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Universality of $\Delta I = 1$ meson mixing and charge symmetry breaking

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The Coleman-Glashow scheme for $\Delta I_z = 1$ charge symmetry breaking (CSB) applied to meson and baryon SU(2) mass splittings suggests a universal scale. This scale can be extended to $\Delta I = 1$ nonstrange CSB transitions $\langle\rho^\circ|H_{\rm em}|\omega\rangle$ and $\langle\pi^\circ|H_{\rm em}|\eta_{NS}\rangle$ of size -0.005 GeV^2 in the energy region 760–780 MeV. The resulting nucleon-nucleon vector meson exchange CSB potential then predicts $\Delta I = 1$ effects which are in approximate agreement with recent data characterizing nuclear charge asymmetry.

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In a series of papers [1–3] we have long advocated the successful Coleman-Glashow [4, 5] tree-level picture of electromagnetic (em) charge symmetry breaking (CSB), for the $\Delta I = 1$ transitions $\langle\pi^\circ|H_{\rm em}|\eta\rangle$ and $\langle\rho^\circ|H_{\rm em}|\omega\rangle$. In fact, it is well known that the observed semistrong and electromagnetic splittings of all hadron masses are related by a hierarchy of interactions which break the underlying SU(3) symmetry. The fundamental dynamical assumption of the Coleman-Glashow picture is that symmetry-violating processes are dominated by symmetry-breaking tadpole diagrams. A specific realization of this dynamics, explored by Coleman, Glashow, and collaborators, is due to an octet of scalar mesons which yields a class of Feynman diagrams with one external line ("tadpoles") which only occur as internal parts of other diagrams (see, for example, Fig. 1). Empirically, these tadpoles dominate the electromagnetic self-energies of hadrons and the transitions $\langle\pi^\circ|H_{\rm em}|\eta\rangle$ or $\langle\rho^\circ|H_{\rm em}|\omega\rangle$. More recently, it was shown how the scalar $\bar{q}q$ meson tadpoles can be coupled with current divergences (i.e., PCAC dynamics) to link the Coleman-Glashow picture with current quark mass differences [6]. From this tree-level picture, em self-energies or em transitions could have no dependence on the four-momentum squared of the hadrons, and the theoretical predictions are simply compared with measured em mass splittings and with (on-mass-shell) CSB violating transitions $\omega \to 2\pi$ and $\eta \to 3\pi^\circ$. In Refs. [1, 3] our main interest was with the exchange of the spacelike ρ° or ω vector meson between two nucleons, so that a Fourier transform of the resulting diagram generates a CSB nucleon-nucleon potential in coordinate space. Because the transition matrix element has no q^2 dependence in this (tadpole) picture, the (updated) on-mass-shell Coleman-Glashow determinations of $\langle\rho^\circ|H_{\rm em}|\omega\rangle$ are correctly employed even in the case of the spacelike momentum transfer of a nuclear force diagram. Recently, however, some physicists have developed alternative CSB schemes based on the different off-shell q^2 dependence of fermion loop (rather than tree) graphs, both for $\langle\rho^\circ|H_{\rm em}|\omega\rangle$ [7–11] and for $\langle\pi^\circ|H_{\rm em}|\eta\rangle$ [12, 13] transitions. Other alternative CSB schemes include approaches based on the phenomenologies of chiral perturbation theory [14] and of QCD(ITEP) sum rules [15]. These latter approaches attempt to connect the microscopic theory of QCD, formulated in terms of degrees of freedom that do not appear in the physical spectrum, to the resonances of the Particle Data Group (PDG) table and of the CSB nucleon-nucleon potential. In these fermion loop models and in the latter semiphenomenological approaches, the transitions become weaker than the on-shell values in the spacelike region and the suggestion is made that the good agreement with charge asymmetry data in nuclear systems from tree-level meson mixing will be greatly lessened.

We attempt to clarify this debate by returning to the fundamental symmetry structures of SU(3) and SU(6). In the text we focus on the off-diagonal $\Delta I = 1$ $\eta \to \pi^0$ and $\omega \to \rho^0$ electromagnetic transitions and characterize them in as model-independent a fashion as is possible. Then we compute these $\eta \to \pi^0$ and $\omega \to \rho^0$ transitions via treelike tadpole graphs first proposed by Coleman and Glashow (CG) in 1964 [4, 5]. The agreement between these two approaches is satisfying, and it involves no arbitrary parameters. Both schemes in fact are compatible with on-mass-shell data and also with the predictions of a simple meson exchange nucleon nucleon poten-

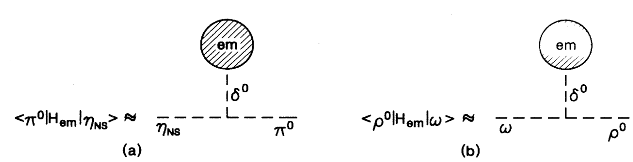

FIG. 1. Delta meson tadpole dominance of the CSB $\Delta I = 1$ (a) $\langle\pi^\circ|H_{\rm em}|\eta_{NS}\rangle$, (b) $\langle\rho^\circ|H_{\rm em}|\omega\rangle$ transitions.

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tial. Finally in the Appendix we review the successful diagonal Coleman-Glashow predictions of electromagnetic splitting of both meson masses and of baryon masses.

We begin with the $\Delta I = 1$ Hamiltonian density, scaled to the strength of second-order electromagnetic theory and therefore labeled by $H_{\rm em}$ as is traditional, which is taken as a current-current part and a CG tadpole part:

$$H_{\rm em} = H^3_{\rm tad} + H_{JJ} \ . \tag{1a}$$

Here the $I_z = 3$ tadpole along with the $I_z = 8$ Hamiltonian contribute to the quark mass matrix as

$$cH_8 + c'H_3 = m_u\bar{u}u + m_d\bar{d}d + m_s\bar{s}s \ . \tag{1b}$$

This in turn requires the SU(2)-breaking part $\bar{q}\lambda^3 q$ to have the $\Delta I = 1$ tadpole form

$$H^3_{\rm tad} = c'H_3 = (m_u - m_d)(\bar{u}u - \bar{d}d)/2 \ . \tag{1c}$$

But since the current-quark mass *difference* $m_u - m_d$ in (1c) is not actually known, we avoid modeling it and instead follow the Coleman-Glashow scheme and exploit only the *model-independent* symmetry properties of the λ_3 tadpole operator in (1c).

The physics of this characterization is that the observed octet pattern of the electromagnetic mass splittings is related to the octet pattern of the semistrong mass splittings (Gell-Mann–Okubo formulas) by a single octet of scalar mesons. That is, with (1a) we can calculate the dominant part ($H^3_{\rm tad}$) of the electromagnetic mass splittings in terms of the semistrong mass splittings and one parameter c'/c, the measure of SU(2) symmetry breaking relative to SU(3) symmetry breaking. (We emphasize, *not* one parameter for every unitary multiplet, but only one parameter for all the hadrons.) In order to compare the Coleman-Glashow tadpole scheme with experiment, it is necessary to have a theory of the next corrections, the H_{JJ}. To this end we follow Coleman-Glashow and note that the electromagnetic mass of any particle can be written in terms of the forward scattering of unphysical photons off that particle, summed over all polarizations, and integrated over all photon four-momenta. One can approximate this amplitude by the dispersion theoretic poles in all three Mandelstam variables s, t, and u. The poles in s and u, summed and integrated give the conventional expression for the self-mass in terms of the experimental electromagnetic form factors, first suggested by Feynman and Speisman [16]. The only t channel poles which survive the summation and integration are those in which the particle exchanged is a scalar meson, i.e., the tadpoles. So one can calculate matrix elements of H_{JJ} as the conventional contributions to the electromagnetic self-mass and add them to the Coleman-Glashow tadpole. The resulting self-energy can then be compared to experimental electromagnetic mass splittings of hadrons. How this all works and the verification that it works very well indeed is reviewed in the Appendix.

In order to extend the above (diagonal) CG mass splittings to the off-diagonal $\eta \to \pi^0$ and $\omega \to \rho^0$ meson mixing transitions, first we invoke SU(3) symmetry to write [17]

$$\langle \pi^\circ | H_{\rm em} | \eta_8 \rangle = (\Delta m^2_K - \Delta m^2_\pi)/\sqrt{3} \ ,$$

where meson data [18] requires $\Delta m^2_K \equiv m^2_{K^+} - m^2_{K^\circ} \approx -0.003\,99$ GeV^2, $\Delta m^2_\pi \approx 0.001\,26$ GeV^2. This $\eta_8 \to \pi^0$ $\Delta I = 1$ transition can be converted to the nonstrange $\Delta I = 1$ scale

$$\langle \pi^\circ | H_{\rm em} | \eta_{NS} \rangle = \Delta m^2_K - \Delta m^2_\pi \approx -0.005\,25 \text{ GeV}^2 \ , \tag{2a}$$

where η_{NS} is the idealized nonstrange $(\bar{u}u + \bar{d}d)/\sqrt{2}$ eta meson. In the same manner, SU(3) also scales the $\omega \to \rho^\circ$ $\Delta I = 1$ transition (taking ω as pure nonstrange)

$$\langle \rho^\circ | H_{\rm em} | \omega \rangle = \Delta m^2_{K^*} - \Delta m^2_\rho \ . \tag{2b}$$

Since Δm^2_ρ is not accurately measured, we compute this $\omega \to \rho^\circ$ transition in (2b) by invoking SU(6) symmetry between the pseudoscalar and vector meson masses, $m^2_K - m^2_\pi = m^2_{K^*} - m^2_\rho$. But because this SU(6) relation is valid to within 5%, it is reasonable to assume the SU(6) mass difference also holds:

$$\Delta m^2_K - \Delta m^2_\pi = \Delta m^2_{K^*} - \Delta m^2_\rho \approx -0.005\,25 \text{ GeV}^2 \ . \tag{2c}$$

Thus SU(3) and SU(6) symmetry implies a universality between these $\Delta I = 1$ transitions given in Eqs. (2):

$$\langle \pi^\circ | H_{\rm em} | \eta_{NS} \rangle \approx \langle \rho^\circ | H_{\rm em} | \omega \rangle \approx -0.005\,25 \text{ GeV}^2 \ . \tag{2d}$$

Indeed, we note that this latter $\rho - \omega$ mixing scale is not too far removed from the observed value of -0.0035 GeV^2 found from early determinations of $\omega \to 2\pi$ decay [1, 2] (later discarded by PDG [18]) and from its decrease to -0.0045 ± 0.0006 GeV^2 obtained from more accurate data [3, 19]. Since $\langle \rho^\circ | H_{\rm em} | \omega \rangle$ in (2b) is the same as the $\langle \pi^\circ | H_{\rm em} | \eta_{NS} \rangle$ SU(3) scale in (2a), one might even suggest that since $\omega(783)$ is known to be almost purely nonstrange and the nonstrange eta mass is [22] $m_{\eta_{NS}} \sim 760$ MeV, the universal strength of charge symmetry breaking (CSB) at energy scale 760–780 MeV is about -0.005 GeV^2. The Appendix contains the demonstration that this universality between these off-diagonal $\Delta I = 1$ transitions is identical with the universality of diagonal mass splittings in the pseudoscalar and vector mesons, as well as the baryon octet and decuplet. While we have identified this "CSB universality" in (2d) with SU(3) and SU(6) symmetry breaking physics and data, in what follows we show that (2) also holds using (tree-level) tadpoles or PCAC dynamics.

To return to the CG tadpole decomposition of $H_{\rm em}$ in (1a), for these off-diagonal transitions we follow Sutherland [23] and Dashen [24] who employ PCAC to deduce that $\langle \pi^\circ | H_{JJ} | \eta \rangle = 0$ (in the chiral limit). We extend this result to $\omega \to \rho^\circ$ by appealing again to SU(6). This gives

$$\langle \pi^\circ | H_{\rm em} | \eta_{NS} \rangle = 0 + \langle \pi^\circ | H^3_{\rm tad} | \eta_{NS} \rangle, \tag{3a}$$

$$\langle \rho^\circ | H_{\rm em} | \omega \rangle \approx \langle \rho^\circ | H^3_{\rm tad} | \omega \rangle \ . \tag{3b}$$

Then we consider the (tree-level) scalar δ° tadpole graphs of Fig. 1 which make up Eqs. (3). The vector current divergence $\langle 0|i\partial V^{1+i2}|\delta^-\rangle = \sqrt{2} f_\delta m_\delta^2$ defines the "scalar decay" constant f_δ in analogy to the pseudoscalar decay constant f_π, and $\delta(983)$ is the narrow width scalar meson now called a_0 by PDG [18]. With an SU(2) rotation of the charge-changing $\Delta I = 1$ δ^- tadpole generated by this divergence, one finds [6, 25]

$$\langle 0|H_{\rm em}|\delta^\circ\rangle = f_\delta m_\delta^2 \ , \tag{4a}$$

for the $\Delta I = 1$ scalar δ° tadpole. The SU(2) charge symmetry-breaking parameter f_δ has been estimated to be [6, 26] $f_\delta \sim 0.5$ to 0.6 MeV. Now we attempt to sharpen that estimate. In the Appendix we show that c'/c, the (universal) measure of SU(2) relative to SU(3) symmetry-breaking is 2%; specifically for meson states, $c'/c = 0.020$. Then the scalar CSB parameter f_δ can be computed as

$$f_\delta = (c'/c) f_\kappa \approx (0.020)(21\,{\rm MeV}) \approx 0.42 \ {\rm MeV} \ , \tag{4b}$$

where the parameter f_κ is the scalar kappa measure of SU(3) symmetry-breaking obtained by Glashow and Weinberg [27] [taking $f_+(0) = 1$]

$$f_\kappa = f_K - f_\pi \approx 21\,{\rm MeV} \ , \tag{4c}$$

for [18] $f_K/f_\pi \approx 1.22$ and $f_\pi \approx 93$ MeV. Finally then one obtains from the (tree-level) tadpole graph of Fig. 1(a), letting the pion four-momentum become soft,

$$\langle \pi^\circ|H_{\rm em}|\eta_{NS}\rangle \approx -\frac{f_\delta m_\delta^2 2 g_{\delta\eta_{NS}\pi}}{m_\delta^2 - m_{\eta_{NS}}^2} = -\frac{f_\delta m_\delta^2}{f_\pi} \approx -0.0044\,{\rm GeV}^2 \ . \tag{4d}$$

Here we have used the effective chiral-invariant meson-meson coupling $g_{\delta\eta_{NS}\pi} = (m_\delta^2 - m_{\eta_{NS}}^2)/2f_\pi$, obtained from the linear σ model [28] $g_{\sigma\pi\pi} = (m_\sigma^2 - m_\pi^2)/2f_\pi$ by the generalization to SU(3) [29]. The former coupling ($g_{\delta\eta_{NS}\pi} \approx 2.1$ GeV) is in good numerical agreement with that obtained from recent $\delta \to \eta\pi$ decay data ($g_{\delta\eta_{NS}\pi} \approx 1.8 \pm 0.2$ GeV) [6, 18]. With regard to the $\omega - \rho$ CSB transition, the analog δ° tadpole graph in Fig. 1(b) representing $\langle \rho^\circ|H_{\rm em}|\omega\rangle$ gives

$$\langle \rho^\circ|H_{\rm em}|\omega\rangle \approx -\frac{f_\delta m_\delta^2 2 g_{\delta\omega\rho}}{m_\delta^2} \approx -\frac{f_\delta m_\delta^2}{f_\pi} \approx -0.0044 \ {\rm GeV}^2 \ , \tag{5}$$

where we have neglected the invariant momentum transfer to the δ ($p^2 = m_\rho^2 - m_\omega^2 \approx 0$) and assumed SU(6) symmetry ($g_{\delta\omega\rho} = g_{\delta\eta_{NS}\pi}$) for the unmeasured coupling $g_{\delta\omega\rho}$. The equality of (4d) and (5) shows that tree-level scalar δ° tadpole graphs recover the results (3) obtained first by SU(3) symmetry-breaking physics and empirical electromagnetic mass splittings. Observe that once the CSB scale $\langle 0|H_{\rm em}|\delta^\circ\rangle$ is determined via (4a) and (4b), the Coleman-Glashow $\Delta I = 1$ tadpole mechanism and SU(6) symmetry requires the (on-shell) universality $\langle \pi^\circ|H_{\rm em}|\eta_{NS}\rangle = \langle \rho^\circ|H_{\rm em}|\omega\rangle \approx -0.005$ GeV^2 as already anticipated in (2d).

The nucleon loop model for $\langle \rho^\circ|H_{\rm em}|\omega\rangle$ as depicted in Figs. 2 and advocated by Ref. [10] does give the $q^2 = m_\omega^2$ on-shell result

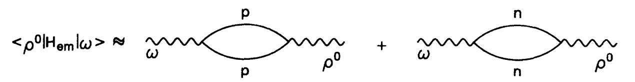

FIG. 2. Nucleon loop graphs contributing to $\langle \rho^\circ|H_{\rm em}|\omega\rangle$.

$$\langle \rho^\circ|H_{\rm em}|\omega\rangle = \frac{g_\rho g_\omega}{4\pi^2} m_N \Delta m_N \times \left[1 - \xi^{-1}(1 + \xi^2 + \kappa_V)\tan^{-1}(\xi^{-1})\right] \tag{6a}$$

$$\approx -0.0043 \ {\rm GeV}^2 \ , \tag{6b}$$

for $\xi^2 = |1 - 4m_N^2/m_\omega^2|$ and $g_\rho \equiv g_{\rho NN}$, $g_\omega \equiv g_{\omega NN}$, and κ_V taken from one-boson-exchange fits to NN data (but not for other, also plausible, values of the coupling constants [30]). A quark loop model with confining quark propagators achieves a similar on-shell result [with the aid of five more parameters than (6)]

$$\langle \rho^\circ|H_{\rm em}|\omega\rangle = \frac{3 g_{\rho qq} g_{\omega qq}}{16\pi^2\lambda^2} e^{m_\omega^2/2\mu^2} \times \left\{ \left[f_{0u}^2[-m_\omega^2 - 4m_u^2 - (4/\lambda)]\right] - [u \to d] \right\} \tag{7a}$$

$$\approx -0.0049 \ {\rm GeV}^2 \ , \tag{7b}$$

where $\lambda \equiv 2[(1/\Lambda^2) + (1/\mu^2)]$, and the parameters, including $m_u = 450$ MeV and $m_d = 454$ MeV, are chosen as explained in Ref. [9]. The fact that the fermion loop CSB scales in (6) and (7) are in approximate agreement with -0.0043 GeV^2 as given by the PDG in [18] or with -0.0045 GeV^2 as suggested in [3] or -0.0044 GeV^2 as found in (5) via the δ tadpole mechanism and SU(6) symmetry is only a constraint on the input parameters. In contrast, the most realistic (as determined from fits to other measured and inferred quantities) of these multiparameter but otherwise confining and self-consistent quark loop models of the $\omega \to \rho^\circ$ transition finds [8] an on-shell value of only 25% of the universal and empirical value of (2).

In some sense, the nucleon loop computations of Refs. [10, 13] and (6) resemble the "lengthy and brutal" (Treiman's words) nucleon loop calculations [31, 32] of the Goldberger-Treiman (GT) relation $f_\pi g_{\pi NN} = m_N g_A$. Treiman [32] notes that this over enthusiastic dispersion-theory exercise would have been for naught were it not for the low energy theorem of Nambu [33] which later put the GT relationship on a solid (and simple) footing. We suggest that the CSB universality as characterized by Eqs. (2) does provide an analogous (on-mass-shell) footing based on SU(3) and SU(6) symmetry, but that the correct dynamics is the tadpole dynamics of Fig. 1 and not the fermion loop dynamics of Fig. 2.

From our treelike SU(6) symmetry principle perspective, the above on-shell $\langle \rho^\circ|H_{\rm em}|\omega\rangle$ scale in (3) or (5) at mass 760–780 MeV is naturally translated to the off-shell spacelike q^2 NN exchange graph of Fig. 3, with CSB amplitude as in Refs. [1, 3, 34]. A partial-fraction identity of the double-pole structure in Fig. 3 leads to a class III (see reviews in [34]) exponential CSB potential

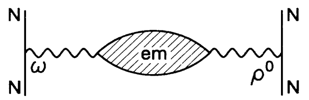

FIG. 3. Nucleon-nucleon CSB vector meson exchange graph.

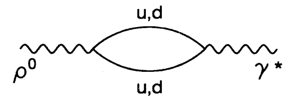

FIG. 4. Quark loop graphs contributing to the $\rho \to \gamma^*$ transition.

$$\Delta V \equiv V_{nn}^{\rho\omega}({}^1S_\circ) - V_{pp}^{\rho\omega}({}^1S_\circ) = \frac{g_\rho g_\omega}{4\pi}\frac{\langle\rho^\circ|H_{\rm em}|\omega\rangle}{2m_V}\left[1+\beta\left(\frac{2}{m_V r}-1\right)\frac{m_V^2}{m_N^2}\right]e^{-m_V r}, \tag{8a}$$

where $m_V = (m_\rho + m_\omega)/2$. The $O(q^2)$ corrections to the leading exponential in (8a) arise from the necessary momentum-dependent Pauli couplings of the vector mesons to nucleons and are not due to any putative q^2 dependence of the CSB $\langle\rho^\circ|H_{\rm em}|\omega\rangle$ transition. These relativistic corrections are small, because $\beta = \frac{1}{2}[\kappa_S\kappa_V + \frac{1}{2}(\kappa_S+\kappa_V)] \approx \kappa_V/4 \approx 0.6-1.2$. Then the class III CSB potential in (8a) is compatible with NN scattering and bound state (the Nolen-Schiffer anomaly) data in mirror nuclear systems [1, 3, 34–36], with Coulomb displacement energies of isobaric analog states [37], and with isospin-mixing matrix elements relevant to the isospin-forbidden beta decays [38]. The mixing of vector mesons also causes a class IV CSB force which is antisymmetric in isospin under the interchange of nucleons. The corresponding class IV CSB potential [39] is linear in β and has the lead terms

$$V_{IV} = (\tau_1-\tau_2)_3(\vec{\sigma}_1-\vec{\sigma}_2)\cdot\vec{L}\frac{\beta g_\rho g_\omega}{4\pi}\frac{\langle\rho^\circ|H_{\rm em}|\omega\rangle}{4m_N^2} \times\left\{\left(1+\frac{\Delta m}{2m_V}\right)\frac{e^{-m_V r}}{r}+\frac{1}{2}\Delta m e^{-m_V r}+\cdots\right\}, \tag{8b}$$

where $\Delta m = m_\omega - m_\rho$ so that $\Delta m/2m_V \approx 0.008$ and the exponential term in (8b) is $\leq 3\%$ of the lead Yukawa. This CSB potential (8b) is also compatible with precise measurements of the elastic scattering of polarized neutrons off of polarized protons [40]. In the above CSB potentials in Eqs. (8), the scale of $\langle\rho^\circ|H_{\rm em}|\omega\rangle$ is predetermined by the mass-shell or tree-level SU(3) symmetry relations in (2)–(5). An additional off-shell q^2 dependence of the fermion loops of Fig. 2 is expected to spoil the agreement with experimental nuclear asymmetry (as also emphasized in [7–10]).

Can one confirm or deny empirically, at the level of particle mixing, an additional off-shell q^2 dependence from the fermion loops of Fig. 2? Miller [21] has recently pointed out that these fermion loop models (see Fig. 4) have an implicit prediction for the q^2 variation of the $\rho - \gamma^*$ coupling constant g_ρ, defined by the vector current matrix element

$$\langle 0|\mathcal{V}_{\rm em}^{\mu}|\rho^\circ(q^2)\rangle = eq^2\epsilon^\mu(q)/g_\rho(q^2)\ . \tag{9}$$

Miller evaluates the q^2 dependence of g_ρ using the fermion loop model of [9] and finds a decrease of g_ρ by a factor of 4 from $q^2 = m_\rho^2$ to $q^2 = 0$. Indeed, an explicit variation of $g_\rho(q^2)$ of about the same magnitude is displayed in [11]. These predictions of an off-shell q^2 variation in g_ρ *can* be confronted with data [21] because this coupling constant is measured on the rho-mass-shell ($q^2 = m_\rho^2$) in the $e^+e^- \to \rho \to e^+e^-$ reaction and on the photon-mass-shell ($q^2 = 0$) from the high energy (3 to 10 GeV) reaction $\gamma + p \to \rho^\circ + p$. A careful review and compilation of experiments at the CEA, DESY, SLAC, and Cornell facilities show no q^2 variation whatsoever; $g_\rho(q^2 = m_\rho^2) = 2.11 \pm 0.06$ and $g_\rho(q^2 = 0) = 2.18 \pm 0.22$ [41]. The fermion loop models for the $\rho - \gamma^*$ transition fail this test, thus casting doubt on their validity in the $\rho - \omega$ mixing case.

Furthermore, the empirical constancy of g_ρ with q^2 also would seem to provide a counterexample to the claims that (i) current conservation imposes a node condition on a scalar polarization function $\Pi(q^2)$ and that (ii) this node condition imposes a q^2 dependence which has implications for physical quantities [42]. That is, one can define a $\Pi(q^2)$ for the $\rho - \gamma^*$ transition by

$$\langle 0|\mathcal{V}_\mu^{\rm em}|\rho_\nu^\circ(q^2)\rangle = eg_\rho\Pi_{\mu\nu}(q^2) = eg_\rho(q^2 g_{\mu\nu} - q_\mu q_\nu)\Pi(q^2)/q^2 \tag{10}$$

which follows the convention of [42]. This explicitly displays the transverse structure of the polarization operator $\Pi_{\mu\nu}(q^2)$; the fermion loop of Fig. 4 is indeed transverse in any covariant regularization scheme such as dimensional regularization [43] or dispersion theory [44]. Multiplying Eq. (10) by the rho meson polarization vector ϵ^ν and using the subsidiary condition $\epsilon^\nu q_\nu = 0$, Eq. (10) reduces to Eq. (9) with the identification

$$\frac{1}{g_\rho^2(q^2)} = \frac{\Pi(q^2)}{q^2}. \tag{11}$$

From (11) it is apparent that $\Pi(q^2)$ (defined in this way) does have a node at $q^2 = 0$, but the concomitant physical observable $\Pi(q^2)/q^2$ has little, if any, q^2 dependence as follows from the data on $g_\rho(q^2)$.

Then there is the issue of possible q^2 dependence in $\langle\pi^\circ|H_{\rm em}|\eta_{NS}\rangle$ via the em fermion loops of Refs. [12, 13]. While we also cannot disprove this scheme off mass shell, we likewise distrust the model even though it explicitly recovers the SU(3) symmetry-breaking for $\langle\pi^\circ|H_{\rm em}|\eta_{NS}\rangle$ in (2a) or the equivalent scale of -0.005 GeV^2 on mass shell [as found from the tadpole graph of Fig. 1(a) and Eqs. (4)]. In Ref. [13] this scale is part of the input to the model. That is, one of the divergences in the proton loop of the $\pi - \eta$ analogue of Fig. 2 cannot be cancelled by subtracting the neutron loop (which was used in [10]), but is removed by subtracting a counterterm chosen so that the scale of (2a), (4d), and Ref. [1] is reproduced. The quark loops with free, constituent quark propagators of the model of Ref. [12] are similarly normalized by setting the cutoff of a monopole form factor for the quark-quark-meson vertex to recover the scale of (2a) and (4d). The model employed in [12] is the Georgi-Manohar (GM) chiral quark model which describes an effective theory of massive constituent quarks and a pseudoscalar octet. It is the coupling of the octet with the quarks which gives rise to the q^2 dependent quark loops and to our greatest concern with this model. From the perspective of our analysis of CSB universality based on SU(3) and SU(6) symmetry, an unfortunate aspect of this GM model is described in Georgi's following remark. "In our (Georgi-Manohar) picture, however, the spin-0 bound state of quark and antiquark is not the pion. The pion is an elementary Goldstone boson. Presumably, what happens is that s-channel pion exchange in the quark-antiquark interaction produces a repulsive force that pushes up the $\bar{q}q$ state. Thus, the connection between ρ and π is lost. I believe that this picture of the meson is consistent but probably not very useful [45]." Based on the above remarks by one of the GM authors, we suggest the CSB results of Ref. [12] could be suspect.

The (on-mass-shell) reduced matrix element $\langle\pi^\circ|H_{\rm em}|\eta_{NS}\rangle$ in (2a) and Eqs. (4) in fact also sets the scale for the measured $\eta\rightarrow 3\pi$ decay amplitudes, long thought to be a puzzle. To support this statement, we follow the "PCAC consistency" (soft-pion theorem) approach already applied to $K_{2\pi}$ and $K_{3\pi}$ decays and only recently extended to $\eta_{3\pi}$ decays [46]. In this PCAC consistency scheme, the soft-pion PCAC limit must be consistently applied to all three final-state pions, leading to the overall on-shell $\eta_{3\pi}$ Feynman amplitude,

$$M_{\eta_{3\pi}}=\frac{1}{2}(M_{CC1}+M_{CC2}+M_{CC3})+0(m_\pi^2/m_\eta^2), \quad (12)$$

where M_{CC1} is the usual equal time charge commutator assuming soft momentum $p_{\pi_1}\rightarrow 0$, etc. The factor of 1/2 in (12) is due to the mismatch between (symmetrized) PCAC consistency and Feynman amplitudes which treat each final-state pion as independent (and instead account for Bose symmetry factors in the resulting decay rate). The analogue "mismatch" factor in $K_{3\pi}$ decays [46] corresponds exactly to the results of the chiral Lagrangian approach of Cronin [47].

Applying standard PCAC and current algebra to the PCAC consistency amplitude (12) then predicts for $\eta_{3\pi^\circ}$ and $f_\pi\approx 93$ MeV, the on-shell decay amplitude [46, 48]

$$|\langle 3\pi^\circ|H_{\rm em}|\eta\rangle|=(3/2f_\pi^2)|\langle\pi^\circ|H_{\rm em}|\eta\rangle|+0(m_\pi^2/m_\eta^2)\ . \quad (13a)$$

To relate $\langle\pi^\circ|H_{\rm em}|\eta\rangle$ in (13a) to our fundamental CSB scale (2a), we employ the $\eta-\eta'$ mixing angle relative to the NS-S flavor basis [2],

$$\phi=\tan^{-1}\left[\frac{(m_{\eta'}^2-m_K^2+m_\pi^2)(m_\eta^2-m_\pi^2)}{(2m_K^2-m_\pi^2-m_\eta^2)(m_{\eta'}^2-m_\pi^2)}\right]^{1/2}\approx 42^\circ\ . \quad (13b)$$

This mixing angle ϕ is related to the usual singlet-octet $\eta'-\eta$ mixing angle Θ as $\Theta=\phi-\tan^{-1}\sqrt{2}\approx 42^\circ-55^\circ\approx -13^\circ$, which is known [49] to be compatible with world data. Then the reduced matrix element

$$\begin{aligned}\langle\pi^\circ|H_{\rm em}|\eta\rangle&=\cos\phi\langle\pi^\circ|H_{\rm em}|\eta_{NS}\rangle\\&=\cos 42^\circ(\Delta m_K^2-\Delta m_\pi^2)\approx -0.0039\ \text{GeV}^2\ ,\end{aligned} \quad (13c)$$

in turn predicts the PCAC consistency $\eta_{3\pi^\circ}$ amplitude in (13a),

$$|M_{\eta_{3\pi^\circ}}|\equiv|\langle 3\pi^\circ|H_{\rm em}|\eta\rangle|=(3/2f_\pi^2)|\langle\pi^\circ|H_{\rm em}|\eta\rangle|\approx 0.68\ . \quad (13d)$$

On the other hand, the observed $\eta_{3\pi^\circ}$ decay rate assuming a constant matrix element integrated over the Dalitz plot is now [18]

$$\Gamma_{\rm exp}(\eta_{3\pi^\circ})=(816\text{eV})|M_{\eta_{3\pi^\circ}}|^2=(380\pm 59)\ \text{eV}\ , \quad (13e)$$

corresponding to the amplitude magnitude

$$|M_{\eta_{3\pi^\circ}}|_{\rm exp}=0.68\pm 0.05\ . \quad (13f)$$

The agreement between (13d) and (13f) once again points to the importance of the charge symmetry-breaking scale $|\langle\pi^\circ|H_{\rm em}|\eta_{NS}\rangle|\approx 0.0053\ \text{GeV}^2$ as obtained from the SU(3) symmetry-breaking formula (2a).

Alternative to the above discussion, chiral perturbation theory (ChPT) has been applied to both the process $\eta\rightarrow 3\pi^\circ$ [50] and to the question of a possible q^2 dependence in the $\eta\rightarrow\pi^\circ$ meson mixing transition [14]. Both calculations were truncated at the one-meson-loop level in an expansion based on an effective chiral Lagrangian expressed with hadronic (rather than quark) degrees of freedom. The former $\eta_{3\pi^\circ}$ calculation does not match the experimental amplitude (13f) as the PCAC consistency amplitude (13d) does. Instead the ChPT rate of 230 eV falls 40% below the observed rate (13e). In spite of its phenomenological difficulty, the ChPT calculation of Ref. [50] has been used to argue that earlier extractions of $\langle\pi^\circ|H_{\rm em}|\eta\rangle$ via pole models of $\eta\rightarrow 3\pi^\circ$ and $\eta'\rightarrow 3\pi^\circ$ are incorrect [14]. We note that the analysis just presented in Eqs. (12) and (13) is not based on a simple pole model and is presumably not subject to that particular criticism, but it also successfully links the mass-shell symmetry-breaking formula (2a) with the measured rate of $\eta\rightarrow 3\pi^\circ$ and that is the central issue.

This brings us to the alternative CSB scheme based upon a one-loop calculation of ChPT which indicated a modest q^2 dependence in the transition $\eta\rightarrow\pi^\circ$ [14]. One-loop calculations of standard ChPT tend not to describe data very well in a variety of processes including $\gamma\gamma\rightarrow\pi\pi$ [51], elastic pion scattering [52], and the aforementioned $\eta\rightarrow 3\pi^\circ$. A variety of suggestions have been put forth for this failure, ranging from truncation at the one loop level, through a lack of unitarity at this level of the expansion [53], to an incorrect choice of the small parameter in this perturbation expansion [54, 55]. In the current situation, we would prefer that guidance for the off-shell behavior of the $\eta \to \pi^\circ$ transition came from fitting data on-mass-shell, as in Eqs. (2), (4), and (5), rather than an approach (standard ChPT truncated at one loop) which has so much trouble explaining those processes most relevant to our topic.

Finally we discuss, by expanding on the remarks of Iqbal and Niskanen [56], the crucial dependence of the QCD(ITEP) sum rule results [15] for the $\omega \to \rho^\circ$ transition on the parameter $(m_d - m_u)/(2\hat{m})$ where $\hat{m} = (m_d + m_u)/2$ is the average nonstrange current quark mass. High energy data allow this parameter to have values in a much wider range than the choice of 0.28 ± 0.03 made in Ref. [15]. What *is* pinned down by the data is the Coleman-Glashow c'/c ratio of SU(2) breaking relative to SU(3) breaking which is shown in the Appendix to be 2%. Using instead the quark mass matrix of Eq. (1), simple SU(3) symmetry requires

$$\frac{c'}{c} = \left(\frac{\sqrt{3}}{2}\right) \frac{m_d - m_u}{m_s - \hat{m}} .$$

Before one can turn this accurately known ratio into the parameter of the QCD sum rule calculation, one must have an estimate of the mass ratio $m_s/\hat{m}$ where m_s is the strange current quark mass, together with a value for the $d-u$ current mass difference. The present data allow independent estimates of the former ratio which range from $m_s/\hat{m} \approx 25$ [50] (the choice of standard ChPT) to $m_s/\hat{m} \leq 10$ [57] to $m_s/\hat{m} \approx 5$ [58]. It is not our intent here to advocate any particular value of $m_s/\hat{m}$ (our results are independent of this ratio), but to reiterate the suggestion made by Iqbal and Niskanen that the QCD sum rule finding of a q^2 dependence in the $\omega \to \rho^\circ$ transition depends upon a parameter which could be as much as a factor of five smaller than the value assumed in Ref. [15].

In summary, from the (mass-shell) symmetry-breaking formulas (2,3) and tree-level tadpole amplitudes of (4), the $\Delta I = 1$ CSB nonstrange transitions $\langle \pi^\circ | H_{\text{em}} | \eta_{NS} \rangle$ and $\langle \rho^\circ | H_{\text{em}} | \omega \rangle$ are universally determined to be -0.005 GeV^2 at the energy scale 760–780 MeV. The Coleman-Glashow $\Delta I = 1$ tadpole scheme summarized in the Appendix supports this picture. It is a dynamical consequence of the tadpole scheme that the transitions $\eta \to \pi^\circ$ and $\omega \to \rho^\circ$ are q^2 independent. Then the natural CSB force generated between two nucleons is induced by the vector meson exchange graph of Fig. 3. The latter (spacelike q^2) amplitude can be Fourier-transformed to form a CSB potential in coordinate space. The consequent Class III and IV CSB potentials due to vector meson mixing are in fact compatible with recent deductions [35–38] and measurements [40] of nuclear charge symmetry breaking.

We suggest fermion loop models [7–13] and other alternative charge symmetry-breaking schemes [14, 15] purporting to find the "true" off-shell q^2 dependence of $\langle \pi^\circ | H_{\text{em}} | \eta \rangle$ and $\langle \rho^\circ | H_{\text{em}} | \omega \rangle$ transitions are quite model dependent and cannot be tested except on the mass shell $q^2 \approx m_V^2$. But most models by construction make sure that they recover the SU(3) symmetry scales of Eqs. (2) and (3) and the tree-level tadpole CSB scales of (4) so this is not much of a new test. The most elaborate and internally consistent example of these fermion loop models, however, finds that the quark-loop mixing-amplitude accounts for less than 25% of the mass-shell mixing strength [8]. On the other hand, the quark loop models can be tested off the mass shell for the transition $\rho \to \gamma^*$, and those which have been so scrutinized [9, 11] fail this test.

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APPENDIX

We begin with the usual Gell-Mann decomposition of the "world" Hamiltonian density H through order e^2

$$H = H_\circ + cH_8 + c'H_3 + H_{JJ} . \tag{A1}$$

Here the first term is the strong (QCD) chiral Hamiltonian $H_\circ$, the semistrong [SU(3) breaking] Hamiltonian H_8 transforms as λ_8 while conserving isospin and hypercharge, and the tadpole Hamiltonian $H^3_{\text{tad}} = c'H_3$ transforms as λ_3. The latter changes isospin by one unit $\Delta I = 1$ in electromagnetic (em) interactions as does the last em current-current Hamiltonian term in (A1) found from the time-ordered LSZ operator

$$H_{JJ} = -\frac{1}{2} ie^2 \int d^4x D^{\mu\nu}(x) T^*[J^{\text{em}}_\mu(x), J^{\text{em}}_\nu(0)]. \tag{A2}$$

In modern language the H_8 and H_3 Hamiltonian terms combine to form the quark mass matrix

$$cH_8 + c'H_3 = m_u \bar{u}u + m_d \bar{d}d + m_s \bar{s}s , \tag{A3}$$

corresponding to the SU(2)-breaking part $\bar{q}\lambda^3 q$ or

$$H^3_{\text{tad}} = c'H_3 = (m_u - m_d)(\bar{u}u - \bar{d}d)/2 . \tag{A4}$$

But since the current-quark mass difference $m_u - m_d$ in (A4) is difficult to determine accurately, we follow the Coleman-Glashow (CG) SU(2) symmetry prescription [4, 5] that the *entire* em interaction is of the form

$$H_{\text{em}} = H^3_{\text{tad}} + H_{JJ} , \tag{A5}$$

where H^3_{tad} is the λ_3 SU(3) extension of the semistrong Hamiltonian transforming like λ_8. While this CG postulate is obvious from the structure of (A3), it was not manifest in the prequark days of 1964.

Nevertheless the CG postulate (A5) leads to important model-independent results, especially when combined with the later observations of Dashen [24] that neutral meson matrix elements vanish [59]

$$(H_{JJ})_{\pi^\circ} = (H_{JJ})_{K^\circ} = (H_{JJ})_{\bar{K}^\circ} = (H_{JJ})_{\pi^\circ\eta} = 0 \tag{A6}$$

along with the nonvanishing charged meson matrix elements

$$(H_{JJ})_{\pi^+} = (H_{JJ})_{K^+} \ . \tag{A7}$$

Then the diagonal em mass splitting measured as [18] $\Delta m_K^2 \equiv m_{K^+}^2 - m_{K^\circ}^2 \approx -0.003\,99$ GeV^2, $\Delta m_\pi^2 \approx 0.001\,26$ GeV^2, in turn predict from the CG decomposition (A5) combined with Dashen's observations (A6) and (A7), [where, e.g., $(H_{\rm em})_{\Delta\pi} = (H_{\rm em})_{\pi^+} - (H_{\rm em})_{\pi^\circ}$]

$$(H_{\rm em})_{\Delta\pi} = (H_{\rm tad}^3)_{\Delta\pi} + (H_{JJ})_{\Delta\pi} = 0 + (H_{JJ})_{\Delta K} = \Delta m_\pi^2 \ . \tag{A8}$$

$$(H_{\rm em})_{\Delta K} = (H_{\rm tad}^3)_{\Delta K} + (H_{JJ})_{\Delta K} = \Delta m_K^2 \ . \tag{A9}$$

Subtracting (A8) from (A9) therefore requires the kaon tadpole scale [60]

$$(H_{\rm tad}^3)_{\Delta K} = \Delta m_K^2 - \Delta m_\pi^2 \approx -0.005\,25\,{\rm GeV}^2 \ . \tag{A10}$$

This in turn leads to a 2% Coleman-Glashow ratio of electromagnetic (em) to semistrong (ss) interaction pseudoscalar (P) mass splittings:

$$\left(\frac{c'}{c}\right)_P = -\left(\frac{\sqrt{3}}{2}\right)\frac{(H_{\rm tad}^3)_{\Delta K}}{m_K^2 - m_\pi^2} \approx 0.020 \ . \tag{A11}$$

Invoking the SU(6) relation $m_K^2 - m_\pi^2 = m_{K^*}^2 - m_\rho^2$ to both charge and neutral masses, it is clear their difference,

$$\Delta m_K^2 - \Delta m_\pi^2 = \Delta m_{K^*}^2 - \Delta m_\rho^2 \approx -0.005\,25\ {\rm GeV}^2 \ , \tag{A12}$$

can be used to fix the imprecisely measured Δm_ρ^2. But the SU(6) squared mass difference in (A12) also scales $(H_{\rm tad}^3)_{\Delta K^*}$ in a manner similar to (A10):

$$\begin{aligned}(H_{\rm tad}^3)_{\Delta K^*} &= \Delta m_{K^*}^2 - \Delta m_\rho^2 \\ &= \Delta m_K^2 - \Delta m_\pi^2 \approx -0.005\,25\,{\rm GeV}^2\end{aligned} \tag{A13}$$

which in turn determines the CG ratio for vector meson (V) em mass splittings,

$$\left(\frac{c'}{c}\right)_V = -\left(\frac{\sqrt{3}}{2}\right)\frac{(H_{\rm tad}^3)_{\Delta K^*}}{m_{K^*}^2 - m_\rho^2} \approx 0.022 \ . \tag{A14}$$

For the four octet baryon (B) em mass splittings in Table I, an SU(3) pattern for the tadpole parts emerges, with ratios $(d/f)_{\rm em} = (d/f)_{\rm ss} \approx -1/3$. Also one finds the fitted scale $(H_{\rm tad}^3)_{\Delta N} \approx -2.5$ MeV for $\Delta N = p - n$ along with $(H_{\rm tad}^8)_N \approx -210$ MeV as the shift from the average octet mass 1149 MeV. Then the Coleman-Glashow ratio for octet baryons (B) is (see, e.g., Refs. [2, 25])

$$\left(\frac{c'}{c}\right)_B = \left(\frac{\sqrt{3}}{2}\right)\left(\frac{f - \frac{1}{3}d}{f + d}\right)_{\rm ss}\frac{(H_{\rm tad}^3)_{\Delta N}}{(H_{\rm tad}^8)_N} \approx 0.0172 \ . \tag{A15}$$

Likewise for decuplet (D) em mass splittings, the SU(3) tadpole parts can be inferred from Table II, with Coleman-Glashow ratio (see, e.g., Refs. [2, 25])

$$\left(\frac{c'}{c}\right)_D = \frac{(H_{\rm tad}^3)_{\Sigma^{\circ *} - \Sigma^{*-}}}{(H_{\rm tad}^8)_{\Delta - \Sigma}} \approx \frac{-2.5\ {\rm MeV}}{-140\ {\rm MeV}} \approx 0.0179 \ . \tag{A16}$$

The approximate equality between the 2% Coleman-Glashow ratios in (A11), (A14), (A15), and (A16) speaks well for the consistency of the Coleman-Glashow scheme assisted by the Dashen observations (A6) and (A7).

The consistency of the Coleman-Glashow scheme can be reexpressed in terms of the magnitude of the tadpoles found in the fit to the eight octet baryon and decuplet em mass splittings of Tables I and II:

$$\begin{aligned}(H_{\rm tad}^3)_{p-n} &= \frac{2}{3}(H_{\rm tad}^3)_{\Sigma^\circ - \Sigma^-} = \frac{1}{3}(H_{\rm tad}^3)_{\Sigma^+ - \Sigma^-} \\ &= \frac{1}{2}(H_{\rm tad}^3)_{\Xi^\circ - \Xi^-} \approx -2.5\ {\rm MeV} \ .\end{aligned} \tag{A17}$$

and

$$\begin{aligned}(H_{\rm tad}^3)_{\Sigma^{*\circ} - \Sigma^{*-}} &= \frac{1}{2}(H_{\rm tad}^3)_{\Delta^{++} - \Delta^\circ} = \frac{1}{2}(H_{\rm tad}^3)_{\Sigma^{*+} - \Sigma^{*-}} \\ &= (H_{\rm tad}^3)_{\Xi^{*\circ} - \Xi^{*-}} \approx -2.5\ {\rm MeV} \ .\end{aligned} \tag{A18}$$

Clearly the latter two parameters are the same in (A17) and (A18) because they represent the *single* $\Delta I = 1$ CSB scale of -2.5 MeV. This baryon CSB scale, with spinors covariantly normalized to $\bar{u}u = 2M_{\rm baryon}$, can be related to the meson pseudoscalar and vector scale of (A10) and (A13). To do this, fold in the factor of $2M_{\rm baryon} \approx 2.3$ GeV so that the baryon CSB scale is also of mass dimension GeV^2 but now with $(H_{\rm tad}^3)_{\rm B\,and\,D} \approx -0.0057$ GeV^2. Since the latter is roughly compatible with the CSB meson splittings in (A10) and (A13), it is clear that there is only *one* universal $\Delta I = 1$ CSB scale for *all* P, V, B, and D em mass splittings.

TABLE I. SU(2) mass splitting for octet baryons (in MeV).

Baryons	H_{JJ} (Ref. [61])	$H_{\rm tad}^3$	Total	Experiment (Ref. [18])
$m_p - m_n$	0.8	-2.5	-1.7	-1.29
$m_{\Sigma^0} - m_{\Sigma^-}$	-1.0	-3.8	-4.8	-4.88±0.06
$m_{\Sigma^+} - m_{\Sigma^-}$	-0.3	-7.5	-7.8	-7.97±0.07
$m_{\Xi^0} - m_{\Xi^-}$	-1.1	-5.0	-6.1	-6.4±0.6

TABLE II. SU(2) mass splitting for decuplet baryons (in MeV).

Baryons	H_{JJ} (Ref. [61])	$H_{\rm tad}^3$	Total	Experiment (Ref. [18])
$\Delta^{++} - \Delta^0$	3.3	-5.0	-1.7	≈-2
$\Sigma^{*0} - \Sigma^{*-}$	-0.8	-2.5	-3.3	-3.5±1.6
$\Sigma^{*+} - \Sigma^{*-}$	0	-5.0	-5.0	-4.4±1.0
$\Xi^{*0} - \Xi^{*-}$	-0.8	-2.5	-3.3	-3.2±0.7

To extend the above diagonal CG results to the off-diagonal $\Delta I = 1$ transitions $\langle \rho^\circ | H_{\rm em} | \omega \rangle$ and $\langle \pi^\circ | H_{\rm em} | \eta_{NS} \rangle$, the matrix elements of the CG form (A5) yield

$$\langle \pi^\circ | H_{\rm em} | \eta_{NS} \rangle = \langle \pi^\circ | H^3_{\rm tad} | \eta_{NS} \rangle + 0, \quad (A19)$$

$$\langle \rho^\circ | H_{\rm em} | \omega \rangle = \langle \rho^\circ | H^3_{\rm tad} | \omega \rangle + 0 \; . \quad (A20)$$

Here the zeros on the right-hand side of (A19) and (A20) correspond to the Dashen observation $\langle \pi^\circ | H_{JJ} | \eta \rangle = 0$ and its obvious extension to $\langle \rho^\circ | H_{JJ} | \omega \rangle = 0$. But it has long been understood [60] on the basis of SU(3) symmetry that (A19) and (A20) obey

$$\langle \pi^\circ | H^3_{\rm tad} | \eta_{NS} \rangle = \Delta m^2_K - \Delta m^2_\pi, \quad (A21)$$

$$\langle \rho^\circ | H^3_{\rm tad} | \omega \rangle = \Delta m^2_{K^*} - \Delta m^2_\rho \; . \quad (A22)$$

Finally then, the SU(6) symmetry squared meson mass difference (A12) converts (A21) and (A22) to the universality relation

$$\langle \pi^\circ | H_{\rm em} | \eta_{NS} \rangle = \langle \rho^\circ | H_{\rm em} | \omega \rangle \approx -0.005\,25 \text{ GeV}^2. \quad (A23)$$

Such a conclusion as (A23) directly follows from calculation of the tadpole graphs in Fig. 1 as found in the body of the text.

We have shown that the single universal SU(6) $\Delta I = 1$ charge symmetry-breaking scale for the electromagnetic mass splitting of the pseudoscalar and vector mesons and the octet and the decuplet baryons can be expressed as the Coleman-Glashow ratio (c'/c) of about 2%. Alternatively universality can be seen from the approximate equality of the four diagonal mass splitting scales $(H^3_{\rm tad})$ in (A10), (A13), (A17), and (A18) (the latter two expressed in dimensions of mass squared [62]) *and* the two off-diagonal scales of (A23).

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