nearly sudden tunneling, where the tunneling process is fast, and discuss how the formula for the tunneling probability in the sudden tunneling limit is modified by the finite excitation energy of the environment. This study is especially important to analyze recent data of heavy-ion fusion reactions, where the excitation energy of nuclear intrinsic motions is small, but not negligible [14,15].

In Sec. II we derive the zero point motion formula for the tunneling probability based on the influence functional method [4] without assuming particular properties for the environment. This is an alternative derivation to Ref. [8], which used a Green's function method. In Sec. III we derive a formula for the barrier penetrability which modifies the zero point motion formula by taking the finite excitation energy of the environment into account. The path integral method is superior to the Green's function method, because it enables us to use a time-dependent perturbation theory in order to discuss the effects of the finite excitation energy. In Sec. IV we apply our general formulas to particular examples, i.e., to the problem of a linear oscillator coupling and to that of a rotational coupling. These examples help us to understand the physical meaning of the modifications. The finite excitation energy leads to a kind of dissipation factor multiplying the zero point motion formula for the tunneling probability in the sudden tunneling limit. The

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dissipation factor suggests that not only the ratio of the time scales of the environmental and the tunneling motions, but also the properties of the coupling form factor govern the validity of the sudden tunneling approximation. We apply our modified formula of the tunneling probability to calculate the fusion cross section between ^{148,152}Sm and ^{16}O and between ^{194}Pt and ^{16}O. We summarize the results in Sec. V.

II. GENERAL DERIVATION OF THE ZERO POINT MOTION FORMULA

We assume the following Hamiltonian for a system consisting of a macroscopic motion R and an environmental degree of freedom ξ:

$$H(R,\xi) = -\frac{\hbar^2}{2M}\frac{\partial^2}{\partial R^2} + U(R) + H_0(\xi) + V(R,\xi) \ , \quad (1)$$

$$H_0(\xi) = -\frac{\hbar^2}{2D}\frac{\partial^2}{\partial \xi^2} + U_0(\xi) \ . \quad (2)$$

M and D are the masses for the macroscopic and internal, i.e., environmental, motions, respectively. $U(R)$, which we call the bare potential, is the potential in the absence of coupling between the macroscopic and internal systems. $H_0(\xi)$ is the Hamiltonian for the internal motion, and $V(R,\xi)$ is the coupling Hamiltonian.

In the path integral formalism, the S matrix of the transition from an initial position R_i on the right side of the barrier with initial internal state n_i to a final position R_f on the left side with final internal state n_f is given by [4]

$$S_{n_f,n_i}(E) = \lim_{\substack{R_i\to\infty \\ R_f\to-\infty}} \left(\frac{P_iP_f}{M^2}\right)^{1/2} e^{\frac{i}{\hbar}(P_fR_f-P_iR_i)} \int_0^\infty dT\, e^{(i/\hbar)ET} \int \mathcal{D}[R(t)] e^{(i/\hbar)[S_t(R,T)]} \langle n_f|\hat{u}(R(t),T)|n_i\rangle \ , \quad (3)$$

where the Green's function for the internal motion along a given path $R(t)$ obeys

$$i\hbar\frac{\partial}{\partial t}\hat{u}(R,t) = [H_0(\xi) + V(R,\xi)]\hat{u}(R,t) \ , \quad (4)$$

with $\hat{u}(R,t=0)=1$. In Eq. (3), E is the total energy of the system, and P_i and P_f are the classical momenta at R_i and R_f, respectively. $S_t(R,T)$ is the action for the macroscopic motion along a path $R(t)$ and is given by

$$S_t(R,T) = \int_0^T dt[\tfrac{1}{2}M\dot{R}(t)^2 - U(R(t))] \ . \quad (5)$$

In many situations, we are interested only in the inclusive process. In that case, the barrier transmission probability is given by

$$P(E) = \sum_{n_f} |S_{n_f,n_i}|^2 \quad (6)$$

$$= \lim_{\substack{R_i\to\infty \\ R_f\to-\infty}} \left(\frac{P_iP_f}{M^2}\right) \int_0^\infty dT\, e^{(i/\hbar)ET} \int_0^\infty d\tilde{T}\, e^{-(i/\hbar)E\tilde{T}}$$

$$\times \int \mathcal{D}[R(t)] \int \mathcal{D}[\tilde{R}(\tilde{t})] e^{(i/\hbar)[S_t(R,T)-S_t(\tilde{R},\tilde{T})]} \rho_M(\tilde{R}(\tilde{t}),\tilde{T};R(t),T) \ . \quad (7)$$

The effects of the internal degree of freedom are here included in the two time influence functional ρ_M, which is defined by

$$\rho_M(\tilde{R}(\tilde{t}),\tilde{T};R(t),T) = \sum_{n_f} \langle n_i|\hat{u}^\dagger(\tilde{R}(\tilde{t}),\tilde{T})|n_f\rangle\langle n_f|\hat{u}(R(t),T)|n_i\rangle \ . \quad (8)$$

We assumed that the energy dissipation is small compared with the total energy and that the potential energy is independent of the channel at R_f. We thus took P_f outside the sum over the final states. Note that the summation over the final states in Eq. (8) can be performed by choosing any convenient complete set.

Let us now introduce the sudden tunneling approximation. In this limit, the excitation energy of the internal motion is set to be zero. Hence we discard $H_0(\xi)$ in Eq. (4). The time evolution operator $\hat{u}$ can then be solved as

$$\hat{u}(R,t) = \exp\left(-\frac{i}{\hbar}\int_0^t dt'\, V(R(t'),\xi)\right) \ . \quad (9)$$

If the coupling Hamiltonian $V(R,\xi)$ does not contain the conjugate momentum operator of ξ, the time evolution operator $\hat{u}$ is diagonal in the coordinate space of the environmental degrees of freedom. Denoting the eigenvalue of ξ by x, the two time influence functional therefore takes the form

$$\rho_M(\tilde{R}(\tilde{t}),\tilde{T};R(t),T) = \int_{-\infty}^{\infty} dx|\langle x|n_i\rangle|^2 \exp\left[-\frac{i}{\hbar}\left(\int_0^T dt\, V(R(t),x) - \int_0^{\tilde{T}} d\tilde{t}\, V(\tilde{R}(\tilde{t}),x)\right)\right] . \tag{10}$$

Inserting this expression into Eq. (7), the inclusive barrier penetrability in the sudden limit is found to be

$$P(E) = \int_{-\infty}^{\infty} dx|\langle x|n_i\rangle|^2 \Bigg\{ \lim_{\substack{R_i\to\infty \\ R_f\to-\infty}} \left(\frac{P_iP_f}{M^2}\right)\Bigg|\int_0^{\infty} dT\, e^{(i/\hbar)ET} \times \int \mathcal{D}[R(t)] \exp\left[(i/\hbar)\int_0^T dt[\tfrac{1}{2}M\dot{R}^2 - U(R) - V(R,x)]\right]\Bigg|^2\Bigg\} \tag{11}$$

$$= \int_{-\infty}^{\infty} dx|\langle x|n_i\rangle|^2 P_0(E,U(R)+V(R,x)) , \tag{12}$$

where $P_0(E,U(R)+V(R,x))$ is the barrier penetrability across a one-dimensional potential barrier $U(R)+V(R,x)$ when the energy is E. This is a general expression of the zero point motion formula. This approach clearly shows that the weight factor in the zero point motion formula is given by the square of the ground state wave function of the environmental degrees of freedom, which was found in Ref. [8] for special environments.

III. EFFECTS OF FINITE EXCITATION ENERGY

In this section, we discuss the effects of a small but finite excitation energy of an internal motion on the tunneling rate. We treat $H_0(\xi)$, which was neglected in the previous section, by a perturbation theory. To this end we first introduce the interaction representation defined by

$$\tilde{u}(R,t) = \exp\left(\frac{i}{\hbar}\int_0^t dt'\, V(R(t'),\xi)\right)\hat{u}(R,t) , \tag{13}$$

$$\tilde{H}_0(R,\xi) = \exp\left(\frac{i}{\hbar}\int_0^t dt'\, V(R(t'),\xi)\right) H_0(\xi) \exp\left(-\frac{i}{\hbar}\int_0^t dt'\, V(R(t'),\xi)\right) . \tag{14}$$

Equation (4) for the time evolution of the internal motion then leads to

$$i\hbar\frac{\partial}{\partial t}\tilde{u}(R,t) = \tilde{H}_0(R,\xi)\tilde{u}(R,t) . \tag{15}$$

The first order solution of this equation reads

$$\tilde{u}^{(1)}(R,t) = 1 + \frac{1}{i\hbar}\int_0^t dt'\, \tilde{H}_0(R(t'),\xi) . \tag{16}$$

If we denote the intrinsic state at the initial time $t=0$ as n_i, then the first order solution of the internal wave function in the coordinate representation at time t is given by

$$\langle x|\hat{u}(R,t)|n_i\rangle = \exp\left(-\frac{i}{\hbar}\int_0^t dt'\, V(R(t'),x)\right) \times\left[1+\frac{1}{i\hbar}\int_0^t dt'\, \exp\left(\frac{i}{\hbar}\int_0^{t'} dt''\, V(R(t''),x)\right) H_0(x)\exp\left(-\frac{i}{\hbar}\int_0^{t'} dt''\, V(R(t''),x)\right)\right]\langle x|n_i\rangle . \tag{17}$$

Note that the second derivative operator in $H_0(x)$ operates on both $\exp\left[-\frac{i}{\hbar}\int_0^{t'} dt''\, V(R(t''),x)\right]$ and $\langle x|n_i\rangle$. Equation (17) therefore becomes

$$
\begin{aligned}
\langle x|\hat{u}(R,t)|n_i\rangle &= \exp\left(-\frac{i}{\hbar}\int_0^t dt'\, V(R(t'),x)\right)\langle x|n_i\rangle \\
&\quad\times\left\{1+\frac{1}{i\hbar}\int_0^t dt'\left[\frac{i\hbar}{2D}\int_0^{t'} dt''\frac{\partial^2 V(R(t''),x)}{\partial x^2}+\frac{1}{2D}\left(\int_0^{t'} dt''\frac{\partial V(R(t''),x)}{\partial x}\right)^2\right.\right. \\
&\quad\left.\left.+\epsilon_i+\frac{i\hbar}{D}\int_0^{t'} dt''\frac{\partial V(R(t''),x)}{\partial x}\frac{d\langle x|n_i\rangle}{dx}\frac{1}{\langle x|n_i\rangle}\right]\right\} \qquad (18) \\
&\sim \exp\left(-\frac{i}{\hbar}\int_0^t dt'\, V(R(t'),x)\right)\langle x|n_i\rangle\exp\left(-\frac{i}{\hbar}\epsilon_i t\right) \\
&\quad\times\exp\left\{\frac{1}{i\hbar}\int_0^t dt'\left[\frac{i\hbar}{2D}\int_0^{t'} dt''\frac{\partial^2 V(R(t''),x)}{\partial x^2}+\frac{1}{2D}\left(\int_0^{t'} dt''\frac{\partial V(R(t''),x)}{\partial x}\right)^2\right.\right. \\
&\quad\left.\left.+\frac{i\hbar}{D}\int_0^{t'} dt''\frac{\partial V(R(t''),x)}{\partial x}\frac{d\langle x|n_i\rangle}{dx}\frac{1}{\langle x|n_i\rangle}\right]\right\}, \qquad (19)
\end{aligned}
$$

where ϵ_i is the eigenvalue of H_0 for the initial internal state n_i. By a procedure similar to the one used to obtain Eq. (12), the barrier penetrability is found to be

$$
P(E)=\int_{-\infty}^{\infty} dx|\langle x|n_i\rangle|^2 P_0(E_{\text{c.m.}},U_{\text{eff}}(R,x)) , \qquad (20)
$$

where $E_{\text{c.m.}}=E-\epsilon_i$ and

$$
\begin{aligned}
U_{\text{eff}}(R,x) &= U(R)+V(R,x)+\frac{i\hbar}{2D}\int_0^t dt'\frac{\partial^2 V(R(t'),x)}{\partial x^2}+\frac{1}{2D}\left(\int_0^t dt'\frac{\partial V(R(t'),x)}{\partial x}\right)^2 \\
&\quad+\frac{i\hbar}{D}\int_0^t dt'\frac{\partial V(R(t'),x)}{\partial x}\frac{d\langle x|n_i\rangle}{dx}\frac{1}{\langle x|n_i\rangle} \qquad (21) \\
&= U(R)+V(R,x)+\frac{i\hbar}{2D}\int_{R_i}^R\frac{dR'}{\left(\frac{dR'}{dt}\right)}\frac{\partial^2 V(R',x)}{\partial x^2}+\frac{1}{2D}\left(\int_{R_i}^R\frac{dR'}{\left(\frac{dR'}{dt}\right)}\frac{\partial V(R',x)}{\partial x}\right)^2 \\
&\quad+\frac{i\hbar}{D}\frac{d\langle x|n_i\rangle}{dx}\frac{1}{\langle x|n_i\rangle}\int_{R_i}^R\frac{dR'}{\left(\frac{dR'}{dt}\right)}\frac{\partial V(R',x)}{\partial x} . \qquad (22)
\end{aligned}
$$

The time dependence of the macroscopic coordinate R in Eq. (22) is obtained by evaluating the integrals over the path and the time in Eq. (7) in stationary phase approximations for each value of x:

$$
\frac{dR}{dt}=-\sqrt{\frac{2}{M}[E_{\text{c.m.}}-U_{\text{eff}}(R,x)]} . \qquad (23)
$$

The sign in front of the square root was chosen to be consistent with Eq. (3), which corresponds to the incident beam from the right-hand side (rhs) of the potential barrier. Since U_{eff} depends on $R(t)$, Eq. (23) should be solved self-consistently. The velocity becomes imaginary in the classically forbidden region. If one keeps R to be real in this region, Eq. (23) leads to an imaginary time. Writing $t=-i\tau$, τ being positive, the time evolution of R is given by

$$
\frac{dR}{d\tau}=-i\frac{dR}{dt}=-\sqrt{\frac{2}{M}[U_{\text{eff}}(R,x)-E_{\text{c.m.}}]} . \qquad (24)
$$

Consequently, the effective potential for R under the barrier reads

$$
\begin{aligned}
U_{\text{eff}}(R,x) &= U(R)+V(R,x)+\frac{\hbar}{2D}\int_{R_0}^R\frac{dR'}{\left(\frac{dR'}{d\tau}\right)}\frac{\partial^2 V(R',x)}{\partial x^2} \\
&\quad+\frac{1}{2D}\left(-i\int_{R_0}^R\frac{dR'}{\left(\frac{dR'}{d\tau}\right)}\frac{\partial V(R',x)}{\partial x}+\int_{R_i}^{R_0}\frac{dR'}{\left(\frac{dR'}{dt}\right)}\frac{\partial V(R',x)}{\partial x}\right)^2 \\
&\quad+\frac{\hbar}{D}\frac{d\langle x|n_i\rangle}{dx}\frac{1}{\langle x|n_i\rangle}\int_{R_0}^R\frac{dR'}{\left(\frac{dR'}{d\tau}\right)}\frac{\partial V(R',x)}{\partial x}+\frac{i\hbar}{2D}\int_{R_i}^{R_0}\frac{dR'}{\left(\frac{dR'}{dt}\right)}\frac{\partial^2 V(R',x)}{\partial x^2} \\
&\quad+\frac{i\hbar}{D}\frac{d\langle x|n_i\rangle}{dx}\frac{1}{\langle x|n_i\rangle}\int_{R_i}^{R_0}\frac{dR'}{\left(\frac{dR'}{dt}\right)}\frac{\partial V(R',x)}{\partial x} , \qquad (25)
\end{aligned}
$$

where R_0 is the outer classical turning point. We have separated the integral over R in Eq. (22) into that in the classically allowed region between R_i and R_0 and in the classically forbidden region between R_0 and R. Equation (25) shows that a channel coupling leads to an imaginary potential in the classically allowed region. On the other hand, it renormalizes the real potential in the classically forbidden region.

Equations (20) and (25) hold in general as long as the tunneling process is fast. We call Eq. (20) a revised zero point motion formula. In the next section, we apply this formula to two concrete examples. We thereby clarify the physical meaning of the correction terms and the conditions for the validity of the revised formula.

IV. EXAMPLES: PHYSICAL INTERPRETATION AND APPLICABILITY OF THE SUDDEN TUNNELING APPROXIMATION

A. Linear oscillator coupling

Let us first consider a linear oscillator coupling. This system has been examined by Esbensen, Wu, and Bertsch in Ref. [16] using the eikonal approximation to discuss the effects of the finite excitation energy of the target nucleus on heavy-ion fusion reactions. The internal and coupling Hamiltonians for this system are

$$H_0(\xi) = -\frac{\hbar^2}{2D}\frac{d^2}{d\xi^2} + \frac{1}{2}D\omega^2\xi^2 , \tag{26}$$

$$V(R,\xi) = f(R)\xi , \tag{27}$$

where $\hbar\omega$ and $f(R)$ are the excitation energy of the oscillator and the coupling form factor, respectively. If the initial state $|n_i\rangle$ is the ground state of the oscillator, as in the case of heavy-ion fusion reactions, then

$$\langle x|n_i\rangle = (2\pi\alpha_0^2)^{-1/4}e^{-\frac{x^2}{4\alpha_0^2}} , \tag{28}$$

where $\alpha_0 = \sqrt{\hbar/2D\omega}$ is the amplitude of the zero point motion of the oscillator.

We can recover the earlier result of Ref. [16] as follows. Provided that there is no excitation in the classically allowed region, Eq. (20) becomes

$$P(E) = \frac{1}{\sqrt{2\pi\alpha_0^2}}\int_{-\infty}^{\infty} dx\, e^{-\frac{x^2}{2\alpha_0^2}} P_0\left[E_{\rm c.m.}, U(R) + xf(R) - \frac{\omega}{\hbar}\alpha_0^2\left(\int_{R_0}^{R}\frac{dR'}{\left(\frac{dR'}{d\tau}\right)}f(R')\right)^2 - \omega x\int_{R_0}^{R}\frac{dR'}{\left(\frac{dR'}{d\tau}\right)}f(R')\right] , \tag{29}$$

where $E_{\rm c.m.} = E - \frac{1}{2}\hbar\omega$. Equation (29) coincides with Eq. (32) in Ref. [16] if the classical velocity in the tunneling region can be approximated by

$$\frac{dR}{d\tau} = -\sqrt{\frac{2}{M}[U(R) + xf(R) - E_{\rm c.m.}]} , \tag{30}$$

although this does not satisfy the self-consistent relation (24).

In this system the effects of the finite excitation energy can be cast into a more transparent form. To this end, we start from the general first order perturbation expression for the influence functional. From Eq. (19), it reads

$$\rho_M(\tilde{R}(\tilde{t}),\tilde{T};R(t),T) = \frac{1}{\sqrt{2\pi\alpha_0^2}}\int_{-\infty}^{\infty} dx\, e^{-\frac{x^2}{2\alpha_0^2}} e^{x[A+\tilde{A}^*+(B+\tilde{B}^*)\omega]} e^{(C+\tilde{C}^*)\omega} e^{-\frac{i}{2}\omega(T-\tilde{T})} , \tag{31}$$

with

$$A = -\frac{i}{\hbar}\int_0^T dt\, f(R(t)), \quad \tilde{A} = -\frac{i}{\hbar}\int_0^{\tilde{T}} d\tilde{t}\, f(\tilde{R}(\tilde{t})) , \tag{32}$$

$$B = -\frac{1}{\hbar}\int_0^T dt\int_0^t dt'\, f(R(t')), \quad \tilde{B} = -\frac{1}{\hbar}\int_0^{\tilde{T}} d\tilde{t}\int_0^{\tilde{t}} d\tilde{t}'\, f(\tilde{R}(\tilde{t}')) , \tag{33}$$

$$C = -\frac{i}{\hbar}\int_0^T dt\frac{\alpha_0^2}{\hbar}\left(\int_0^t dt'\, f(R(t'))\right)^2, \quad \tilde{C} = -\frac{i}{\hbar}\int_0^{\tilde{T}} d\tilde{t}\frac{\alpha_0^2}{\hbar}\left(\int_0^{\tilde{t}} d\tilde{t}'\, f(\tilde{R}(\tilde{t}'))\right)^2 . \tag{34}$$

Using the Gauss integration formula

$$\int_{-\infty}^{\infty} dx\, e^{-[ax^2+(b+c)x]} = \sqrt{\frac{\pi}{a}}e^{(b+c)^2/4a} \tag{35}$$

$$= e^{(2bc+c^2)/4a}\int_{-\infty}^{\infty} dx\, e^{-(ax^2+bx)} , \tag{36}$$

Eq. (31) becomes

$$\rho_M(\tilde{R}(\tilde{t}),\tilde{T};R(t),T) = \rho_M^S(\tilde{R}(\tilde{t}),\tilde{T};R(t),T)\chi_\omega \ , \tag{37}$$

with

$$\rho_M^S(\tilde{R}(\tilde{t}),\tilde{T};R(t),T) = \frac{1}{\sqrt{2\pi\alpha_0^2}}\int_{-\infty}^{\infty} dx\, e^{-\frac{x^2}{2\alpha_0^2}} e^{x(A+\tilde{A}^*)} \tag{38}$$

and

$$\chi_\omega = e^{[2(A+\tilde{A}^*)(B+\tilde{B}^*)\omega+(B+\tilde{B}^*)^2\omega^2]\alpha_0^2/2} e^{(C+\tilde{C}^*)\omega} e^{-\frac{i}{2}\omega(T-\tilde{T})} \ . \tag{39}$$

Here ρ_M^S is the influence functional in the sudden tunneling limit and χ_ω is the correction factor due to the finite excitation energy $\hbar\omega$. If we assume a constant coupling form factor, i.e., $f(R) = F$, then χ_ω becomes

$$\chi_\omega = e^{\frac{1}{6}i\omega(T-\tilde{T})^3 F^2 \left(\frac{\alpha_0}{\hbar}\right)^2} e^{-\frac{i}{2}\omega(T-\tilde{T})} \tag{40}$$

up to the first order of $\hbar\omega$.

The parameters T and $\tilde{T}$ in Eq. (40) are integration variables in calculating the tunneling probability based on Eq. (7). They turn out to be the tunneling time in semiclassical evaluations of the integrals over the path and the time. Denoting the tunneling time by T_0, we obtain [17]

$$P(E) = P_S(E_{\text{c.m.}})e^{-\frac{4}{3}\left(\frac{F\alpha_0 T_0}{\hbar}\right)^2 \omega T_0} \ , \tag{41}$$

where $P_S(E_{\text{c.m.}})$ is the transmission probability in the limit of sudden tunneling with energy $E_{\text{c.m.}} = E - \frac{1}{2}\hbar\omega$. Equation (41) clearly shows that the finite excitation energy of the environment leads to a kind of dissipation factor which reduces the barrier penetrability estimated in the sudden limit.

Note that the tunneling probability in the limit of sudden tunneling is given by an average of the tunneling probability over a distribution of potential barriers [see Eq. (12)]. In deriving Eq. (41), we assumed that a common tunneling time exists for all the barriers. This is the case in the constant coupling model if one can approximate the potential barrier by a parabola. In general, one should calculate the penetrability according to Eq. (29) or modify Eq. (41) by taking the variation of the tunneling time into account for different potential barriers.

We can derive from Eq. (41) an interesting conclusion concerning the validity of the sudden tunneling approximation. The tunneling time T_0 can be approximated by π/Ω, Ω being the barrier curvature of the bare potential $U(R)$, if the coupling is not too large. Therefore Eq. (41) implies that the applicability of the sudden tunneling approximation is governed by two parameters. The one is the ratio of the energy scales for the intrinsic and the macroscopic motions, $s_{1v} = \omega/\Omega$. In other words, it is the ratio of the time scales of the tunneling motion and of the intrinsic motion. The other is the ratio of the coupling strength to the curvature of the bare potential barrier, $s_{2v} = F\alpha_0/\hbar\Omega$ [6].

We now apply our results to heavy-ion fusion reactions. Figure 1 shows the excitation function of the fusion cross section in the reactions between ^{148}Sm and ^{16}O. We chose this system in order to illustrate the effect of a typical vibrational coupling. The same system was previously studied by Esbensen, Wu, and Bertsch [16]. In this case, the first excited state of ^{148}Sm gives $\hbar\omega = 0.55$ MeV. The bare potential is assumed to be a parabolic function with curvature $\hbar\Omega$ being 3.5 MeV to mimic realistic heavy-ion fusion reactions. The value of α_0 was estimated to be $0.0636R_T$, R_T being the radius of ^{148}Sm, based on the $B(E2)$ value for the transition from the first excited 2^+ state to the ground state in ^{148}Sm. We assumed the charge radius parameter to be 1.2 fm. In heavy-ion collisions, the coupling form factor $f(R)$ consists of nuclear and Coulomb parts. We determine them following the collective model. In our applications, we further assume a constant coupling model [6,18] and replace $f(R)$ by the value at the position of the potential barrier, $F = -15.5$ MeV$/R_T$. We plot the fusion cross section calculated by several methods as a function of the energy relative to the height of the bare potential barrier. The dotted line is the fusion cross section in the absence of chan-

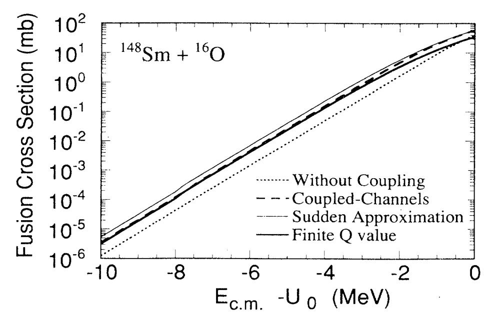

FIG. 1. Excitation function of the fusion cross section for ^{148}Sm + ^{16}O scattering. A linear oscillator coupling is assumed. The energy is taken relative to the height of the bare potential barrier. The dotted and dashed lines are the fusion cross section in the absence of channel coupling and the results of the exact numerical solution of the coupled-channels equations, respectively. The thin solid line was obtained by the zero point motion formula. The thick solid line was calculated with Eq. (41) by taking the dissipation factor due to the finite excitation energy of the intrinsic motion into account. See text for the values of the parameters.

nel coupling. The dashed line shows the results of the direct numerical solution of the coupled-channels equations. The thin solid line was obtained by the zero point motion formula. It clearly overestimates the fusion cross section compared to the exact coupled-channels calculation (dashed line). The thick solid line was calculated with Eq. (41) by taking the *dissipation factor* due to the finite excitation energy of the intrinsic motion into account. We observe that it reproduces very well the results of the exact coupled-channels calculations. Note that at low energies the dashed and thick solid lines are indistinguishable. Because of the effects of multiple reflection, our simple formula (41) does not work near the fusion barrier. This is because the tunneling time T_0 cannot be determined unambiguously if multiple reflection plays a significant role.

B. Rotational coupling

We next consider the effects of coupling of the relative motion to the rotational motion of a static quadrupole deformed nucleus. If we assume axial symmetry and introduce the no-Coriolis approximation [19–21], the internal and coupling Hamiltonians for this problem are given in the rotating frame by [8]

$$H_0(\theta) = \frac{\vec{I}^2\hbar^2}{2\mathcal{J}} = -\frac{\hbar^2}{2\mathcal{J}}\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) , \tag{42}$$

$$V(R,\theta) = \sqrt{\frac{5}{4\pi}}\beta P_2(\cos\theta)f(R) , \tag{43}$$

where $\mathcal{J}$, β, and $f(R)$ are the moment of inertia for the rotational motion, deformation parameter, and coupling form factor, respectively.

The initial wave function in this problem is given by

$$\langle\Omega|n_i\rangle = Y_{00}(\Omega) . \tag{44}$$

The inclusive tunneling probability up to the first order of the excitation energy of the first excited 2^+ state, $E^*_{2+} = 6\hbar^2/2\mathcal{J}$, is obtained as

$$P(E) = \int d\Omega|Y_{00}(\Omega)|^2 P_0\left[E, U(R) + \sqrt{\frac{5}{4\pi}}\beta f(R)P_2(\cos\theta)\right.$$
$$\left. -\frac{E^*_{2+}}{\hbar}\int_{R_0}^{R}\frac{dR'}{\left(\frac{dR'}{d\tau}\right)}\sqrt{\frac{5}{4\pi}}\beta f(R')P_2(\cos\theta) - \frac{3}{2}E^*_{2+}\left(\frac{1}{\hbar}\int_{R_0}^{R}\frac{dR'}{\left(\frac{dR'}{d\tau}\right)}\sqrt{\frac{5}{4\pi}}\beta f(R')\right)^2 \sin^2\theta\,\cos^2\theta\right] . \tag{45}$$

We assume that there is no excitation in the classically allowed region.

Similarly to the case of vibrational coupling, the effects of the finite excitation energy of the rotational motion can be expressed in terms of a dissipation factor on the penetration probability in the limit of sudden tunneling. The equations corresponding to Eqs. (31)–(34) in this problem read

$$\rho_M(\tilde{R}(\tilde{t}),\tilde{T};R(t),T) = \int d\Omega|Y_{00}(\Omega)|^2 e^{P_2(\cos\theta)(A+\tilde{A}^*)}$$
$$\times\{1 + P_2(\cos\theta)(B+\tilde{B}^*)E^*_{2+} + \sin^2\theta\,\cos^2\theta(C+\tilde{C}^*)E^*_{2+}\} \tag{46}$$
$$= \int_0^1 dx\, e^{a(3x^2-1)}[1 + b(3x^2-1)E^*_{2+} + cx^2(1-x^2)E^*_{2+}] , \tag{47}$$

with

$$A = -\frac{i}{\hbar}\int_0^T dt\sqrt{\frac{5}{4\pi}}\beta f(R(t)) , \tag{48}$$

$$\tilde{A} = -\frac{i}{\hbar}\int_0^{\tilde{T}} d\tilde{t}\sqrt{\frac{5}{4\pi}}\beta f(\tilde{R}(\tilde{t})) , \tag{49}$$

$$B = -\frac{i}{\hbar}\int_0^T dt\left(-\frac{i}{\hbar}\int_0^t dt'\sqrt{\frac{5}{4\pi}}\beta f(R(t'))\right) , \tag{50}$$

$$\tilde{B} = -\frac{i}{\hbar}\int_0^{\tilde{T}} d\tilde{t}\left(-\frac{i}{\hbar}\int_0^{\tilde{t}} d\tilde{t}'\sqrt{\frac{5}{4\pi}}\beta f(R(\tilde{t}'))\right) , \tag{51}$$

$$C = -\frac{i}{\hbar}\int_0^T dt\left(-\frac{3}{2}\right)\left(-\frac{i}{\hbar}\int_0^t dt'\sqrt{\frac{5}{4\pi}}\beta f(R(t'))\right)^2 , \tag{52}$$

$$\tilde{C} = -\frac{i}{\hbar}\int_0^{\tilde{T}} d\tilde{t}\left(-\frac{3}{2}\right)\left(-\frac{i}{\hbar}\int_0^{\tilde{t}} d\tilde{t}'\sqrt{\frac{5}{4\pi}}\beta f(R(\tilde{t}'))\right)^2 , \tag{53}$$

$$a = (A+\tilde{A}^*)/2,\quad b = (B+\tilde{B}^*)/2,\quad c = (C+\tilde{C}^*) . \tag{54}$$

Using

$$\int_0^1 dx\, x^2 e^{a(3x^2-1)} = \frac{1}{6a}\left(e^{2a} - \int_0^1 dx\, e^{a(3x^2-1)}\right) , \tag{55}$$

$$\int_0^1 dx\, x^4 e^{a(3x^2-1)} = \frac{1}{6a}\left[e^{2a} - \frac{1}{2a}\left(e^{2a} - \int_0^1 dx\, e^{a(3x^2-1)}\right)\right] , \tag{56}$$

the influence functional can be rewritten as

$$\rho_M(\tilde{R}(\tilde{t}),\tilde{T};R(t),T) = \left[1 - \left(b + \frac{3b+c}{6a} + \frac{c}{12a^2}\right)E^*_{2^+}\right]\rho^S_M(\tilde{R}(\tilde{t}),\tilde{T};R(t),T) + \left(\frac{b}{2a} + \frac{c}{12a^2}\right)e^{2a}E^*_{2^+} , \tag{57}$$

with

$$\rho^S_M(\tilde{R}(\tilde{t}),\tilde{T};R(t),T) = \int_0^1 dx\, e^{a(3x^2-1)} . \tag{58}$$

ρ^S_M is the influence functional in the sudden tunneling limit. We now assume a constant coupling form factor, i.e., $f(R) = F$. Following the same procedure used to go from Eq. (38) to (41), we obtain

$$P(E) = \left(1 - \frac{1}{3}\frac{\sqrt{\frac{5}{4\pi}}\beta F E^*_{2^+} T_0}{\hbar}\frac{T_0}{\hbar} + \frac{E^*_{2^+}T_0}{6\hbar}\right)P_S(E) - \frac{E^*_{2^+}T_0}{6\hbar}P_{\text{bare}}\left(E - \sqrt{\frac{5}{4\pi}}\beta F\right) . \tag{59}$$

An equation similar to Eq. (41) can be obtained by noticing that the penetrability in the sudden approximation and the bare penetrability are related by

$$P_S(E) = \left(1 + \frac{2}{5}\frac{\frac{5}{4\pi}\beta^2F^2T_0^2}{\hbar^2}\right)P_{\text{bare}}(E) \tag{60}$$

in the case of weak coupling. Equation (60) is obtained by expanding the exponential on the rhs of Eq. (58) up to the second order of the coupling strength a. Moreover, we can write

$$P_{\text{bare}}\left(E - \sqrt{\frac{5}{4\pi}}\beta F\right) = \exp\left(-\frac{2}{\hbar}\sqrt{\frac{5}{4\pi}}\beta F T_0\right)P_{\text{bare}}(E) \tag{61}$$

if the incident energy is well below the barrier. We then obtain

$$P(E) = \left[1 - \frac{4}{15}E^*_{2^+}\left(\frac{\frac{5}{4\pi}\beta^2F^2T_0^3}{\hbar^3} + O(F^3)\right)\right]P_S(E) \tag{62}$$

$$\sim \exp\left[-\frac{4}{15}\left(\frac{\sqrt{\frac{5}{4\pi}}\beta F T_0}{\hbar}\right)^2\frac{E^*_{2^+}T_0}{\hbar}\right]P_S(E) . \tag{63}$$

This corresponds to Eq. (41) in the previous subsection for a vibrational coupling. Note that the dissipation factor in Eq. (63) has the same dependence on various parameters as that for the oscillator coupling. The only difference is the factor of 5 in the exponent, which originates from the difference of the original coupling matrix in the two cases.

The conditions for Eq. (59) to be a good approximation can be expressed in simple terms if one introduces

$$s_{1r} = E^*_{2^+}T_0/\hbar , \tag{64}$$

$$s_{2r} = \sqrt{\frac{5}{4\pi}}\beta F T_0/\hbar . \tag{65}$$

They correspond to the parameters s_{1v} and s_{2v} in the case of vibrational coupling. Since $(3x^2-1)$ and $x^2(1-x^2)$ vary between -1 and 2 and between 0 and $\frac{1}{4}$, respectively, Eq. (47) suggests that our perturbative procedure works only when the following conditions are satisfied:

$$|s_{1r}s_{2r}| \ll 1 , \tag{66}$$

$$|s_{1r}|s^2_{2r} \ll 4 . \tag{67}$$

Equations (60)–(63) further require

$$|s_{2r}| \ll \sqrt{\tfrac{5}{2}} . \tag{68}$$

Let us now use our formulas for rotational coupling to describe heavy-ion collisions. Figure 2 shows the fusion cross section in the reaction between ^{152}Sm and ^{16}O as a function of the bombarding energy relative to the height of the bare potential barrier. ^{152}Sm is a typical deformed nucleus with prolate shape. The excitation energy of the first 2^+ state $E^*_{2^+}$ of this nucleus is 0.12 MeV. The deformation parameter β was estimated to be 0.3 from the $B(E2)$ value for the transition from the first excited 2^+ state to the ground state. The bare potential is assumed to be a parabolic function with curvature $\hbar\Omega$ being 3.5 MeV as in the case of the collision between

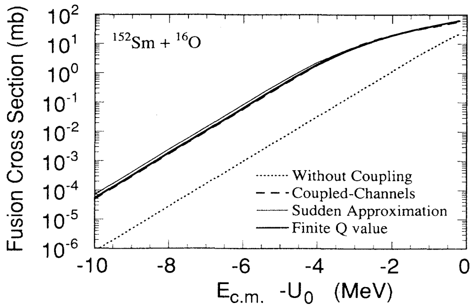

FIG. 2. Excitation function of the fusion cross section for ^{152}Sm + ^{16}O scattering as an example of rotational coupling. The meaning of each line is the same as in Fig. 1 except that the thick solid line was obtained with Eq. (59) instead of Eq. (41).

^{148}Sm and ^{16}O in Fig. 1. The coupling form factor F was chosen to be -22.16 MeV. This is the value of the sum of the nuclear and Coulomb coupling form factors of the collective model at the position of the potential barrier. The meaning of each line in Fig. 2 is the same as in Fig. 1, except that the thick solid line was calculated with Eq. (59). The condition (68) does not hold in realistic heavy-ion reactions, because the deformation parameter is too large. Hence one should use Eq. (59) rather than Eq. (63) for quantitative estimation of the effect of the finite excitation energy. By taking into account the effect of the finite excitation energy, the results of the exact coupled-channels calculations are well reproduced.

As an example of the rotational coupling for an oblate nucleus, we next consider the reaction between ^{194}Pt and ^{16}O. Although ^{194}Pt lies between the γ-unstable and rigid rotational limits in the interacting boson model [22], we assume that ^{194}Pt has a static oblate shape. The deformation parameter β, coupling form factor F, and excitation energy of the first 2^+ state $E^*_{2^+}$ of ^{194}Pt are obtained as -0.2, -26.7 MeV, and 0.3 MeV, respectively. Figure 3 shows the fusion cross section as a function of the bombarding energy relative to the bare potential barrier. We used the same bare potential as in Fig. 2. The meaning of each line is the same as in Fig. 2. We again observe that the results of the exact coupled-channels calculations are reproduced very well with our formula (59) except for the near-barrier region where we must consider the effect of multiple reflection.

Before we conclude this section, we would like to mention the role of high-lying states in our calculations. We derived our formulas (41), (59), and (63) by assuming that the vibrational excitation and the rotational band are not truncated at any excited states. Therefore, in order to be consistent, one has to perform the coupled-channels calculations and the calculations of $P_S(E)$ by including all the members of the vibrational or rotational excitations which significantly affect the barrier penetration.

In the example shown in Fig. 2, the contribution from the 6^+ and the higher states is negligible. Accordingly, the coupled-channels calculations and the estimate of $P_S(E)$ have been performed by including up to the 4^+ member of the rotational band. This was essential to obtain a good agreement between the calculations of our revised zero point motion formula and the exact numerical solutions of the coupled-channels equations. The situation is similar in the case of the vibrational coupling discussed in Fig. 1 and for the oblate deformation in Fig. 3, though in these cases the contribution from the two-phonon state and from the 4^+ state is less important.

In order to see the contribution from the second excited states, Fig. 4 shows the ratio of $P_S(E)$ for the s-wave barrier penetrabilities calculated by truncating at the second and first excited states. The solid, dotted, and dashed lines correspond to the cases of Figs. 1, 2, and 3, respectively. We see that the second excited states contribute significantly, especially at low energies [21]. These ratios were calculated by the zero point motion formulas corresponding to each truncation [9]. Integrals over the internal coordinates are thus replaced by the Gauss-Hermite quadrature or the Gauss quadrature for the vibrational and rotational coupling cases, respectively. The exact coupled-channels calculations almost converge at the first excited state. This would seem to indicate that the second excited and higher excited states do not affect the tunneling probability. Our calculations show, however, that this convergence is caused by the cancellation between the additional enhancement of $P_S(E)$ due to the second excited state and the effect of dissipation discussed in this paper. Accordingly, if we estimate the results of the coupled-channels calculations and the $P_S(E)$

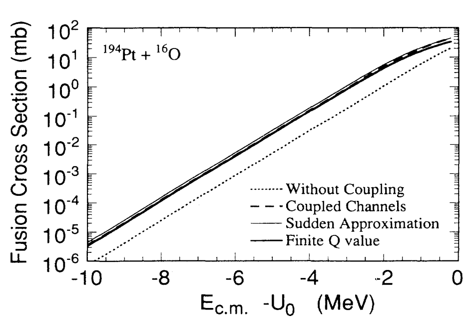

FIG. 3. Same as Fig. 2, but for ^{194}Pt + ^{16}O scattering. ^{194}Pt is assumed to have a static oblate shape. The meaning of each line is the same as in Fig. 2.

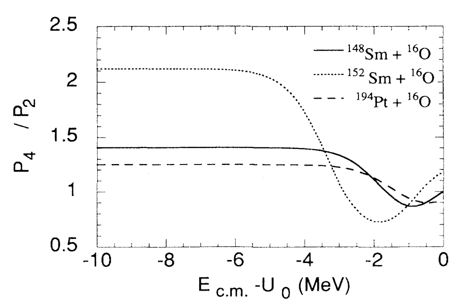

FIG. 4. Ratio of $P_S(E)$ for the s-wave barrier penetrabilities estimated by the zero point motion formula truncated at the second and first excited states. The solid, dotted, and dashed lines are for the systems in Figs. 1, 2, and 3, respectively.

by including only up to the first excited state, then our revised zero point formula does not agree with the result of the exact coupled-channels calculations. This example shows that one should study the convergence with respect to the role of high-lying states of the environment based on the zero point motion formula without including the effects of dissipation.

V. SUMMARY

We derived a new formula for the barrier penetration probability for a nearly sudden quantum tunneling. It modifies the so-called zero point motion formula by taking the effects of a finite excitation energy of environments into account. Our formula is applicable to a very wide range of problems without requiring specific properties for the environmental degrees of freedom. Moreover, the formula can be applied to cases where the initial state of the internal motion is not necessarily in a given state, but is distributed over a number of possible states as in the case of thermal equilibrium with a heat bath. The formula keeps the intuitive structure of the zero point motion formula in the limit of sudden tunneling. We used the influence functional formalism in the path integral approach to derive the general formula and showed that the weight factor there is given by the eigenfunction of the internal, i.e., environmental, Hamiltonian.

Using the examples of linear oscillator coupling and rotational coupling, we showed that the effects of the finite excitation energy can be represented by a dissipation factor which reduces the barrier penetrability estimated in the sudden tunneling approximation. We also discussed the applicability of the sudden tunneling approximation and showed that it is governed by both the ratio of the time scales in the internal and tunneling motions and the coupling strength.

There are many interesting applications of these formulas. One of them is the problem of the barrier distribution in heavy-ion fusion reactions [23]. The finite excitation energy will affect especially the barrier distribution associated with a nuclear surface vibration, because in many cases the excitation energy is not negligible compared with the curvature of the fusion barrier. As we mentioned in the Introduction, the fusion reaction between ^{58}Ni and ^{60}Ni, where the important role of two-phonon states in the barrier distribution has been claimed [14], is one of such systems. Another example is the fusion reaction where a light deformed nucleus is involved. The fusion reaction between ^{28}Si and ^{144}Sm recently measured at Canberra [15] is an example. The large excitation energy of the rotational band of light deformed nuclei will require one to take the finite excitation energy into account in determining the barrier distribution.

Another interesting application of our study is scattering problems [24]. We can derive a similar formula for the S matrix, which modifies that in the limit of sudden approximation. Light could be shed on the threshold anomaly of the optical potential in heavy-ion collisions discussed in Ref. [25] and the problem of excitations of valence electrons in Auger transitions of atoms [26] by using such a revised zero point motion formula for the scattering matrix.

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[1] P. Hänggi, P. Talkner, and M. Borkovec, Rev. Mod. Phys. **62**, 251 (1990), and references therein.
[2] *Proceedings of the Fourth International Symposium on Foundations of Quantum Mechanics*, edited by M. Tsukada *et al.*, Japanese Journal of Applied Physics Series Vol. 9 (Publication Office of Japanese Journal of Applied Physics, Tokyo, 1993).
[3] M. Beckerman, Rep. Prog. Phys. **51**, 1047 (1988).
[4] A. B. Balantekin and N. Takigawa, Ann. Phys. (N.Y.) **160**, 441 (1985).
[5] N. Takigawa, K. Hagino, M. Abe, and A. B. Balantekin, Phys. Rev. C **49**, 2630 (1994).
[6] N. Takigawa, K. Hagino, and M. Abe, Phys. Rev. C **51**, 187 (1995).
[7] K. Hagino and N. Takigawa, report, Tohoku University, 1995.
[8] N. Takigawa, Y. Alhassid, and A. B. Balantekin, Phys. Rev. C **45**, 1850 (1992).
[9] M. A. Nagarajan, A. B. Balantekin, and N. Takigawa, Phys. Rev. C **34**, 894 (1986).
[10] H. Esbensen, Nucl. Phys. **A352**, 147 (1981).
[11] P. M. Jacobs and U. Smilansky (unpublished).
[12] N. Rowley, G. R. Satchler, and P. H. Stelson, Phys. Lett. B **254**, 25 (1991).
[13] J. X. Wei *et al.*, Phys. Rev. Lett. **67**, 3368 (1991).
[14] A. M. Stefanini *et al.*, Phys. Rev. Lett. **74**, 864 (1995).
[15] M. Dasgupta *et al.*, in *Proceedings of the International Workshop on Heavy-Ion Fusion: Exploring the Variety of Nuclear Properties*, edited by A. M. Stefanini *et al.* (World Scientific, Singapore, 1995), p. 115.
[16] H. Esbensen, J. Q. Wu, and G. F. Bertsch, Nucl. Phys. **A411**, 275 (1983).
[17] N. Takigawa, in *Proceedings of the Tsukuba International Symposium on Heavy-Ion Fusion Reactions*, edited by K.

Furuno and T. Kishimoto (World Scientific, Singapore, 1984), p. 20.
[18] C. H. Dasso, S. Landowne, and A. Winther, Nucl. Phys. **A405**, 381 (1983); **A407**, 221 (1983).
[19] K. Hagino, N. Takigawa, A. B. Balantekin, and J. R. Bennett, report, Tohoku University, 1995.
[20] D. M. Brink, B. Buck, R. Huby, M. A. Nagarajan, and N. Rowley, J. Phys. G **13**, 629 (1987).
[21] N. Takigawa and K. Ikeda, in *Proceedings of the Symposium on The Many Facets of Heavy Ion Fusion Reactions* (Argonne National Laboratory Report No. ANL-PHY-1, 1986), p. 613.
[22] V. -S. Lac and S. Kuyucak, Nucl. Phys. **A539**, 418 (1992).
[23] A. T. Kuruppa, P. Romain, M. A. Nagarajan, and N. Rowley, Nucl. Phys. **A560**, 845 (1993).
[24] N. Takigawa, F. Michel, A. B. Balantekin, and G. Reidemeister, Phys. Rev. C **44**, 477 (1991).
[25] J. Gomez-Camacho and M. A. Nagarajan, Phys. Lett. B **300**, 303 (1993).
[26] V. N. Kondratyev, Phys. Lett. A **190**, 465 (1994).