Energies and widths of low-lying levels in ^{11}Be and ^{11}N

H. T. Fortune, D. Koltenuk, and C. K. Lau*
Department of Physics, University of Pennsylvania, Philadelphia, Pennsylvania 19104
(Received 7 February 1994)

We use a simple potential model to calculate low-lying mirror states of ^{11}Be and ^{11}N. We make predictions for energies and widths in ^{11}N.

PACS number(s): 21.60.Cs, 21.10.Dr, 21.10.Jx, 27.20.+n

In light nuclei with large neutron (or proton) excess, intruder states can come quite low in excitation energy. For example, ^{11}Be has [1] a $\frac{1}{2}^+$ (g.s.), and a ($5/2^+$, $3/2^+$) level (almost certainly $5/2^+$) at $E_x = 1.778$ MeV. Both states have large spectroscopic factors (Table I) in ^{10}Be(d,p) [2,3], and hence consist primarily of an *sd*-shell neutron coupled to ^{10}Be(g.s.). Such *sd*-shell intruder states are quite common in other nuclei. For example, ^{11}Li [4] has a low-lying positive-parity state, and the g.s. of ^{12}Be possesses a large amount of the $^{10}\mathrm{Be}\otimes(sd)^2$ configuration [5].

The nucleus $^{11}_{7}\mathrm{N}_4$ is the mirror of $^{11}_{4}\mathrm{Be}_7$ and should have approximately the same level scheme. Only one state is known in ^{11}N, from the reaction ^{14}N(^{3}He,^{6}He) [6]. Indirect arguments were used there to suggest that it is the $\frac{1}{2}^-$ first excited state, not the $\frac{1}{2}^+$ (g.s.).

Coulomb energies for lightly bound (or unbound) levels can be quite different for nucleons in different orbitals. Hence, excitation energies (and absolute g.s. energy) in ^{11}N are not easy to estimate. In the present work, we use a simple model to calculate them. The use of the isobaric multiplet mass equation (IMME) is inappropriate for two principal reasons: (1) Inspection of $T{=}3/2$ states in ^{11}B and ^{11}C reveals that assignments are unknown and/or incorrect, or that some isospin mixing is present; (2) the IMME does not work for lightly bound (or unbound) $2s\frac{1}{2}$ states. We do not attempt to calculate Coulomb energies from first principles. Rather, we seek to use a reasonable model to produce reliable estimates of energies and widths. We address the uncertainties as we go along.

We use the program ABACUS [7] and a Woods-Saxon [plus Coulomb plus spin-orbit (when appropriate)] potential to describe relative motion of $^{10}\mathrm{Be}+n$ and $^{10}\mathrm{C}+p$. Geometrical parameters are $r_0 = 1.26$ fm, $a = 0.60$ fm. Spin-orbit potential is $V_{\mathrm{s.o.}} = 6.0$ MeV. The Coulomb potential is that of a uniformly charged sphere of the same r_0. These parameters give a satisfactory account of the $2s\frac{1}{2}$ single-particle energies in ^{17}O and ^{17}F. For a given state in ^{11}Be we use its known energy to determine the appropriate Woods-Saxon depth. We then use this potential, with Coulomb added, to compute the mirror energy in ^{11}N. Of course, this technique is strictly appropriate only for pure single-particle (sp) states. However, the first three states of ^{11}Be are dominated by sp configurations, and the current method should be reasonable. Based on experience with a large number of such calculations in many light nuclei, we estimate the uncertainty in the present approach to be 50–100 keV in energy.

We start first with the $(\frac{5}{2}^+, \frac{3}{2}^+)$ state at 1.778 MeV, which is unbound by 1.275 MeV. We assume a $1d\frac{5}{2}$ sp state; but it is primarily only the l value that is important, and ^{10}Be(d,p) establishes $l = 2$. With our parameters, the well depth for this state is 55.9 MeV, and the sp neutron width is 174 keV. This width is evaluated from the expression $4/\Gamma = d(\mathrm{Im}\eta)/dE$, where $\eta = e^{2i\delta}$ and δ is the phase shift. [If we use, instead, the energy interval over which the phase shift changes from $\pi/4$ to $3\pi/4$, the result is 180 keV.] In a shell-model calculation, Teeters and Kurath [8] predict a spectroscopic factor of 0.67 for

TABLE I. Energies, widths, and neutron spectroscopic factors in ^{11}Be.

E_x[a] (MeV ± keV)	J^π [a]	E (MeV)	Γ (keV)[a]	S $E_d = 12$ MeV[b]	 25 MeV[c]
0.0	$\frac{1}{2}^+$	-0.503	bound	0.73 ± 0.06	0.77
0.3200 ± 0.1	$\frac{1}{2}^-$	-0.183	bound	0.63 ± 0.15	0.96
1.778 ± 12	$(\frac{5}{2},\frac{3}{2})^+$	1.275	100 ± 20		0.50

[a]Reference [1].
[b]Reference [2].
[c]Reference [3].

*Present address: Physics Department, University of California at Davis, Davis, CA 95616.

TABLE II. Width as a function of energy for ^{11}N($\frac{1}{2}^-$).

			$\Gamma_{pred} = S\Gamma_{sp}$(MeV)	
E_{unb} (MeV)	Γ_{sp}[a] (MeV)	$\frac{\Gamma_{exp}}{\Gamma_{sp}}$[b]	$S = 0.96$[c]	$S = 0.63 \pm 0.15$[d]
1.50	0.368	2.01 ± 0.17	0.35	0.23 ± 0.05
2.00	0.820	0.90 ± 0.12	0.79	0.52 ± 0.12
2.24	1.118	0.66 ± 0.09	1.07	0.70 ± 0.17
2.58	1.67	0.44 ± 0.06	1.60	1.05 ± 0.25

[a]Present calculation.
[b]Using $\Gamma_{exp} = 0.74 \pm 0.10$ (Ref. [6]).
[c]Reference [3].
[d]Reference [2].

this state. The measured width (which is all neutron width) is 100±20 keV, giving $\Gamma_n/\Gamma_{sp} = 0.57 \pm 0.11$, to be compared with $S = 0.50$ [3] for the spectroscopic factor in ^{10}Be(d,p). Thus, the agreement is good, and this state has at least 50% of the $1d\frac{5}{2}$ single-particle strength.

With this potential, we compute the expected position of the $1d\frac{5}{2}$ proton single-particle state in ^{11}N (as ^{10}C+p). The result is that it should be unbound by 3.906 MeV, with a sp width of 0.89 MeV. Putting in the ^{11}Be width (or spectroscopic factor) implies $\Gamma(\frac{5}{2}^+) = 0.50 \pm 0.10$ MeV in ^{11}N. The second largest component (18%) of this state in Ref. [8] is $2s\frac{1}{2}$ coupled to the 2^+, T=1 first-excited A=10 state. Because this configuration involves a $2s\frac{1}{2}$ nucleon, its presence will influence the Coulomb energy. However, such a change should be within the 50–100 keV mentioned above. Given the energy of the 2^+ state in ^{10}C, the extra width from this component is negligible.

We repeat the process for the $\frac{1}{2}^-$ state (assumed to be $1p\frac{1}{2}$), which is bound in ^{11}Be and unbound in ^{11}N. The result in ^{11}N is E = 2.46 MeV above the ^{10}C+p threshold, with Γ_{sp} = 1.45 MeV. The ^{11}N state observed [6] in ^{14}N(^{3}He,^{6}He) and tentatively assigned as the mirror of ^{11}Be($\frac{1}{2}^-$) lies 0.24 ± 0.10 MeV below this (i.e., less unbound) and has a smaller width, Γ = 740 ± 100 keV. The identification is probably correct, as this reaction is more likely (as pointed out in [6]) to populate a p-shell state than one having a neutron in the $2s\frac{1}{2}$ orbit [of which the ^{14}N(g.s.) has very little].

Is our procedure inaccurate, or is the experiment wrong, or both? The discrepancy (0.24 ± 0.10 MeV) is not that great, and the $\frac{1}{2}^-$ state is certainly not a pure $1p\frac{1}{2}$ sp state. (As we see below, it is probably just an unkind coincidence that the discrepancy is comparable to the g.s. first excited state splitting in ^{11}Be.)

Because the calculated sp width is a rapid function of the assumed energy, if the $\frac{1}{2}^-$ spectroscopic factor were reliably known, we could use the relation $\Gamma = S\Gamma_{sp}(E)$ to determine E. Unfortunately, two different values of S exist for the $\frac{1}{2}^-$ state. However, they can be used to place limits on the energy. We can use the measured S values and the calculated $\Gamma_{sp}(E)$ to compare with the measured width of 740 ± 100 keV. Or, equivalently, we can use the measured width and the calculated $\Gamma_{sp}(E)$ to compute S to compare with the measured spectroscopic factor. The two are indeed equivalent, except that the ±0.10 MeV uncertainly in the measured energy [6] corresponds to an uncertainty of 30% (±15%) in Γ_{sp}. The results are (Table II) as follows. If $S = 0.96$, then we get $S\Gamma_{sp} = \Gamma_{exp}$ at $E = 1.96^{+0.08}_{-0.10}$ MeV — unacceptably low; if $S = 0.63 \pm 0.15$, then $S\Gamma_{sp} = \Gamma_{exp}$ for $E = 2.29^{+0.33}_{-0.28}$ MeV, well within the measured range. If we use the measured energy and width (see Fig. 1), together with our $\Gamma_{sp}(E)$, we get $\Gamma_{exp}/\Gamma_{sp} = 0.64 \pm 0.12$, to be compared with $S = 0.63 \pm 0.15$ [2]. The authors of [6] state that their width indicates $S = 0.7 \pm 0.1$. The theoretical spectroscopic factor [9] is 0.60. Our calculated $1d\frac{5}{2} - 1p\frac{1}{2}$ splitting in ^{11}N is 1.42 MeV, compared with the $\frac{5}{2}^+ - \frac{1}{2}^-$ difference of 1.458 MeV in ^{11}Be. Configuration mixing will have a minor effect on this energy, because any expected configuration impurities also involve a p-shell nucleon whose Coulomb energy is relatively insensitive to the excitation energy.

Continuing this process, we find that the bound $\frac{1}{2}^+$ state (g.s. of ^{11}Be) should have its mirror in ^{11}N at 1.60 ± 0.22 MeV above the ^{10}C+p threshold, i.e., 0.88

TABLE III. Energies and widths (both in MeV) calculated for low-lying states of ^{11}N.

E[a]	J^π	Γ_{sp}	$\Gamma_{pred} = S\Gamma_{sp}$	Γ_{meas}	E_{meas}
1.60 ± 0.22	$\frac{1}{2}^+$	$2.1^{+1.0}_{-0.7}$	$1.58^{+0.75}_{-0.52}$		
2.48	$\frac{1}{2}^-$	1.45	0.91 ± 0.22,[b] 0.73 ± 0.17[c]	0.74 ± 0.10[d]	2.24 ± 0.10[d]
3.90	$\frac{5}{2}^+$	0.88	0.50 ± 0.10		

[a]These are energies above ^{10}C+p threshold.
[b]At our calculated energy.
[c]At the measured energy.
[d]Reference [6].

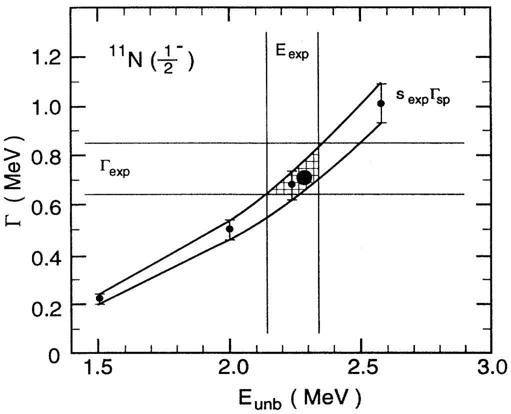

FIG. 1. Plot of Γ vs energy above ^{10}C+p threshold for ^{11}N($\frac{1}{2}^-$). Measured $\Gamma_{\rm exp}$ and $E_{\rm exp}$ [6] are shown as bands. The curve is $\Gamma = S_{\rm exp}\Gamma_{\rm sp}$, with $\Gamma_{\rm sp}$ calculated as described in the text, and using $S_{\rm exp} = 0.63 \pm 0.15$.

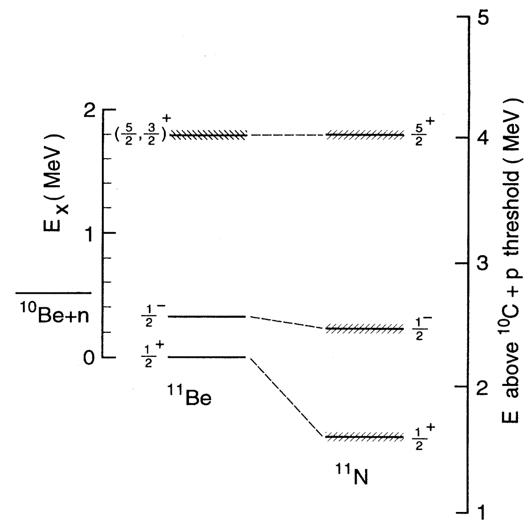

FIG. 2. Calculated energies of $\frac{1}{2}^+$, $\frac{1}{2}^-$, and $\frac{5}{2}^+$ states in the mirror nuclei ^{11}Be and ^{11}N.

MeV below the calculated $p\frac{1}{2}$ energy and 0.64 MeV below the measured ^{11}N($\frac{1}{2}^-$) state. The single-particle width of a $2s\frac{1}{2}$ state unbound by this much is difficult to calculate. However, we estimate $\Gamma_{\rm sp} = 2.1^{+1.0}_{-0.7}$ MeV, with nearly half of the uncertainty coming from the uncertainty in the expected energy. With the measured spectroscopic factor [2,3] of 0.75, we thus expect the $s\frac{1}{2}$ ground state of ^{11}N to have an experimental width of $1.58^{+0.75}_{-0.52}$ MeV. Results of these calculations are summarized in Table III. Teeters and Kurath predict $\mathcal{S}$=0.82 for the ^{11}Be g.s., with a 12% admixture of $d\frac{5}{2}$ coupled to ^{10}Be (2_1^+). Any uncertainty in energy arising from such an admixture is swamped by the above uncertainty of 0.22 MeV that comes from the fact that the state is nearly too unbound to compute.

We do not extend this procedure to higher-lying levels for several reasons: the J^π in ^{11}Be are not known, the next set of levels is certainly not primarily single-particle, and the corresponding levels of ^{11}N would be too unbound for our method of work. We have, however, computed energies and widths of the three lowest levels of ^{11}N. They are displayed in Fig. 2. It would be interesting to do an experiment to try to find these states. The $\frac{1}{2}^+$ g.s. is narrow enough and far-enough separated from the first-excited state that the two should be practically resolvable. We note that, even with the rather large uncertainties associated with the present procedure for the $2s\frac{1}{2}$ orbital, our predicted ^{11}N(g.s.) is much further below the $\frac{1}{2}^-$ state than is assumed in the latest compilation.

As a by-product of the current work, we find agreement between calculated and measured widths and the experimental spectroscopic factors $S = \Gamma_{\rm exp}/\Gamma_{\rm sp}$ for the $\frac{5}{2}^+$ state of ^{11}Be. Our calculated width for ^{11}N($\frac{1}{2}^-$) is consistent with the measured width if we take the lower of the two experimental spectroscopic factors ($S = 0.63 \pm 0.15$) for its mirror in ^{11}Be. However, because both uncertainties are large, this is not a severe test.

We acknowledge financial support from the National Science Foundation.

[1] F. Ajzenberg-Selove, Nucl. Phys. **A506**, 1 (1990).
[2] D.L. Auton, Nucl. Phys. **A157**, 305 (1970).
[3] B. Zwieglinski, W. Benenson, R.G.H. Robertson, and W.R. Coker, Nucl. Phys. **A315**, 124 (1979).
[4] T. Kobayashi *et al.* (unpublished).
[5] See, e.g., F.C. Barker, J. Phys. G **2**, L45 (1976).
[6] W. Benenson, E. Kashy, D.H. Kong-A-Siou, A. Moalem, and H. Nann, Phys. Rev. C **9**, 2130 (1974).
[7] S. Zawadski (unpublished).
[8] W.D. Teeters and D. Kurath, Nucl. Phys. **A275**, 61 (1977).
[9] S. Cohen and D. Durath, Nucl. Phys. **A101**, 1 (1967).